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# Beam Combining and Atmospheric Propagation of High Power Lasers

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### Beam Combining and Atmospheric Propagation of High Power Lasers

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#### Abstract

Solid state lasers are one of the candidates for efficient and compact directed energy systems. To achieve the necessary power levels many laser beams must be combined to form a single focused beam. We discuss coherent and incoherent combining of multi-kW, high quality lasers and the effects of atmospheric turbulence and aerosols on the propagation of the combined beams. We analyze the beam centroid wander and spreading contributions to the spot size and obtain the power on target for a range of turbulence levels. We find that there is virtually no difference in the power on target between coherently combined and incoherently combined beams. We also find that for km-range propagation in moderate turbulence, there is a maximum intensity that can be propagated to the target which is independent of initial beam size and beam quality. In addition, due to turbulence, it is not necessary to have extremely high quality beams, i.e.,  $M^2 < 3$  is sufficient. For low levels of turbulence, tip-tilt corrections can be used to reduce the beam centroid wander contribution to the spot size. The HELCAP laser propagation code is used to compare the propagation efficiency of coherently and incoherently combined beams for various levels of turbulence and propagation ranges.

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#### **I. Introduction**

Advances in solid state lasers, i.e., slab and fiber lasers, have made them candidates for directed energy applications [1, 2]. To achieve the necessary power levels needed for these applications, it is necessary to combine a large number of lasers and propagate the beam many kilometers in a turbulent atmosphere.

In the following we discuss laser propagation in a maritime environment and compare the propagation characteristics of coherently and incoherently combined beams, see Fig. 1. Based on our analysis and simulations we find that for typical levels of turbulence  $(C_n^2 > 5 \times 10^{-15} \text{ m}^{-2/3})$  and multi-km propagation ranges, there is virtually no difference, on target, between coherently and incoherently combined laser beams. In strong turbulence, beam centriod wander and spreading are not distinct, i.e., the beam breaks up into multiple beamlets. Therefore, employing tip-tilt correction is less effective in strong turbulence than in weak turbulence. We find that when adaptive optics are not effective there is no advantage in having excellent quality laser beams; a beam quality of  $M^2 < 3$  is sufficient. There is a maximum intensity that can be placed on a target which is independent of the initial beam spot size (aperture) and laser beam quality  $M^2$ .

#### **II.** Atmospheric Propagation (Spot Size, Intensity, Extinction)

Propagation through atmospheric turbulence results in spreading of the laser beam radius and wandering of the laser beam centroid which lead to fluctuations of the intensity on target. *a) Laser Beam Radius* 

The time averaged laser spot size R(L) at range L is given by

$$R^{2}(L) = (\lambda L / \pi R_{o})^{2} (M^{4} + 2.9(R_{o} / r_{o})^{2}) + \theta_{jitter}^{2} L^{2} + R_{o}^{2} (1 - L / L_{f})^{2}, \qquad (1)$$

where  $R_o$  is the initial spot size,  $\lambda$  is the laser wavelength,  $M^2$  is the intrinsic laser beam quality,  $L_f$  is the focal length,  $r_o$  is the transverse coherence length associated with turbulence and  $\theta_{jitter}$  is the mechanical jitter angle of the beam. Equation (1) is valid for the parameters considered here [3,4]. The strength of the turbulence is characterized by the structure parameter  $C_n^2$  [4,5]. Typically  $C_n^2$  is in the range of  $10^{-15} - 10^{-13}$  m<sup>-2/3</sup> at ground level [4, 5]. For a constant value of  $C_n^2$  the transverse coherence length used here is  $r_o = 0.184 (\lambda^2 / C_n^2 L)^{3/5}$ . However, as far as laser beam propagation is concerned the effective strength of turbulence is not determined by  $C_n^2$  alone, but also depends on the range of propagation and the wavelength. The relevant combination is the Rytov variance, defined by  $\sigma_R^2 = 10.5 C_n^2 \lambda^{-7/6} L^{11/6}$ . For weak turbulence ( $\sigma_R^2 \ll 1$ ), beam wander and spreading are distinct and tip-tilt compensation can be used to correct for beam centroid wander. On the other hand, for strong turbulence ( $\sigma_R^2 > 1$ ) the beam breaks up into multiple beams and beam wander and spreading are not distinct, hence, tip-tilt compensation is less effective. The Rytov variance is a measure of the intensity scintillation level on axis at the target, i.e.,  $\sigma_R^2 = \langle (I - \langle I \rangle)^2 \rangle / \langle I \rangle^2$  for weak turbulence.

The laser spot size due to diffraction and turbulence,  $R = (R_s^2 + R_w^2)^{1/2}$ , consists of a contribution due to spreading  $R_s$  and a contribution due to centroid wander  $R_w$  as shown in Fig. 2. The beam centroid wander is given by

$$R_W^2 = 1.7 \left(\lambda L / \pi R_o\right)^2 \left(R_o / r_o\right)^{5/3},\tag{2}$$

and the spot size spreading is given by  $R_s^2 = R^2 - R_w^2$ .

