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A MODEL FOR THE GROWTH OF NETWORK SERVICE PROVIDERS

by

David Tou Wei Chiam

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Thesis Advisor:	David L. Alderson
Second Reader:	Nedialko B. Dimitrov

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Consider a set of points and an associated demand for traffic between each pair of points. In this thesis, we consider the perspective of a notional Network Service Provider (NSP) who has to decide on the connections to build and the demands to satisfy in order to maximize its profits. The NSP makes these decisions based on the demand for connectivity and the constraints on their resources needed to provide the connections. We perform numerical experiments to study the tensions faced by the NSP in its decisions to structure its service network. Through the results generated, we infer how demand, revenue and cost influence the decisions of the NSP.

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A MODEL FOR THE GROWTH OF NETWORK SERVICE PROVIDERS

David Tou Wei Chiam
Civilian, ST Engineering, Singapore
B.Eng.(Hons), National University of Singapore, 2004

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NAVAL POSTGRADUATE SCHOOL
December 2011

Author: David Tou Wei Chiam

Approved by: David L. Alderson
Thesis Advisor

Nedialko B. Dimitrov
Second Reader

Robert F. Dell
Chair, Department of Operations Research

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ABSTRACT

Consider a set of points and an associated demand for traffic between each pair of points. In this thesis, we consider the perspective of a notional *Network Service Provider* (NSP) who has to decide on the connections to build and the demands to satisfy in order to maximize its profits. The NSP makes these decisions based on the demand for connectivity and the constraints on their resources needed to provide the connections. We perform numerical experiments to study the tensions faced by the NSP in its decisions to structure its service network. Through the results generated, we infer how demand, revenue and cost influence the decisions of the NSP.

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LIST OF ACRONYMS AND ABBREVIATIONS

BCG	Bilateral Connection Game
GAMS	General Algebraic Modeling System
Gbps	Gigabits per second
ISP	Internet Service Provider
NSP	Network Service Provider
O-D	Origin-Destination
POP	Point of Presence
UCG	Unilateral Connection Game

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EXECUTIVE SUMMARY

A *network* is a system of entities, called *nodes*, that are connected together to perform a service. We refer to the connections as *arcs* or *links*. The *network service* typically involves the exchange of information, physical goods, or other commodities.

Many systems that affect our daily lives rely upon networks in order to function. For example, the Internet is the fundamental global information structure, a critical element in the interactions between people, businesses and other organizations. Many civilian and military systems are dependent on it to function.

Over the last decade, there has been considerable effort devoted to understanding the structure and function of many different types of networks. Much of this effort has focused on graph connectivity properties and random generation mechanisms that give rise to them. However, there can be many generation mechanisms that give rise to the same network structure. Thus, the ability of a model to produce connectivity statistics similar to those in the real world does not by itself prove that the model explains that real system.

The Internet provides an interesting case study, where simple models of preferential attachment have been argued to be the cause of power-law distributions in network connectivity. A closer look at the details of the real Internet reveals the specious nature of these models and argues instead for models of network formation that capture the economic and technical drivers of real network growth.

In this thesis, we consider the perspective of a notional *Network Service Provider (NSP)* who has to decide on the connections to build and the demands to satisfy in order to maximize its profits. The NSP makes these decisions based on the demand for connectivity and the constraints on their resources needed to provide the connections. We model the decisions of the NSP, through a *traffic engineering model* and a *network provisioning model*, and we perform numerical experiments to study the tensions faced by the NSP in its decisions to structure its service network.

We make use of the Abilene dataset as input to the network provisioning model and assume that the NSP is new to the market and is building an entirely new network.

Our model includes two types of costs: fixed costs for building new arcs and variable costs for adding capacity to the arcs. When there are no fixed costs associated with building new arcs, we see that flows of commodities are through direct arcs, which are the least costly to transport the commodities. We also identify that the priority is to build arcs to satisfy the demand of the commodity with the greatest marginal profit. If the marginal profit for a commodity is negative, the direct arc will not be built. This gives a simple greedy algorithm heuristic.

When we introduce fixed costs, the flows of commodities are no longer necessarily via direct arcs. Even when the fixed costs are low, we observe that there are commodities that are not transported even though the network has capacity to do so. By increasing the marginal revenue for the unfulfilled demand, we can make it profitable for the NSP to service that demand. We also illustrate a case where increasing marginal revenue leads to a new network construction that also reduces the costs of servicing other demands.

We generalize the results in a network provisioning decision rule that provides a simple heuristic of constructing the NSP's network. We observe that when fixed costs get higher, more nodes will be disconnected and nodes with larger populations tend to remain connected.

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I. INTRODUCTION

A. BACKGROUND

A *network* is a system of entities, called *nodes*, that are connected together to perform a service. We refer to the connections as *arcs* or *links*. A *network service* typically involves the exchange of information, physical goods, or other commodities over the shared network.

Many systems that affect our daily lives rely upon networks in order to function. For example, the Internet is the fundamental global information structure, a critical element in the interactions between people, businesses and other organizations. Many civilian and military systems are dependent on it to function.

Over the last decade, there has been considerable effort devoted to understanding the structure and function of many different types of networks, (e.g., Newman, Barabási, and Watts, 2007; Newman, 2003; and Dorogovtsev and Goltsev, 2008). Much of this effort has focused on graph connectivity properties and random generation mechanisms that give rise to them.

However, there can be many generation mechanisms that give rise to the same network structure. Thus, the ability of a model to produce connectivity statistics similar to those in the real world does not by itself prove that the model explains that real system. The internet provides an interesting case study, where simple models of preferential attachment have been argued to be the cause of power-law distributions in network connectivity (Barabási and Albert, 1999). But a closer look at the details of the real Internet reveals the specious nature of these models and argues instead for models of network formation that capture the economic and technical drivers of real network growth (Alderson, Li, Willinger, and Doyle, 2005; Alderson, 2008; Willinger, Alderson and Doyle, 2009).

In this thesis, we consider the perspective of a notional *Network Service Provider (NSP)* who has to decide on the connections to build and the markets to serve in order to maximize its profits. The NSP makes these decisions based on the market demand for connectivity and the constraints on available resources to provide the connections. These demands and resource constraints will vary by environment; hence, the resultant networks formed may also vary.

In particular, this thesis looks into the following question: How does a Network Service Provider decide to which locations it should provide connectivity?

We build simple models and perform numerical experiments to study the tensions faced by the NSP in its decisions to build its service network. Our results allow us to infer how demand, revenue and cost influence the decisions of the NSP.

B. RELATED WORK

Understanding the decisions of a NSP touches on several important topics.

1. Network Design Problem

In the network design problem, the network planner decides how to connect nodes to form a network and how to route the traffic through the network in order to satisfy performance criteria such as cost, capacity and resiliency. Much research has been conducted in the fields of Telecommunications and Transportation.

In the field of telecommunications, Resende and Pardalos (2006) cover a wide range of topics on the use of optimization techniques in the design of telecommunications networks. In particular, Forsgren and Prytz (2006) cover the important classes of network design problems, and they discuss solution strategies to these classes of problems using optimization models.

Lederer and Nambimadom (1998) study the effect of distance, demand rate and number of cities served on the performance of four types of airline networks for a profit maximizing airline, and they report that each type of network can be optimal for selected parameters.

In this thesis, the NSP essentially is posed with the network design problem, and we formulate an optimization-based model to capture the essential features which affect the NSP’s decision.

2. Network Economics

Network Economics is the field that analyzes the provision, distribution and utilization of services through a network. In the context of networks, economists are interested in the important factors determining what services are offered, and how they are priced to give incentive to the provider to offer that service.

Relating back to the network design problem, the services offered manifest in the form of commodity flows in the network – if a commodity is serviced, there must be a non-zero flow of that commodity through the network, and this implies that at least one path must exist between the origin of the commodity, and the destination of the commodity.

Economides (2008) notes that a fundamental property of networks is that they exhibit network externalities. The presence of positive network externalities signifies that the value of a unit of the good increases with the number of units sold. This gives rise to the presence of network effects, a unique feature in the context of economics, whereby the value of a connection to a network increases with the size of the network. He also covers in detail the consequences of network effects on the market structure in network industries.

Odlyzko (2004) presents a discussion of the evolution of pricing in early transportation industries, and shows that price discrimination is an important factor in the development of those industries. He uses these historical precedents to explain the behavior of the telecommunications industry, and explores the implications for the evolution of the internet.

Zhang, Nabipay, Odlyzko and Guerin (2010) present an economic model for studying competition and innovation in a complex system of network providers, users and service providers. The model combines Cournot and Bertrand games to model competition among the service providers and network providers respectively. The service providers determine the optimal amount of services to offer to meet demand, while the network providers determine the optimal price to charge the service providers.

In this thesis, the results generated based on our models tell us the connections or services offered. Through the variation of input parameters to the model, we also observe the impact of pricing of the services.

3. Models of Network Formation as a Game

Fabrikant et al. (2003) propose a network formation game called the *Unilateral Connection Game (UCG)* that models the creation of networks by selfish acting nodes without coordination. Each node is a player, and a player's strategy is the choice of the subset of nodes to which it wants to form a connection with through an arc. In the UCG, an arc is formed either of the incident nodes' strategies contain that arc.

Corbo and Parks (2005) describe an extension of the UCG called the *Bilateral Connection Game (BCG)*. Similar to the UCG, the players of the BCG are also nodes. In the BCG, however, an arc is formed only if both incident nodes' strategies contain that arc. Any connection cost is shared equally between the two nodes.

The UCG and BCG can be considered forms of the *Local Connection Game* (Tardos & Wexler, 2007). In contrast, in a *Global Connection Game* (Tardos & Wexler, 2007), each player has a specified source node and sink node, and the goal for each player is to build a path from his source node to sink node while paying as little as possible to do so. Costs on an arc are shared by the number of players whose paths contains that arc.

McPherson (2009) contrasts networks that grow randomly with those that result from design. The probabilistic graph formation models include the classic Erdős-Rényi models, geometric random graphs and preferential attachment models. Using the minimum spanning tree problem as the basis, the behavior of the random models are contrasted with optimization-based models that deliberately grow network structure to achieve a stated performance objective.

In our thesis, the player is not constrained to a single source and destination node pair, but instead considers a collection of source nodes and destination nodes to build paths to derive the highest possible profit.

4. Models of Internet Design

Derosier (2008) compares heuristics and optimization-based approaches to the design of *Internet Service Provider (ISP)* network topologies. He infers the underlying key design principle from observing and reverse-engineering a national ISP, and develops heuristics and optimization models to generate ISP networks based on the observations of the national ISP. He then compares the performance of the networks created from using the heuristic models and the optimization models, using cost and throughput as performance measures.

Sanchez (2010) develops a traffic engineering model, a network provisioning model and a multi-period network provisioning model for the ISP whose objective is to minimize cost. Through experiments using the three models, he focuses on explaining the factors that lead to changes in network performance and extracts investment policies for ISPs to maximize the effectiveness of limited resources over a multi-period planning horizon.

Adopting a similar approach, we develop optimization models framed in the economics of a notional NSP who is motivated by profits. Through numerical experiments using these models, we attempt to gain insight on the effects of changes in values of input parameters.

C. STRUCTURE OF THESIS AND CHAPTER OUTLINE

In this thesis, we formulate an optimization-based model that focuses on network design. To incorporate economic drivers in the model, we endow the NSP with a profit maximizing motive, and include elements of cost and revenue in the objective function and the constraints. We then perform numerical experiments to identify the impact of changes in the input parameters. Through the experiments, we illustrate the tensions faced by the NSP in deciding which demands to satisfy in the face of revenue and cost.

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II. MODELLING THE GROWTH OF A SINGLE NSP

A. MODELING THE NSP NETWORK

We assume the main drivers of the growth of a network are the demand from customers, cost of providing links, the revenue generated from fulfilling demand, and the capacity of the links. The NSP has to decide which demands to satisfy and how to route the commodities in order to receive maximum revenue. When given the ability to enhance the network, the NSP also has to decide if it makes economic sense to invest in enhancements to derive more profits.

In an unconstrained environment, we expect that a NSP will grow to obtain more customers provided that the marginal cost of servicing them is less than the marginal revenue. However, we identify three main factors that can also restrict the growth of the service network.

1. All demands are satisfied.
2. The budget for network expansion is exhausted.
3. The maximum capacities of critical links are reached.

In this thesis, we focus on routing commodities in order to maximize profit. The models introduced here belong to the general class of multi-commodity flow models.

