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Asset Allocation to Cover a Region of Piracy

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ABSTRACT

Piracy in the Gulf of Aden and the waters around Somalia has increased in recent years, with international naval assets allocated to patrol at-risk areas. This paper compares measures of effectiveness of area coverage in situations where there is a uniform piracy risk and where some areas are more vulnerable than others. Simulated annealing was used to allocate the patrolling naval assets. The novel problem of positioning a coalition of ships whilst incorporating the transit of a non-coalition ship that may respond to a piracy incident but can not be prepositioned is addressed.

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Executive Summary

With a rising number of piracy incidents off the coast of Somalia, multinational naval coalitions have been formed to patrol Somali waters. Various countries are also acting independently in the region. This paper addresses the problem of positioning a coalition of ships across a region with various risk levels. The issue of how to incorporate the transit of a non-coalition ship that provides some extra coverage is also addressed. The incorporation of a moving asset into a covering problem in this way is novel in the literature.

Simulated annealing was used to position the assets and various Measures of Effectiveness (MOEs) were compared under different levels of piracy risk. The problem space was discretised and it was found that an MOE which positions the ships to maximise the probability of a ship responding to a piracy incident, compared to MOEs that minimise the average or maximum distance each point in the region is to its closest responding asset, performed best when there was risk involved in the scenario. In the case where each point in the grid had the same level of risk, the MOEs performed similarly.

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Marc joined the Surface and Air Warfare Group of MOD in 2002 and since 2005 has worked on Maritime Security studies, including being an Australian member of TTCP Action Group 5 on Force Protection and Action Group 10 on Maritime Security. Marc received the University Medal for Chemistry at the University of Sydney in 2000, completed a Graduate Diploma in Science Communication at ANU in 2001 and a Masters of Operations Research and Statistics at ADFA in 2004.

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1. Introduction

1.1 Somali piracy

Piracy has risen dramatically in the waters around Somalia in recent years. Naval warships, helicopters and surveillance aircraft provide some level of protection to merchant shipping however piracy activity has increased and analysis of recent shipping patterns has showed marked changes in shipping traffic. Ships may reroute up to 800 nautical miles from the Somali coast. To counter this, pirate tactics have evolved to operate further into the Indian Ocean to both avoid naval patrols and intercept rerouted shipping [1].

Figure 1 shows the number of piracy attacks around Somalia and the Gulf of Aden, as detailed by the International Maritime Bureau. These results suggest that the naval presence in the Gulf of Aden is helping to reduce the number of attacks there, but have had little effect on attacks off the Somali east coast [2].

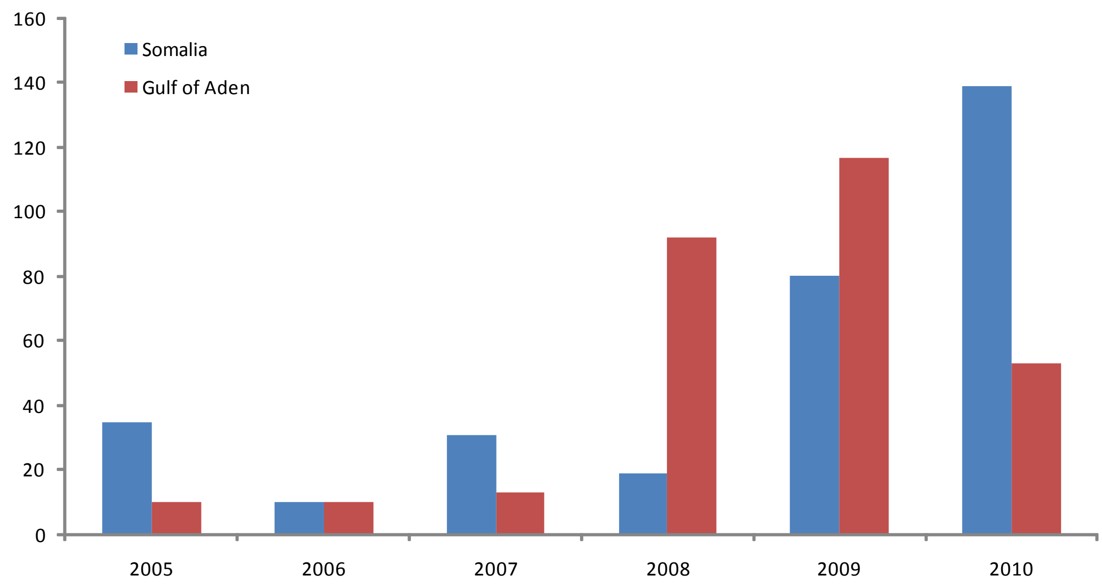

Figure 1: Number of attempted piracy attacks in the Gulf of Aden and off the coast of Somalia

1.2 Methodology

The current challenge concerns how to best allocate naval assets within an area such as the Gulf of Aden or Indian Ocean in order to stop piracy attacks. This is known as a *covering location* problem. The optimal positioning of naval assets within a patrol area is a well-studied problem space within DSTO [3, 4], however there are often no analytical (that is, closed form) solutions.

The technique chosen in this study was that of simulated annealing. The technique as applied in this paper was introduced in [5] and has been chosen because there are no analytical solutions to the problem, and simulated annealing is a well-known heuristic that converges to a solution quickly and allows its measures of effectiveness to be easily modified, which is a natural approach for the problem as posed.

Annealing is the process of heating a solid past its melting point, then cooling it back down. The resultant solid crystals will have few imperfections if the cooling is slow – the molecules that make up the crystals will be in low energy states as the cooling is slow enough for the molecules to optimally arrange themselves. However, if the cooling is too quick, there may be a number of molecules frozen into higher energy levels and therefore not in their optimum state [6]. An algorithm to describe annealing was first published in 1953 [7], and the application of simulated annealing to optimisation problems was first described in 1983 [8]. The technique has been widely used for solving location problems [9-13] and also within DSTO to study the packing of cargo onto watercraft [14]. It has even been used to generate music playlists adapted to the listener's taste [15].

The measure of effectiveness (MOE) chosen for this study was the average minimum distance of all patrolling ships to all possible points in the patrol zone – that is, all possible piracy locations. In this particular circumstance, distance is analogous to time, and time is important as we wish to optimally locate the ships in order to react quickly to a piracy event. The model makes the assumption that the ship that is closest to the piracy event will respond, and that the piracy event is equally likely to occur at any point at any time. Throughout this report, this MOE is called the "Average MOE".

The heuristic for the simulated annealing process is:

1. generate a valid initial configuration of naval vessels;
2. randomly move one of the ships;
3. calculate the new value of the objective function;
4. accept the new configuration if the value of the objective function is reduced, or accept the increase with probability:
   $$P = e^{-\frac{\Delta E}{T}};$$
   where ΔE is the change in the objective function and T is the control parameter (temperature); and
5. iterate towards convergence by cooling the temperature (when the difference between successive objective function values is within some tolerance), or stop after a given number of iterations.

In general, there is no particular method for determining the best starting values for each parameter in a simulated annealing problem; some trial and error is involved. For this problem, a starting temperature of 0.05 degrees was used and cooled by a factor of 0.999 at each step. This allowed for quick and accurate convergence. A higher starting temperature was found unnecessary, whilst a smaller cooling factor cooled the scenario too quickly for optimal configurations to be found. The model assumes that piracy events can occur at equally spaced grid points in the patrol zone. The space used throughout the modelling was

100x100 units with a grid point each 5 units, although both these are variable. The use of grids as an aid to this type of problem is well established as continuous solutions can often not be found [16-18]. When the ships are randomly moved (step 2 above), they can move to a position within 10 units of their current position. The ships themselves are not bound to grid points and can position themselves anywhere in the space. The initial configuration of ships was randomly generated. Moving all ships in the space at once at step 2 was trialled, but it was found that moving them one by one gave better results. The model was written in Visual Basic for Applications (VBA) in Microsoft Excel.

Figure 2 shows sample simulated annealing results for 4 ships in a square grid, and Figure 3 shows how the objective function quickly converges. The chart has been truncated at 2000 runs.

