Ship Response Capability Models for Counter-Piracy Patrols in the Gulf of Aden

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Abstract

This work examines the capability-models of ships performing counter-piracy patrols in the International Recommended Transit Corridor, located in Gulf of Aden (GOA). Specifically, it considers possible approaches to predict the response time of assets patrolling long and thin regions and to facilitate coordination between multiple assets.

For situations where the pirate attacks occur randomly, the across-channel location of the ship prior to the attack has only a limited impact on the response-time probability distribution, supporting the notion that the problem can be examined in a one-dimensional (1D) context. It is demonstrated that the 1D approach is extremely effective at reproducing the cumulative distributions of higher fidelity models. The 1D approach is then used to demonstrate that coordinating patrol ship positions is a key factor, while coordinating the rotation of helicopter crews between ships may be less important. It is also shown that for the types of wind fields in which pirates will operate in the GOA, a ship can reposition itself in a manner such that the winds will not heavily affect response capabilities.

Finally, a description of how the results from this work can be applied in the development of an asset positioning model are presented, and a description of the way forward is provided.

Résumé

Ce travail porte sur les modèles de capacité des navires qui effectuent des patrouilles de lutte contre la piraterie dans le couloir de transit international recommandé (IRTC), situé dans le golfe d'Aden. Plus précisément, il examine les façons possibles de prévoir le délai d'intervention des ressources qui patrouillent dans des régions longues et étroites et de faciliter la coordination entre de multiples ressources. Dans les cas où les attaques de pirates ont lieu au hasard, l'emplacement du navire de l'autre côté du chenal avant l'attaque n'a qu'une incidence limitée sur la distribution probabiliste du délai d'intervention, ce qui corrobore l'idée selon laquelle le problème peut être examiné dans un contexte unidimensionnel (1D). Il est démontré que l'approche 1D est extrêmement efficace pour reproduire les distributions cumulatives de modèles de plus grande fidélité. Cette approche est ensuite utilisée pour démontrer que la coordination des positions des navires de patrouille constitue un facteur clé, tandis que la coordination de la rotation des équipages d'hélicoptère entre les navires peut s'avérer moins importante. Il est également établi que pour les types de champs de vent dans lesquels les pirates opéreront dans le golfe d'Aden, un navire peut se repositionner de manière à ce que les vents ne nuisent pas beaucoup aux capacités d'intervention. Enfin, le document renferme une description de la façon dont les résultats de ce travail peuvent être appliqués à l'élaboration d'un modèle de positionnement des ressources, ainsi qu'une description de la voie à suivre.

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Executive summary

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Ramzi Mirshak; DRDC CORA TM 2011-139; Defence R&D Canada – CORA; September 2011.

Background: The Gulf of Aden is an approximately 450 nm-long section of water connecting the Gulf of Oman to the Arabian Sea, and is bordered by Yemen to the north and Somalia to the south. It is a major commercial shipping corridor, with 12% of the world's daily oil supply traveling through the region. In recent years, Somali-based piracy has flourished, posing a threat to international trade and freedom of the seas. In 2007, the International Maritime Bureau (IMB) reported 43 incidents of piracy around near Somalia, increasing to 111 in 2008. By the end of September 2009, over 300 events had taken place in that year so far. To help control piracy in the area, coalition forces responded to the area. To improve the effectiveness of patrol coverage, the 12 nm-wide International Recommended Transit Corridor (IRTC) was introduced, with the purpose of focusing traffic into an easier to manage and better defined region. However, few planning tools are presently available to assist analysts and planners when it comes to allocating ships to patrol this region in an optimal fashion.

Principal Results: The work presented here identifies the probability that an arbitrary ship can respond to an act of piracy at some random location within its area of responsibility, or patrol area. Four patrol strategies are presented for a two-dimensional (2D) model and a one-dimensional (1D) simplification of that model. Both the 2D and 1D models can produce the probability of a ship responding to an incident of piracy in a specific time frame, using the ship profile, the size of the patrol region, and the patrol strategy as inputs. The 1D model is computationally simpler than its 2D counterpart, however, which makes it more amenable for considering more complex multiple-ship problems.

It is shown that ships that remain at the centre of their patrol region will consistently outperform those that travel along its length, and that the effects of moving across its (narrow) width can be accounted for while essentially restricting the problem to a 1D-nature. Simple two-ship and three-ship problems are presented to examine how ships with various patrol strategies (representing various degrees of cooperation with coalition command) can be coordinated. It is shown that coordinated efforts that optimize the spacing between ships will consistently out-perform uncoordinated efforts. A simple model also demonstrates that a ship can position itself within a patrol area to accommodate the effects of wind on helicopter range in a manner that permits wind to be neglected in initial calculation of ship positions. These findings lead to the following recommendations:

a. Patrol Areas should be defined in a manner that considers the response capabilities of the ship. For example, a ship with a fast helicopter may be able to cover a larger area effectively than a ship with a slower helicopter or no helicopter.
b. Ships should remain as close to the centre of their Patrol Area as possible to maximize response effectiveness. This strategy will minimize the number of ships required to patrol the entire corridor.
c. In cases where the operations of one ship do not permit it to remain at the centre of its patrol box, adjacent ships that alter their position and Patrol Area in a dynamic fashion based on the real-time position of the other ship can improve patrol coverage effectiveness.
d. In windy conditions, ships should reposition themselves upwind by a distance that is based on helicopter speed, wind speed, and time delay for helicopter launch.

Significance of Results: The modeling presented here lays forth several recommendations for allocating assets to patrol a long and thin region such as the IRTC.

It is worth noting that, while being used to address problems related to counter-piracy operations, the results presented are not restricted to that domain. They can be applied to any situation where multiple assets with individual capabilities are patrolling or searching a long and thin region.

