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THESIS

ION TRAP QUANTUM COMPUTING

by

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December 2011

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13. ABSTRACT (maximum 200 words)

Richard Feynman first proposed the idea of quantum computers thirty years ago. Since then, efforts have been undertaken to realize large-scale, fault-tolerant quantum computers that can factor large numbers much more quickly than classical computers, which would have significant implications for computer security. While there is no universally agreed upon technology for experimentally realizing quantum computers, many researchers look to ion traps as a promising technology. This thesis focuses on ion traps, how they fulfill the Divincenzo criteria, what obstacles must be overcome, and recent achievements in this field. We examine the physical principles of a linear Paul trap, including the confining potential and its quantum dynamics. In addition, we built a mechanical analogue of an ion trap for pedagogical purposes, and we provide an analysis of its trapping potential and compare it to a real ion trap, the Paul trap. Furthermore, we provide guidance for building a course module on ion trap based quantum computing; our guidance is based on course materials from several institutions.

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ION TRAP QUANTUM COMPUTING

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ABSTRACT

Richard Feynman first proposed the idea of quantum computers thirty years ago. Since then, efforts have been undertaken to realize large-scale, fault-tolerant quantum computers that can factor large numbers much more quickly than classical computers, which would have significant implications for computer security. While there is no universally agreed upon technology for experimentally realizing quantum computers, many researchers look to ion traps as a promising technology. This thesis focuses on ion traps, how they fulfill the Divincenzo criteria, what obstacles must be overcome, and recent achievements in this field. We examine the physical principles of a linear Paul trap, including the confining potential and its quantum dynamics. In addition, we built a mechanical analogue of an ion trap for pedagogical purposes, and we provide an analysis of its trapping potential and compare it to a real ion trap, the Paul trap. Furthermore, we provide guidance for building a course module on ion trap based quantum computing; our guidance is based on course materials from several institutions.

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I.   INTRODUCTION

> Trying to find a computer simulation of physics seems to me to be an excellent program to follow out. . . . the real use of it would be with quantum mechanics. . . . Nature isn't classical . . . and if you want to make a simulation of Nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy. Feynman, 1981, [1].

The idea of quantum computers is not novel; quantum computers were envisioned decades ago by Richard Feynman and others. In an inspiring speech at the MIT Physics of Computation 1st Conference in 1981, Feynman proposed the development of a computer that would obey the laws of quantum mechanics in order to be able to simulate quantum physics [1]. He understood that this attempt would not be easy, but success would yield great benefits. Thirty years later, there is great interest in quantum computing, but many challenges remain.

Four years after Feynman's speech, David Deutsch described the first universal quantum computer "*Q*," a new way to think about quantum bits and their interactions [2]. He introduced the idea of quantum parallel processing and showed that, by using interference through quantum entanglement, it is possible to compute a problem that simultaneously acts on a superposition of all 2^N input states and results in a single coherent output state that depends on all the input states.

In 1994, Peter Shor, following the work of Dan Simon, developed a quantum algorithm that could factor a large integer exponentially faster than any other algorithm for classical computers [3]. This was a major breakthrough for quantum computing because it was the first application of quantum computers to cryptanalysis of asymmetric cryptography. That discovery led to serious initiatives by defense and intelligence agencies to pursue the development of quantum computers.

In 1995, Peter Zoller and Ignacio Cirac [4] were the first to propose a quantum computer based on trapping ions in a linear electromagnetic trap. In

their system, they used laser beams, a scheme that is still considered the most reliable for building scalable quantum computers [5].

Eight years later, a group of researchers at the University of Innsbruck was able to demonstrate the Deutsch–Jozsa algorithm on a single ion trap quantum processor [6]. They were able to use Calcium ions as quantum bits (qubits) for the processor by taking advantage of the electronic and motional states. One year later, at NIST, another group of researchers lead by David Wineland performed the first teleportation using three Beryllium ions as qubits in an ion trap [7]. More recently, in March 2011, the latest development from the Innsbruck group was the success of a controlled entanglement of 14 Calcium ions as qubits, which holds the record for the largest quantum register ever produced [8].

In this thesis, we examine ion traps, the leading architecture scheme for building quantum computers. In Chapter II, we provide background on quantum computing and explain why quantum computers are more powerful compared to classical computers for certain problems (and less powerful for other problems). In Chapter III, we briefly examine other schemes for the physical implementation of quantum computers and describe the advantages and disadvantages of each approach. In Chapter IV, we provide background on ion traps. In Chapter V, we analyze the confining potential of the ion trap and in Chapter VI its quantum dynamics. In Chapter VII, we describe a mechanical analogue of an ion trap, an apparatus we built for educational demonstrations, and we show that its behavior is consistent with the behavior predicted by the equations, presented in Chapters V and VI, of a real ion trap. In Chapter VIII, we describe educational resources for teaching about ion traps, addressing the physical implementation of quantum computers.

II. QUANTUM COMPUTING

A. QUANTUM MECHANICS

Before exploring quantum computing, we provide background on quantum mechanics. The basic principles of quantum mechanics are:

1. For every physical system there exists a function $\Psi(r,t)$ called a wave function in a suitable Hilbert space. The wave function contains all the information that could be extracted from the system.

2. To every observable phenomenon in classical mechanics there corresponds a linear, Hermitian operator in quantum mechanics, whose eigenvalues are the only possible outcomes of a measurement.

3. The evolution of the wave function is determined by the Schrödinger equation, $\hat{H}\Psi(r,t) = i\hbar\dfrac{\partial}{\partial t}\Psi(r,t)$.

4. The wave function is such that its absolute square value is the probability density.

5. The measurement of the size and the determination of an eigenvalue of the corresponding operator change the system, so that immediately after the measurement, the system is described by the corresponding eigenvectors of the eigenvalue measured.

B. QUANTUM BITS

In addition to simulating quantum physics more efficiently than classical computers, quantum computers can perform a variety of computational tasks more efficiently than classical computers. However, there are some tasks for which a quantum computer does not provide an advantage in terms of time or space complexity over a classical computer (and actually may be worse).

A quantum algorithm exists for integer factorization. Finding the prime factors of a very large integer is thought to require computational resources beyond the capabilities of most attackers, which is why the strength of asymmetric ciphers like Advanced Encryption Standard (AES) and Elliptic Curve Cryptography (ECC) relies on the difficulty of factoring. The computational complexity of factoring on a classical computer is exponential in the number of

digits. Using Shor's factoring algorithm, a quantum computer could factor an integer in polynomial time, which could jeopardize the strength of asymmetric ciphers.

Another quantum algorithm, Grover's algorithm, performs a type of "search," but it has a variety of other applications, including boolean satisfiability. The time complexity of Grover's algorithm on an input of size N is $O(\sqrt{N})$; using a classical computer would require a time complexity of O(N).

The previous two examples of quantum algorithms explain the motivation behind the ongoing quest to build a quantum computer. One might say that the first country to own one could influence world events, should a quantum computer capable of efficiently factoring large integers be built, and thus break RSA, ECC, and other widely used ciphers.