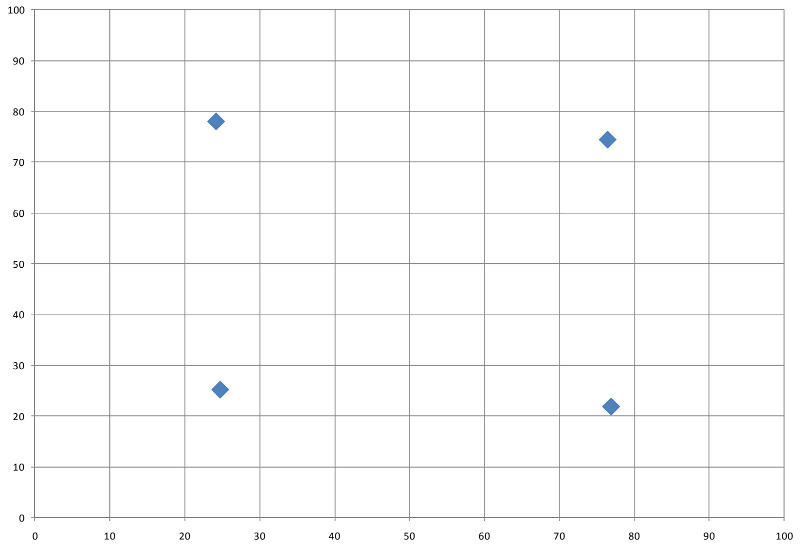

Figure 2: Sample simulated annealing results for 4 ships in a square grid

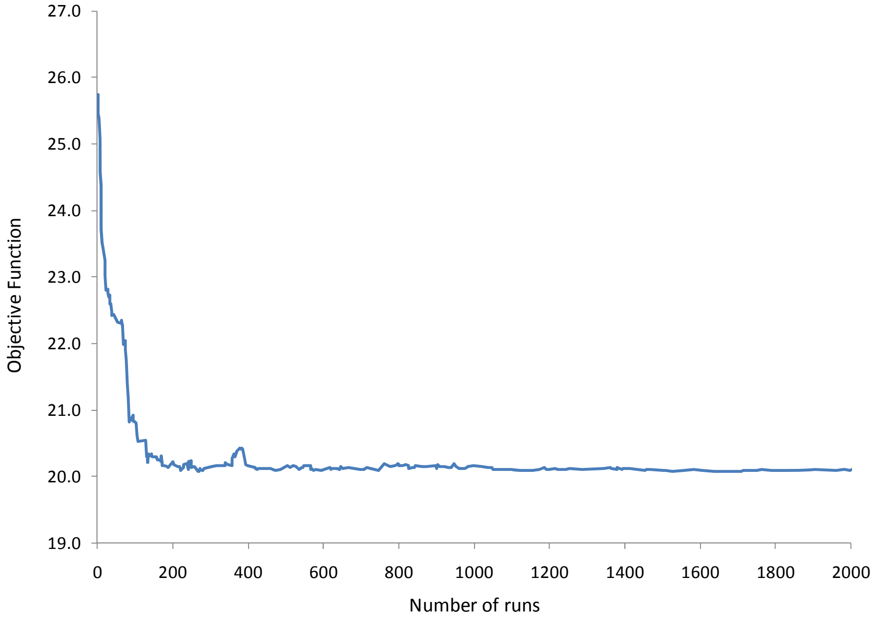

Figure 3: Objective Function convergence for 4 ships

1.3 Inclusion of risk

Risk can be included in the objective function as a multiplying factor for the distance between the grid point and the closest ship. Scaling distance in this way to incorporate risk is a common technique [19, 20]. An example of the incorporation of risk is shown in Figure 4 where red is a high risk and green a low risk. As can be seen in the image on the right, the four ships are pulled towards areas of high risk.

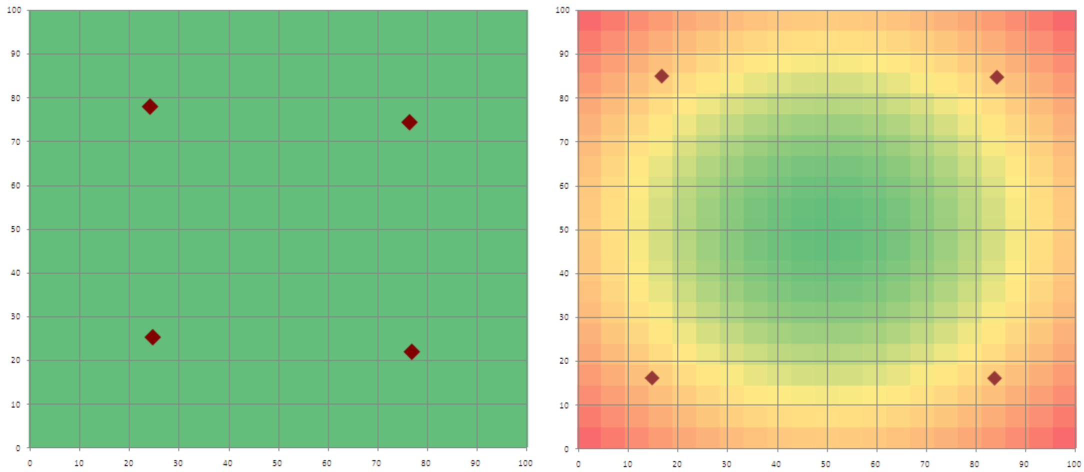

Figure 4: Risk incorporation – red is an area of high risk whilst green is an area of low risk. The red dots represent 4 ships.

Conversely, this approach can be used to show the most unprotected areas in a scenario. Using the 4 ships as placed in the left of Figure 4, Figure 5 shows the most unprotected areas, with red areas the furthest away from their closest ship.

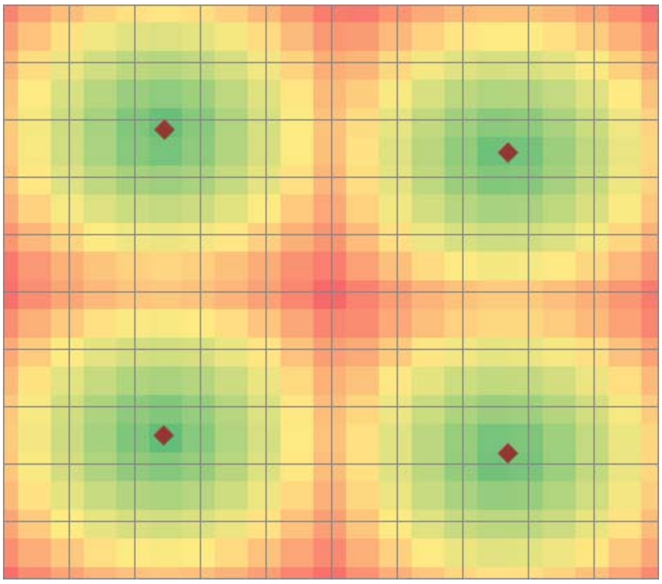

Figure 5: Unprotected areas of the space are shown in red

2. Modelling the passage of a ship

In the Gulf of Aden, patrol vessels may operate as members of a coalition or as individual countries. The problem that arises is how to position coalition ships in advance of a non-coalition naval transit. The non-coalition ship will respond to a piracy incident if it is the closest but otherwise it will not dwell in the area.

2.1 Coalition prepositioning

This scenario has multiple stationary coalition ships with a moving non-coalition ship providing some extra coverage. The stationary ships are modelled as points in the plane and one approach is to model the moving ship as a line. This situation where the line is providing some of the coverage itself is novel. While coverage of points in a plane is a well studied problem, the incorporation of lines into covering problems is relatively new [21]. Simulated annealing has been used in this context to model the positioning of mobile telephone towers to provide telecommunication coverage to a road [16]. The problem of locating a line such that it minimises the distance from a set of points in the plane [22] or maximises the distance from a number of regions (known as an obnoxious route) [19] has also been studied.

The first approach used was to determine the average distance from the moving ship line to any grid point and use this distance in the objective function. This essentially considers the moving ship as an extra stationary ship by treating the line as a point whose distance from each grid point is the average distance between the grid point and the line. This distance is of course different for each grid point and you can not uniquely position this new point in the space. The mathematics of this approach is included in Appendix A as it could form part of future work.