B. NOTATION

In this thesis, we adopt the notation and conventions in Ahuja, Magnanti, and Orlin (1993). Let N be a set of *nodes*, each representing a city that can potentially be connected in the NSP network. We define a *path* as a sequence of nodes in a network without repetition. Here, a node can correspond to the origin, intermediate point, or destination along a particular path. Let $n = 1, 2, ..., |N|$ index set N. Aliases for n include i, j, s and t. The set $A \subseteq N \times N$ represents the set of *arcs* that connect nodes in a network. We use (i, j) to denote a directed arc from node i to node j.

C. THE DEMAND MATRIX

The demand between each source and destination city can be summarized in the form of the Origin-Destination (O-D) demand matrix. We use the *gravity model* (Roughan, 2005) to generate the demand matrix. This is the same method for the synthesis of the demand matrices adopted in Sanchez (2010). The gravity model is conceptually similar to Newton's law of gravitation. The key idea in the gravity model is that the demand for network traffic between two cities is proportional to the product of their customer populations.

1. The Gravity Model

Let b_{st} be the demand for service for commodities from origin *s* bound for destination *t*. Let $p(i)$ be the population of city *i* in millions of people. Let θ_i be the proportion of population that desires to be serviced. Then the customer population for city *i* is $\theta_i p(i)$.

The general formulation for the demand between two cities *s* and *t* is $b_{st} = \theta_s p(s)\theta_t p(t)$, where $\theta_s p(s)$ is the customer population at origin city *s*, and $\theta_t p(t)$ is the customer population at destination city *t*.

D. TRAFFIC ENGINEERING TO MAXIMIZE PROFIT

We first consider a single NSP whose objective is to maximize its revenue through routing commodities from source node s to destination node t to satisfy the demand, given constraints on the network. The constraints appear in the form of maximum load capacities that each arc can carry. We assume that the network topology is given and that the NSP knows the demand matrix. We also assume that there is no cost associated with routing the commodity over any existing arcs.

The NSP generates revenue from each unit of commodity (s,t) delivered, which we denote as Q_{st}. Let ρ_{st} represent the revenue generated per unit of commodity (s,t). The NSP has to decide how to route each commodity (s,t) through a path in the network connecting s and t. Let X_{ij}^{st} represent the amount of commodity (s,t) routed over arc (i, j). There is also a capacity limit cap_{ij}, which denotes the maximum amount of total flow that can be carried over arc (i, j). We assume that Q_{st} is also limited by the demand b_{st}.

We formulate the Traffic Engineering (TE) model as follows:

Sets

N	Nodes
A	Directed Arcs $A \subseteq N \times N$

Index Use

$n \in N$	Index of Node (alias *i*, *j*, *s*, *t*)
$(i, j) \in A$	Directed Arcs from node *i* to node *j*

Parameters [units]

ρ_{st}	Revenue per unit of commodity delivered between source node *s* and destination node *t* [$/Gbps]
cap_{ij}	Existing capacity over arc (i, j) [Gbps]
b_{st}	Demand between source node *s* and destination node *t* [Gbps]

Decision Variables [units]

X_{ij}^{st}	Quantity of commodity from source node *s* and destination node *t* routed on arc (i, j) [Gbps]
Q_{st}	Quantity of commodity from source node *s* delivered to destination node *t*, ($Q_{st} = \sum_{i \in N} X_{it}^{st}$) [Gbps]

Formulation (Traffic Engineering)

$$\max_{X,Q} \quad Z_{TE} = \sum_{s\in N}\sum_{t\in N} \rho_{st} Q_{st} \qquad (TE0)$$

$$s.t. \quad \sum_{j\in N} X_{nj}^{st} - \sum_{i\in N} X_{in}^{st} = \begin{Bmatrix} Q_{st}, & n = s \\ -Q_{st}, & n = t \\ 0, & otherwise \end{Bmatrix} \qquad \forall n,s,t \in N \qquad (TE1)$$

$$\sum_{s\in N}\sum_{t\in N} X_{ij}^{st} \le cap_{ij} \qquad \forall (i,j) \in A \qquad (TE2)$$

$$Q_{st} \le b_{st} \qquad \forall s,t \in N \qquad (TE3)$$

$$X_{ij}^{st} \ge 0 \qquad \forall (i,j) \in A, \forall s,t \in N \qquad (TE4)$$

$$Q_{st} \ge 0 \qquad \forall s,t \in N \qquad (TE5)$$

Discussion

The objective function (TE0) gives the total revenue generated by the NSP, which we compute from the amount of commodity actually delivered and the revenue for each unit of commodity delivered. Constraints (TE1) ensure the balance of commodity flow at every node *n*. Constraints (TE2) ensure that the commodity flow along every arc (i, j) does not exceed the available capacity. Constraints (TE3) ensure that the amount of commodity does not exceed the amount demanded. Constraints (TE4) ensure the network flows are nonnegative. Constraints (TE5) ensure that the demands fulfilled are nonnegative.

E. NETWORK PROVISIONING TO MAXIMIZE PROFIT

Next, we consider the case where the NSP is given a budget to enhance the capacity of the network, potentially allowing it to generate more profit. The NSP can do so by increasing the capacity of existing arcs or by building additional arcs where none previously existed.

We assume that increasing the capacity of existing arcs incurs a variable cost, while building additional arcs incurs both the variable cost and a fixed cost. The NSP now decides which option to take on each of the arcs (i, j). However, there is also a physical limit on how much total capacity can exist on each arc (i, j).

The Network Provisioning (NP) model builds upon the Traffic Engineering model, with the following modifications:

Additional Parameters [units]

γ_{ij}	Cost per unit distance per unit of additional capacity provided over arc (i, j) [\$/Gbps/unit distance]
λ_{ij}	Fixed cost of building new arc (i, j) [\$]
d_{ij}	Distance between node i and node j [unit distance]
$budget$	Total budget given to enhance capacity of network [\$]
$maxcap_{ij}$	Maximum capacity over arc (i, j) [Gbps]

Derived Data

$$min\_pos\_cap = \min_{(i,j)\in A}\{cap_{ij} \mid cap_{ij} > 0\}$$

Additional Decision Variables [units]

Y_{ij}	Additional capacity provided between node i and node j [Gbps]
V_{ij}	Binary variable {0,1}, 1 if new arc (i, j) is built, and 0 otherwise [binary]
R_{ij}	Binary variable {0,1}, 1 if arc (i, j) exists, and 0 otherwise [binary]

Formulation (Network Provisioning)

$$\max_{X,Q,Y,V,R} Z_{NP} = \sum_{s\in N}\sum_{t\in N}\rho_{st}Q_{st} - \sum_{i\in N}\sum_{j\in N}(\gamma_{ij}d_{ij}Y_{ij} + \lambda_{ij}V_{ij}) \qquad (NP0)$$

$$s.t. \qquad (TE1),(TE3),(TE4),(TE5)$$

$$\sum_{s\in N}\sum_{t\in N}X_{ij}^{st} \le cap_{ij} + Y_{ij} \qquad \forall(i,j)\in A \qquad (NP1)$$

$$\sum_{i\in N}\sum_{j\in N}(\gamma_{ij}d_{ij}Y_{ij} + \lambda_{ij}V_{ij}) \le budget \qquad (NP2)$$

$$Y_{ij} \le R_{ij}(maxcap_{ij} - cap_{ij}) \qquad \forall s,t\in N \qquad (NP3)$$

$$R_{ij} \ge \frac{cap_{ij}}{maxcap_{ij}} \qquad \forall(i,j)\in A \qquad (NP4)$$

$$R_{ij} \le \frac{cap_{ij}}{min\_pos\_cap} + V_{ij} \qquad \forall(i,j)\in A \qquad (NP5)$$

$$Y_{ij} = Y_{ji} \qquad \forall(i,j)\in A \qquad (NP6)$$

$$Y_{ij} \ge 0 \qquad \forall(i,j)\in A \qquad (NP7)$$

$$V_{ij} = \{0,1\} \qquad \forall(i,j)\in A \qquad (NP8)$$

$$R_{ij} = \{0,1\} \qquad \forall(i,j)\in A \qquad (NP9)$$

Discussion

The objective function (NP0) is the extension of (TE0) which gives the profit generated by also considering the costs incurred for the expansion of the network. Constraints (NP1) ensure that the commodity flow along each arc (i, j) does not exceed the enhanced capacity of the arc. Constraint (NP2) ensures that the costs do not exceed the budget. Constraints (NP3) ensure that the enhanced capacity does not exceed the maximum capacity allowable over arc (i, j) when arc (i, j) exists. Constraints (NP4) force R_{ij} to value 1 for arc (i, j) when there is any capacity on the arc. Constraints (NP5) force R_{ij} to value 0 for arc (i, j) when there is no capacity on the arc and the arc is not built. Constraints (NP6) ensure symmetry between arcs built. Constraints (NP7) ensure that additional arc capacity on any arc is nonnegative. Constraints (NP8) and (NP9) ensure that V_{ij} and R_{ij} are binary variables.

When Y, V, and R are fixed and feasible, the NP model reduces to traffic engineering on the resultant enhanced network incorporating Y, V, and R. Also note that when $budget = 0$, the Network Provisioning model is the same as the Traffic Engineering model. This is because when $budget = 0$, $Y_{ij} = 0$ and $V_{ij} = 0$ for all arcs (i, j). When $Y_{ij} = 0$, constraints (NP1) are the same as constraints (TE2), and constraint (NP0) is the same as constraint (TE0). Then, constraints (NP3), (NP4) and (NP5) are unnecessary as R_{ij} does not affect the objective function constraint (NP0).

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III. NUMERICAL EXPERIMENTS

In this chapter, we conduct numerical experiments to derive key insights from the Network Provisioning model. We first give an overview of the underlying data, and present the input parameters to our model. We then solve a sample traffic engineering problem, and through the presentation of the result, we introduce some useful visualization and interpretation aids. We then present the results of the application of the Network Provisioning models with select input parameters. For some key results, we offer explanation on the underlying reason for the results.

We implement the models using the *General Algebraic Modeling System* (GAMS, 2011), and use CPLEX as the solver on a personal notebook computer with an Intel CORE i5 CPU at 2.53 GHz. The runtimes to generate the solutions vary from a few seconds in cases without fixed costs, and up to thirty minutes when fixed costs are included.

A. THE ABILENE NETWORK

Abilene is a high-speed research and education network providing Internet services to universities throughout the United States. The network is used by hundreds of universities in the United States for services such as “tele-immersion, virtual laboratories, distance learning, distributed performing arts, tele-medicine, grid computing and digital libraries” (Qwest, 2011).

The topology of the Abilene network (Qwest, 2011) is shown in Figure 1. Abilene has Points of Presence (POPs) in 11 cities throughout the United States, connected via 28 arcs, each with 10 Gbps of capacity.

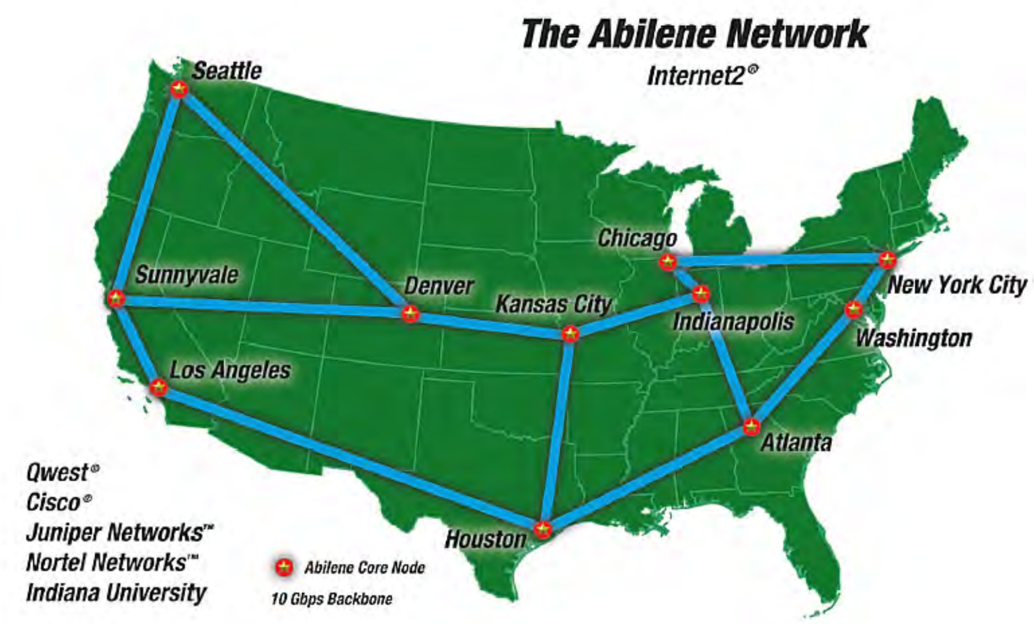


Figure 1. Abilene Topology (From Qwest, 2011).