What is quantum computing, and does it differ from classical computing? We know that classical computing uses bits of information that represent a "yes" or a "no" to an argument, or ones and zeros. A classical bit has two states 1 and 0. In quantum computing, the basic unit of information is a qubit (quantum bit). A qubit is a quantum system whose basis is two states:

$$|0\rangle \text{ and } |1\rangle \text{ or } \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The state of a qubit is represented by:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle,$$

where α and β are complex numbers that obey the normalization condition

$$|a|^2 + |\beta|^2 = 1.$$

The difference between a classical bit and a quantum bit is shown in Figure 1, where a classical bit can either be 1 or 0, and a qubit has possible states that lie along a sphere. It is possible for the qubit to form arbitrary linear combinations of states, which are called superpositions.
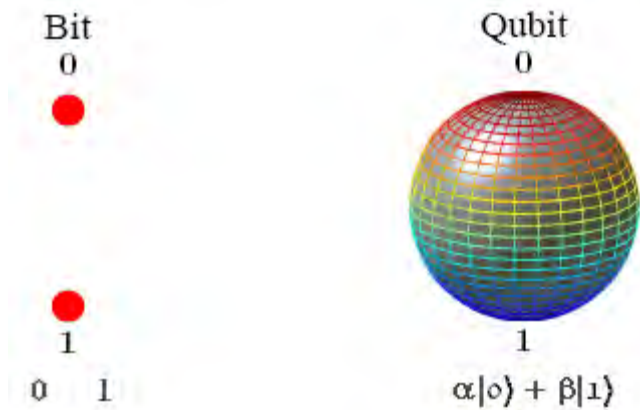
Figure 1.   Possible states of a bit and a qubit (From [9]).

Several quantum two-state systems can be used to physically realize qubits. For example, the state of the spin of a particle with spin $\frac{1}{2}$ could be defined as a qubit, where spin $+\frac{1}{2}$ corresponds to $|0\rangle$ and spin $-\frac{1}{2}$ to state $|1\rangle$. The polarization of a photon could be a qubit, where the horizontal polarization corresponds to $|0\rangle$, and the vertical polarization corresponds to $|1\rangle$. Also, qubits could be represented by two oscillating ions in an ion trap.

C.   QUANTUM REGISTER

In classical computers, we use a series of bits to form a memory register. In registers we store information or variables. Respectively, in quantum computers multiple qubits comprise a quantum register. While a classical register of N bits can represent a single value at a time (a unique assignment of each of the N bits to zero or one), a quantum register can represent 2^N values simultaneously, if each of the N qubits in the quantum register is in an equal superposition of zero and one. For example, for a register of 2 bits there are four possible states, $|00\rangle, |01\rangle, |10\rangle$ and $|11\rangle$. In a quantum register, the possible states are a linear combination $\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$, where $|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$. In a classical register we would be able to store

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only one of the four possible binary numbers 00, 01, 10, or 11, but in a quantum register of two qubits, we would be able to store all four possible states because of the superposition of the two qubits. Similarly for a quantum register with three qubits, we would be able to store eight states, sixteen states with four qubits, and 2^n states with n qubits. Once the qubits in the register are put into superposition, computation is applied to the register and simultaneously to each component of the superposition. This behavior follows from the linearity of operators on quantum mechanical systems and is called "quantum parallelism," which provides the speedup of quantum algorithms over classical algorithms.

D.   ENTANGLEMENT

Entanglement is the phenomenon where the state of two or more quantum bits cannot be described as a combination of the states of each qubit. Entanglement could be formed by several quantum transformations applied to more than one qubit.

When we make a measurement of one of the entangled qubits, we force it to take a definite value, and then we are able to predict the state of the other half of the pair. Until then (measurement) their state remains indefinite. An example of two entangled qubits could be:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

The difference between superposition and entanglement is that in superposition the measurement of a qubit doesn't determine the state of the other, while in entanglement the measurement of one qubit determines the state of the other.

E.   QUANTUM GATES

Classical computers have logic gates, e.g., AND, OR, and NOT, for building combinational circuits that help process information. For quantum computers, gates represent actions that are performed on qubits or quantum

registers. Also, the information is stored in the qubits and doesn't run through the gate. The quantum gates act on the qubits one after another until they reach the desired outcome. The most common quantum gates are:

- Hadamard gate
- CNOT gate
- Phase Shift gate
- SWAP gate
- Toffoli (CCNOT) gate
- Fredkin (CSWAP) gate

Information for each of the above quantum gates is shown in Table 1.

GATE	REPRESENTATION	ACTION	DETAILS
Hadamard	$H = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$\lvert 0 \rangle \rightarrow \dfrac{\lvert 0 \rangle + \lvert 1 \rangle}{\sqrt{2}}$ $\lvert 1 \rangle \rightarrow \dfrac{\lvert 0 \rangle - \lvert 1 \rangle}{\sqrt{2}}$	Acts on a single qubit.
CNOT	$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\lvert 00 \rangle \rightarrow \lvert 00 \rangle$ $\lvert 01 \rangle \rightarrow \lvert 01 \rangle$ $\lvert 10 \rangle \rightarrow \lvert 10 \rangle$ $\lvert 11 \rangle \rightarrow \lvert 10 \rangle$	Acts on 2 qubits and perfoms the NOT operation on the second qubit only when the first qubit is $\lvert 1 \rangle$.
Phase Shift	$R_{\theta} = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$	$\lvert 0 \rangle \rightarrow \lvert 0 \rangle$ $\lvert 1 \rangle \rightarrow e^{i\theta} \lvert 1 \rangle$	Modifies the phase of the quantum state, where θ is the phase shift.

SWAP	$\text{SWAP}=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\lvert 00\rangle \rightarrow \lvert 00\rangle$ $\lvert 01\rangle \rightarrow \lvert 10\rangle$ $\lvert 10\rangle \rightarrow \lvert 01\rangle$ $\lvert 11\rangle \rightarrow \lvert 11\rangle$	Swap gate acts on 2 qubits and swaps them.
Toffoli	$\text{CCNOT}=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	$\lvert 000\rangle \rightarrow \lvert 000\rangle$ $\lvert 001\rangle \rightarrow \lvert 001\rangle$ $\lvert 010\rangle \rightarrow \lvert 010\rangle$ $\lvert 011\rangle \rightarrow \lvert 011\rangle$ $\lvert 100\rangle \rightarrow \lvert 100\rangle$ $\lvert 101\rangle \rightarrow \lvert 101\rangle$ $\lvert 110\rangle \rightarrow \lvert 111\rangle$ $\lvert 111\rangle \rightarrow \lvert 110\rangle$	Toffoli gate acts on 3 bits only if the first 2 bits are in the state $\lvert 1\rangle$.
Fredkin	$F=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\lvert 000\rangle \rightarrow \lvert 000\rangle$ $\lvert 001\rangle \rightarrow \lvert 001\rangle$ $\lvert 010\rangle \rightarrow \lvert 010\rangle$ $\lvert 011\rangle \rightarrow \lvert 011\rangle$ $\lvert 100\rangle \rightarrow \lvert 100\rangle$ $\lvert 101\rangle \rightarrow \lvert 110\rangle$ $\lvert 110\rangle \rightarrow \lvert 101\rangle$ $\lvert 111\rangle \rightarrow \lvert 111\rangle$	Fredkin gate acts on 3 qubits and swaps the two last qubits if the first qubit is in the state $\lvert 1\rangle$

Table 1.   Most common quantum gates

F.   NO-CLONING THEOREM

For a register of qubits, we could assume that the amount of information that could be stored is exponential in the number of qubits in the register. However, this information cannot be extracted because measurement collapses the superposition, yielding a single value, i.e., a needle-in-a-haystack. Also, that

information cannot be copied, i.e., cloned. Wooters, Zurek and Dieks proved the no-cloning theorem, which applies to all quantum systems due to linearity in quantum mechanics [10].