Future Work: At present, executing the recommendations listed above is a challenge as there is no tool that is readily available to analysts and officers setting the patrol schedules. The next step in development will be to synthesize the findings presented in this report to produce an operationally useful asset position and allocation model. The model should include the ability to consider multiple ships with individual capabilities that are covering a region where the object of interest (spatiotemporal index of the risk of a pirate attack, in this case) varies in space and time. The models discussed in this report consider response capabilities exclusively, while ignoring reconnaissance requirements. Future efforts that also considered such requirements would also be beneficial.

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Ship Response Capability Models for Counter-Piracy Patrols in the Gulf of Aden

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Contexte : Le golfe d'Aden est une étendue d'eau d'environ 450 NM de longueur qui relie le golfe d'Oman à la mer d'Arabie et qui est bordée au nord par le Yémen et au sud par la Somalie. Il s'agit d'un important couloir de navigation commerciale, et 12% des approvisionnements mondiaux en pétrole passent par cette région tous les jours. Ces dernières années, les actes commis par des pirates basés en Somalie se sont multipliés, posant ainsi une menace pour le commerce international et la liberté des mers. En 2007, le Bureau maritime international (BMI) a signalé 43 incidents de piraterie près de la Somalie, et ce nombre est passé à 111 en 2008. À la fin de septembre 2009, plus de 300 incidents avaient eu lieu cette année-là. Des forces coalisées sont intervenues pour aider à enrayer la piraterie dans la région. Afin d'accroître l'efficacité de la couverture du patrouille, on a créé le couloir de transit international recommandé (IRTC), d'une largeur de 12 NM, en vue de concentrer la circulation dans une région mieux définie et plus facile à gérer. Toutefois, il existe actuellement peu d'outils de planification pour aider les analystes et les planificateurs lorsqu'il s'agit d'affecter des navires pour patrouiller dans cette région de manière optimale.

Résultats principaux : Le présent document indique la probabilité qu'un navire arbitraire puisse réagir à un acte de piraterie commis quelque part au hasard dans son secteur de responsabilité ou de patrouille. Quatre stratégies de patrouille sont présentées à l'égard d'un modèle bidimensionnel (2D) et d'une simplification unidimensionnelle (1D) de ce modèle. En utilisant comme intrants le profil du navire, la superficie de la région de patrouille et la stratégie de patrouille, les modèles 2D et 1D peuvent tous deux produire la probabilité qu'un navire réagisse à un incident de piraterie dans un délai précis. Le modèle 1D est cependant plus simple que le modèle 2D sur le plan du calcul, ce qui facilite son utilisation dans le cas de problèmes plus complexes mettant en cause plusieurs navires. Il est démontré que les navires qui restent au centre de leur région de patrouille auront toujours de meilleurs résultats que ceux qui se déplacent le long du bord du secteur, et que les effets de la traversée du chenal (étroit) peuvent être justifiés tout en limitant essentiellement le problème à un contexte unidimensionnel. Des problèmes simples à deux navires et à trois navires sont présentés afin d'examiner comment l'on peut coordonner des navires ayant diverses stratégies de patrouille (qui représentent divers degrés de coopération avec le commandement de la coalition). Il est établi que des efforts coordonnés qui optimisent l'espacement entre

les navires donneront toujours de meilleurs résultats que des efforts non coordonnés. Un modèle simple démontre également qu'un navire peut se positionner dans un secteur de patrouille afin de tenir compte des effets du vent sur le rayon d'action de l'hélicoptère, ce qui permet de négliger le vent dans le calcul initial de la position des navires. Ces constatations donnent lieu aux recommandations suivantes :

a. Il faudrait définir les secteurs de patrouille de façon à prendre en considération les capacités d'intervention du navire. Par exemple, un navire doté d'un hélicoptère rapide peut être capable de couvrir efficacement un secteur plus vaste qu'un navire dont l'hélicoptère est moins rapide ou qui ne possède pas d'hélicoptère.

b. Les navires devraient rester le plus près possible du centre de leur secteur de patrouille afin de maximiser l'efficacité de l'intervention. Cette stratégie réduira au minimum le nombre de navires qui doivent patrouiller dans tout le couloir.

c. Dans les cas oû les opérations d'un navire ne lui permettent pas de rester au centre de sa zone de patrouille, l'efficacité de la couverture peut être améliorée si des navires adjacents modifient leur position et leur secteur de patrouille de façon dynamique en fonction de la position en temps réel de l'autre navire.

d. Par temps venteux, les navires devraient se repositionner au vent selon une distance fondée sur la vitesse de l'hélicoptère, la vitesse du vent et le délai de décollage de l'hélicoptère.

Portée des résultats : La modélisation présentée ici permet de formuler plusieurs recommandations concernant l'affectation des ressources nécessaires pour patrouiller dans une région longue et étroite comme l'IRTC. Il convient de noter que, même s'ils servent à traiter des problèmes liés aux opérations de lutte contre la piraterie, les résultats présentés ne sont pas limités à ce domaine. En effet, ils peuvent être appliqués à toute situation oû de multiples ressources aux capacités individuelles effectuent des patrouilles ou des recherches dans une région longue et étroite.

Recherches futures : À l'heure actuelle, la mise en œuvre des recommandations susmentionnées pose des difficultés, car aucun outil n'est mis à la disposition des analystes et des officiers qui établissent les calendriers de patrouille. La prochaine étape dans le développement consistera à faire la synthèse des constatations formulées dans le présent rapport afin de produire un modèle d'affectation et de positionnement des ressources qui soit utile sur le plan opérationnel. Ce modèle devrait inclure la capacité de considérer plusieurs navires aux capacités individuelles qui couvrent une région oû l'objet d'intérêt (dans le cas présent, l'indice spatio-temporel du risque d'attaque de pirates) varie dans l'espace et dans le temps. Les modèles dont il est question dans ce rapport considèrent exclusivement les capacités d'intervention, sans tenir compte des exigences en matière de reconnaissance. Il serait également avantageux de consacrer de futurs travaux à l'étude de ces exigences.