Ultimately, this approach was not adopted as it does not take into account the fact that the objective function uses only the distance to the closest ship. Once the transiting ship is no longer the closest ship, it should not be used in the objective function. However, when using the average distance across the whole transit, even when the transiting ship is not the closest ship to a grid point, its distance will influence the objective function.

An approach to address this deficiency is to incorporate time into the model through the use of simulated annealing. The approach as defined in section 1.2 was modified so that after step 2:

2a) Increment the moving ship along its path in one unit lots and determine the objective function at each position.
2b) Average the objective function for that transit and use this value in the simulated annealing process.

Figure 6 shows the results of this process when the non-coalition ship moves through the centre of the space from position (0, 50) to (100, 50) – see Figure 4. The figure compares the scenario where there is no moving ship (blue) with those where the moving ship is incorporated into the scenario with the coalition ships prepositioned using the above process (green), and where the coalition ships are not prepositioned using this process (red).

It can be seen that the moving ship provides only marginal improvement when incorporated into the scenario, and as expected, it does not provide as much benefit as an additional ship. It can also be seen that the above prepositioning process provides little to no improvement over simply including a moving ship into the space without the coalition ships prepositioned for its transit – the only improvement is in the case of odd numbers of ships. The reason is that the moving ship spends so little time at each point along the line that there is little to no benefit for coalition ships in the area to shift positions to allow for the transit. Even if the moving ship provides some extra coverage of a section of the space at some time, for the majority of the transit the coalition ships will be the responding ships. One conclusion is that, using this measure of effectiveness, there is little to be gained in trying to reposition coalition ships to take advantage of a non-coalition ship that may move through the area. Modification of the measure of effectiveness will be discussed in Section 3.

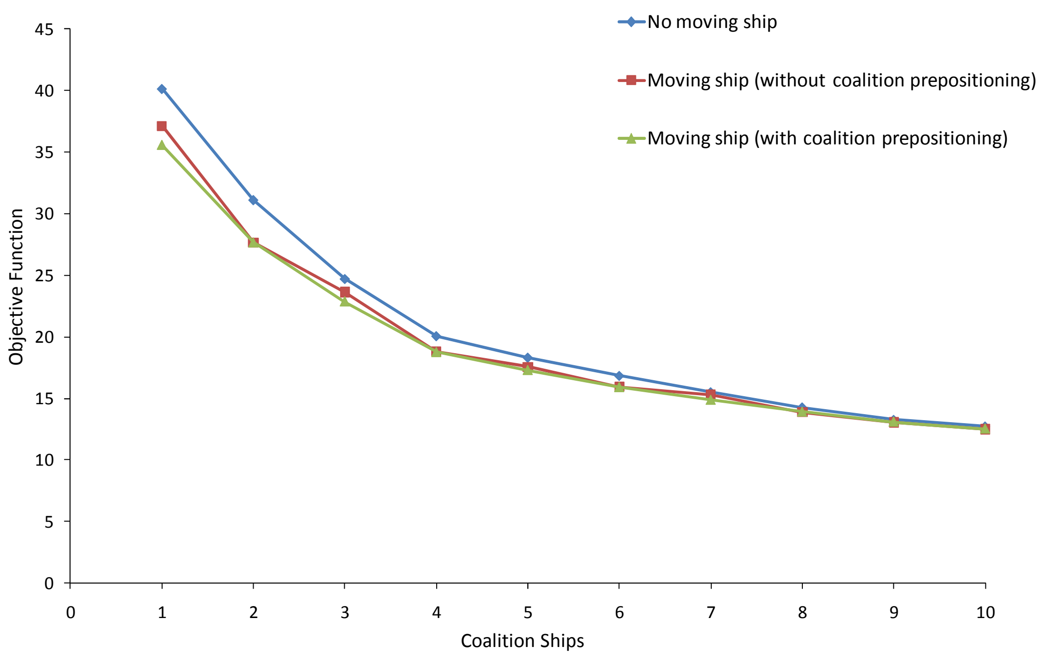

Figure 6: The objective function for no moving ship (blue), the scenario where the moving ship is incorporated with the coalition ships prepositioned using the above process (red), and where the coalition ships are not prepositioned using this process but simply to cover the space on their own (green).

2.2 Dynamic readjustment

An alternative approach is to allow coalition ships to adjust their position throughout the passage of the non-coalition ship, rather than fixing them in their pre-determined positions as in the previous sections. In this case, the simulation is time-stepped with the moving ship's position varied along a line between its start and end points. At each time step, the coalition ships use the simulated annealing process to find a new position optimised for that slightly different configuration of ships, however this new position must be physically possible to achieve – that is, the coalition ships can only search for new positions within a certain distance of their current position. In the following scenarios, it was assumed that all ships, coalition and non-coalition, were the same so if in one time step the non-coalition ship moved forward one unit, then in that same time step the coalition ships could only search for new positions within one unit of themselves.

Figure 7 shows the objective function as a position of the moving non-coalition ship when the moving ship travels along the centre line of a 100x100 grid – that is, from position (0, 50) to (100, 50) – and where two coalition ships have been preposition at the start of the model run at (50, 25) and (50, 75). The blue curve shows the objective function where the coalition ships are fixed to their starting positions, whilst the red curve shows the situation where the coalition ships are allowed to move at each time step.

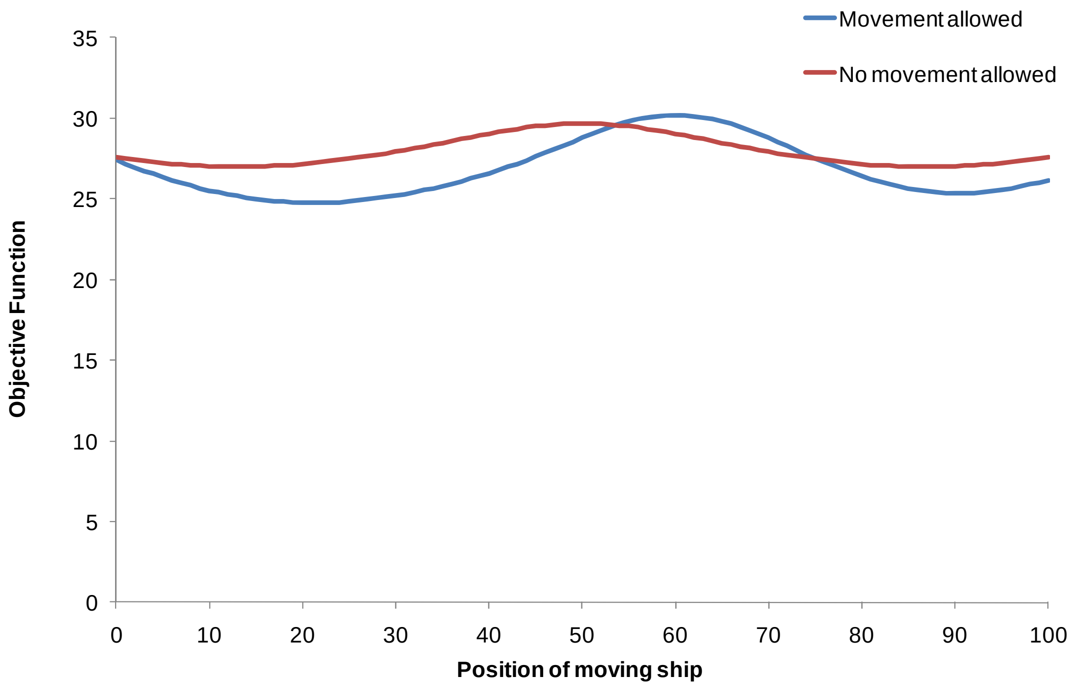

Figure 7: The objective function for two pre-positioned coalition ships and one moving non-coalition ship. The blue curve shows the situation where the coalition ships are fixed to their starting position whilst the red curve allows the coalition ships to move at each time step.