The no-cloning theorem is the main principle behind quantum cryptography. Also known as quantum key distribution, it is not to be confused with quantum computing. It holds only for the general case where we have a qubit in an unknown state different than the two basis states ($|0\rangle$ and $|1\rangle$). However, there has been much effort [11],[12] in the last few years to perform optimal cloning with high fidelity.

G. QUANTUM ERROR CORRECTION (QEC)

One of the major implications of the no-cloning theorem is the field of error correction in quantum computing. In classical computing, error correction is used mainly in data transmission and is based on encoding information using data bits along with redundant bits that help to detect and correct errors. In quantum computing, measurement is impossible because it collapses the superposition. The need for correcting errors in quantum computing comes from the fact that qubits are very sensitive to noise, and they interact with the environment (decoherence) [13]. Errors that can occur during qubit processing are bit-flip errors and phase-flip errors. Without error correction, large-scale quantum computers could never be realized. However, using techniques from classical computation and aspects from quantum mechanics, Peter Shor and Steane in 1995 and 1996 respectively were the first to introduce quantum error correction codes. Since then there has been much effort in developing new codes [14],[15],[16].

H. WHY QUANTUM COMPUTERS

Quantum computers cannot efficiently perform every computation that is intractable on classical computers, e.g., algorithms, whose time complexity grows exponentially in the problem size. However, Shor's factoring algorithm runs in

polynomial time on a quantum computer and in exponential time on a classical computer [3]. Another important example is Grover's quantum algorithm [18] which has a time complexity of $O(N^{0.5})$ on a quantum computer and a time complexity of $O(N)$ on a classical computer for a problem size of N.

Despite the existence of these quantum algorithms, there are many problems for which quantum computing will not provide advantages over classical computers. For example, Scott Aaronson discusses how quantum computers need more than polynomial time to solve NP and NP-complete problems [19].

III.   PROMISING SCHEMES AND TECHNIQUES

In classical computing, the basic elements are software and hardware. In the same way, in quantum computing we need the hardware (physical realization of quantum bits and quantum gates) in order to implement the software (quantum algorithms such as Shor's algorithm and Grover's algorithm) discussed in the previous chapter. In this chapter, we will introduce some of the most promising quantum computer architectures.

A.   DIVINCENZO CRITERIA

Building a large-scale computer requires overcoming many challenges. In 2000, a researcher at IBM labs, David DiVincenzo, set the requirements for any physical implementation for a large-scale quantum computer [20]. These criteria are:

- Implementing qubits requires a scalable physical system whose physical characteristics are well known.
- Qubits should be easily initialized to their arbitrary values.
- Decoherence time should be much longer than the time needed for gate operation.
- There should exist an adequate number of quantum gates in order to cover every possible computation.
- Accurate measurement of each of the qubits.

B.   PHYSICAL IMPLEMENTATION

There has been much research in this area during the last ten years resulting in various experimental physical implementations of a quantum computer such as:

- Ion traps
- Nuclear Magnetic Resonance
- Cavity Quantum Electrodynamics (QED)
- Optical Photon quantum computer

and several more (quantum dots, solid state spin-based quantum computation, Josephson Junctions, linear optics) [21].

C. NUCLEAR MAGNETIC RESONANCE (NMR)

Nuclear Magnetic Resonance (NMR), is a technology used in various fields like medicine and chemistry. The discovery of Shor's algorithm gave researchers the momentum to experiment with NMR due the technology's quantum aspects. NMR uses radiofrequency electromagnetic waves to manipulate and detect nuclear spin states. This technology is so advanced that it allows a very large number of nuclei to be used in a single experiment. NMR uses the spin of the nucleus as qubits.

The actual apparatus consists of two main parts: the sample molecule, which could contain a number of protons n with spin ½, and the NMR spectrometer. Typical molecules are: ^{1}H, ^{13}C, ^{19}F, ^{19}N. The molecule is placed in a glass tube in the center of a superconducting magnet combined with the radiofrequency electronics, which are the parts of the spectrometer. Pulses are applied to the system to manipulate the spin states of the nuclei, which must have the appropriate phase, power, and frequency, which represent the gate function in quantum computing.

There have been successful experiments [22], [23] that showed that NMR has a long decoherence time and is able to work at room temperature. However, this scheme shows poor scalability and slow gate operation. Another major drawback of NMR is the difficulty in preparing the molecules into a pure state (initialization). The largest demonstration so far is a liquid state NMR with 12 qubits [24].

D. CAVITY QUANTUM ELECTRODYNAMICS (QED)

Another promising scheme is the use of Cavity Quantum Electrodynamics (QED). QED is the quantum theory that describes the interaction between an optical field and single atoms. Atoms that are trapped in a cavity, which interact

with single photons reproduced from an optical resonator, could represent qubits. Additionally, the polarization and the state of the photons coming out of the cavity after the interaction with the trapped atom could also represent a qubit. A proposed apparatus is shown in Figure 2. Various atoms have been used in this scheme such as Cs, Ca, Rb and Ba. A low–loss build up cavity with spherical mirrors is used in order to achieve a strong coupling. Photons are considered perfect because they can carry their state for a long time and distance. The basic idea behind cavity QED is that we are able to transfer the state of a photon to a trapped atom back and forth with the concurrent ability to control the atom's interactions with the photon.
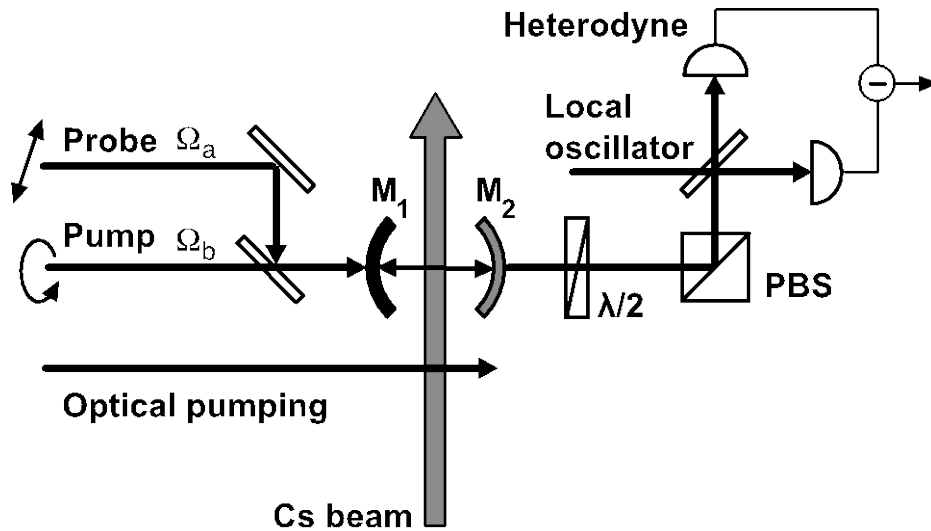
Figure 2.   An elementary quantum logic gate using optical QED, from [25].

The apparatus shown in Figure 2 was demonstrated by researchers using photons as "flying" qubits [26].