The topology of the Abilene network is well documented, but the O-D demand matrices are unknown. Feeding the customer population data shown in Table 1 into the gravity model (Roughan, 2005) described in Chapter II, we generate the O-D demand matrix in Table 2. Summing across all entries in Table 2, the total demand from all the nodes is 34.866 Gbps.

Table 3 shows the nominal distances used in the computation in this thesis. Each nominal distance value is the Euclidean distance computed based on the latitude and longitude coordinates of the source and destination cities.

Table 1. Population and assumed customer population of the 11 cities with POPs in the Abilene network; Population data from U.S. Census Bureau (2011)

City	**City Initials**	**Population (millions)**	**Customer Population (millions)**
Atlanta	ATL	0.50	0.16
Chicago	CHI	2.90	0.93
Denver	DEN	0.60	0.19
Houston	HOU	2.20	0.70
Indianapolis	IND	0.80	0.26
Kansas City	KSC	0.50	0.16
Los Angeles	LAX	3.80	1.22
New York	NYC	8.40	2.69
Seattle	SEA	0.60	0.03
Sunnyvale	SUN	0.10	0.19
Washington DC	WDC	0.60	0.19

Table 2. Abilene O-D demand matrix based on gravity model, using the customer population data in Table 1. The units of demand are in Gbps.

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SEA	SUN	WDC
ATL	-	0.148	0.031	0.113	0.041	0.026	0.195	0.430	0.031	0.005	0.031
CHI	0.148	-	0.178	0.653	0.238	0.148	1.128	2.494	0.178	0.030	0.178
DEN	0.031	0.178	-	0.135	0.049	0.031	0.233	0.516	0.037	0.006	0.037
HOU	0.113	0.653	0.135	-	0.180	0.113	0.856	1.892	0.135	0.023	0.135
IND	0.041	0.238	0.049	0.180	-	0.041	0.311	0.688	0.049	0.008	0.049
KSC	0.026	0.148	0.031	0.113	0.041	-	0.195	0.430	0.031	0.005	0.031
LAX	0.195	1.128	0.233	0.856	0.311	0.195	-	3.269	0.233	0.039	0.233
NYC	0.430	2.494	0.516	1.892	0.688	0.430	3.269	-	0.516	0.086	0.516
SEA	0.031	0.178	0.037	0.135	0.049	0.031	0.233	0.516	-	0.006	0.037
SUN	0.005	0.030	0.006	0.023	0.008	0.005	0.039	0.086	0.006	-	0.006
WDC	0.031	0.178	0.037	0.135	0.049	0.031	0.233	0.516	0.037	0.006	-

Table 3. Distance Matrix for cities in Abilene. The distances are in nominal units calculated from the Euclidean distance based on the latitudes and longitudes of the nodes.

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SEA	SUN	WDC
ATL	-	8.86	21.34	11.53	6.36	11.76	33.98	12.64	40.32	38.17	9.03
CHI	8.86	-	17.35	14.20	2.57	7.53	31.77	13.72	35.09	34.99	11.04
DEN	21.34	17.35	-	13.65	18.60	10.16	14.73	30.91	19.06	17.64	27.85
HOU	11.53	14.20	13.65	-	13.33	9.37	23.39	23.94	32.12	28.09	20.35
IND	6.36	2.57	18.60	13.33	-	8.46	32.65	12.33	36.85	36.17	9.27
KSC	11.76	7.53	10.16	9.37	8.46	-	24.29	20.79	28.75	27.71	17.69
LAX	33.98	31.77	14.73	23.39	32.65	24.29	-	44.94	14.07	5.43	41.65
NYC	12.64	13.72	30.91	23.94	12.33	20.79	44.94	-	48.78	48.50	3.61
SEA	40.32	35.09	19.06	32.12	36.85	28.75	14.07	48.78	-	9.83	46.07
SUN	38.17	34.99	17.64	28.09	36.17	27.71	5.43	48.50	9.83	-	45.36
WDC	9.03	11.04	27.85	20.35	9.27	17.69	41.65	3.61	46.07	45.36	-

B. ILLUSTRATION OF THE TRAFFIC ENGINEERING MODEL

With the network topology of the Abilene network and the derived O-D demand matrix in Table 2, we can now use the traffic engineering model to determine the optimal plan for the NSP to route the traffic demands to maximize its profits. We assume that the capacity of each arc is 10 Gbps. We also assume that the marginal revenue for each Gbps of demand satisfied is $50.

With the input parameters, the solution to the traffic engineering problem indicates that the maximum profit achieved is $1743.30. With the marginal revenue at $50 for each Gbps of demand satisfied, we know that all the demand is satisfied, since the total demand from all the nodes is 34.866 Gbps.

In order to help in understanding the result outputs, we use several visualization and interpretation aids, namely the *Network Utilization Graph*, the *Demand Satisfaction Matrix*, and the *Arc Utilization Matrix*.

1. Network Utilization Graph

We incorporate visual enhancements to the graphical representation of the network topology to create the *Network Utilization Graph*. The thickness of each arc scales proportionally to represent the capacity of that arc. The color (shade) of the arc represents the utilization on the arc. Utilization on an arc is the ratio of the sum of all flows through that arc to the capacity of that arc. A darker shade represents a higher utilization. The size of each node scales with the population of the respective city the node represents and the position of the nodes represent the relative location of the nodes. These enhancements help us to quickly identify the relative capacities of the arcs and also the extent the capacities are used in the arcs, together with information on the relative populations and relative positions of the nodes.

The *Network Utilization Graph* based on the results from traffic engineering is shown in Figure 2. The capacities of all the existing arcs are the same at 10 Gbps, hence they appear to have the same thickness. It can be seen that SUN has the smallest population. We see that the arc utilization is high on HOU-KSC, ATL-WDC and NYC-WDC, while DEN-SEA, SEA-SUN and ATL-IND have relatively low utilization.

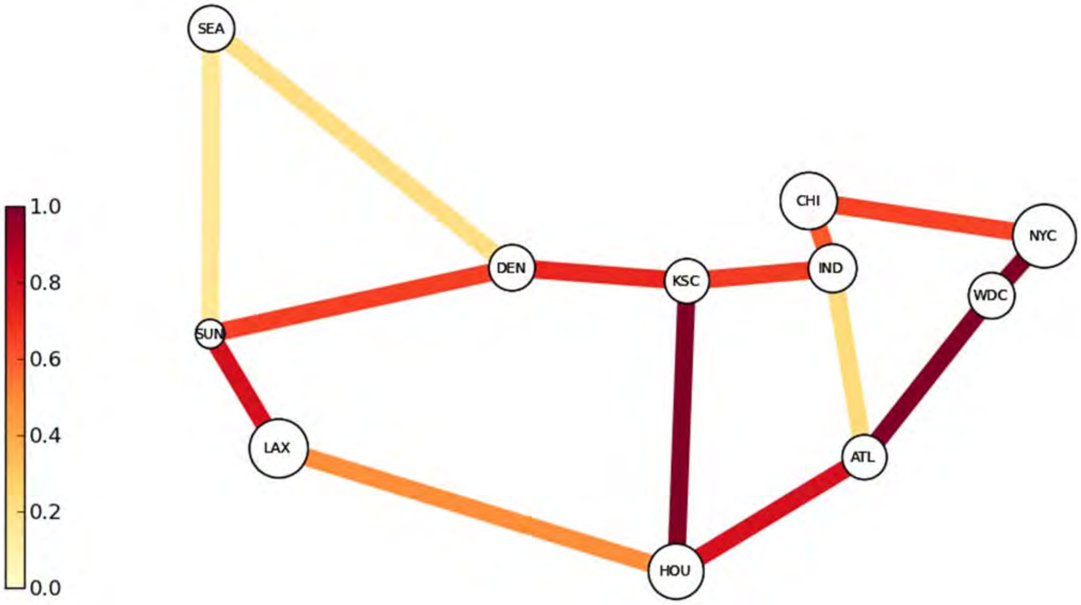
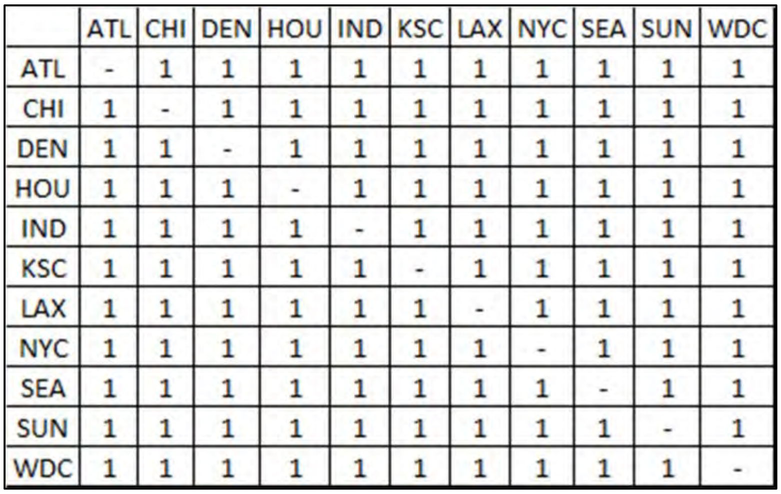


Figure 2. The Network Utilization Graph representation of the results of traffic engineering. The utilization of DEN-SEA, SEA-SUN and ATL-DEN is relatively low compared to the other arcs.

2. Demand Satisfaction Matrix

The *Demand Satisfaction Matrix* shows in matrix form the proportion of demand for each O-D node pair that is satisfied by the network. The matrix helps us to identify quickly the extent of demand satisfaction for all the node-pairs.

The *Demand Satisfaction Matrix* based on the results of traffic engineering is in Figure 3. We observe that all of the demands are satisfied.



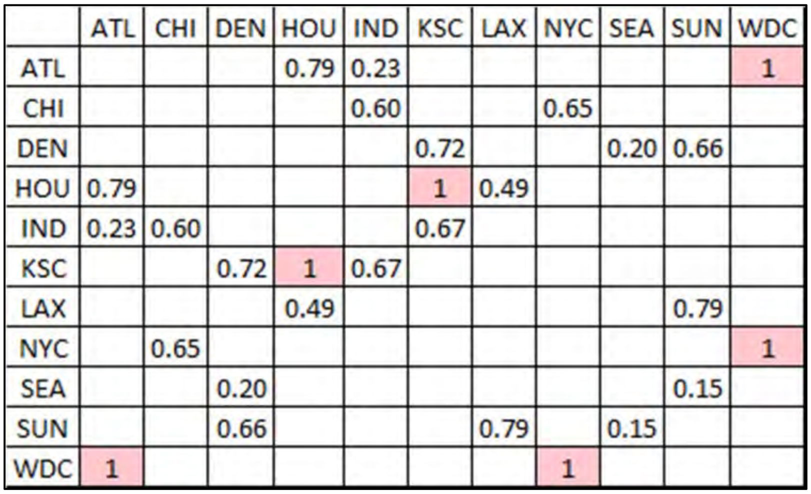
	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SEA	SUN	WDC
ATL	-	1	1	1	1	1	1	1	1	1	1
CHI	1	-	1	1	1	1	1	1	1	1	1
DEN	1	1	-	1	1	1	1	1	1	1	1
HOU	1	1	1	-	1	1	1	1	1	1	1
IND	1	1	1	1	-	1	1	1	1	1	1
KSC	1	1	1	1	1	-	1	1	1	1	1
LAX	1	1	1	1	1	1	-	1	1	1	1
NYC	1	1	1	1	1	1	1	-	1	1	1
SEA	1	1	1	1	1	1	1	1	-	1	1
SUN	1	1	1	1	1	1	1	1	1	-	1
WDC	1	1	1	1	1	1	1	1	1	1	-

Figure 3. Demand Satisfaction Matrix showing results of traffic engineering. The demands for all O-D node-pairs are fully satisfied.