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1 Introduction

1. In 2009, the Maritime Systems Group (MAR) of The Technical Cooperation Program (TTCP) granted a 1-year extension to Action Group 10 of MAR (TTCP MAR AG-10), expanding its mandate to consider counter-piracy (CP) operations. Motivation for the extension was based on the increasing pirate activity in the Gulf of Aden (GOA) and Somali Basin. In a given year, over 20,000 commercial vessels transit the waters targeted by pirates off Somalia, with captured ships often fetching ransoms of several million US dollars as the endangered crews are held hostage.

2. To counter the problem, the international naval community has put forth considerable effort to reduce and control maritime piracy in the region. This effort has included the creation of several international coalitions or task forces (TFs). These TFs work together in scheduling patrols and allocating assets [1] in a coordinated effort. However, there are other vessels performing counter-piracy operations that are not a part of any coalition and may not cooperate closely with them. This provides a challenge of allocating assets in a manner that minimizes the duplication of effort between taskings from the above-mentioned task forces, and the less coordinated ships of other nations.

3. One tool provided to planners at Combined Maritime Forces Headquarters (CMF HQ) was the GOA Anti-Piracy Planning Tool (GAPP, see Figure 1) [1]. In its earlier versions, GAPP used sets of Monte Carlo simulations to determine the time required for a ship with a helicopter to respond to an act of piracy within a rectangular patrol area like those assigned to ships in the GOA. In its most recent iteration, GAPP treats the problem analytically [2]. The tool illustrates how ship speed, helicopter speed, and helicopter launch delay affect likely response times for a single ship. A primary reason for developing this tool was that CMF HQ set the objective of TF ships patrolling shipping lanes within the GOA to be able to respond to pirate incidents within 30 minutes of a merchant vessel broadcasting that it was under attack [3]. Existing estimates did not include probabilistic effects, something that GAPP provides. However, GAPP is a simple and limited tool that is not amenable to examining the use of multiple ships to collectively cover a region of interest.

4. This report presents analyses that follow-up on those provided by GAPP, discussing how patrol strategies in the GOA can be managed to improve the probability of responding to a pirate attack within a specified time frame. It presents several probability modeling approaches that can be used to measure response capabilities, then uses the models to examine the likelihood that assets with various capabilities and patrol strategies, working either together or independently, will respond to events within a particular time period. It also discusses the way forward for developing a multi-ship asset allocation tool, part of the ongoing development under the auspices of TTCP MAR AG-14.