E.   OPTICAL PHOTON QUANTUM COMPUTER

This scheme, like the previous one, is based on the use of photon states or polarization as qubits. In the past, for classical computing, photons were considered as replacements for electrons for developing circuits because their

transmission uses less energy. Photons can be generated by an attenuated laser source. Other elements of this scheme include a phase shifter, a beam splitter, and a photo-detector.

The main strength of this scheme is the use of photons as qubits. Recent approaches [26], [27], [28] showed that this scheme would be viable for scalable quantum computing. The major drawback is that the best available technology to handle interacting photons is very week, making two-qubit gates very difficult to utilize with a sufficient success rate [29].

F.   ION TRAPS

The ion-trap architecture scheme, which will be described in detail in the next chapters, is based in the use of ions as qubits. Specifically, it is the spin state of the ion's nucleus and its outermost, or valence, electron that represent the qubit. The ion trap scheme is considered one of the most scalable quantum computer architecture schemes, but there are several problems that will be discussed later.

IV. ION TRAPS

Wolfgang Paul was the first researcher [30] that introduced the concept of ion traps in the 1950s. His experiments focused on separating atoms with different masses in order to observe them. He received the Nobel Prize in 1989 for his contribution to the development of atomic precision spectroscopy. As discussed in the introduction, Cirac and Zoller [4] were the first to propose the use of the "Paul" trap as a way to implement qubits using trapped ions in order to build scalable quantum computers. The NIST group took advantage of this proposal, and in 1995, they introduced a CNOT quantum gate with a Be^+ ion [31]. Another major achievement was made by the Innsbruck group, who demonstrated the Deutch-Josza algorithm with a Ca+ ion, and in the same year (2003), the first implementation of a set of universal gates using a string of two ions [6], [32]. Furthermore, there have been several experiments in the fields of teleportation [7], quantum error correction [33], and entanglement [34], [35] using ion traps.

Figure 3.   Qubit states of an ion, (From [36]).
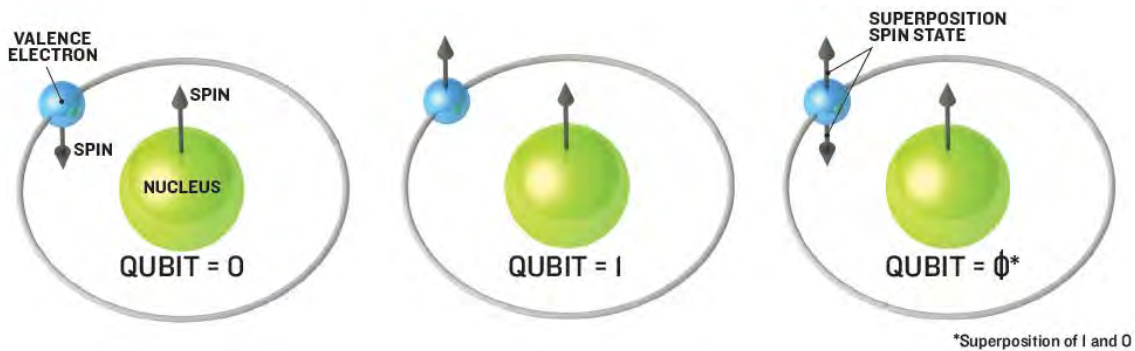
In ion-trap based quantum computers, the spin state of the particle determines the value of the qubit. The quantum information is stored in the ground state levels. The spin state could be up or down based on the angular momentum of the electron. The states of the qubit depend on the spin of both the nucleus and its valence electron. Therefore, when the spins have the same

direction, the qubit is in state $|0\rangle$; when they have opposite directions, the qubit is in in state $|1\rangle$. When the spin of the valence electron is in a superposition state, the directions of the spins are up and down at the same time, as shown in Figure 3.

There are several different approaches for implementing ion traps; however, the main components are an ultra-high vacuum so that other particles do not collide with the ion; a radio frequency resonator to achieve the desirable high trapping electric field; and a source where the ions would enter the vacuum.

Another matter that has to be addressed is the temperature. At room temperature, collisions that could result in the loss of quantum information occur in large numbers over a short time, eliminating any hope of storing quantum information. In order to perform quantum computation, a laser source must cool the ions. Photons bombard the ions at the right *resonance* frequency to slow them down in the center of trap (or axis for multiple ions) at very low temperature. Lasers are also used to initialize the qubits by bombarding ions with photons at the required energy as shown in Figure 4. The ions which are in state $P_{3/2}$ decay either in state $D_{5/2}$ ($|1\rangle$) or in state $S_{1/2}$ ($|0\rangle$).

In Chapter III, we outlined the DiVincenzo criteria, which must be met in order to build a scalable quantum computer. The ion-trap based quantum computer is considered the leading scheme for satisfying the DiVincenzo criteria as follows:

1.   *A scalable physical system with well characterized qubits.* The internal levels (superpositons) between the ground state and an excited stated of several ions have been used to represent a qubit [37], [37]. By forming a linear trap with strings of ions, a scalable system could be built. However, other approaches which involve ions moving among multiple traps have also been proposed [39], [40].

2.   *Ability to initialize the state of the qubits.* Lasers are used to initialize the qubits by pumping the ion with photons at the right amount of energy in order to force the ion to an exited state from which it decays to the ground state as shown for a 40 Ca+ ion in Figure 4.

3.   *Decoherence time much longer than the operation time.* Recent work showed that a decoherence time of 100 ms can be achieved for a qubit, which is 1000 times longer than the operation time of a quantum gate [8].

4.   *A universal set of quantum gates: single-qubit and two-qubit gates.* Focused laser beams on individual qubits act as single qubit-gates. Several two-ion gates, such as the controlled-NOT gate [32], and more recently the Toffoli gate [41], have been demonstrated.

5.   *A qubit-specific measurement.* The readout of a qubit's state is achieved by measuring the fluorescent light emitted after the irradiation of the ions with a laser beam with a frequency in resonance for the transition from $S_{1/2}$ to $P_{1/2}$. If the ion is in state $|0\rangle$ ($S_{1/2}$), it absorbs a photon, excites to state $P_{1/2}$ and then decays to state $|0\rangle$ again by emitting a photon. If the ion is in state $|1\rangle$ ($D_{5/2}$) then the transition to $P_{1/2}$ is not allowed, so no photon will be emitted.
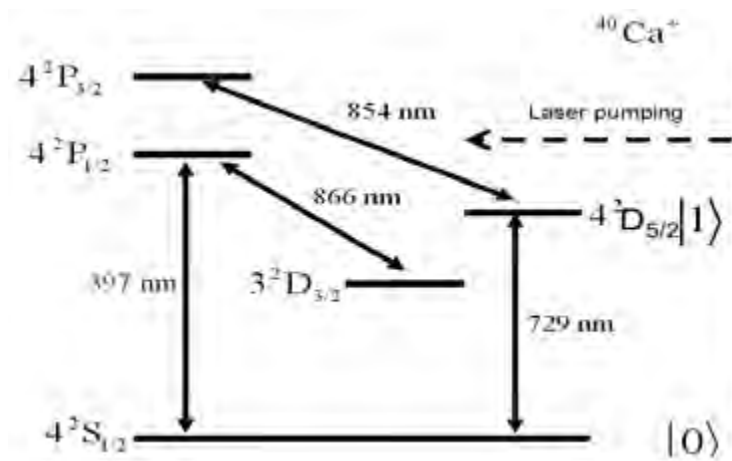
Figure 4.   An example of a Ca$^+$ ion showing its levels. The numbers in nm are the wavelengths of the transition frequencies. The ion is initialized by a laser beam in resonance with the frequency required for the transition from $D_{5/2}$ to $P_{3/2}$. (From [42]).