Whilst the red curve in Figure 7 has a smaller average objective function over the whole passage of the moving ship, the objective function is higher between x = 55 and 75. This is because the coalition ships are optimising their position at each time step, rather than over the whole timeline. Figure 8 shows an example of how optimising at each time step results in poor coalition ship positioning. The coalition ships are shown in red whilst the non-coalition ship is shown in blue. The coalition ships have moved to the right throughout the non-coalition ship's journey to this point, however the left of the space is now poorly covered.

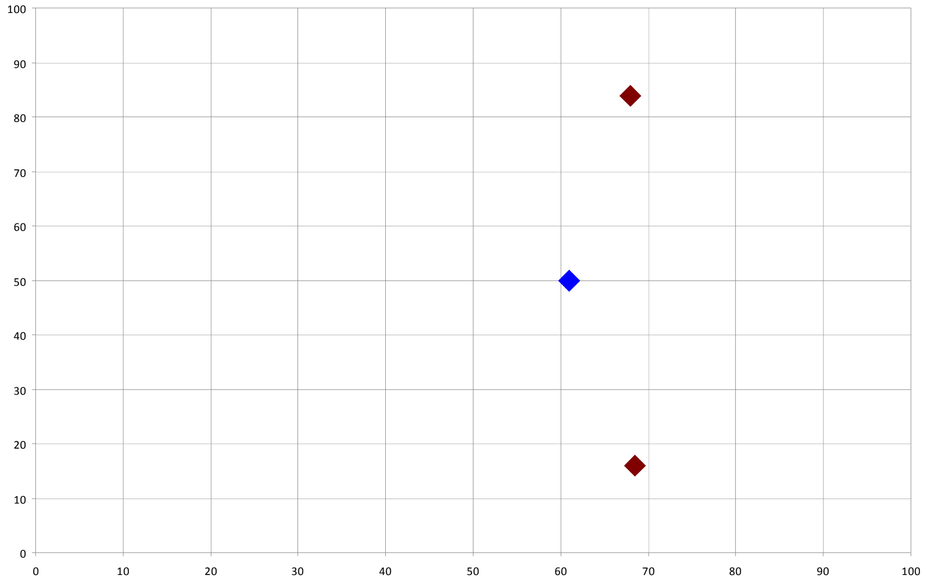

Figure 8: Poor positioning as a result of optimising at each time step. The moving non-coalition ship is shown in blue whilst the coalition ships are red.

This situation can also be analysed by looking at the percentage of the space to which the moving ship would respond – that is, the percentage of the space it is closest to at each time step. Figure 9 shows these results when the coalition ships are fixed to their predetermined positions and when they are allowed to move at each time step. Whilst the moving ship's coverage remains relatively constant when the coalition ships are fixed, when the coalition ships can adjust their positions, the moving ship covers more of the space and therefore would be required to respond to more piracy incidents. On average, when the ships are allowed to adjust, each ship covers a third of the space, whilst when the coalition ships are fixed, the moving ship only covers 23% of the space. It can also be seen that at some stages, when the coalition ships are allowed to move, the non-coalition ship would be required to respond to almost 50% of the space. It is unlikely that a coalition would plan their patrols in such a way that a non-coalition ship would be required to cover such an area.

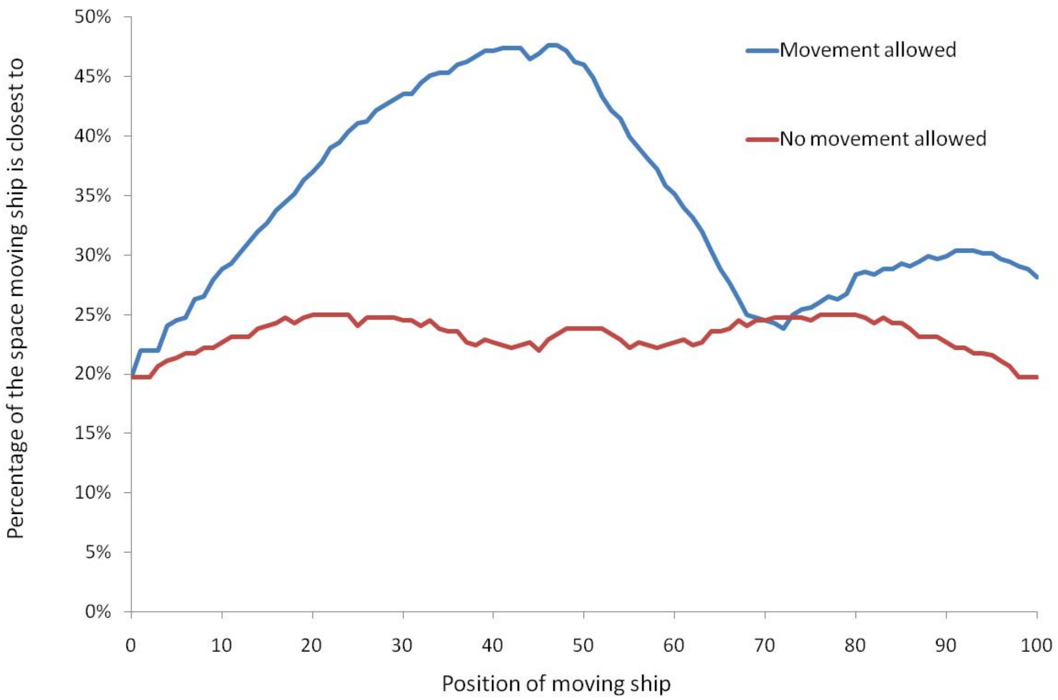

Figure 9: The percentage of the space covered by the moving ship for fixed and dynamic coalition positioning

It should also be noted that this method of time-stepped optimisation does not necessarily leave the coalition ships back at their starting positions. One method to do this would be to start the model run with the moving ship at x=-50 and end it at x=150. This puts the non-coalition ship far enough away that it is not the closest ship to any grid point and so does not affect the objective function – hence the coalition ships can adjust their position back to their starting position, assuming the starting position was optimal. As simulated annealing is a heuristic however, for large numbers of ships, various optimal starting positions may exist so even this method will not necessarily position the coalition ships back at their starting position at the completion of the model run.

This method does not always produce a smaller average objective function over the passage of the moving ship when we allow the coalition ships to adjust their position. Figure 10 shows the absolute difference between the average objective function over the trip of the moving ship for the situation where coalition ships are allowed to adjust their positions, and when they are not. It can be seen for 1, 8, 9 and 10 coalition ships, it is better that the coalition ships do no move at all. For this reason, and because it is unlikely naval assets would change their position so dynamically in a patrol operation without great benefit, it is unlikely that this approach would be adopted by a coalition force.

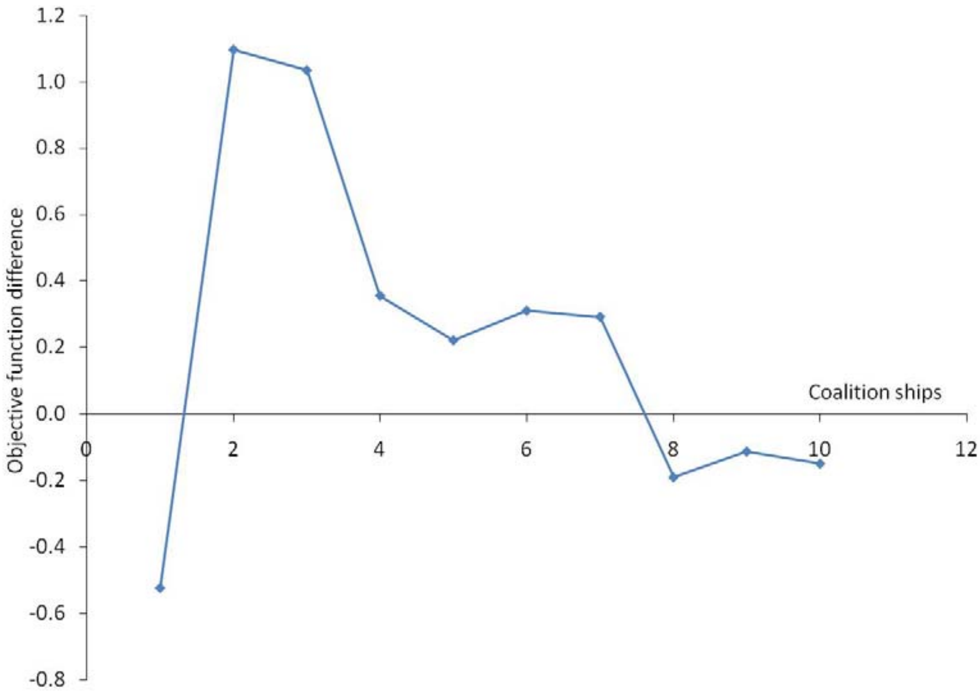

Figure 10: The difference for the average objective function over the trip of the moving ship between the situation where coalition ships are allowed to adjust their positions and when they are not.