3. Arc Utilization Matrix

The *Arc Utilization Matrix* presents numerically in matrix form for each arc the ratio of the sum of all flows through that arc to the capacity of that arc. A value of 1 indicates that the arc is at capacity and saturated, while a value of 0 indicates that there is no flow over that arc even though there is capacity on the arc. Where an entry is blank, it means that the arc does not exist at all. The *Arc Utilization Matrix* complements the *Network Utilization Graph* by giving a more precise numerical indication of the utilization of the arcs.

The *Arc Utilization Matrix* from the traffic engineering results is shown in Figure 4. We observe that ATL-WDC, HOU-KSC and NYC-WDC are utilized fully.



	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SEA	SUN	WDC
ATL				0.79	0.23						1
CHI					0.60			0.65			
DEN						0.72			0.20	0.66	
HOU	0.79					1	0.49				
IND	0.23	0.60				0.67					
KSC			0.72	1	0.67						
LAX				0.49						0.79	
NYC		0.65									1
SEA			0.20							0.15	
SUN			0.66				0.79		0.15		
WDC	1							1			

Figure 4. Arc Utilization Matrix for results of traffic engineering. Shaded entries indicate that the arcs are saturated. Other non-blank entries indicate the fraction of arc capacity utilized. Blank entries indicate the arc does not exist.

C. NETWORK PROVISIONING BASED ON ABILENE NODES

With the existing Abilene network as a basis, we now look into how to build a network from scratch, as in the case of a new NSP coming into the market. This can be implemented by using the Network Provisioning model with the initial capacities of all arcs set to zero.

Although a NSP might desire a fully connected network with infinite capacity, there are many reasons why this is not possible in practice. First of all, there are costs associated with building each arc, and the total cost of building the arcs cannot exceed the limited amount of budget. Even if the NSP has a sufficiently large budget, it will only spend up to the amount needed to build arcs that satisfy all the profitable demand. Specifically, the costs of building on the arcs in the path required for a unit of commodity cannot exceed the revenue associated with delivering that unit of commodity; otherwise, the NSP will be making a loss on that commodity. Finally, each arc can have a limit on how much capacity can be built. In the case where there the desired traffic is greater than the maximum capacity, the excess flow will have to be re-directed along other arcs on alternative economically profitable paths to the destination.

We consider first the case where the fixed cost of building an arc is zero, before moving into cases where there are fixed costs.

1. Case 1: Building a New Network with Zero Fixed Costs

We first look into the case where building any new arc does not incur any fixed costs. In the first portion of this study, we assume the revenue for each unit of O-D traffic delivered is uniform at $50. We assume the maximum capacity that can be built on each arc is 10 Gbps. We assume the variable cost per unit distance per unit capacity of each arc built is fixed at $1. This implies that between two arcs having the same capacity, the arc connecting cities that are farther apart will be more costly than the arc connecting cities that are closer together. We vary the budget allowed in each run.

The resultant network topology and demand satisfaction matrix for given budget of $200, $400, $600, $800 and $1000, shown in Figure 5. The corresponding arc utilization matrices are also shown in Figure 6.

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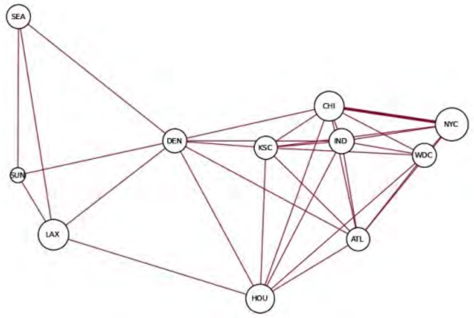
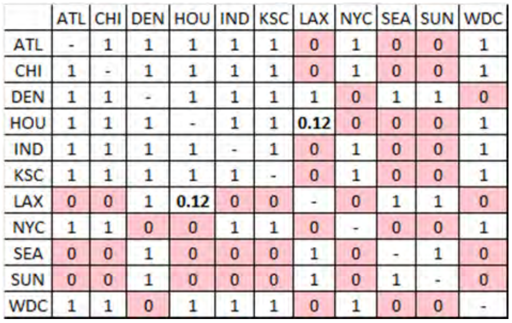
<tr><td>$200</td><td> </td><td>
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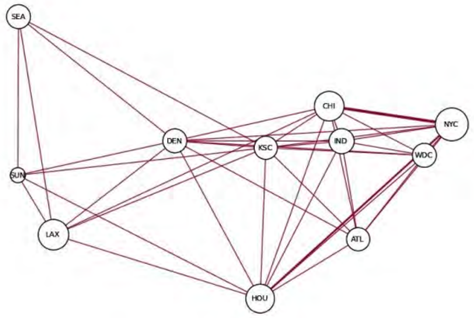
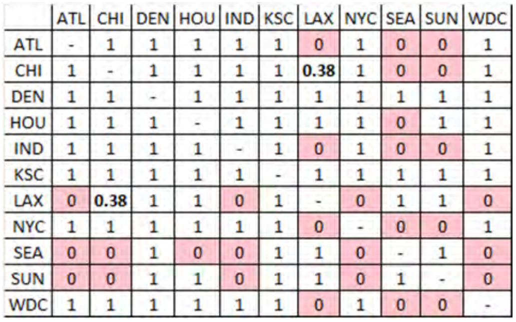
<tr><td>$400</td><td> </td><td>
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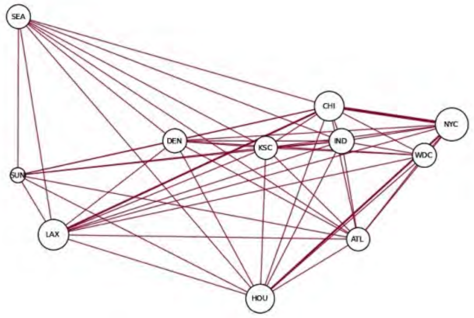
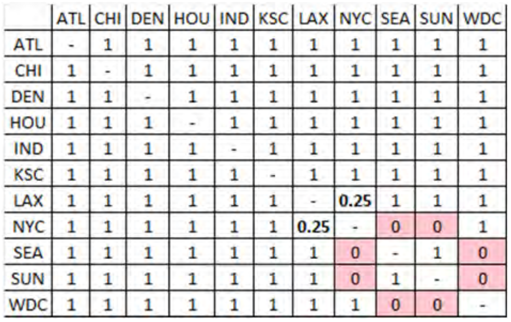
<tr><td>$600</td><td> </td><td>
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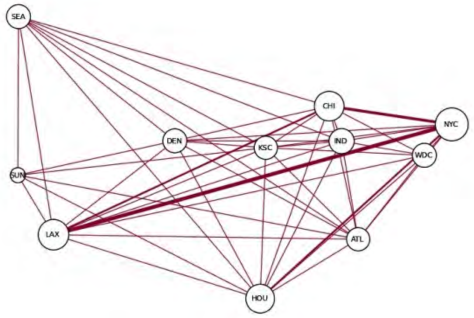
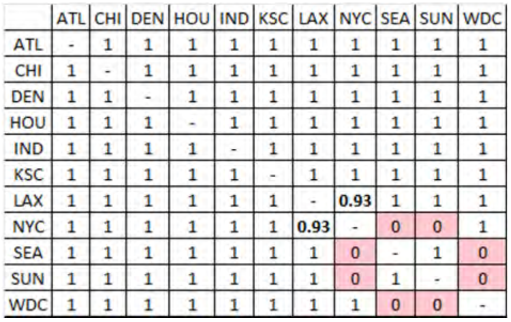
<tr><td>$800</td><td> </td><td>
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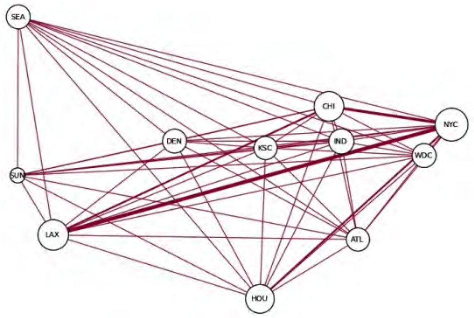
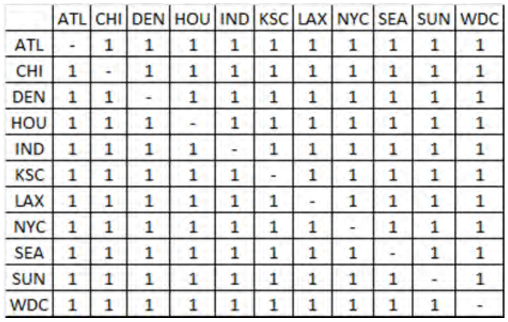
<tr><td>$1000</td><td> </td><td>
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<tr><th></th><th>ATL</th><th>CHI</th><th>DEN</th><th>HOU</th><th>IND</th><th>KSC</th><th>LAX</th><th>NYC</th><th>SEA</th><th>SUN</th><th>WDC</th></tr>
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<tr><td>DEN</td><td>1</td><td>1</td><td>-</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>HOU</td><td>1</td><td>1</td><td>1</td><td>-</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>IND</td><td>1</td><td>1</td><td>1</td><td>1</td><td>-</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
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<tr><td>LAX</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>-</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
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</table></td></tr>
</table>

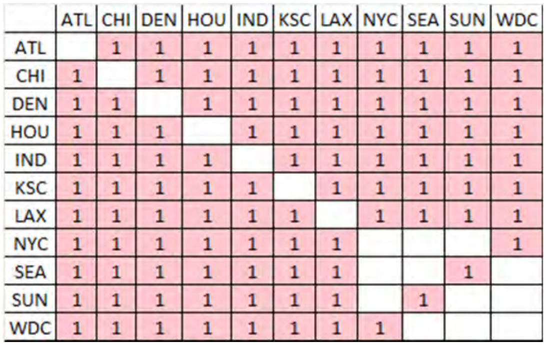
Figure 5. Resultant Network Topology and Demand Satisfaction Matrix in Case 1, for given budget $200, $400, $600, $800 and $1000, when building a new network with no fixed costs.

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SEA	SUN	WDC
ATL		1	1	1	1	1		1			1
CHI	1		1	1	1	1		1			1
DEN	1	1		1	1	1	1		1	1	
HOU	1	1	1		1	1	1				1
IND	1	1	1	1		1		1			1
KSC	1	1	1	1	1			1			1
LAX			1	1					1	1	
NYC	1	1			1	1					1
SEA			1				1			1	
SUN			1				1		1		
WDC	1	1		1	1	1		1			

Budget = $200

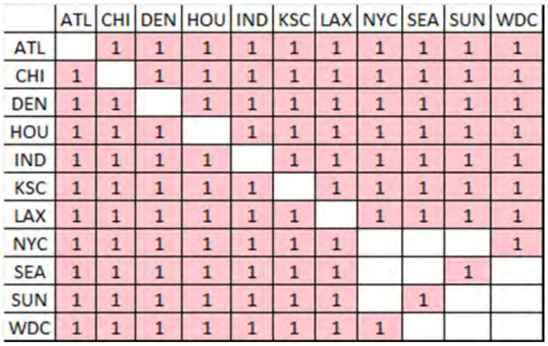
	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SEA	SUN	WDC
ATL		1	1	1	1	1		1			1
CHI	1		1	1	1	1	1	1			1
DEN	1	1		1	1	1	1	1	1	1	1
HOU	1	1	1		1	1	1	1		1	1
IND	1	1	1	1		1		1			1
KSC	1	1	1	1	1		1	1	1	1	1
LAX		1	1	1		1			1	1	
NYC	1	1	1	1	1	1					1
SEA			1			1	1			1	
SUN			1	1		1	1		1		
WDC	1	1	1	1	1	1		1			

Budget = $400



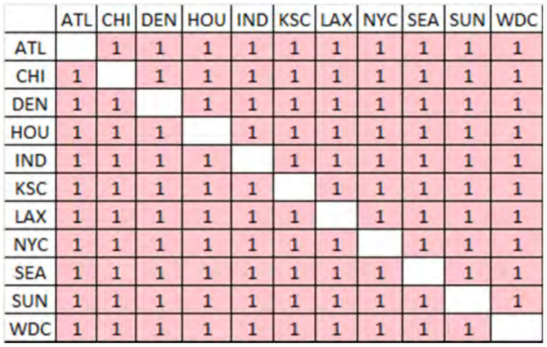
	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SEA	SUN	WDC
ATL		1	1	1	1	1	1	1	1	1	1
CHI	1		1	1	1	1	1	1	1	1	1
DEN	1	1		1	1	1	1	1	1	1	1
HOU	1	1	1		1	1	1	1	1	1	1
IND	1	1	1	1		1	1	1	1	1	1
KSC	1	1	1	1	1		1	1	1	1	1
LAX	1	1	1	1	1	1		1	1	1	1
NYC	1	1	1	1	1	1	1				1
SEA	1	1	1	1	1	1	1			1	
SUN	1	1	1	1	1	1	1		1		
WDC	1	1	1	1	1	1	1	1			