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There are several groups that are experimenting with ion trap quantum computing. In 2004 there were around a dozen, but in the last few years the number has grown to over 50 [43], [44]. That clearly shows the interest in building scalable quantum computers through implementing ion trap techniques.

V. CONFINING POTENTIAL

There are several variations of ion traps, including (1) the cylindrically symmetric 3D ring trap; (2) the linear trap with a combination of cavity QED; (#) the symmetric quadrupole linear trap; and (4) the asymmetric quadupole linear trap, shown in Figure 5. In this chapter, we will analyze the confining potential of the linear radio frequency quadrupole trap. This trap, shown in Figure 5, consists of four cylindrical electrodes, two of which (diametrically opposite) produce a steady DC voltage U_o that holds the ion in the center of the line between the four electrodes and an RF voltage that protects the ion from static.

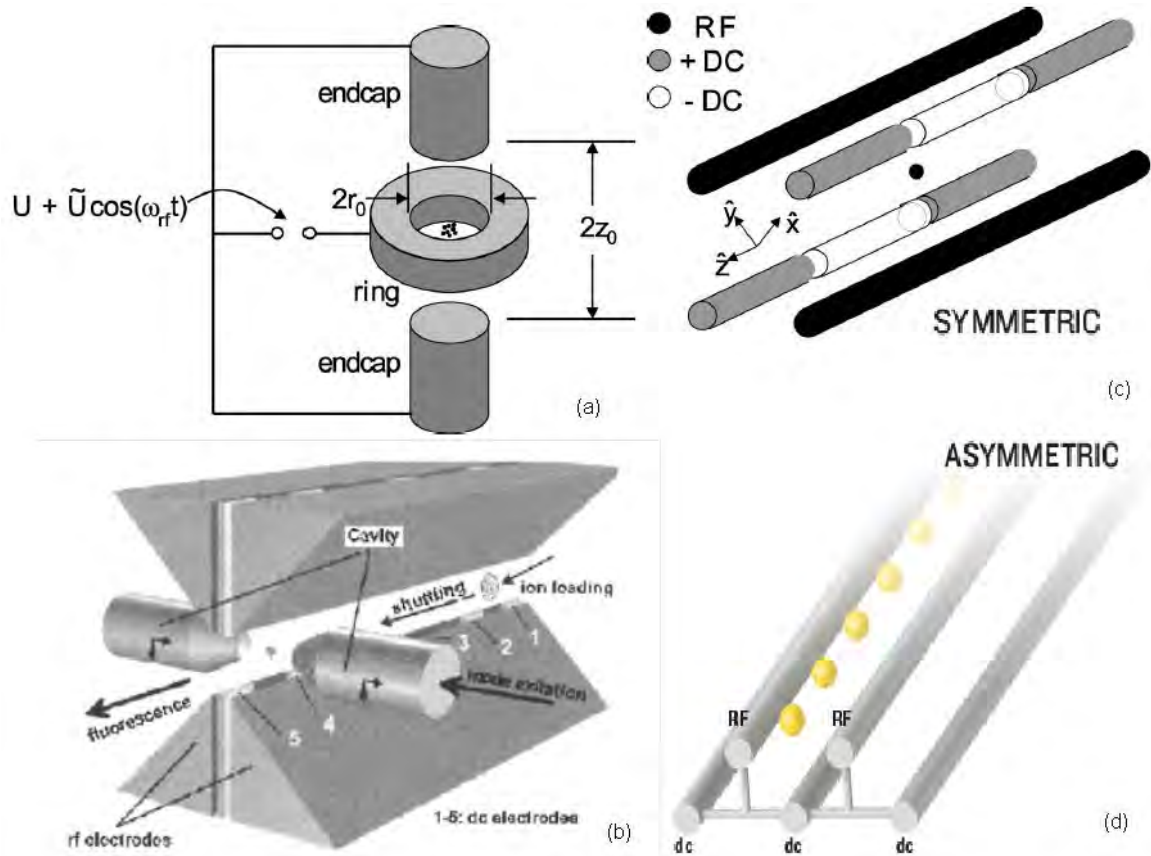Figure 5.   Examples of ion traps from [30], [36], [45], [46]: (a) a cylindrically symmetric 3D trap; (b) a linear RF ion trap with cavity QED; (c) a symmetric linear RF trap; and (d) an asymmetric linear RF trap.

We will discuss the trapping potential in a linear quadrupole RF trap. The dynamics of an ion in a linear radio frequency ion trap are established by the classical motion equation. The trapping potential for an electric quadrupole field is [30]:

$$\Phi = \Phi_o\left(\frac{\kappa x^2 + \lambda y^2 + \mu z^2}{2r_o}\right) \tag{5.1}$$

where r_o is the distance between the center of the trap and the rod, and κ, λ and μ are parameters that have to satisfy the relation $\kappa + \lambda + \mu = 0$ so that $\nabla^2\Phi = 0$. For $\kappa = -\lambda = 1, \mu = 0$ we get a two dimensional field, and Eq. 5.1 becomes:

$$\Phi = \Phi_o\left(\frac{x^2 - y^2}{2r_o}\right) \tag{5.2}$$

In the quadrupole RF trap, the applied voltage is equal to:

$$\Phi_o = U + V\cos(\Omega t) \tag{5.3}$$

where Ω is the driving frequency, which changes Eq.5.2 to:

$$\Phi(x,y,t) = (U + V\cos(\Omega t))\left(\frac{x^2 - y^2}{2r_o}\right) \tag{5.4}$$

We derive the equations of an ion's motion with mass m and charge e in the x-y plane by applying Newton's laws and Maxwell's equations:

$$\frac{d^2x}{dt^2} = -\frac{e(U + V\cos(\Omega t))x}{mr_o^2} \tag{5.5a}$$

$$\frac{d^2y}{dt^2} = \frac{e(U + V\cos(\Omega t))y}{mr_o^2} \tag{5.5b}$$

By setting the unitless parameters т, ξ, and q, which respectively are equal to:

$$\tau = \frac{\Omega t}{2}, \quad \xi = \frac{4eU}{mr_o^2\Omega^2}, \quad q = \frac{2eV}{mr_o^2\Omega^2} \tag{5.6}$$

we get the following Mathieu equations from Eq. 5.5a and b:

$$\frac{d^2x}{d\tau^2} + \left[\xi + 2q\cos(2\tau)\right]x = 0,$$
$$\frac{d^2y}{d\tau^2} - \left[\xi + 2q\cos(2\tau)\right]y = 0 \tag{5.7}$$

Mathieu equations have two types of solution, one that represents stable motion where the particles oscillate with limited amplitude and another that is unstable where the oscillating amplitude grows exponentially. The stability depends on the parameters ξ and q with $\xi < q^2 \ll 1$. A stability diagram is presented in [45]. The solution of Eq. 5.7 for x is:

$$x(\tau) = Ae^{i\beta\tau} \sum_{n=-\infty}^{\infty} C_{2n} e^{i2\beta\tau} + Be^{-i\beta\tau} \sum_{n=-\infty}^{\infty} C_{2n} e^{-i2\beta\tau} \tag{5.8}$$

with $\beta \approx \sqrt{\xi + \dfrac{q}{2}}$.