2.3 Modelling the passage using risk multipliers

In this report it has been assumed that piracy events can occur at any grid point at any time, but another approach is to assume that the moving ship clears the area of piracy for some time after its passage. In this case, it would be more likely that coalition ships could find benefit in changing positions in advance of a non-coalition transit. One way to model this would be to use the risk scaling factor. The path of the moving ship could be designated as having low risk and therefore contribute less to the objective function. This also introduces physical dimensions into the scenario – that is, a moving ship will only clear the area within visual path of its transit.

The simplest way to model this is to not explicitly model time, but to simply model the path of the moving ship as being through a region of low risk. Figure 11 shows the results for numerous ship numbers when the swath through the middle of the space between y=40 and y=60 has risk equal to zero, with the rest of space having risk equal to one.

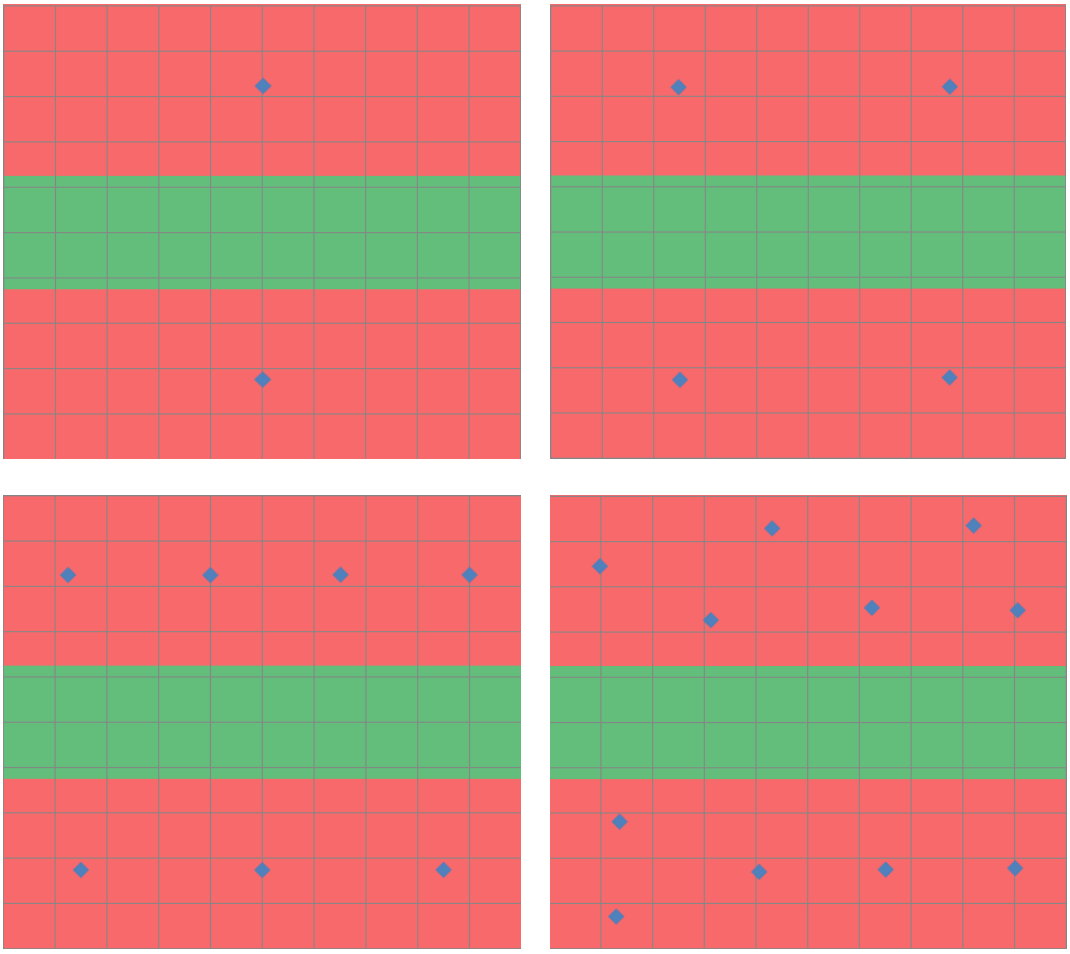

Figure 11: Configurations allowing for an area of low risk (green) which represents the passage of a transiting ship

One difficulty encountered with this method is that, even though the simulated annealing technique accepts less-than-optimal configurations as part of its process, ships can get stuck on one side of the passage and so optimal results are not always obtained. One method to overcome this is to use a higher starting temperature and it was found that using a starting temperature of 1 gave improved results without significantly increasing the run time.

To accurately represent the risk level in Figure 11, the charts can be transformed to reduce the risk level around the stationary ships to zero. This would then show how much of the space lies outside 10 units of each coalition ship and the passage of the non-coalition ship. Figure 12 shows this result with blue representing less than 10 units from the coalition ship or within the transit swath of the non-coalition vessel.

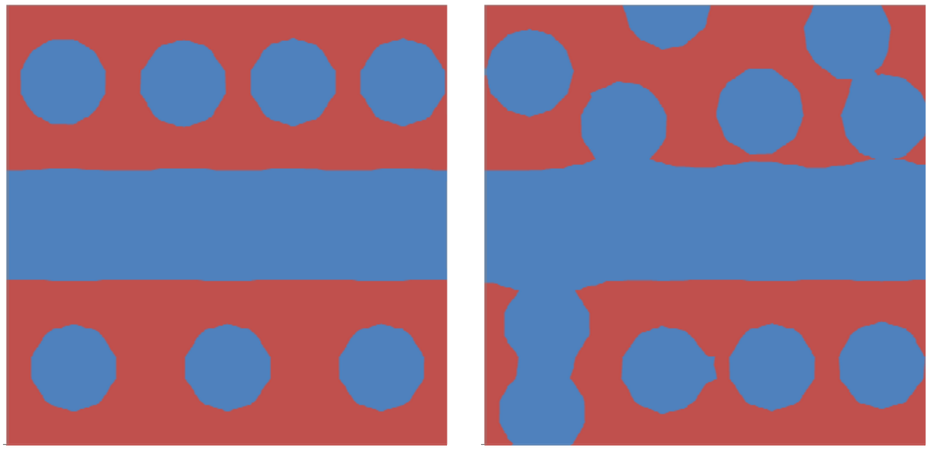

Figure 12: Risk map for 7 and 11 ships – blue shows area within 10 units a ship or transiting ship

3. Measures of effectiveness

3.1 Minimax MOE

Whilst the simplicity of the current MOE is attractive, it may not be the most operationally sound alternative. For instance, the space is dimensionless, and so the same results would be obtained for a 1km x 1km grid as a 100 km x 100 km grid. It is assumed that the closest ship will respond to an incident, but if that incident is over the radar-horizon, then it may not get noticed at all. Secondly, as the average minimum distance from all the grid points is used, overly large distances could be compensated by very small distances. Tactically however, a coalition of ships may wish to make sure that no point in the zone is more than, say, 30 minutes away by helicopter. Therefore, an MOE minimising the maximum distance between the grid points and their closest ship may seem like an appropriate modification to the model (a minimax MOE). There is more variability in the ship positioning under the minimax MOE as the maximum distance for the whole space is determined by a single grid point / ship pair and therefore ships not a member of this pair can move without changing the objective function – that is, there is no unique solution. Grid points towards the edge and corners of the space are the most likely to be contributing to this maximum, and therefore positioning of the ships near the centre of the space is variable. For this reason, a non-coalition moving ship moving through the centre of the space makes no difference to the coverage using this MOE.