Budget = $600



	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SEA	SUN	WDC
ATL		1	1	1	1	1	1	1	1	1	1
CHI	1		1	1	1	1	1	1	1	1	1
DEN	1	1		1	1	1	1	1	1	1	1
HOU	1	1	1		1	1	1	1	1	1	1
IND	1	1	1	1		1	1	1	1	1	1
KSC	1	1	1	1	1		1	1	1	1	1
LAX	1	1	1	1	1	1		1	1	1	1
NYC	1	1	1	1	1	1	1				1
SEA	1	1	1	1	1	1	1			1	
SUN	1	1	1	1	1	1	1		1		
WDC	1	1	1	1	1	1	1	1			

Budget = $800



	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SEA	SUN	WDC
ATL		1	1	1	1	1	1	1	1	1	1
CHI	1		1	1	1	1	1	1	1	1	1
DEN	1	1		1	1	1	1	1	1	1	1
HOU	1	1	1		1	1	1	1	1	1	1
IND	1	1	1	1		1	1	1	1	1	1
KSC	1	1	1	1	1		1	1	1	1	1
LAX	1	1	1	1	1	1		1	1	1	1
NYC	1	1	1	1	1	1	1		1	1	1
SEA	1	1	1	1	1	1	1	1		1	1
SUN	1	1	1	1	1	1	1	1	1		1
WDC	1	1	1	1	1	1	1	1	1	1	

Budget = $1000

Figure 6. Arc Utilization Matrix in Case 1, for given budget $200, $400, $600, $800 and $1000, when building a new network with no fixed costs.

We see from Figure 5 that, as budget is increased, more of the demands will be satisfied. This is because the NSP is able to build and enhance the capacity of more arcs when there is more budget available.

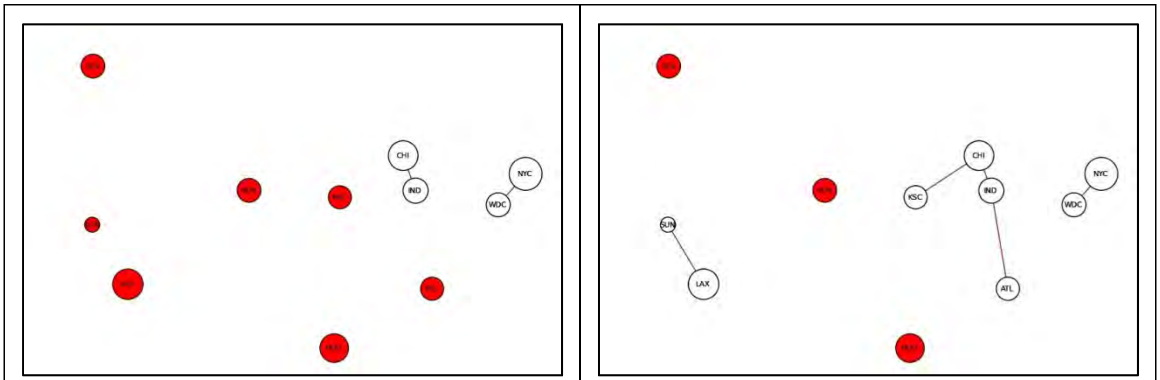
We also observe that in the absence of fixed costs, the NSP will build direct arcs between node *s* and node *t* to satisfy demand between the pair of nodes, as this direct arc represents the shortest path and hence the least costly path to build.

We note from Figure 6 that the NSP fully utilizes the capacities of the arcs built. This makes economic sense as the NSP will not spend to increase capacity that will be left unused, since all the arcs are direct.

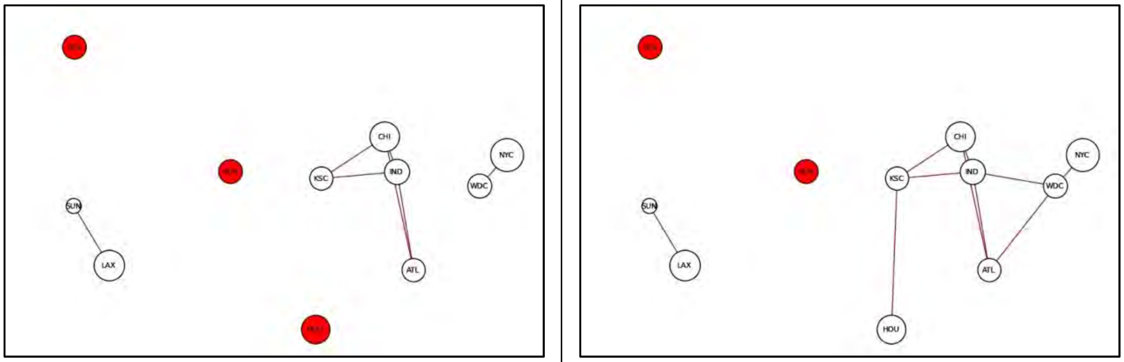
We note that a budget of $883.924 facilitates the fulfillment of all demand. We see from the demand satisfaction matrices that at any given budget less than $883.924, at most two arcs, connecting across the same pair of nodes (arising from constraint NP6), are partially built. This indicates that the underlying principle in building the network is to concentrate on expanding the network to satisfy the demands between a pair of nodes before building to service the demands for another pair of nodes.

Since the amount of capacity built for a pair of nodes is equal in both directions, from this point on, when we refer to the arcs of a node-pair, we refer to arcs of both directions. We also define the duplex demand of a node-pair as the total demand regardless of direction between the node-pair

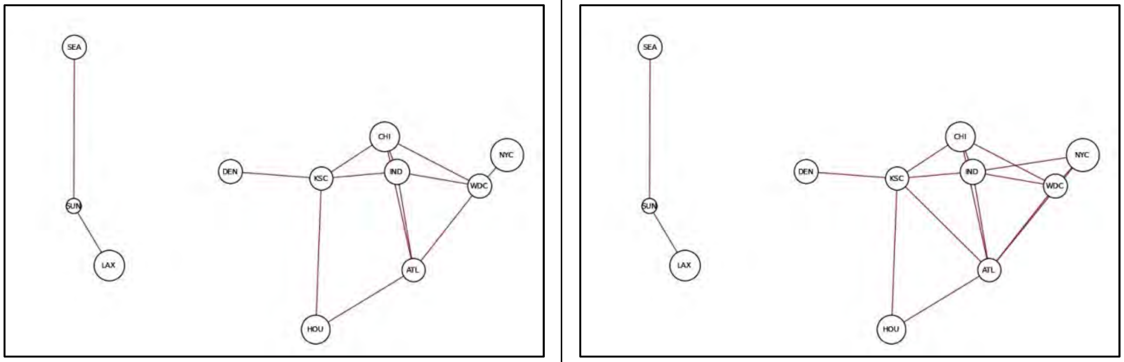
We now examine in detail the topology of the network when the budget given is relatively small in order to deduce how a NSP should decide which arcs to build. The resultant network topologies for values for the budget at $2, $6, $10, $15, $20 and $50 are given in Figure 7.



Budget = $2 | Budget = $6



Budget = $10 | Budget = $15



Budget = $20 | Budget = $50

Figure 7. Resultant Network Topology for given budget $2, $6, $10, $15, $20 and $50, when building a new network with no fixed costs.

Table 4. Node-pair data in Case 1, sorted by decreasing order of marginal profit.

Node-Pair	Duplex Demand	Marginal Revenue	Distance	Marginal Cost	Marginal Profit	Full Duplex Link Cost	Cumulative Cost
CHI-IND	0.476	50	2.572	2.572	47.428	1.224	1.224
NYC-WDC	1.032	50	3.612	3.612	46.388	3.728	4.952
LAX-SUN	0.078	50	5.427	5.427	44.573	0.423	5.375
ATL-IND	0.082	50	6.355	6.355	43.645	0.521	5.896
CHI-KSC	0.296	50	7.526	7.526	42.474	2.228	8.124
IND-KSC	0.082	50	8.46	8.46	41.540	0.694	8.818
ATL-CHI	0.296	50	8.86	8.86	41.140	2.623	11.440
ATL-WDC	0.062	50	9.028	9.028	40.972	0.560	12.000
IND-WDC	0.098	50	9.272	9.272	40.728	0.909	12.909
HOU-KSC	0.226	50	9.371	9.371	40.629	2.118	15.026
SEA-SUN	0.012	50	9.83	9.83	40.170	0.118	15.144
DEN-KSC	0.062	50	10.159	10.159	39.841	0.630	15.774
CHI-WDC	0.356	50	11.04	11.04	38.960	3.930	19.705
ATL-HOU	0.226	50	11.533	11.533	38.467	2.606	22.311
ATL-KSC	0.052	50	11.758	11.758	38.242	0.611	22.922
IND-NYC	1.376	50	12.334	12.334	37.666	16.972	39.894
ATL-NYC	0.86	50	12.637	12.637	37.363	10.868	50.762
HOU-IND	0.36	50	13.331	13.331	36.669	4.799	55.561
DEN-HOU	0.27	50	13.648	13.648	36.352	3.685	59.246
CHI-NYC	4.988	50	13.717	13.717	36.283	68.420	127.666
LAX-SEA	0.466	50	14.071	14.071	35.929	6.557	134.223
CHI-HOU	1.306	50	14.199	14.199	35.801	18.544	152.767
DEN-LAX	0.466	50	14.729	14.729	35.271	6.864	159.631
CHI-DEN	0.356	50	17.354	17.354	32.646	6.178	165.809
DEN-SUN	0.012	50	17.639	17.639	32.361	0.212	166.021
KSC-WDC	0.062	50	17.686	17.686	32.314	1.097	167.117
DEN-IND	0.098	50	18.6	18.6	31.400	1.823	168.940
DEN-SEA	0.074	50	19.055	19.055	30.945	1.410	170.350
HOU-WDC	0.27	50	20.35	20.35	29.650	5.495	175.845
KSC-NYC	0.86	50	20.791	20.791	29.209	17.880	193.725
ATL-DEN	0.062	50	21.34	21.34	28.660	1.323	195.048
HOU-LAX	1.712	50	23.388	23.388	26.612	40.040	235.088
HOU-NYC	3.784	50	23.944	23.944	26.056	90.604	325.692
KSC-LAX	0.39	50	24.286	24.286	25.714	9.472	335.164
KSC-SUN	0.01	50	27.712	27.712	22.288	0.277	335.441
DEN-WDC	0.074	50	27.845	27.845	22.155	2.061	337.502
HOU-SUN	0.046	50	28.092	28.092	21.908	1.292	338.794
KSC-SEA	0.062	50	28.753	28.753	21.247	1.783	340.576
DEN-NYC	1.032	50	30.907	30.907	19.093	31.896	372.472
CHI-LAX	2.256	50	31.766	31.766	18.234	71.664	444.137
HOU-SEA	0.27	50	32.122	32.122	17.878	8.673	452.809
IND-LAX	0.622	50	32.649	32.649	17.351	20.308	473.117
ATL-LAX	0.39	50	33.981	33.981	16.019	13.253	486.370
CHI-SUN	0.06	50	34.993	34.993	15.007	2.100	488.469
CHI-SEA	0.356	50	35.092	35.092	14.908	12.493	500.962
IND-SUN	0.016	50	36.172	36.172	13.828	0.579	501.541
IND-SEA	0.098	50	36.848	36.848	13.152	3.611	505.152
ATL-SUN	0.01	50	38.167	38.167	11.833	0.382	505.534
ATL-SEA	0.062	50	40.315	40.315	9.685	2.500	508.033
LAX-WDC	0.466	50	41.652	41.652	8.348	19.410	527.443
LAX-NYC	6.538	50	44.944	44.944	5.056	293.844	821.287
SUN-WDC	0.012	50	45.357	45.357	4.643	0.544	821.831
SEA-WDC	0.074	50	46.07	46.07	3.930	3.409	825.240
NYC-SUN	0.172	50	48.502	48.502	1.498	8.342	833.583
NYC-SEA	1.032	50	48.78	48.78	1.220	50.341	883.924

We observe from Figure 7 that the resultant networks are nested, meaning that arcs built under a given budget will also be built when the budget allocated is higher. To explain this, we refer to Table 4, the list of data of the node-pairs in Case 1. The marginal revenue is the revenue earned per unit of commodity delivered, which is ρ_{st}. Likewise, the marginal cost is the cost per unit commodity for building on arcs in the path to deliver that commodity, and is equal to $\sum_{(i,j)\in path(s,t)} \gamma_{ij} d_{ij}$. The marginal profit is calculated from subtracting the marginal cost from the marginal revenue. In particular, we bring to attention that the data is sorted according to decreasing marginal profit. The Cumulative Cost indicates the sum of the cost of building the arc to fully satisfy its demand, and the cost of all other arcs with higher marginal profits.