Ion trap based methods are attractive options for the realization of quantum computers, due to the increased lifespan, i.e., coherence, of the qubit and later-mediated interaction of Coulomb force between multiple ions in a string [4], [45], [39]. In other words, the decoherence time is low compared to other technologies. Several techniques for implementing quantum gates for ion trap based quantum computers have been proposed [39], and some ion traps have been demonstrated. The challenge remains to increase the number of ions in the trap to realize the benefits of quantum parallelism. The linear radio frequency ion trap, which uses laser cooling and is detained in a 1-D crystal, is the working platform for ion trap based quantum computing. Nonetheless, adjusting the linear radio frequency ion trap to measurable numbers of ions still poses crucial difficulties for the researchers.

The combination of DC and RF fields in the trap puts the ion in radial motion, which is complex, and is described by differential equations. These equations when solved lead to the stability diagram which enables one to evaluate the efficiency of trap and provides values of various critical parameters like voltage components, RF amplitude, trap size, and ion mass [48]. Until these values are fulfilled, the ion retains its position at the device's axis. Moreover, the magnitude of the pseudo-potential of the restoring forces, responsible for holding ions in the proper direction, has a direct relationship in proportion to the distance from the center. The secular motion of the ion becomes distorted in the event of

misalignment of the pseudo-potential and the radio frequency field minima in the trap. Misalignments arising from the asymmetry in the construction of the trap or a patch of minimal DC potentials on the surface of the electrodes lead to the ion getting displaced from the radio frequency field. This is unrelated to the cause of misalignment. Along with micro-motion of the ions causing them to heat up and the secular motion of ion, there is also the vibration of the ion in the axial direction.

In the case where the number of confined ions is few, they align themselves in a linear configuration along the axis; however, increasing the DC voltage or increasing the number of ions causes instability in the trap. This happens due to the close association of ions with each other, leading to ions squeezing against each other. The radial restoring force becomes weak in comparison to the Coulomb repulsion between adjacent ions causing ions to move in zig-zag pattern. The situation worsens with the addition of more ions in the trap, and the zig-zag pattern takes the complex structure of a three-dimensional helix [49]. Ions move farther from the axis, and the micro-motion heating initiates, which must be avoided in the experiment.

The major problem of radio frequency ion traps is that the charged particles like ions are comparatively sensitive to stray electric fields in their vicinity. Such fields adversely impact the motion of these ions and become time-dependent, resulting in the heating of ions. The heating rates for two ions in the trap are on the order of 1 ms. The rate of heating increases with the increase in the number of ions and is dependent on the number of particles coupled with stray fields along with the increase in the quantum of the degree of freedom. Ion traps must perform a sufficient number of gate operations within the decoherence time limit, and to bring the ions to their excited state for the purpose of implementing qubits, it is extremely crucial that they be metastable in order to avoid impulsive emission. This involves having these states driven by weak optical transitions and low Rabi frequencies. Increasing the number of ions in the

trap is hindered by the frequency of the trap and by the fact that heating increases with the number of ions in the trap.

Despite its limitations, linear RF ion traps have demonstrated several fundamental concepts of quantum information. The major demonstration has been the test of a Bell inequality as demonstrated by Rowe et al. [50] and a decoherence-free subspace demonstration by Kielpinski et al. [51]. As the number of ions in the trap increases, it becomes more difficult to trigger the string of ions with photons. Four techniques have been proposed to overcome these difficulties, including: (1) splitting the ion string in minute portions to facilitate the movement of ions [40]; (2) coupling of ions through photons and cavities [52]; (3) wiring of ion traps through the use of image chargers [53], and (4) utilizing the radial modes of a string of ions [54]. Currently, the first proposal appears to be the most promising as demonstrated by [51]. Segmented ion traps permit movement by changing voltages on the electrodes of the trap. The ion strings can also be split and merged, which provides the flexibility to tailor the ion string size according to the register size. Rowe et al. [50] and Barrett et al. [7] demonstrated this procedure successfully.

Despite progress to address the difficulties of quantum computing using linear radio frequency ion traps, several challenges remain. To achieve large-scale quantum computing, the fault tolerance must be addressed. This requires applying quantum error correction methods to ion traps. In addition to improving the inherent reliability of ion traps to a minimum requisite threshold (and increasing the decoherence time), researchers must also use redundant physical quantum bits to implement quantum error correction. Furthermore, the encoding and decoding process of quantum error correction must occur within the decoherence time of the technology and must not introduce additional errors. Transverse gate operation is one technique for minimizing the introduction of errors during the encoding and decoding process. This involves designing quantum gates that operate on quantum bits in their encoded state rather than decoding, applying the gate, and re-encoding, a cycle that could introduce further

errors. The decoherence time of the qubit must be one or two orders of magnitude greater than the time required for quantum gate operation. Several studies have demonstrated decoherence times as much as five orders of magnitude larger than gate operation time [55]. Cooling ion trap electrodes to cryogenic temperature allows strong suppression of motional decoherence [56]. The accuracy of qubit initialization of 0.999 must be improved upon. As demonstrated by Knill et al., the operation of a single qubit can carried out with fidelities beyond 0.995 [57], and further improvement in fidelity can be achieved using increasingly stable laser fields at the position of the ions [58]. The achievement of a fidelity of 0.9 for two-qubit gate operation by Benhelm et al. is a significant achievement [59], [60]. However, the fidelity of the operation is dependent on the type of gate. The major sources of error are (1) off-resonant scattering, (2) insufficient cooling, (3) error in addressing standalone qubits, (4) intensity of noise, and (5) the frequency of the laser. Myerson et al. demonstrated that single-qubit readout can be performed with a fidelity of 0.999. Moreover, as demonstrated by Blakestad et al., ion strings can be merged, split, and shuttled with low decoherence and high fidelity [61].

Benhelm et al. [59] established two-qubit operations in a fault tolerant manner based on the method of Knill [57]. Linear radio frequency ion traps are useful for performing quantum computations, and their continued development enables further progress. For example, the single-qubit gates have been evolved significantly over the last decade; however, the two-qubit gates require further research.

VI. QUANTUM DYNAMICS BEHIND ION TRAPS

The quantum dynamics of ion traps are thoroughly described in [45], [39]. In this section, we will review the quantum dynamics behind a two-level trapped ion with light fields in a quadrupole trap.

The Hamiltonian for an ion that corresponds to the internal electronic level structure is given by:

$$H_e = -\frac{\hbar\omega_o}{2} S_z \tag{6.1}$$

where $\hbar\omega_o = \hbar\omega_{|1\rangle} - \hbar\omega_{|0\rangle}$, which is the energy difference between the ground and exited states.