Figure 13 compares the results obtained for the two MOEs. The average value refers to the average distance from each grid point to its closest ship, whilst the maximum value refers to the maximum distance between a grid point and its closest ship in the space. It should be noted that as the minimax does not find a unique solution, its average value is variable. Despite this, the minimax MOE gives very similar results for the average distance and an improvement in the maximum distance.

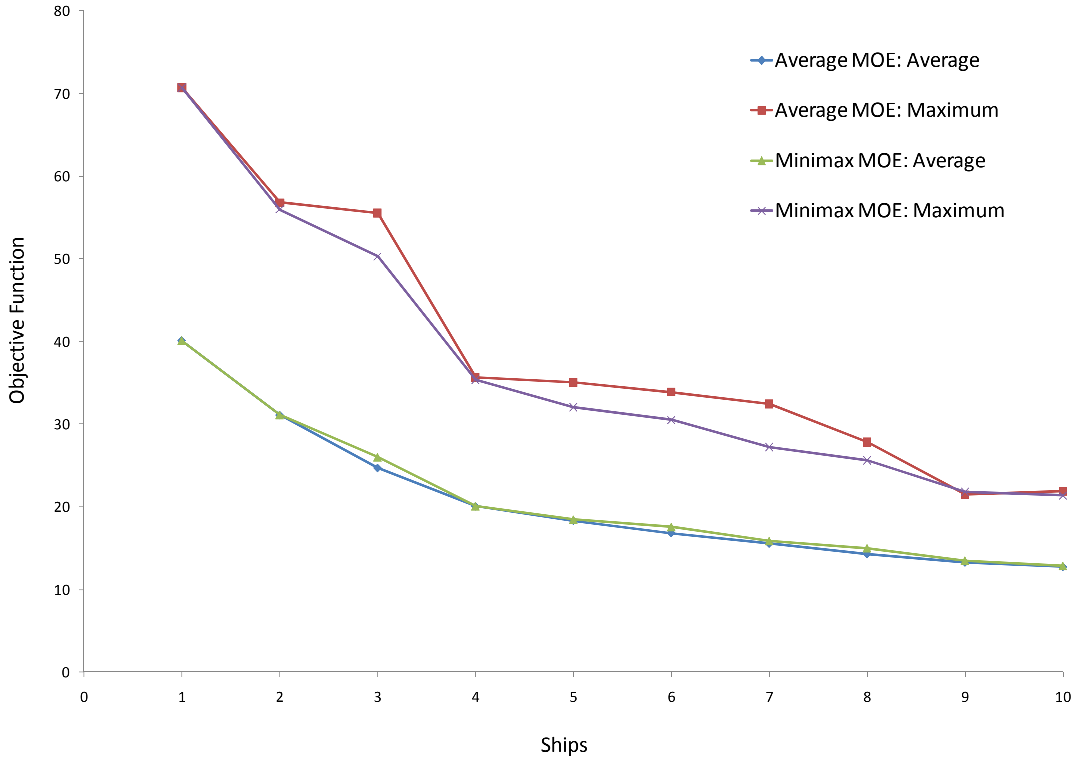

Figure 13: The average and maximum results for the two MOEs

3.2 Exponential probability MOE

The average and minimax MOEs are less than satisfactory in some cases where risk is involved. For example, when there are two regions of high risk separated by a region of low risk with only one patrol vessel available, both MOEs position the lone ship between the two high risk regions. In reality, such a positioning may mean that the ship is too far from both regions of piracy to be effective.

An example of this problem is shown in Figure 14. Three ships have been positioned using the average MOE, with red areas having a risk value of one and dark green areas a risk value of zero. The figure on the right shows the areas where the risk-scaled distance from the closest ship is less than 10. It is evident that the three ships do a poor job of covering the regions of high risk and are completely covering the regions of low risk. This is because there are fewer ships than high risk areas and therefore one ship gets stationed between these areas, effectively covering neither of them.

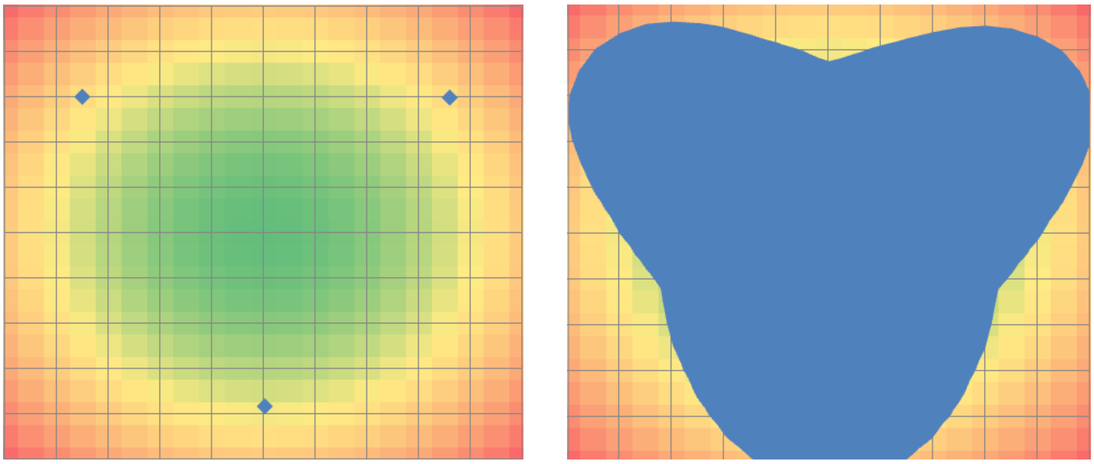

Figure 14: Average MOE positioning of 3 ships under risk. The blue area on the right shows the positions where the risk-scaled distance is less than 10.

An alternate MOE is one that models the distance between a grid point and its closest ship as a probability that that ship will react in time to stop the piracy incident. This can be modelled as an exponential function, with a large distance creating a small probability of preventing attack. The MOE would then maximise the overall probability of preventing piracy attack – or in our modelling framework, minimising the negative of this function. This value can be then scaled by risk.

Using the risk map from Figure 14, the following results are obtained for the exponential MOE, where the MOE is defined as:

$$MOE = -\sum_{grid} r_{grid} e^{-d_{grid}/\lambda} ,$$

where d_{grid} is the distance between each grid point and its closest ship, and r_{grid} is the risk at each grid point.

A value of 10 has been chosen for λ. This means at a distance of 10 units, there is a 37% chance of preventing the piracy event. Figure 15 shows that under this MOE, three of the four regions of highest risk are now within 10 risk-scaled units of their closest ship.

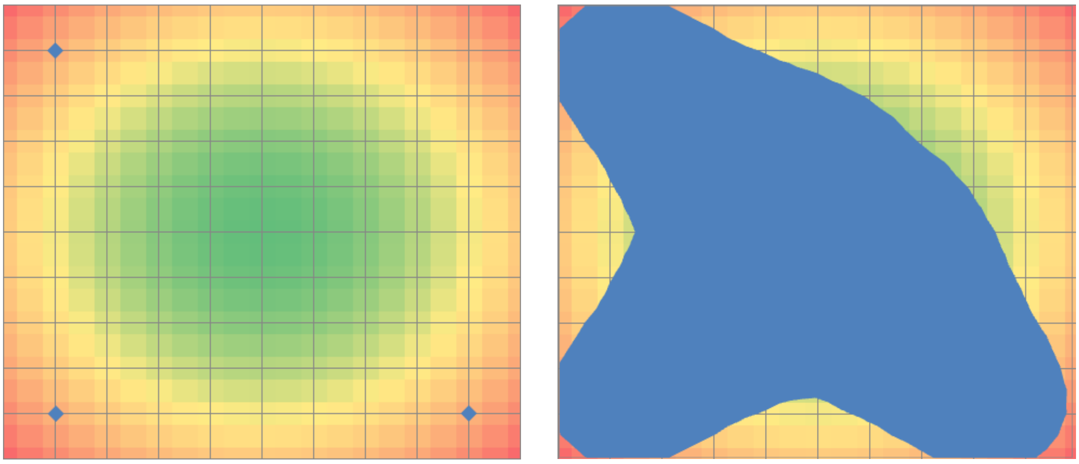

Figure 15: The exponential MOE places the ships in the highest risk regions, rather than between them

Figure 16 compares the exponential MOE with the Average and Minimax MOEs for a scenario with no risk. It produces an equal or higher maximum distance for each number of ships, however the average distance almost exactly matches the Average MOE. Figure 17 shows the probability of piracy attack for each of the MOEs. The exponential MOE provides no additional benefit over the Average MOE when there is no risk and only a slight improvement on the Minimax MOE.