#### b) Laser Intensity

The peak laser intensity on axis, at the target, is

$$I_{peak} = \left(\frac{6.28 P_T}{M^4 + 2.9 (R_o/r_o)^2 + (\pi R_o \theta_{jitter}/\lambda)^2}\right) \left(\frac{R_o}{\lambda L}\right)^2 \exp(-\gamma L), \quad (3)$$

where  $P_T$  is the total transmitted laser power and  $\gamma = \alpha + \beta$  is the atmospheric extinction (absorption and scattering) coefficient. In a maritime environment, aerosol scattering dominates molecular extinction. Typical values for aerosol absorption and scattering are  $\alpha_A \approx 1 \times 10^{-3} \text{ km}^{-1}$ and  $\beta_A \approx 0.1 \text{ km}^{-1}$ , respectively, for a laser wavelength of 1µm [6]. When turbulence is sufficiently strong, i.e.,  $r_o < 1.7 R_o / M^2$ , the intensity on target reaches a maximum and Eq. (3) reduces to

$$I_{\max} = 2.17 P_T (r_o / \lambda L)^2 \exp(-\gamma L).$$
 (4)

where mechanical jitter has been neglected. It is important to note that this maximum intensity is independent of the initial spot size and beam quality. Increasing the aperture size or improving

beam quality will not increase the intensity on target. Note also that for typical parameters this maximum intensity is obtained for modest initial laser spot sizes of  $R_o \ge 10$  cm.

#### c) Propagation Efficiency

The propagation efficiency is defined as the ratio of the power on target to the total transmitted power and for a Gaussian laser beam is given by

$$\eta_{prop} = (1 - \exp(-2R_{target}^2 / R^2)) \exp(-\gamma L), \qquad (5)$$

where  $R_{\text{target}}$  is the radius of the target.

#### **III. Beam Combining**

In this section we give a brief overview of the two laser beam combining configurations.

#### a) Coherent Combining

Coherent beam combining [7] relies on constructive interference of many lasers to produce high intensities on target, Fig. 1(a). It achieves this by effectively making the aperture size larger which, in vacuum, results in the peak intensity on target to be  $N^2 I_o$ , where  $I_o$  is the peak intensity of a single beam at the target and N is the number of combined beams. The diffraction length of the combined laser array is ~ N times longer than that of the individual beams. However, as Eq. (4) indicates, even for relatively moderate turbulence, the peak intensity is essentially independent of aperture size so that the benefit of coherent combining is lost. In addition, coherent combining can be difficult to achieve. It requires phase locking and polarization matching of the lasers, narrow linewidths ( $\delta \lambda / \lambda < 10^{-5}$ ) and good optical beam quality. The propagation efficiency for coherent combining can also be limited by the filling factor of the laser array. A filling factor less than unity results in some of the laser energy residing in side lobes outside of the central lobe.

#### b) Incoherent Combining

For incoherent combining [8] each beam is focused and directed to the target by individually controlled steering mirrors, Fig. 1(b). Incoherent beam combining does not require phase locking, polarization matching or narrow linewidths. It should therefore result in a simpler and more robust system compared to the coherently combined configuration. The diffraction length is determined by the diffraction length of the individual beams. This is of little

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consequence in moderate to strong turbulence since the intensity on target becomes independent of the diffraction length, see Eq. (4).

#### **IV.** Comparison of Coherent and Incoherent Combining

In comparing the two laser combining architectures we keep the size of the beam director and total transmitted power the same, Fig. 3. We compare the laser spot size, propagation efficiency and intensity on target for the two configurations in the presence of various levels of turbulence. In order to minimize the number of parameters in the comparison, Gaussian beams are used in both beam combining configurations. We assume that the laser linewidths are sufficiently small to be neglected. In both cases N beams are combined, either coherently or incoherently. A single Gaussian of initial spot size  $R_{BD}$  is used to model the N coherently combined beams [Fig.3(a)] while for the incoherently combined beams [Fig.3(b)] the initial spot sizes of the N beams is  $R_{BD} / \hat{N}^{1/2}$  where  $\hat{N} = (\sqrt{2} (\sqrt{N} - 1) + 1)^2$ . Although idealized models are used in the comparison the conclusion is independent of whether the beams have a Gaussian, rectangular or other profile. We find that for typical target ranges and levels of turbulence the spot size on target is essentially the same for both beam combining architectures.