1. The term “asset” or “Task Force asset” describes a naval ship that may or may not have a helicopter.

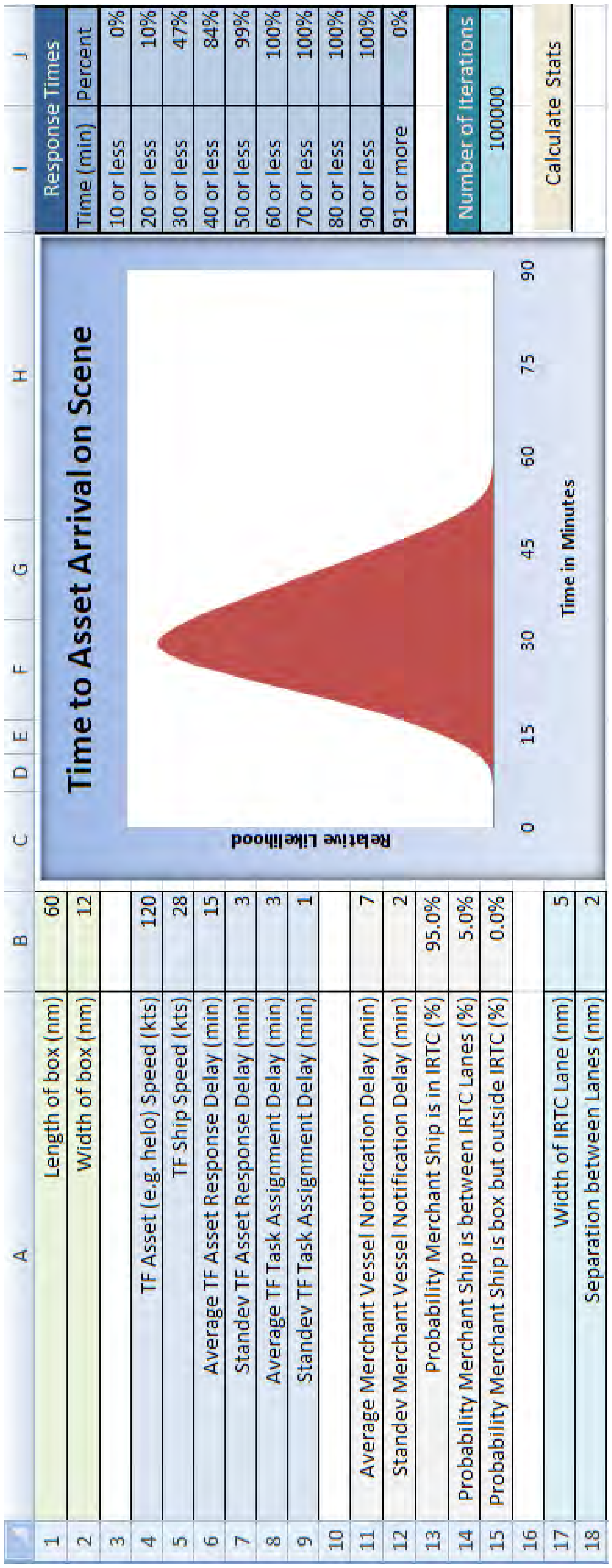

Figure 1: *Screenshot of GAPP version 2.2.*

1.1 Background: Shipping, Piracy and Counter-Piracy in the Gulf of Aden

5. Bordered by Yemen to the north and Somalia to the south, the GOA is an approximately 450 nm-long body of water that connects the Gulf of Oman to the Arabian Sea (Figure 2). It is a major commercial corridor for ships wishing to use the Suez Canal to travel from the Indian Ocean to the Mediterranean Sea (or vice-versa). Estimates of the number of ships transiting the GOA are as high as 20,000 per year, with the cargo including 12% of the world's daily oil supply [4]. In addition to providing a challenge for commercial shipping and international trade, the acts of piracy in the GOA threaten the delivery of humanitarian aid and ransom payments are thought to be financing regional conflict and possibly terrorism [5].

6. Somali-based piracy has flourished in recent years. In 2007, the International Maritime Bureau (IMB) reported 43 incidents of piracy in the waters near Somalia, increasing to 111 in 2008 [6]. By the end of September 2009, over 300 events had taken place in that year [7]. Industry has put forth considerable effort to mitigate the piracy problems. Some larger companies such as Maersk have altered some shipping routes, choosing to avoid the GOA and take the long route around Africa (at a cost of up to $1 Million per transit) [4]. In addition, the IMB has consulted with major international commercial partners to develop a Best Management Practices guide [8] to give ship Masters advice and information on how to avoid being pirated.

7. Military partners have also put forth considerable effort, creating several joint task forces with counter-piracy mandates. The North Atlantic Treaty Organization (NATO) has commissioned several operations, the European Union (EU) created EU NAVFOR, and the Combined Maritime Forces (CMF) created Combined Task Force (CTF) 151. These commands work in a coordinated way to manage counter-piracy operations off Somalia, but other navies not affiliated with these groups also deploy ships to the region.

8. Findings from operational analysts deployed to CMF headquarters have demonstrated that if naval forces are able to arrive on the scene of an attempted act of piracy before the pirates have been able to board the ship, then the pirates will abort their attack and attempt to dispose of any weapons or other pirate-related paraphernalia before they are captured [3]. This practice makes the objective of deterring pirates a question of arriving on scene as quickly as possible, rather than necessarily capturing suspected pirates. Experience suggests that when a commercial vessel comes under attack by pirates and sends out a call for assistance, naval forces have at most thirty minutes to respond [3], although it may be considerably less. The naval response usually involves a ship transiting towards the site of an attack and launching its helicopter (if available) to interdict more quickly.

9. The GOA is a vast region, making it difficult to perform effective CP patrols over its entirety. To make patrols more manageable, the International Community sanctioned

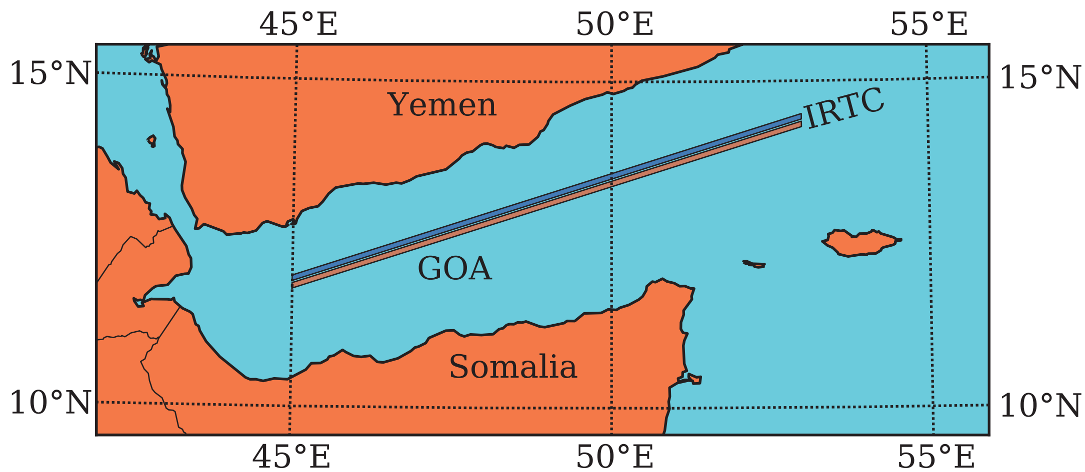

Figure 2: *Map of Gulf of Aden (GOA) with International Recommended Transit Corridor (IRTC).*

the International Recommended Transit Corridor, or IRTC (Figure 2). The IRTC is 12 nm across and about 480 nm long, consisting of two 5 nm-wide shipping lanes (one for each direction of travel) that are separated by a 2 nm buffer zone. By concentrating merchant traffic in the IRTC, the area that needs to be patrolled by TF assets is reduced considerably. However, this concentration of traffic also creates a "honey-pot" effect for would-be pirates if patrols are not effective. In addition, with piracy increasing in the Somali Basin, there is an additional pressure to use fewer ships in this enclosed region so that TF assets may be deployed to cover the larger region further offshore.