When the budget given is \$2, the two arcs built are CHI-IND and NYC-WDC. This corresponds to the two arcs with the shortest distances in Table 4. The demand for CHI-IND is completely fulfilled while the demand for NYC-WDC is partially fulfilled. Examining the cumulative cost of fully building these two shortest arcs, we see a corresponding relation that a budget of \$2 is sufficient to build on CHI-IND, the shortest arc, to fully satisfy the demand of this node-pair, and build on NYC-WDC to partially satisfy that demand.

When the budget given is \$6, three more arcs are built – LAX-SUN, ATL-IND and CHI-KSC. This again corresponds with the node-pairs with the next three shortest distances.

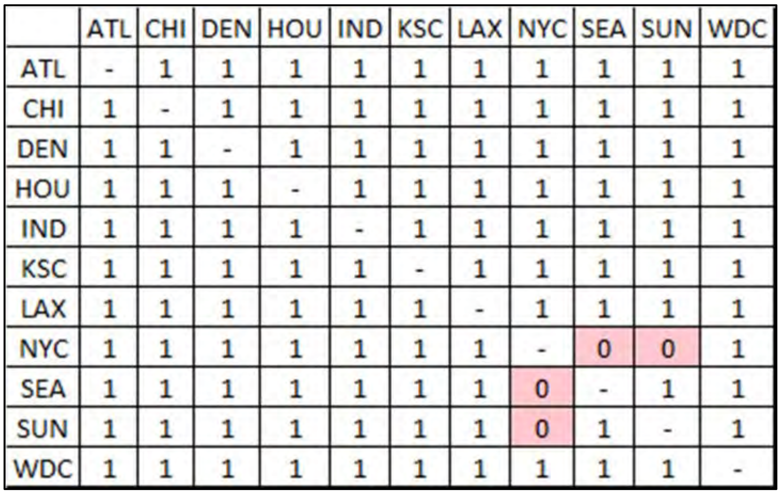
We see the same relationship pattern emerging from the rest of the cases seen in Figure 7.

From this example, we see that the NSP builds arcs in the order of their distances. More precisely, the order to build is based on the marginal profit, starting from the highest.

2. Case 1a: Building a New Network with Zero Fixed Costs, with Variation of Revenue

In Case 1, we observe that when given a sufficiently large budget, all demands between the nodes will be serviced. However, this is based on the premise that there is profit to be reaped from servicing the demand of a node-pair. We now demonstrate this condition by lowering the marginal revenue.

We now decrease the marginal revenue from \$50/Gbps to \$48/Gbps, while allocating a sufficiently large budget of \$2000. The demand satisfaction matrix of the resulting network is shown in Figure 8. The corresponding marginal revenue, marginal cost and the marginal profits for the node-pairs are shown in Table 5. From the demand satisfaction matrix, it can be seen that the traffic between NYC-SEA and NYC-SUN is dropped. This is because the marginal revenue, at \$48/Gbps, is less than the marginal costs for building on NYC-SEA and NYC-SUN, meaning that it is not profitable for the NSP to provide service for the demands of these node-pairs.



	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SEA	SUN	WDC
ATL	-	1	1	1	1	1	1	1	1	1	1
CHI	1	-	1	1	1	1	1	1	1	1	1
DEN	1	1	-	1	1	1	1	1	1	1	1
HOU	1	1	1	-	1	1	1	1	1	1	1
IND	1	1	1	1	-	1	1	1	1	1	1
KSC	1	1	1	1	1	-	1	1	1	1	1
LAX	1	1	1	1	1	1	-	1	1	1	1
NYC	1	1	1	1	1	1	1	-	0	0	1
SEA	1	1	1	1	1	1	1	0	-	1	1
SUN	1	1	1	1	1	1	1	0	1	-	1
WDC	1	1	1	1	1	1	1	1	1	1	-

Figure 8. Demand Satisfaction Matrix when per-unit revenue (ρ_{st}) is \$48/Gbps. Note that demands for NYC-SEA and NYC-SUN are not satisfied at all.

Table 5. Tabulation of Marginal Revenue, Marginal Cost and Marginal Profit for each City-Pair under Case 1a. Note that the Marginal Profit for NYC-SUN and NYC-SEA is negative, meaning that these demands are not profitable.

City-Pair	Marginal Revenue	Marginal Cost	Marginal Profit	City-Pair	Marginal Revenue	Marginal Cost	Marginal Profit
CHI-IND	48	2.572	45.428	HOU-WDC	48	20.35	27.65
NYC-WDC	48	3.612	44.388	KSC-NYC	48	20.791	27.209
LAX-SUN	48	5.427	42.573	ATL-DEN	48	21.34	26.66
ATL-IND	48	6.355	41.645	HOU-LAX	48	23.388	24.612
CHI-KSC	48	7.526	40.474	HOU-NYC	48	23.944	24.056
IND-KSC	48	8.46	39.54	KSC-LAX	48	24.286	23.714
ATL-CHI	48	8.86	39.14	KSC-SUN	48	27.712	20.288
ATL-WDC	48	9.028	38.972	DEN-WDC	48	27.845	20.155
IND-WDC	48	9.272	38.728	HOU-SUN	48	28.092	19.908
HOU-KSC	48	9.371	38.629	KSC-SEA	48	28.753	19.247
SEA-SUN	48	9.83	38.17	DEN-NYC	48	30.907	17.093
DEN-KSC	48	10.159	37.841	CHI-LAX	48	31.766	16.234
CHI-WDC	48	11.04	36.96	HOU-SEA	48	32.122	15.878
ATL-HOU	48	11.533	36.467	IND-LAX	48	32.649	15.351
ATL-KSC	48	11.758	36.242	ATL-LAX	48	33.981	14.019
IND-NYC	48	12.334	35.666	CHI-SUN	48	34.993	13.007
ATL-NYC	48	12.637	35.363	CHI-SEA	48	35.092	12.908
HOU-IND	48	13.331	34.669	IND-SUN	48	36.172	11.828
DEN-HOU	48	13.648	34.352	IND-SEA	48	36.848	11.152
CHI-NYC	48	13.717	34.283	ATL-SUN	48	38.167	9.833
LAX-SEA	48	14.071	33.929	ATL-SEA	48	40.315	7.685
CHI-HOU	48	14.199	33.801	LAX-WDC	48	41.652	6.348
DEN-LAX	48	14.729	33.271	LAX-NYC	48	44.944	3.056
CHI-DEN	48	17.354	30.646	SUN-WDC	48	45.357	2.643
DEN-SUN	48	17.639	30.361	SEA-WDC	48	46.07	1.93
KSC-WDC	48	17.686	30.314	NYC-SUN	48	48.502	**-0.502**
DEN-IND	48	18.6	29.4	NYC-SEA	48	48.78	**-0.78**
DEN-SEA	48	19.055	28.945				

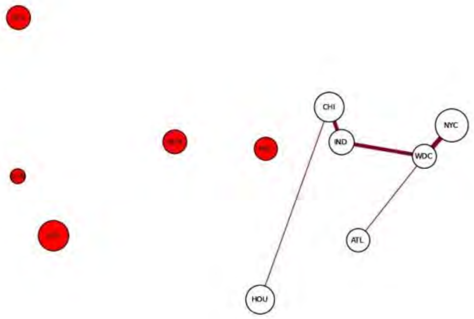
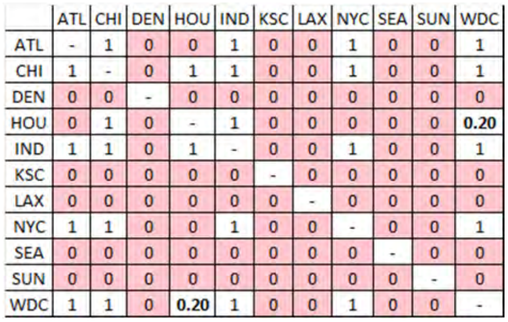
Continuing this experiment to the other extreme, when the per-unit revenue is only $5/Gbps, only CHI-IND and NYC-WDC are profitable and will be serviced by the NSP.

3. A Heuristic Derived from Case 1 and Case 1a

From the results in Case 1 and Case 1a, we can infer that the optimal solution resembles the heuristic greedy algorithm. The NSP will build direct arcs, starting from that with the highest marginal profit. The NSP will continue increasing the capacity of that arc until the demand for that node-pair is satisfied. The NSP will then start to build on increasing the capacity of the arcs that gives the next highest marginal profit until the demand for that node-pair is also satisfied. This is repeated as long as the NSP has budget to continue building until there are no longer any profitable demands. A pre-condition to this heuristic is that the demand does not exceed the maximum allowable capacity.

4. Case 2: Building a New Network with Fixed Costs for New Edges

We next consider the case where there are fixed costs associated with building each new edge. Figure 9 shows the network topology and demand satisfaction matrix when the fixed cost is $5 and the budget given is varied from $200 to $1000.

<table>
<tr><th>Budget</th><th>Resultant Network</th><th>Demand Satisfaction Matrix</th></tr>
<tr><td>$200</td><td>
</td><td>
<table>
<tr><th></th><th>ATL</th><th>CHI</th><th>DEN</th><th>HOU</th><th>IND</th><th>KSC</th><th>LAX</th><th>NYC</th><th>SEA</th><th>SUN</th><th>WDC</th></tr>
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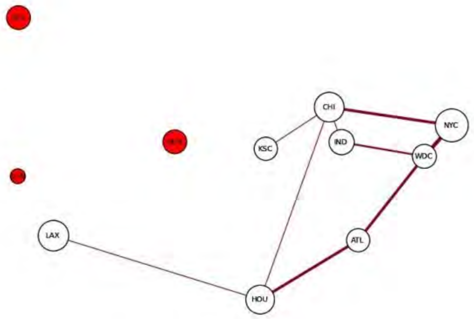
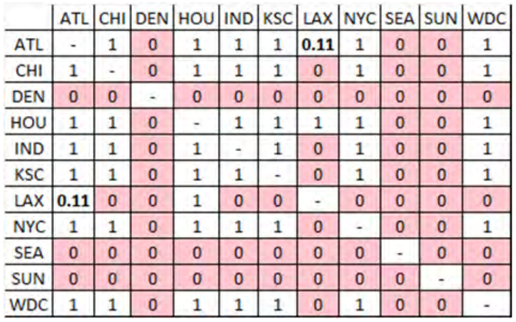
<tr><td>$400</td><td>
</td><td>
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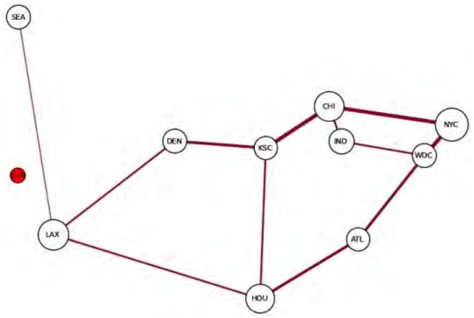
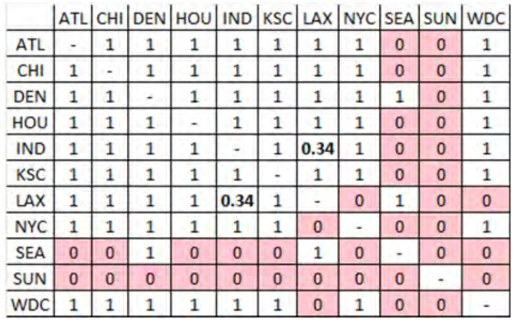
<tr><td>$600</td><td>
</td><td>
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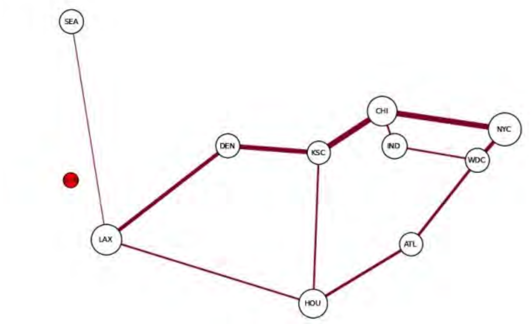
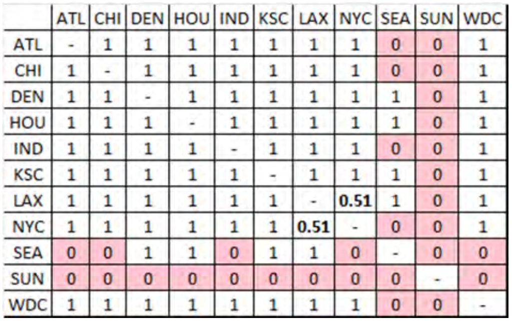
<tr><td>$800</td><td>
</td><td>
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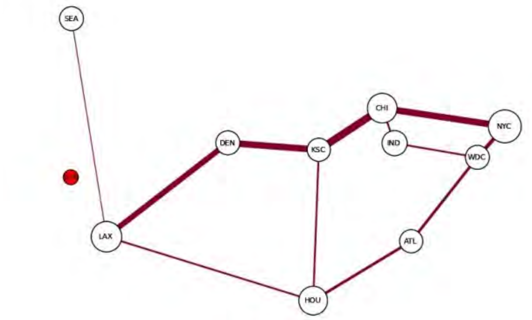
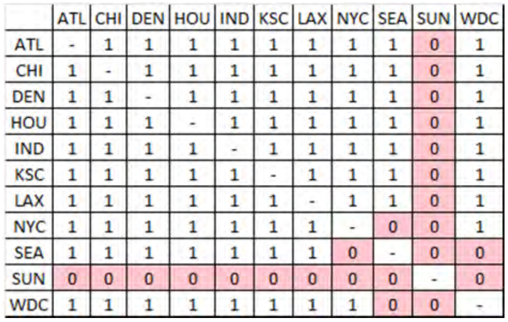
<tr><td>$1000</td><td>
</td><td>
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Figure 9. Resultant Network Topology and Demand Satisfaction Matrix for given budget $200, $400, $600, $800 and $1000, when building a new network with fixed costs of $5.