As an example consider the case where all beams in both architectures are focused on the target at range L. The spot size on target from Eq. (1), for the coherently combined and incoherently combined beams are respectively given by

$$R_{CC} = (\lambda L / \pi R_{BD}) (M^4 + 2.9 (R_{BD} / r_o)^2)^{1/2},$$
 (6a)

$$R_{IC} = (\lambda L / \pi R_{BD}) (\hat{N} M^4 + 2.9 (R_{BD} / r_o)^2)^{1/2},$$
 (6b)

where mechanical jitter has been neglected and the beam quality associated with both configurations are taken be the same and equal to  $M^2$ . In obtaining Eq. (6) from Eq. (1), the initial spot sizes for the coherently and incoherently combined beam configurations were taken to be  $R_o = R_{BD}$  and  $R_o = R_{BD} / \hat{N}^{1/2}$ , respectively. The only difference in Eqs. (6a) and (6b) is in the beam quality terms, i.e., terms proportional to  $M^4$ . The spot size on target is the same for both combining architectures when the level of turbulence is high enough so that the transverse coherence satisfies

$$r_o < 1.7 R_{BD} / (\hat{N}^{1/2} M^2).$$
<sup>(7)</sup>

This condition is easily satisfied for typical ranges and levels of turbulence.

As a specific example, we use the following parameters in the comparison; total transmitted power  $P_T = 100 \,\mathrm{kW}$ , beam director spot size  $R_{BD} = 25 \,\mathrm{cm}$ , target range  $L = 5 \,\mathrm{km}$ , turbulence level  $C_n^2 = 5 \times 10^{-15} \,\mathrm{m}^{-2/3}$  ( $r_o = 2.7 \,\mathrm{cm}$ ,  $\sigma_R^2 = 3.2$ ), target area  $\pi R_{\text{target}}^2 = 50 \,\mathrm{cm}^2$  ( $R_{\text{target}} = 4 \,\mathrm{cm}$ ), total extinction coefficient  $\gamma = 0$ , number of combined beams N = 9, initial spot size of coherently combined beam  $R_{BD} = 25 \,\mathrm{cm}$ , initial spot size of incoherently combined beams  $R_{BD} / \hat{N}^{1/2} = 6.5 \,\mathrm{cm}$  and intrinsic beam quality  $M^2 = 1$ . The result of the comparison is summarized in Table I. Note that the maximum intensity from Eq. (4) is  $I_{\text{max}} = 617 \,\mathrm{W/cm}^2$ .

|  | <b>Coherently Combined</b> | Incoherently Combined |
|--|----------------------------|-----------------------|
| Spot size on target, <i>R</i> [cm]                   | 10.2                       | 10.4                  |
| Centroid wander, $R_W$ [cm]                          | 5.4                        | 6.7                   |
| Propagation efficiency, $\eta_{\it prop}$ [%]        | 26                         | 25                    |
| Intensity on target, $I_{peak}$ [W/cm <sup>2</sup> ] | 614                        | 583                   |

Table I. Shows the comparison between the two combining architectures.

We now present a more detailed comparison using the atmospheric laser propagation code HELCAP. HELCAP is a 3-dimensional, fully time-dependent, nonlinear atmospheric propagation code, the details of which are discussed in Ref. 6. Atmospheric turbulence is modeled by using phase screens distributed along the propagation path which modify the phase of the laser beam [6]. The phase perturbations on each screen are characterized by a Kolmogorov spectrum with a strength parameter  $C_n^2$ .

For the comparison the laser wavelength is chosen to be  $\lambda = 1 \mu m$ , the spot size of the beam director is  $R_{BD} = 25 \text{ cm}$ , the range is L = 5 km, the target area is  $50 \text{ cm}^2$  and the total transmitted power is  $P_T = 100 \text{ kW}$ . Figure 3 shows the cross-sectional area of the laser beams at the source. For the coherent combining case the intensity is Gaussian with a 1/e radius of

 $R_{BD}/\sqrt{2}$  while for the incoherent combining case N = 9 laser beams are distributed in a square array pattern, with  $P_o = 11.1$ kW in each beam.

In Fig. 4 the spot size on target is plotted as a function of the refractive index structure constant  $C_n^2$ . The curves are obtained from Eq. (6) and the dots are the results of HELCAP simulations. The coherent combining results are shown in red and the incoherent combining results are shown in blue. Figure 4 shows that virtually the same spot size is obtained on the target irrespective of the combining architecture.

Figure 5 is a plot of the on-axis intensity at the target, from Eq. (3), versus the refractive index structure constant  $C_n^2$ . In the simulations, the intensity is averaged over a small 0.5 cm<sup>2</sup> area to reduce the variations due to scintillation. The coherent combining results are shown in red and the incoherent combining results are in blue. The peak intensity on target is essentially the same for both configurations at moderate levels of turbulence or higher, i.e.,  $C_n^2 > 5 \times 10^{-15} \text{ m}^{-2/3}$ .