10. Patrolling the IRTC is managed by dividing it into patrol areas (PAs). Identifying the length of each PA is based on how quickly the helicopter of a generic asset can transit from the centre of its patrol box to one of its corners [3]. (The generic asset consists of a ship and a helicopter, each with a given transit speed. It is assumed the helicopter will launch after some time delay.) Then, depending on the size of the box that is deemed manageable by the generic asset, the IRTC is divided into an appropriate number of zones, each requiring a ship. An advantage of this approach is that taskings can be defined and managed quickly. However, questions remain as to whether asset allocation could be further optimized in a manner that would improve response times. For example: Is there a significant advantage to varying the size of each PA to match the capabilities of the ship being assigned to it? Is there a significant advantage to having the ships work in a coordinated manner rather than each patrolling a particular, statically defined PA? Can TF assets be managed in a way to optimally leverage off efforts of other national taskings in the GOA that are not coordinated with the international groups?

1.2 Scope of Work

11. This report contains three main sections. The first discusses various simple models that can be used to estimate TF asset response times in the IRTC. The second uses the results from the models to identify key points that should be considered when scheduling single and multiple-ship patrols, including how multiple assets can coordinate with each other to improve overall coverage. In the Findings and Recommendations section that follows, a proposed optimization model that will assist in solving the multiple-ship patrol problem in the IRTC is described. A more detailed description of each of these three sections is described below.

12. In the first part of this report, a Monte Carlo algorithm and three other models (two 2D models and a 1D model) are described that can be used to identify the portion of the IRTC that can be effectively patrolled by a particular asset. One 2D approach uses the analytical work of Marsaglia et al. [10] to generate the probability distribution of the square of the distance between two random points in two separate rectangles. The other uses what is defined here as a "ripple propagation algorithm" to track the area within a patrol area (PA) as a function of distance from the asset. These models give results equivalent to those

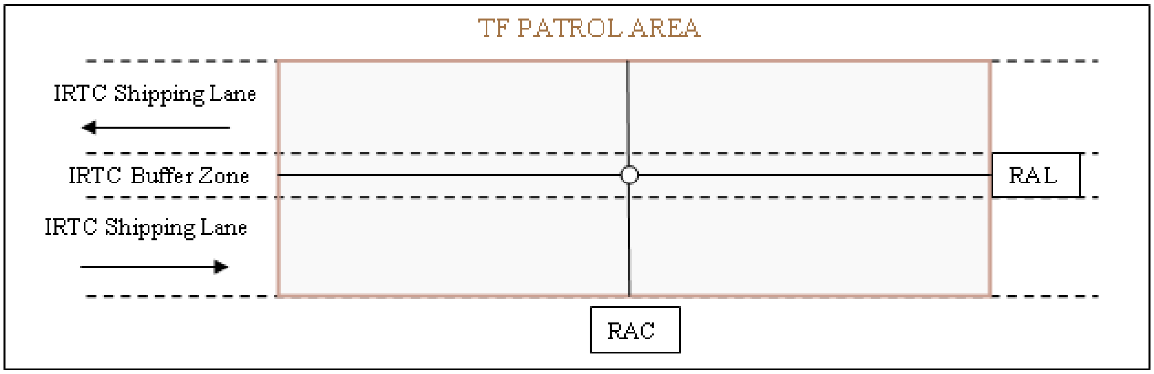

Figure 3: *Depiction of TF Patrol Strategies*

produced by the earlier versions of GAPP, but since they are based on analytical calculations rather than Monte Carlo simulations they deliver an answer more quickly. Results from the 2D models suggest that in most circumstances the across-channel positioning of an asset does not strongly affect response times, suggesting that a simpler 1D model might provide considerable insight at lower computational cost. Following this expectation, a one-dimensional model is developed and a correction factor is identified that allows estimates from the 1D model to match the 2D models closely. Reducing the problem to one dimension provides a modeling tool that is significantly faster than the 2D models, making it useful for solving multiple-ship optimization problems quickly enough to permit the result to be useful in an operational setting.

13. The second portion of the document considers four simple patrol strategies that may be considered by an asset patrolling a long thin region like the PAs in the GOA (Figure 3):

a. Random - the asset could be anywhere within its PA at any given time with equal probability (entire filled region in Figure 3);
b. Centred - the asset remains at the centre of its PA at all times (circle at centre of PA in Figure 3);
c. Random Along (RAL) - the ship remains in the middle of the transit lanes but could be anywhere along its PA at any given time with equal probability (anywhere along the line labeled RAL in Figure 3); and
d. Random Across (RAC) - the asset remains centred relative to the length of its PA but can be anywhere across the width of the PA with equal probability (anywhere along the line labeled RAC in Figure 3).

14. To constrain the parameter space presented in this document, a "generic fleet" of TF assets is used. The capabilities of this fleet are listed in Table 1. None of the ships listed is meant to match the ship of any particular nation. They are presented only to provide a variety of ship capabilities, permitting an illustration of the methodologies and results.

Table 1: *Profile of generic ships used in modeling efforts.*

Ship	Vessel Speed	Helicopter Speed	Time to Helicopter Launch
A	25 kts	90 kts	10 min
B	20 kts	100 kts	8 min
C	30 kts	110 kts	15 min

15. Using the 2D models, it is shown that the optimal strategy is to follow Option 2 (Centred), although Option 4 (RAC) is almost equally effective. Similarly, it is found that Options 1 (Random) and 3 (RAL) share comparable effectiveness. However, these latter Options are less advantageous than the previous ones. The pairings (2 & 4, and 1 & 3) are compared to results from the 1D problem, which is able to match them quite well. This result is promising for future work that will aim to optimize the positioning of multiple ships.

16. After examining these patrol strategies, the benefit of having multiple ships work in coordination with one another is considered. For the most part, two-ship problems are presented to provide a general proof-of-concept, but for one specific problem related to coordinating helicopters, a three-ship problem is presented. In addition, due to concerns expressed during planning meetings, the problem of repositioning ships to consider the effects of wind on helicopter transit are also considered.