With a fixed cost of $5 per arc, we observe that a budget of $1000 yields a network in which not all demands are satisfied. Specifically, demand between NYC-SEA and SEA-WDC is not fulfilled, and SUN is not connected to the network.

In the following two sections, we investigate the reasons for these observations, and through the variation of input parameters, attempt to address the observations.

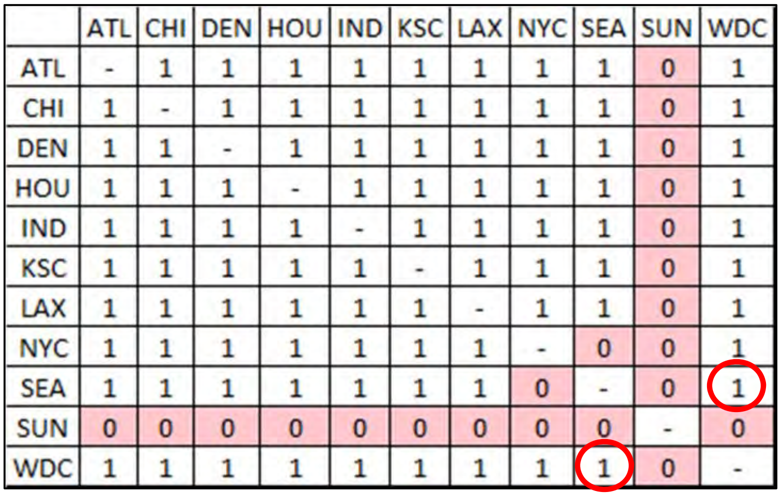
5. Case 2a: Building a New Network with Fixed Costs for New Edges, with Variation of Revenue for NYC-SEA and SEA-WDC

We first investigate why demand for NYC-SEA and SEA-WDC is not fulfilled when the given budget is $1000, by examining the distances of the shortest paths along the built arcs between NYC-SEA and SEA-WDC. We refer to the network topology graph for given budget of $1000 in Figure 9.

The shortest path from between SEA and WDC is SEA-LAX-HOU-ATL-WDC, with a path distance of 58.02 units. This relates to a marginal cost of $58.02/Gbps for delivering commodity SEA-WDC, which is less than the marginal revenue of $50/Gbps. Hence, this demand for SEA-WDC is not serviced.

The shortest path from between SEA and NYC is SEA-LAX-DEN-KSC-CHI-NYC, with a path distance of 60.202 units. This relates to a marginal cost of $60.202/Gbps for delivering commodity SEA-NYC, which is less than the marginal revenue of $50/Gbps. Hence, this demand for SEA-NYC is also not serviced.

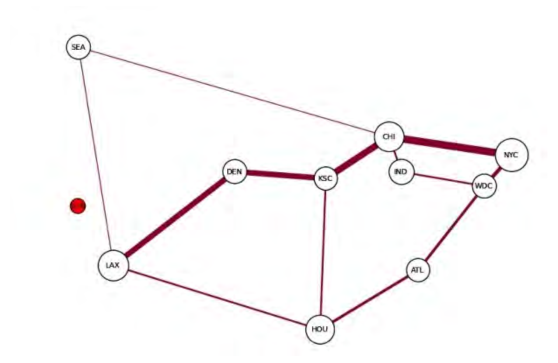
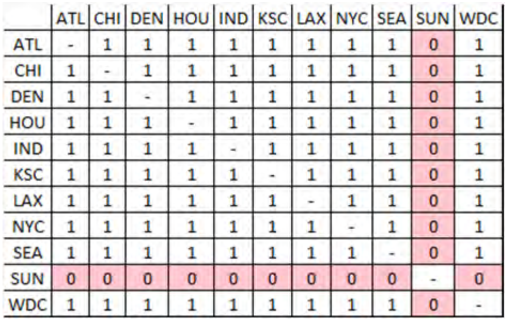
Next, we run Case 2 with a budget of $2000 and have the marginal revenue for SEA-WDC be $60/Gbps instead of $50/Gbps. Since this marginal revenue is greater that the marginal cost, we expect demand for SEA-WDC to be fulfilled. The resulting demand satisfaction matrix in Figure 10 confirms this to be the case.



	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SEA	SUN	WDC
ATL	-	1	1	1	1	1	1	1	1	0	1
CHI	1	-	1	1	1	1	1	1	1	0	1
DEN	1	1	-	1	1	1	1	1	1	0	1
HOU	1	1	1	-	1	1	1	1	1	0	1
IND	1	1	1	1	-	1	1	1	1	0	1
KSC	1	1	1	1	1	-	1	1	1	0	1
LAX	1	1	1	1	1	1	-	1	1	0	1
NYC	1	1	1	1	1	1	1	-	0	0	1
SEA	1	1	1	1	1	1	1	0	-	0	1
SUN	0	0	0	0	0	0	0	0	0	-	0
WDC	1	1	1	1	1	1	1	1	1	0	-

Figure 10. Demand Satisfaction Matrix when marginal revenue for NYC-WDC is increased to $60/Gbps. The demand between SEA and WDC in the case is all fulfilled.

Next, we similarly increase the marginal revenue for SEA-NYC from \$50/Gbps to \$60/Gbps. Based purely on the marginal cost of building along the shortest existing path between SEA and NYC, we do not expect to demand for SEA-NYC to be fulfilled, since the marginal cost at \$60.202/Gbps is still greater than the marginal revenue. The results for this case, in Figure 11, however show otherwise.

Resultant Network	**Demand Satisfaction Matrix**
	

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SEA	SUN	WDC
ATL	-	1	1	1	1	1	1	1	1	0	1
CHI	1	-	1	1	1	1	1	1	1	0	1
DEN	1	1	-	1	1	1	1	1	1	0	1
HOU	1	1	1	-	1	1	1	1	1	0	1
IND	1	1	1	1	-	1	1	1	1	0	1
KSC	1	1	1	1	1	-	1	1	1	0	1
LAX	1	1	1	1	1	1	-	1	1	0	1
NYC	1	1	1	1	1	1	1	-	1	0	1
SEA	1	1	1	1	1	1	1	1	-	0	1
SUN	0	0	0	0	0	0	0	0	0	-	0
WDC	1	1	1	1	1	1	1	1	1	0	-

Figure 11. Resultant Network Topology and Demand Satisfaction Matrix when marginal revenue for NYC-SEA is increased to 60. Note that the arc between SEA and CHI is built, and demand for NYC-SEA and SEA-WDC is now all fulfilled.

The interesting result is that although the marginal revenue of \$60/Gbps does not cover the marginal cost of \$60.202/Gbps for expanding the capacity of the pre-existing shortest path for SEA-NYC, the revenue is enough to compensate for the fixed costs of building the direct arc between SEA and CHI. This also benefits the flow of commodity SEA-WDC as the shortest path for this commodity is now SEA-CHI-IND-WDC, with a path distance of 46.936 units. In this case, the marginal revenue at \$50/Gbps for SEA-WDC is now more than the marginal cost; hence we see the demand for this commodity fulfilled completely.

Previously, all commodity flows in and out of SEA must pass through LAX, the only node connected to SEA. With the new arc SEA-CHI built, some of the commodities will find more economical to pass through CHI instead. The flows from SEA when SEA-

CHI is built are shown in Figure 12. We observe that the NSP benefits from the lower marginal costs for commodities between CHI-SEA, IND-SEA, NYC-SEA and SEA-WDC, which outweighs the fixed cost of building arc CHI-SEA.

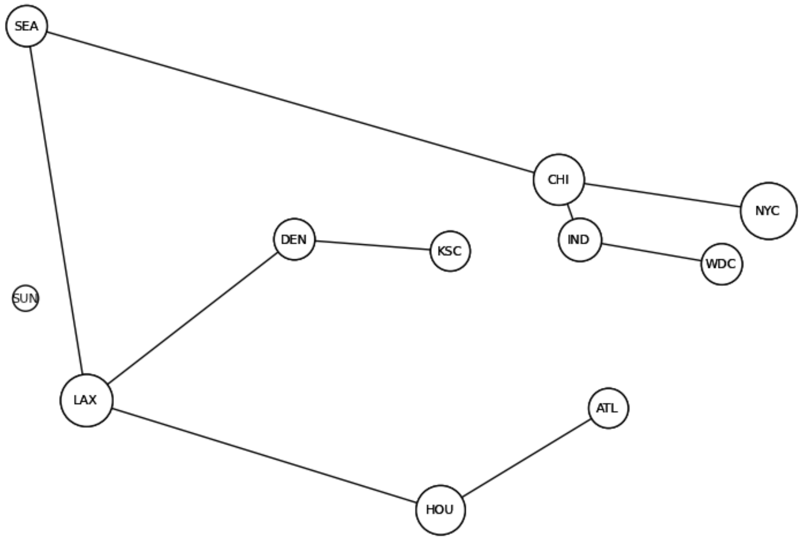
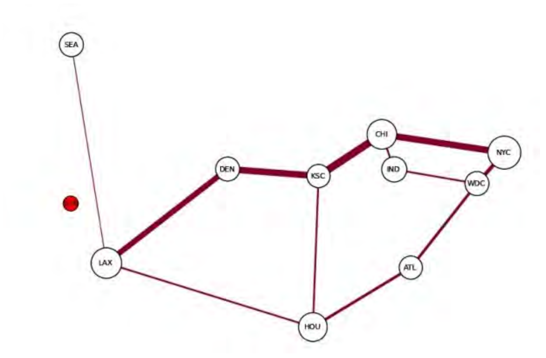
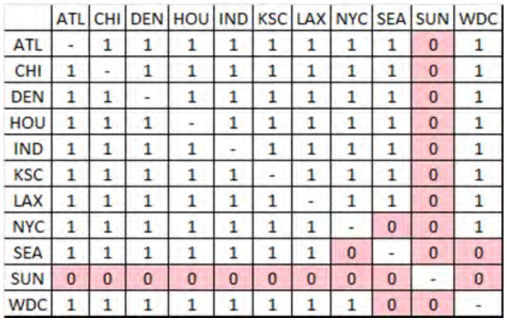


Figure 12. Network flows for commodities originating from SEA. Note that with arc SEA-CHI built, commodities from SEA to CHI, IND, WDC and NYC now need not pass through LAX.