The Hamiltonian that corresponds to the motion of the ion (see Eq. 5.8) along the trap axis is given by:

$$H_m = \frac{p^2}{2m} + \frac{m\Omega^2[\xi + 2q\cos(\Omega t)]}{8} \tag{6.2}$$

The Hamiltonian for the interaction is a result of an ion with spin **S** which interacts with an electric field **E** and is defined as:

$$H_{int} = -\boldsymbol{\mu}\mathbf{E}(z,t) \tag{6.3}$$

where **μ** is the electric dipole operator equal to $\boldsymbol{\mu} = \mu\mathbf{S}$. For spin we have $\mathbf{S}_\pm = \dfrac{S_x \pm iS_y}{2}$ and $\mathbf{E} = \mathbf{E}_o\sin(\mathbf{k}x - \omega t - \varphi)$. Thus Eq. 6.1 becomes

$$H_{int} = \hbar\Omega\cos(kz\cos\theta - \omega t - \varphi)(S_+ + S_-) \tag{6.4}$$

where
$$\hbar\Omega = e_-\left\langle 0\left|(\mathbf{kr})(\mathbf{E_o r})\right|1\right\rangle \tag{6.5}$$

where θ is the angle between the **k** and z axis, **k** is the wave vector of the laser beam that propagates, and **r** is the position of the valence electron from the nucleus.

By applying the main part of the Hamiltonian, which is H_e+H_m, as the interaction picture to the interaction Hamiltonian, which is $U_o = e^{-i\hbar(H_e+H_m)t}$, the transformed interaction Hamiltonian for this interaction picture is:

$$H'_{int} = U^\dagger H_{int} U = \frac{\hbar\Omega}{2}(S_+ e^{i\omega_o t} + S_- e^{-i\omega_o t})e^{iH_m t/\hbar}[e^{i(kx-\omega t-\varphi)} + e^{-i(kx-\omega t-\varphi)}]e^{-iH_m t/\hbar} \quad (6.6)$$

By introducing the Lamb-Dicke parameter $\eta = \kappa\sqrt{\dfrac{\hbar}{2mv}}$, where m is the mass and v is the frequency, the interaction Hamiltonian is:

$$H'_{int} = \frac{\hbar\Omega}{2}\left[S_+ e^{i\eta\left(\xi e^{-ivt}+\xi^\dagger e^{ivt}\right)}e^{i(\varphi-\delta t)} + \text{Hermitian Conjugate} \right] \quad (6.7)$$

where δ is the frequency difference between the laser and the ion.

There exists the possibility of coupling a motional state to an ionic state by controlling the detuning δ [45]. The coupling exists between states $|0\rangle|n\rangle$ and $|1\rangle|n+s\rangle$ when the condition $\delta \approx sv$ is met, with $s = k - l$, where k and l are the number of ξ and $\xi^\dagger$ operators, respectively.

For s=0, we have the carrier transition, with the Hamiltonian:

$$H_{CT} = \frac{\hbar\Omega}{2}\left(S_+ e^{i\varphi} + S_- e^{-i\varphi}\right) \quad (6.8)$$

which corresponds to $|0\rangle|n\rangle \leftrightarrow |1\rangle|n\rangle$.

For s>0, we have the red sideband transition and, for example, if s=-1, the Hamiltonian is:

$$H_{RST} = \frac{i\hbar\Omega}{2}\left(\xi S_+ e^{i\varphi} + \xi^\dagger S_- e^{-i\varphi}\right) \quad (6.9)$$

which corresponds to transitions $|0\rangle|n\rangle \leftrightarrow |1\rangle|n-1\rangle$.

For s>0, we have the red sideband transition and, for example, if s=1, the Hamiltonian which corresponds to transitions $|0\rangle|n\rangle \leftrightarrow |1\rangle|n+1\rangle$ is:

$$H_{BST} = \frac{i\hbar\eta\Omega}{2}\left(\xi^\dagger S_+ e^{i\varphi} + \xi S_- e^{-i\varphi}\right) \quad (6.10)$$

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VII. MECHANICAL ANALOGUE OF THE CONFINING POTENTIAL

In this chapter, we will discuss a pedagogical device we built to explain the principles behind ion traps. Our device is based on a famous physics experiment [62]. Wolfgang Paul demonstrated a similar apparatus during his Nobel Prize speech [63]. This device is hyperbolic-parabolic surface that rotates in order to simulate an ion trap, and a ball represents the ion.
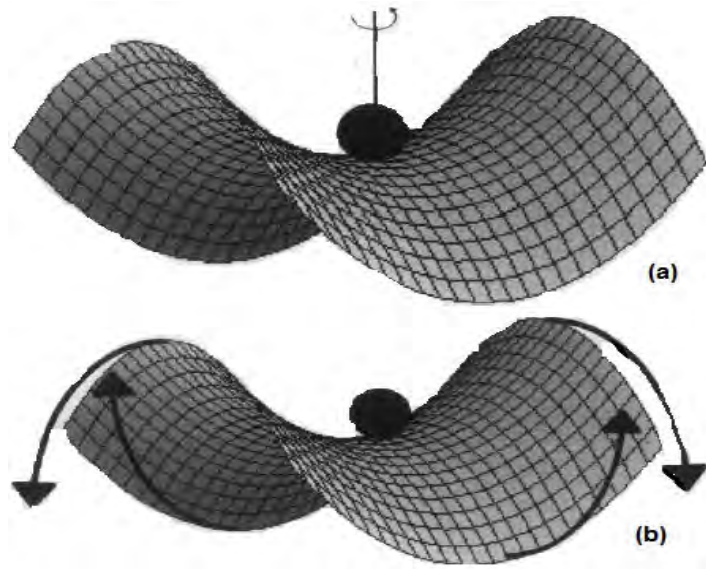
Figure 6.   Illustration of the difference between the potential in the mechanical analogue and the electric potential in linear RF ion traps. In (a) the trapping potential rotates, whereas in (b) the electric potential flaps (up and down). (From [62]).

This mechanical analogue can be described by the gravitational potential:

$$\Phi(x, y) = \gamma(x^2 - y^2) \tag{7.1}$$

where γ is a geometrical factor that is equal to:

$$\gamma = \frac{h}{r^2} \tag{7.2}$$

where h is the maximum height of the rotating surface with radius r.
There are two basic differences between the mechanical analogue and the electric potential of an ion trap [62]:

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1.   The mechanical analogue potential isn't a flapping potential; instead, it rotates (see Figure 6).

2.   In the mechanical analogue, friction and rolling of the ball along the surface play a significant role, but this role is very difficult to quantify [62].

The potential of the apparatus is described in [62]:

$$\Phi(x,y,t) = mg\gamma[(x^2 - y^2)\cos(2\Omega t) + 2xy\sin(2\Omega t)] \qquad (7.3)$$

where m is the mass of the ball, g is the gravitational acceleration, Ω is the angular drive frequency of the spinning surface, and γ is a geometrical constant with units m^{-1} that determines the shape of the saddle surface. We can determine the Lagrangian of the system as:

$$L = \frac{1}{2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right] - g\gamma\left[(x^2 + y^2)\cos(2\Omega t) + 2xy\sin(2\Omega t)\right] \qquad (7.4)$$