17. In the final section, a number of findings and recommendations are presented, and a proposed asset allocation model is described as a potential avenue for future work. The proposed model will likely consider situations where some assets working on CP are outside of the TF command structure, suggesting how cooperating assets can work around other vessels in a manner that optimizes the overall coverage and deterrence capabilities of all naval vessels in the region.

18. It is worth noting that, while being used to address the problem of CP, the results and proposed model presented are not restricted to that domain. They can be applied to any situation where multiple assets with individual capabilities are patrolling or searching a long and thin region where the density of the item of interest varies in space and time. For example, the approaches developed here could be applied to other situations where multiple assets with individual capabilities are patrolling or searching a long and thin region.

2 Ship Response Models

19. This section gives an overview of several patrol coverage models developed in support of counter-piracy operations. Results from the various models are presented in the following section. The Monte Carlo and theoretical approaches used to examine the 2D problem are presented, followed by a 1D model that is useful for multiple-ship optimization problems. Note that models are presented in order of decreasing computational requirement.

20. For all models, the asset response is assumed to be as follows. Upon being notified of a piracy attack in progress, the ship begins transiting towards the incident location while its helicopter prepares for launch. During this time, the asset transit speed is the ship speed, v_{ship}. After some time delay t_h has passed, the helicopter launches and the asset transit speed becomes the average helicopter transit speed v_{helo}. Combining asset transit speeds with the time delay and the range r that must be covered, the response time τ is calculated.

21. Depending on the circumstance, it may be desirable to calculate the helicopter launch delay t_h as a constant value or as an unknown value that fits some known probability distribution. While incorporating probability distributions is easily accommodated by Monte Carlo simulations, the approach becomes considerably more involved for the other models presented. This section discusses coverage capability models under the assumption that t_h is constant. Details on how to include uncertainties in response times are covered in Annex A.

2.1 2D Patrol Coverage Models

22. A Monte Carlo approach and two analytical approaches are presented for considering patrol coverage capabilities. While the Monte Carlo model was initially used for the GAPP model delivered to CMF, it now serves as a verification and validation tool for the theoretical models. Two theoretical models are presented. The first, entitled the Radial Propagation Algorithm, is relatively straightforward to visualize, but solving for the problem is computationally expensive. The second, entitled the Marsaglia et al. Algorithm, is more complex mathematically and difficult to conceptualize, but it is faster to solve numerically. Thus, the Marsaglia et al. model should be considered the theoretical model of choice.

23. Both theoretical models employ a two-step approach to solving the problem. First, the probability distribution of the distance separating a TF ship and a pirate attack is identified based on the patrol strategy being employed by the vessel. (This distribution will be assigned the value $f(r)$ throughout the report.) Then, with the spatial distribution calculated, the response time distribution (assigned $g(t)$ throughout) can be calculated using standard statistical methods, provided below and expanded upon in statistical textbooks, e.g. [11].

2.1.1 Monte Carlo Approach

24. Determination of response times with a Monte Carlo approach involved the following steps:

a. Based on patrol strategy, extract the position of TF ship from a random distribution.
b. Extract the location of the pirate attack from a random distribution.
c. Calculate the range separating the TF ship and the piracy event.
d. Based on anticipated probabilities, determine the delay until helicopter launch, and assume the ship travels towards the event while the helicopter prepares for launch.
e. If the ship has not arrived on scene by the time the helicopter is ready to launch, launch the helicopter and assume the helicopter covers the remaining distance to the site of the pirate attack.
f. Identify the total time required for the TF asset to interdict the act of piracy.
g. Include any other possible delays, such as the delay of the merchant ship in notifying command that they are under attack or the delay of command to notify a particular ship that it is to respond.

This procedure is repeated a large number of times (e.g. 100,000 simulations) to identify the probability distribution of response times for a given ship covering a particular PA with a specified patrol strategy. For more details the algorithms used to calculate the statistics, see [1].

2.1.2 Ripple Propagation Algorithm

25. The Radial Propagation Algorithm is inspired by an expanding ripple on a pond. In essence, one can imagine a circular front propagating out from the ship - much like ripples on a pond after a stone has broken the water surface - where the proportion of the shipping lanes that are a given distance away are identified by overlaying the radial spreading pattern on a map that shows the asset and the area it is patrolling (Figure 4). Mathematical details of this algorithm are provided in Annex B. To covert from the PDF of radial location $f(r)$ to $g(t)$, one applies

$$g(t) = f(r)\left|\frac{dr}{dt}\right| = v\, f(r). \tag{1}$$

2.1.3 Marsaglia et al. Algorithm

26. Marsaglia et al. [10] consider the probability distribution function (PDF) of the square of the distance, r^2, between random points in two arbitrary rectangles that may or may not overlap partially or completely. (The theory is rather involved and not reproduced here, but the key results are given in Annex C.)