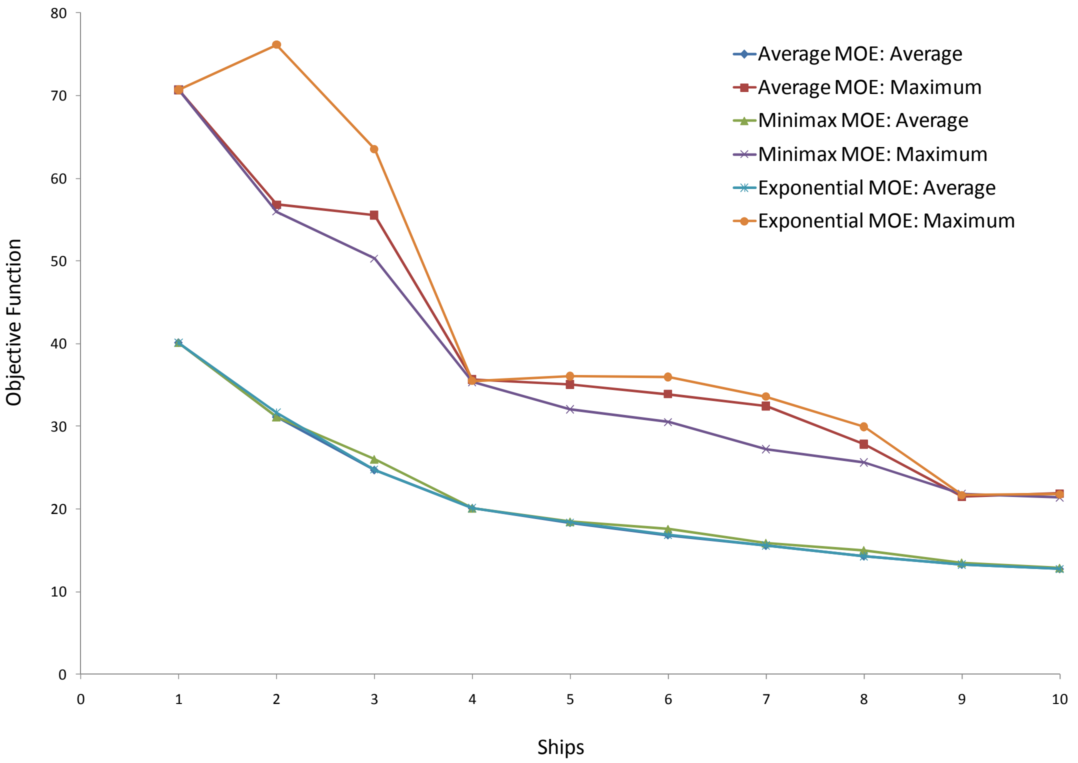

Figure 16: The average and maximum results for the three MOEs

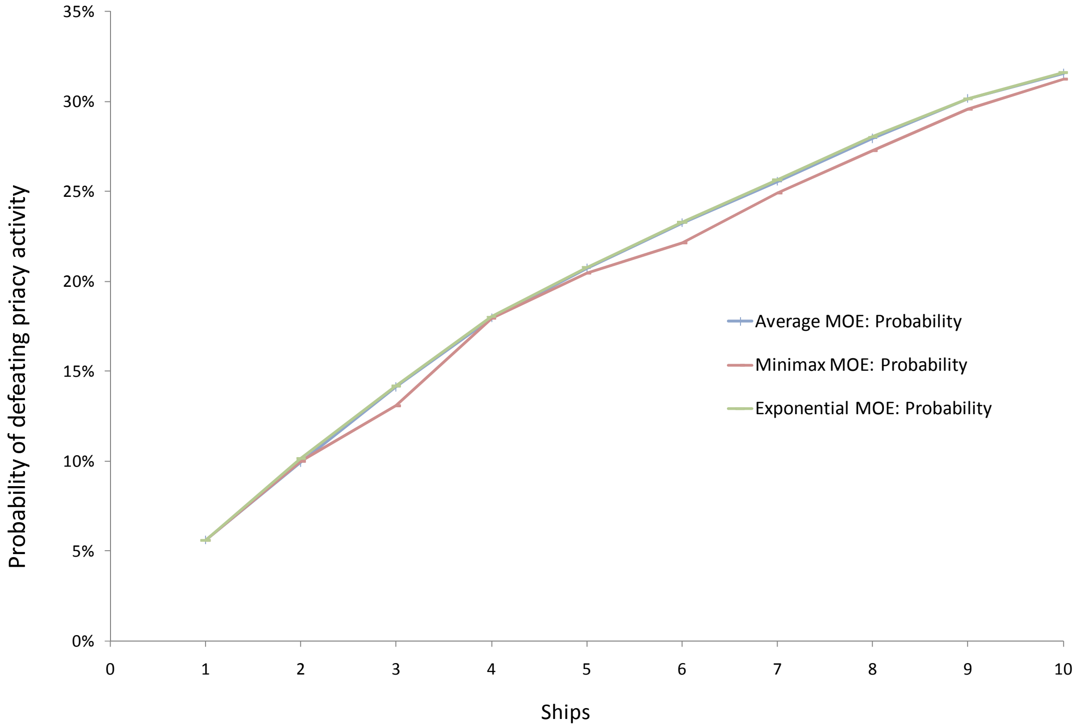

Figure 17: The probability of piracy attack for the three MOEs

Given that the Exponential MOE gives almost the exact same results as the Average MOE in scenarios with no risk, and produces more intuitive results under risk, it seems to be a good choice of MOE for ship placement.

4. Discussion and Conclusions

The approaches detailed in this report are useful for positioning a coalition of ships to cover a patrol area. The incorporation of a moving asset providing some extra coverage is novel and our approach suggests that the non-coalition ship provides little extra coverage if the coalition ships are already well positioned to cover the region. Allowing the coalition ships to reposition themselves after the non-coalition ship has entered the space, as in section 2.2, gives not only unpredictable results but would be incredibly difficult to incorporate into a coalition tactic. The MOEs trialled in this report perform similarly in situations where there is no risk, however the Exponential MOE performs the best of the MOEs under risk.

4.1 Risk and measures of effectiveness

Including risk on a discretised grid is not an easy problem. Risk can be assumed to be the probability of attack at each grid point. Consider the scenario where there are two regions, one small (a single grid point), and one large (multiple grid points), where both have the same high chance of piracy attack somewhere within them. The single grid point will be given a high risk value, however the risk of piracy attack needs to be spread over the larger area as no particular grid point is more vulnerable than the next. Hence each point will have a low risk value. Using the exponential MOE, the single grid point with a high risk value will be a strong attractor and move ships close to it. Under the average MOE, depending on geography, both areas may balance. It is not clear in this situation what the appropriate MOE should be, especially in the case where a ship could physically cover the larger area, but not both areas.

Risk is included in the current model as a scaling factor in the objective function, however accurate data on risk is needed. This could be driven by shipping density data or known shipping lanes [5]. A dense area of shipping could be considered a region of high risk, or conversely, safety-in-numbers might suggest it is an area of low risk. Isolated shipping routes of low density may also be at risk. Data from where piracy attacks have previously occurred could also be used. Similar data could be used to back the exponential MOE.