We assume that z is a dependent coordinate due to the limited movement along the zxaxis resulting in no $\dot{z}$ term in Eq. 7.4. Also, we assume that the ball on the apparatus is not rolling, so the Langrange's equation of motion becomes:

$$\frac{d^2x}{dt^2} = -2g\gamma[x\cos(2\Omega t) + y\sin(2\Omega t)] \qquad (7.5)$$

$$\frac{d^2y}{dt^2} = -2g\gamma[x\sin(2\Omega t) - y\sin(2\Omega t)] \qquad (7.6)$$

$$z = \gamma(x^2 - y^2) \qquad (7.7)$$

Using the same approach as in the confining potential in Chapter V, $\tau = \Omega t, \; q = \dfrac{g\gamma}{\Omega^2}$, Eq. 7.5 and 7.6 become:

$$\frac{d^2x}{d\tau^2} = -2q[x\cos(2\tau) + y\sin(2\tau)] \qquad (7.8)$$

$$\frac{d^2x}{d\tau^2} = -2q[x\sin(2\tau) - y\sin(2\tau)] \qquad (7.9)$$

In order to solve these two coupled differential equations we use the trick $z = x + iy$ to get:

$$\frac{dz}{d\tau} + 2qz^* e^{2i\tau} = 0 \tag{7.10}$$

The solution of this differential equation is in the form of:

$$z(\tau) = f(\tau)e^{i\tau} \tag{7.11}$$

Now, if we combine Eq. 7.10 and 7.11 we are able to eliminate $f^*(\tau)$ and get the following fourth-order equation:

$$\frac{d^4 f}{d\tau^4} + 2\frac{d^2 f}{d\tau^2} + (1 - 4q^2)f = 0 \tag{7.12}$$

The solution to Eq. 7.12 is a linear combination of four exponential terms:

$$f(\tau) = Ae^{+\beta_+ \tau} + Be^{-\beta_+ \tau} + Ce^{+\beta_- \tau} + De^{-\beta_- \tau} \tag{7.13}$$

where coefficients $\beta_\pm$ are equal to:

$$\beta_\pm = \sqrt{\pm 2|q| - 1} \tag{7.14}$$

In order for a particle to be trapped, all the exponential terms must be complex, which leads to the requirement:

$$2q = \frac{2g\gamma}{\Omega^2} \le 1 \tag{7.15}$$

Eq. 7.15 is the trapping parameter for the apparatus; it is equivalent to Eq. 5.8 and shows the relationship to the actual ion trap.

We built apparatus for demonstration using the above equations. We used a Rapid Prototyping / Additive Manufacturing (3D Printer) from NPS Space Systems Academic Group to construct the saddle surface. We placed it on a rotating table where we could vary the frequency of the rotation as shown in Figure 6. We used various types of balls to demonstrate the trapping potential. The best results were with a steel ball, which resulted in trapping lifetimes of ~15 seconds.

Figure 7. Side view of the apparatus used for demonstration of the trapping potential. The saddle surface has a diameter of 15cm, and the steel ball is 1.9cm in diameter.

Our current model needs various enhancements. We should add a photodiode so that we can make more accurate measurements of the frequency. In addition, we must try different materials for the saddle surface, since the material used from the 3D printer was rigid and rough due to the fact that is was "printed" in layers, resulting in a short trapping time.

VIII. TEACHING TOOLS

In this section, we will review teaching materials, such as lectures, tutorials, and presentations from various institutions, that are useful for constructing a course module on ion trap based quantum computing. Most of the institutions not only have courses for teaching quantum computing based on ion traps, but also experimental initiatives. These groups make their research achievements, e.g., academic papers, and course materials available online.

A good place to start is University of Maryland, which is the home of the Trapped Ion Quantum Information Group (TIQIG) led by Christopher Monroe. An introductory tutorial on ion traps is available at this group's website; the tutorial explains the physics behind ion traps, how to make quantum bits with ion traps, and how lasers interact with ion traps [64]. This tutorial is written for a general audience and does not require a background in physics. The tutorial also includes a periodic table of ions that are preferred for use in ion traps. Useful slides contain interesting photos of their laboratory activities. The website also has a list of recent articles on ion traps and a list of upcoming conferences on Quantum Information Science and Technology. However, the website does not include a list of courses on ion traps offered at UMD.

A visualization of an ion trap is another useful teaching tool. MIT has a tool that simulates the interaction between charged particles in a trap [65]. Although it does not apply to linear traps, it is useful to understand the interaction between the Coulomb force and the repulsive quantum-mechanical Pauli force. The interface allows the user to make changes to the particles and the environment and observe the result of their actions. MIT also has a well-organized course on Quantum Information Science [66]. Its website includes well-written lecture notes as well as exercises and their solutions. The course does not focus specifically on ion traps, but it is still a useful resource for materials regarding quantum computing in general.

In addition to conducting research on the simulation of ion traps, Sandia National Laboratories developed software to prepare, visualize, and manipulate trapped charged particles [67]. The result is a 3D representation of ion trap geometry, electrode voltages, and ion motion.

Courses on quantum computing are offered at several institutions, including Portland State University [68]. This course devotes a week to the realization of quantum computers with a module on ion traps. There are slides, lecture notes, and video recordings of lectures for each subject. For example, there is an interesting introductory video lecture from David Deutsch about the qubit [69].

University of California, Berkeley also offers a course on Qubits, Quantum Mechanics, and Computers [70]. This course is very well organized, providing reading material and well-written presentations and lecture notes. In addition, there is a guest lecture on Quantum Computing with Ion Traps by Hartmut Häffner, a researcher form the Institute for Quantum Optics and Quantum Information of Innsbruck, Austria.

California Institute of Technology offers a course titled Quantum Computation. This course is divided into two parts. The first part, taught by Alexei Kitaev, focuses on the basics of quantum mechanics, complexity theory, quantum circuits, and algorithms. The second part covers quantum error correction codes. The course uses very good lecture notes written by the instructor of the second part of the course, John Preskil [71].

The University of Washington also offers courses on quantum computing. Lecture notes by Mark Oskin, a famous computer architect, are available online [72]; these notes were created from [29] and can be used for a Quantum Computing course. The University of Washington website also offers slides and presentations on quantum computer architecture. Several other courses on quantum computing are also available [73], [74] and [75].

Finally, a group at Colgate University produced a set of laboratories [76]. These laboratories were designed to teach the basics of quantum mechanics with correlated photons.

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IX.   CONCLUSION

In the previous chapters, we reviewed the history behind quantum computing. We provided background on the basic principles of quantum computing, and we discussed the importance of quantum computing and its advantages over classical computing for certain problems.

Also, we outlined the requirements for building a scalable quantum computer and presented some of the leading schemes for satisfying these requirements and some of the most challenging obstacles that each scheme must overcome.

In this thesis, we focused on the ion trap scheme, which many researchers believe is the most promising scheme for scalable quantum computers. Ion traps fulfill all five of the DiVincenzo criteria for building scalable computers, although more research is needed to address problems associated with trapping multiple ions.

We also analyzed the confining potential of a linear Paul ion trap and reviewed its quantum dynamics. We compared the trapping potential of the linear ion trap to the confining potential of a mechanical analogue, a pedagogical apparatus we built in the lab to demonstrate the ion's motion.

Finally, we identified several teaching tools, including lecture notes, presentations, tutorials, video lectures, and labs that several institutions and groups have made available online. These materials are useful for building a course module on ion traps.

There are several opportunities for future work. Our apparatus could benefit from refinements, including a photo-detector for precisely measuring the rotation of the saddle and different types of materials for the saddle surface and the ball. We also leave to future work the development of a course module based on the materials we identified, together with the apparatus we built, which could be used for demonstrations and labs. A course module on ion trap based

quantum computing for physics and computer science students would include lecture notes, slides, lesson plans, a syllabus, reading lists, videos, demonstrations, and laboratories.

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