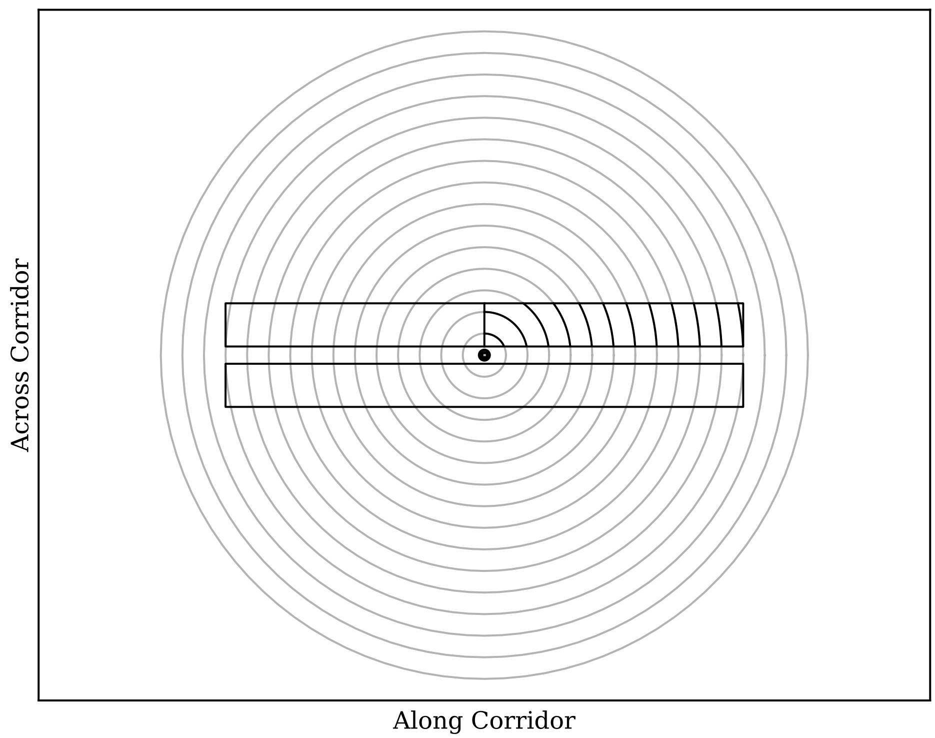

Figure 4: *Depiction of radial spreading algorithm showing the asset position (dot) and section of IRTC (boxes) within PA.*

27. For counter-piracy operations, however, the quantity of interest is response time, t. The PDF $f(r^2)$ is converted to $g(t)$ as [11]

$$g(t) = f(r^2)\left|\frac{dr^2}{dt}\right| = 2rv\, f(r^2) \tag{2}$$

where $v = v(r)$ is the speed of the responding vessel. For small r-values, v will be the ship speed but it will transition to the helicopter speed for higher r-values in a manner that depends on the helicopter launch delay.

2.1.4 Comparison of 2D Algorithms

28. To ensure the theoretical models are correct, their probability distributions are compared to those of GAPP. Figure 5 compares the theoretical distributions to those calculated with GAPP (with 100,000 repeats). The figure considers a PA that is 50 nm long, showing the response-time PDFs of the three ships in Table 1 for each of the four patrol strategies presented in Figure 3, with a 30-minute cut-off shown as a red line. A 3-minute standard deviation in helicopter launch delay is included for all cases. Since the Marsaglia et al. approach and ripple propagation algorithm produced identical results, one curve is simply labeled “Analytical”. Although this curve can be thought to represent either theoretical model, the radial propagation algorithm requires numerical integration and as a result is less computationally efficient than the Marsaglia et al. approach. The figure validates the theoretical models as they reproduce the Monte Carlo results.

2.2 1D Patrol Coverage Model

2.2.1 Why a 1D Model?

29. The 2D model has the capability to take a rectangular region of interest and produce a PDF of the response times for the TF asset to cover that region. While this approach can give high-resolution probabilities for specific ship profiles covering specific regions, it is not currently designed to consider multiple-ship problems. For example, consider the situation presented in Figure 6 where two ships are working in concert to patrol a larger region. If the region is subdivided into two PAs, then the analytical 2D algorithms presented would not correctly predict the time required to interdict an attack in one PA if the ship from the adjoining area is in fact the one that can respond the fastest.

30. Another limitation in considering multiple-ship problems is that of optimization. If the 2D models were extended to consider multiple-ship situations where the positioning of ships required optimization, then the time required to perform the optimization may become too lengthy.