Another issue is that the risk of piracy attack will decrease when a patrolling vessel is nearby. Ideally, the risk function at each point should change depending on how far away the nearest patrolling vessel is. This could perhaps be introduced as a further multiplying factor in the objective function. The use of piracy decoys to draw patrolling vessels away from an area further complicates matters. [23]

4.2 Modelling the moving ship

The work on modelling the transit of a non-coalition ship as a line is novel as similar work has not been found in the literature. If the MOE were modified such that ships could only respond to incidents within a certain distance of them leaving much of an area uncovered, the coverage provided by a transiting ship could be quite significant. In this case, any intermittent coverage provided by a transiting ship would be better than none at all. Modelling the transit as a line could uncover broad areas where the transit provides some coverage, particularly in large areas with few or no coalition assets. This could be true for large at-risk areas such as the Indian Ocean.

4.3 Geography

Another area for modification of the current model is that is only works for simply connected shapes – it would not work if there was a peninsula in the patrol area as whilst the distance between a ship on one side of a peninsula and a grid point on the other may be small, the time taken for the ship to transit around the peninsula may be large. It's possible through use of a grid to define the distances between grid points such that these distances take into account obstacles in the way. This would need to be done before the model run, in which case the current MOE could be used. It should be noted that while in the current model piracy events occur at grid points, the ships themselves can be placed anywhere. If obstacles were introduced, it would be easier to constrain both the ships and piracy events to grid points to reduce the amount of pre-processing of distances around obstacles.

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Appendix A: The average distance from a point to a line

This section details the mathematics of determining the average distance from a point to a line. This could be used when looking at the path of a moving ship.

Figure 18 shows a schematic of the problem, where the ship travels along the line from (0, 0) to (A, 0), where point (p, q) represents a possible piracy attack.

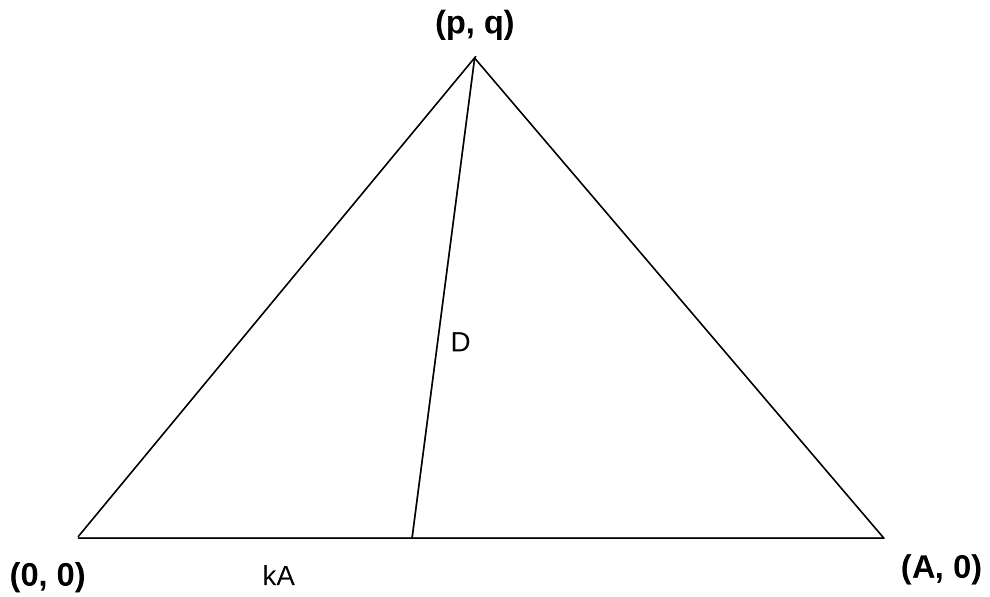

Figure 18: Measuring the distance from point (p, q) to line joining (0, 0) and (A, 0)

For a ship travelling in a straight line between points (0, 0) and (A, 0), the distance between any point along to line (kA, 0) and point (p, q) is:

$$
\begin{aligned}
D(k) &= \sqrt{(p-kA)^2+q^2} \\
&= \sqrt{kA^2-2pkA+p^2+q^2}
\end{aligned}
$$

To find the average distance between the line and point (p, q), we perform the following integration:

$$
\begin{aligned}
D_{average} &= \int_{k=0}^{1} D(k)dk \\
&= \int_{0}^{1} \sqrt{kA^2-2pkA+p^2+q^2}\,dk
\end{aligned}
$$

From a table of integrals [24]:

$$\int \sqrt{ax^2+bx+c}\,dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| + C$$

for a > 0.

In our case:

$$a = A^2$$
$$b = -2pA$$
$$c = p^2+q^2$$

Therefore:

$$\int_{k=0}^{1} D dk = \left[\frac{-p+Ak}{2A}\sqrt{(Ak-p)^2+q^2} + \frac{q^2}{2A}\ln\left|2A\left((Ax-p)+\sqrt{(Ak-p)^2+q^2}\right)\right|\right]_0^1$$
$$= \frac{1}{2A}\left(p\sqrt{p^2+q^2} + (A-p)\sqrt{(A-p)^2+q^2} + q^2\ln\left|\left(\frac{(A-p)+\sqrt{(A-p)^2+q^2}}{-p+\sqrt{p^2+q^2}}\right)\right|\right)$$

The generic triangle which appears in our problem requires transformation to look like Figure 18:

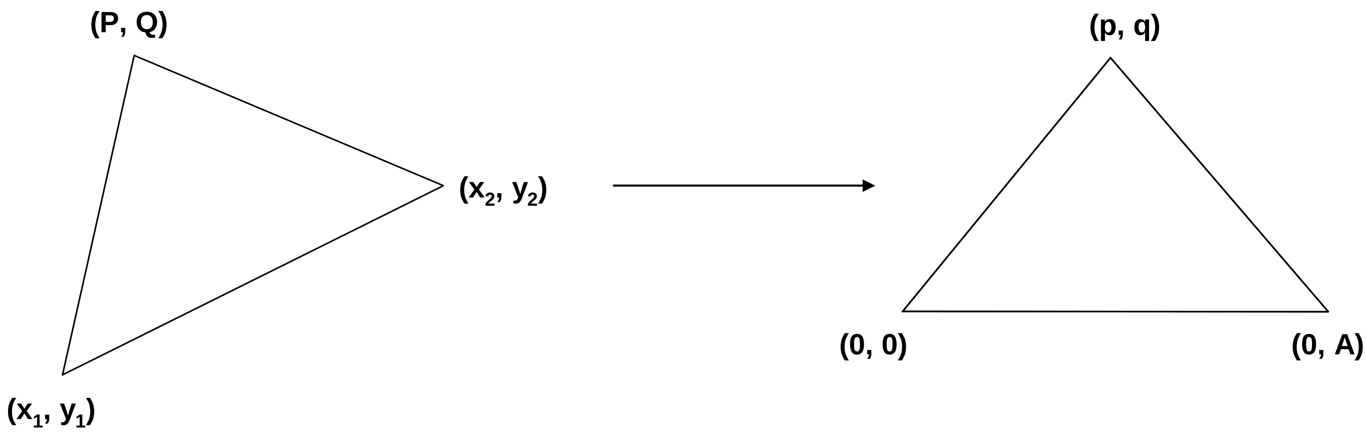

Figure 19: Triangle transformation

This transformation is shown below:

$$(x_1, y_1) \to (0,0)$$
$$(x_2, y_2) \to \left(\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}, 0\right) \to (A,0)$$
$$(P,Q) \to \left(\frac{(P-x_1)(x_2-x_1)-(Q-y_1)(y_2-y_1)}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}, \frac{(P-x_1)(y_2-y_1)+(Q-y_1)(x_2-x_1)}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}\right) \to (p,q)$$

This substitution is simple in mathematical software and not shown here.

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Simulated Annealing, Operations Analysis, Piracy, Mathematics, Measures of Effectiveness

19. ABSTRACT
Piracy in the Gulf of Aden and the waters around Somalia has increased in recent years, with international naval assets allocated to patrol at-risk areas. This paper compares measures of area coverage in situations where there is a uniform piracy risk and where some areas are more vulnerable than others. Simulated annealing was used to allocate the patrolling naval assets. The novel problem of positioning a coalition of ships whilst incorporating the transit of a non-coalition ship that may respond to a piracy incident but can not be prepositioned is addressed.