Figure 6 compares the transverse profiles of the laser beams on target. Qualitatively similar patterns are observed on the target for (a) coherent combining and (b) incoherent combining cases, with power on target of 28 kW and 25 kW, respectively.

Figure 7 shows the beam profiles on target when tip-tilt correction is applied to remove beam centroid wander. For the incoherent combining case, tip-tilt correction changes the orientation of the entire array, and not the relative orientations of the individual beams. Comparison with Fig. 7 shows that tip-tilt correction reduces the spot size and nearly doubles the power on target. With tip-tilt correction the power on target is 50 kW for (a) coherent combining and 44 kW for (b) incoherent combining.

Figure 8 shows the spot size on target as a function of turbulence level  $C_n^2$  for three values of beam quality,  $M^2 = 1$  (red),  $M^2 = 3$  (green) and  $M^2 = 5$  (blue). The curves are obtained from Eq. (6) and the dots are the results of HELCAP simulations. Figure 8 indicates that the spot size on target is essentially the same, independent of initial beam quality, when the turbulence level exceeds  $C_n^2 \ge 5 \times 10^{-15} \text{ m}^{-2/3}$ .

#### **V.** Conclusions

We have presented a comparison of coherent and incoherent laser beam combining configurations that are potential candidates for efficient and compact directed energy applications. The power levels required for these applications require that beams from many lasers be combined. In the absence of turbulence, coherent combining has distinct advantages which include extended propagation range and higher intensity on target. However, in the presence of typical levels of atmospheric turbulence these advantages may not be realized. For example, for typical horizontal propagation, i.e., uniform  $C_n^2$ , ~ 5 km range, and moderate levels of turbulence, the peak intensity and spot size on target are virtually identical for the two combining configurations. We also find that there is a maximum intensity that can be propagated to the target which is independent of initial beam size and beam quality for kmranges and moderate turbulence. Hence, it is not necessary to have extremely high quality beams, i.e.,  $M^2 < 3$  is sufficient. For weak to moderate turbulence levels, tip-tilt corrections and adaptive optics can reduce beam wander and spreading for both coherently and incoherently combined beams. The effects of turbulence can also be significantly reduced for vertical or slanted propagation paths. Simulation results from the propagation code HELCAP support these findings.

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(b) Incoherent combining



Figure 1: Schematic of (a) coherently and (b) incoherently combined laser beams.



Figure 2: Schematic showing the effects of turbulence on beam propagation,  $R_w$  is the centroid displacement (wander), and  $R_s$  is the increase in spot size (spreading). The long-term average spot size is  $R = \sqrt{R_w^2 + R_s^2}$ .



Figure 3. Cross sectional view of (a) coherent and (b) incoherent combining configurations at the source. To keep the beam director dimensions the same the spot size of incoherent beams (square array) is  $R_{IC} = R_{BD} / (\sqrt{2}(N^{1/2} - 1) + 1)$  for N beams.



Figure 4: Spot size on target [Eq. (6)] vs.  $C_n^2$  for coherently combined (red curve) and incoherently combined (blue curve) beams. Red and blue dots denote HELCAP simulation results for coherently and incoherently combined beams, respectively. The beam director diameter is 50 cm and the range to the target is 5 km for both cases. The total transmitted power is 100 kW. For the incoherent case, we assume 9 fibers with 11.1 kW per fiber.



Figure 5: Peak intensity on target [Eq. (3)] vs.  $C_n^2$ , for coherently combined (red) and incoherently combined (blue) beams for the same parameters as Fig. 4.



Figure 6: Transverse profiles of average intensity on target for (a) coherently combined beams and (b) incoherently combined beams. Aerosol and molecular absorption and scattering have been neglected. The turbulence level is  $C_n^2 = 5 \times 10^{-15} \text{ m}^{-2/3}$ . The total transmitted power is 100 kW. For the incoherent case, we assume 9 fibers with 11.1 kW per fiber. For a target with an area of 50 cm<sup>2</sup>, the power on the target is 28 kW and 25 kW for the coherently and incoherently combined beams, respectively.



Figure 7: Transverse profiles of average intensity on target for (a) coherently combined beams and (b) incoherently combined beams with tip-tilt correction applied, i.e., no wander, for the same parameters as Fig. 5. For a target with an area of  $50 \text{ cm}^2$ , the power on the target is 50 kW and 44 kW for the coherently and incoherently combined beams, respectively.



Figure 8: Spot size on target vs.  $C_n^2$  for initial beam quality factors  $M^2 = 1$  (red), 3 (green), and 5 (blue). Solid curves denote values from Eq. (6), points denote simulation results. The initial beam spot size is 6.5 cm and the range to the target is 5 km for both cases. Results show that spot size on target is relatively independent of beam quality for  $C_n^2 > 10^{-14} \text{ m}^{-2/3}$ .

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