6. Case 2b: Building a New Network with Fixed Cost for New Edges, with Population Increase for SUN

We now proceed to increase the population of SUN to illustrate that one way for SUN to be "worthwhile" to the NSP is to increase the customer population of SUN beyond a critical population mass. When the population of SUN is small, it does not make sense to service the demands to and from SUN. A critical population mass is required to overcome the associated fixed costs to add an arc connecting to SUN. When the population of SUN is increased, the corresponding O-D demand matrix derived from the gravity model changes, with entries involving SUN increasing in demand.

Figure 13 shows the resultant network topologies and demand satisfaction matrices when we allow the NSP a sufficiently large budget of $2000, and consider increased populations of SUN by increments 50% and 60%.

<table>
<tr><th>SUN Population</th><th>Resultant Network</th><th>Demand Satisfaction Matrix</th></tr>
<tr><td>50% increase</td><td> </td><td>
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<tr><th></th><th>ATL</th><th>CHI</th><th>DEN</th><th>HOU</th><th>IND</th><th>KSC</th><th>LAX</th><th>NYC</th><th>SEA</th><th>SUN</th><th>WDC</th></tr>
<tr><td>ATL</td><td>-</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>1</td></tr>
<tr><td>CHI</td><td>1</td><td>-</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>1</td></tr>
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<tr><td>HOU</td><td>1</td><td>1</td><td>1</td><td>-</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>1</td></tr>
<tr><td>IND</td><td>1</td><td>1</td><td>1</td><td>1</td><td>-</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>1</td></tr>
<tr><td>KSC</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>-</td><td>1</td><td>1</td><td>1</td><td>0</td><td>1</td></tr>
<tr><td>LAX</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>-</td><td>1</td><td>1</td><td>0</td><td>1</td></tr>
<tr><td>NYC</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>-</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td>SEA</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>-</td><td>0</td><td>0</td></tr>
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<tr><td>WDC</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>-</td></tr>
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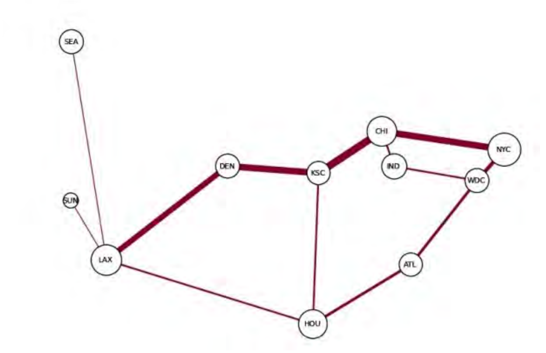
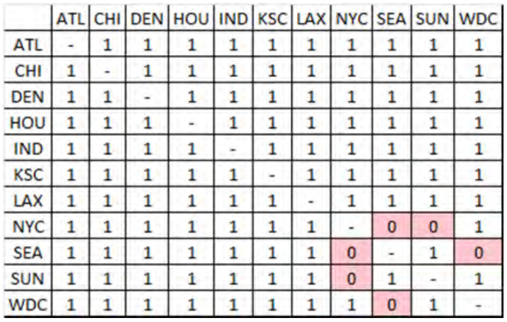
<tr><td>60% increase</td><td> </td><td>
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<tr><td>SEA</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>-</td><td>1</td><td>0</td></tr>
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Figure 13. Resultant Network Topology and Demand Satisfaction Matrix for Case 2b. SUN will be connected to the network if its population increases by at least 60%.

We observe that when the population of SUN increases to 60%, it crosses the critical mass where it is economically viable built arcs from SUN to fulfill some of the demands associated with SUN. Note that none of the demands between SUN-NYC are fulfilled as the marginal cost of building along the shortest path between SUN and NYC, at $51.558/Gbps, is more than the marginal revenue of $50/Gbps.

Another observation of interest is that the flow of commodity SEA-SUN is not via a direct arc, but instead passes through LAX as an intermediate node. When there are fixed costs to build a new arc, it may be cheaper to enhance the existing capacities along the arcs along the shortest built path.

The distance of this path SEA-LAX-SUN is 19.498 units, while the distance of SEA-SUN is 9.83 units. The total demand for SEA-SUN is 0.02 Gbps. The cost of provisioning on existing arcs on path SEA-LAX-SUN to fulfill all demand for commodity SEA-SUN is $0.38996, and this is less than the revenue of $1 (= 0.02 x $50) from fulfilling all that demand. The cost of building new direct arcs SEA-SUN to fulfill all demand for the same commodity is $0.1966 plus the fixed cost of 5 for each direction of the link, leading to the total cost of $10.1966 which will cause the NSP to incur losses delivering this commodity this way.

7. A Network Provisioning Decision Rule

In order to service the unfulfilled demand for a particular commodity (s,t), the NSP has to decide whether to enhance the capacity of the existing arcs in the network or to build a new arc (s,t). To help with this decision, we refer to the marginal costs for each option.

We make use of the same definitions for parameters specified in Chapter III. We assume that γ_{ij} is the same for all arcs (i, j), hence for notational convenience, we drop the subscripts in this section.

The lowest marginal cost for enhancing the capacity of the existing arcs comes from the marginal costs ($MC_{existing}$) of enhancing arcs along the shortest path from source to destination for the commodity. The distance along this shortest path is $d_{shortestPath}$. Based on our cost model, we have $MC_{existing} = \gamma d_{shortestPath}$.

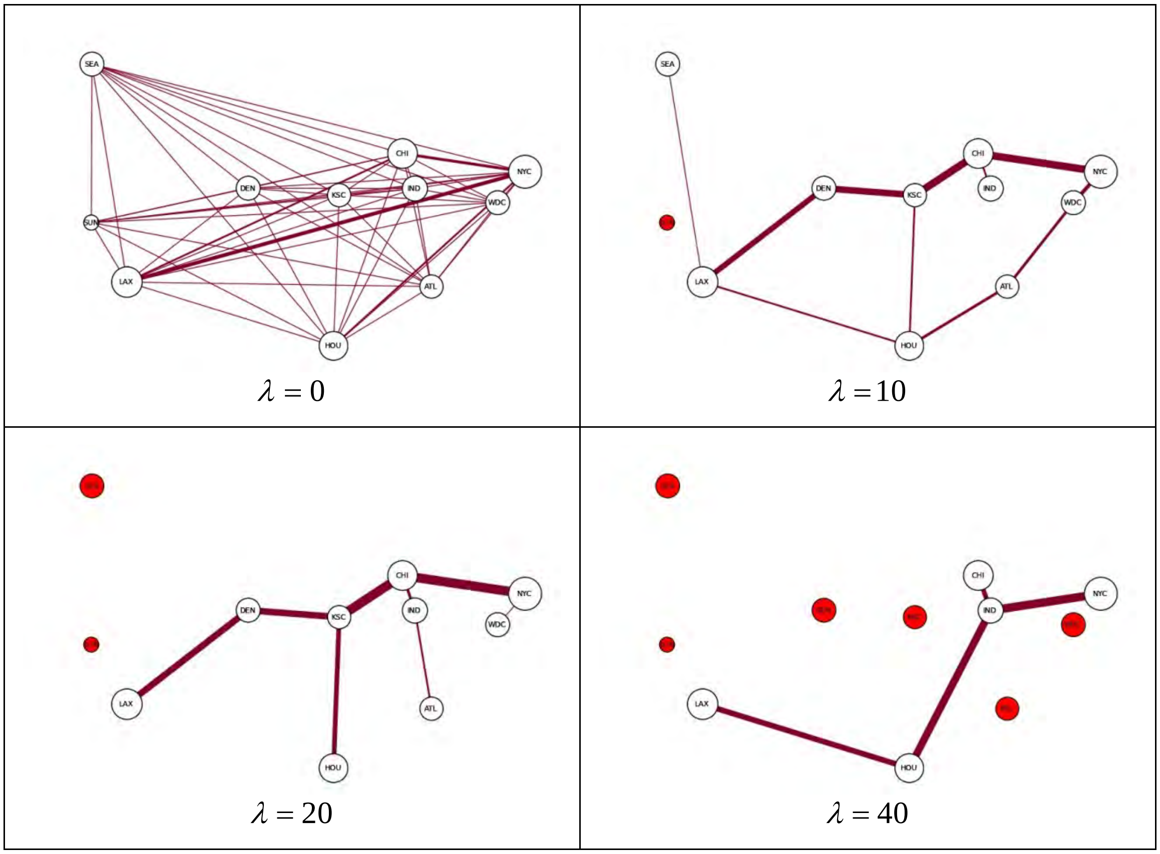
The marginal cost for the new direct arc (MC_{new_arc}) depends on both the arc length, and the fixed cost divided by the capacity built to service the unfulfilled demand. Specifically, $MC_{new\_arc} = \gamma d_{st} + \frac{\lambda}{Y_{st}}$.

When $MC_{existing} > MC_{new\_arc}$, the NSP should build the direct link, otherwise, the NSP should enhance the capacity along the existing shortest path.

Based on this decision rule, we see that when there are no fixed costs ($\lambda = 0$), the new arc should be built, since the direct arc (s,t) will be the shortest distance path possible.

With all other parameters held constant, as the fixed cost increases, MC_{new_arc} increases, and the NSP will be inclined to build on the existing shortest path. Conversely, if the capacity to be built increases arising from higher demand, the fixed cost is spread over the increased capacity built (Y_{st}), and this decreases MC_{new_arc}.

We illustrate this by contrasting the resultant network topology at various levels of fixed cost in Figure 14. We observe that at $\lambda = 40$, the nodes that remain connected are those of the five largest cities (NYC, LAX, CHI, HOU and IND). By extension, the demand between these nodes are also the largest, hence there are sufficiently large flows over the arcs built to spread the fixed costs.



$\lambda = 0$

$\lambda = 10$

$\lambda = 20$

$\lambda = 40$

Figure 14. Resultant topologies at $\lambda = 0, 10, 20, 40$. Note that as λ increases, more nodes will be disconnected from the network, and node-pairs with the highest demands will continue to be serviced.

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IV. CONCLUSIONS

A. SUMMARY

In this thesis, we model the decisions of the NSP, whose aim is to maximize its profit, through the traffic engineering model and the network provisioning model. The NSP decides how to link the nodes and how to route commodities through the resultant network to derive maximum profit. Under different cost structures, revenues, budget allocation and the demand for commodities, the resultant networks are different.

We make use of the Abilene dataset as input to the network provisioning model and consider the case of a new NSP who is building up a new network. We then examine the resultant networks to identify the underlying logic.

Our model include two types of costs: fixed costs for building new arcs and variable costs for adding capacity to the arcs. In the case where there are no fixed costs associated with building new arcs, we see that flows of commodities are through direct arcs, which are the least costly to build to transport the commodities. We also identify that the priority is to build arcs to satisfy the demand of the commodity with the greatest marginal profit. If the marginal profit for a commodity is negative, the direct arc will not be built. This gives a simple greedy algorithm heuristic.

When we introduce fixed costs, the flows of commodities are no longer necessarily via direct arcs. Even when the fixed costs are low, we observe that there are commodities that are not transported even though the network has capacity to do so. Some cities might be ignored, even when there is budget to service them. By increasing the marginal revenue for the unfulfilled demand, we can make it profitable for the NSP to service that demand. We also illustrate a case where increasing marginal revenue leads to a new network construction that also reduces the costs of servicing other demands. Through increasing the population of the disconnected node, we demonstrate that there exists a threshold or critical population mass before it becomes economically viable for the NSP to build arcs to that node.

We generalize the results in a network provisioning decision rule that provides a simple heuristic of constructing the NSP's network. We observe that when fixed costs get higher, more nodes will be disconnected and nodes with larger populations tend to remain connected.

B. SUGGESTIONS FOR FUTURE WORK

This thesis assumes a static set of nodes of which the NSP bases its decisions on over a single time period. A possible area of future research will be to gradually increase the set of nodes over a multi-time period model to determine how the increasing set of nodes will affect decisions on network expansion.

The marginal revenues in this thesis give rise to a linear revenue model, but we observe that changes in marginal revenue can lead to the adding and dropping of demands serviced by the NSP. By considering revenues which change with the amount of commodity delivered, the results could shed some light on the price differentiation or price discrimination that we observe in real-world networks.

This thesis models the decisions of a single NSP in isolation. The natural next step is to model the interaction of multiple NSPs targeting the same pool of customers by considering, for example, the change in marginal revenue arising from the overall amounts of commodities delivered.

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