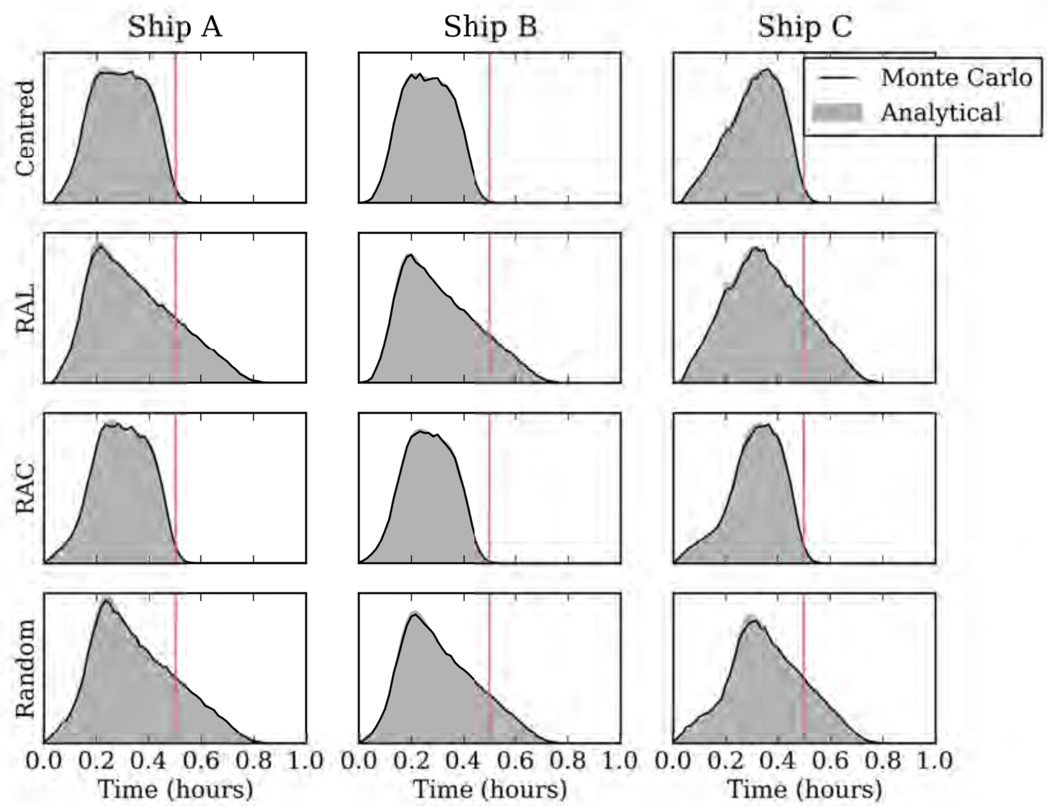

Figure 5: *Comparison of Monte Carlo and 2D Theoretical models.*

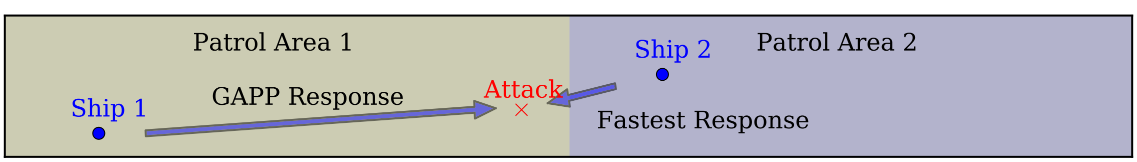

Figure 6: *Limitation of GAPP when dealing with multiple-asset response capabilities.*

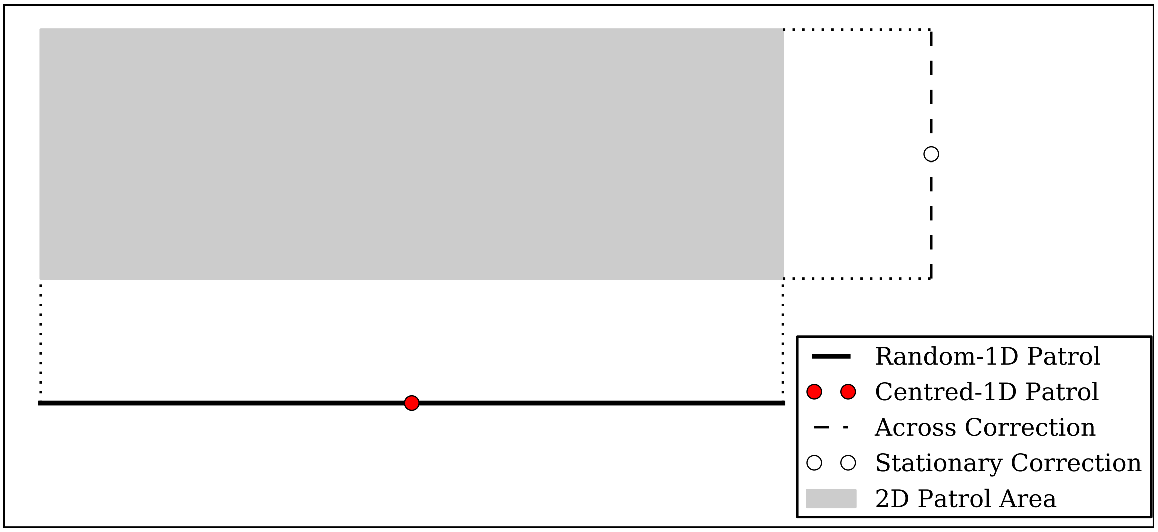

Figure 7: *Decomposition of 2D space into 1D patrol strategies (Random-1D and Centred-1D) and corrections (Across and Stationary).*

31. To address these issue, a model is proposed that is more amenable to multiple-ship optimization problems. The proposed method neglects the across-channel direction and treats the region as a 1-dimensional (1D) corridor, with correction terms introduced to account for the across-channel variability. This treatment decreases computational time considerably. In addition, errors can be identified analytically, which means they can be managed in a manner that does not decrease the accuracy of the results.

2.2.2 Model Description

32. The 1D patrol coverage model ignores the across-channel dimension of the IRTC, considering only the distance along the channel. For the CP problem in the GOA, this approach is effective because the region of the IRTC covered by a particular asset is likely to be long and thin, with the along channel direction considerably longer than the across-channel direction. Figure 7 illustrates the decomposition of the 2D field into sets of along-corridor strategies and across-corridor correction terms.

33. With the 1D model, two possible patrol strategies exist: Centred-1D and Random-1D (Figure 7). The probability of responding to an event in a given time for each of these strategies is identified later in this section.

2.2.3 Calculating Patrol Coverage

34. In the centred-1D scheme, for a patrol region of length L and a known helicopter launch delay t_h, the probability that a piracy event can be intercepted in some time τ is

$$P_C(L,\tau) = \begin{cases} \dfrac{2r_\tau}{L} & r_\tau < \dfrac{L}{2} \\ 1 & r_\tau \geq \dfrac{L}{2} \end{cases} \tag{3}$$

where r_τ is the distance that will be covered in a time τ and is defined as

$$r_\tau = \begin{cases} v_{ship}\tau & \tau \leq t_h \\ v_{ship}t_h + v_{helo}(\tau - t_h) & \tau > t_h \end{cases} \tag{4}$$

Similarly for the Random-1D scheme, the relationship is

$$P_R(L,\tau) = \begin{cases} \dfrac{2r_\tau}{L^2}\left(L - \dfrac{r_\tau}{2}\right) & r_\tau \leq \dfrac{L}{2} \\ \dfrac{r_\tau}{L}\left(2 - \dfrac{r_\tau}{L}\right) & \dfrac{L}{2} \leq r_\tau < L \\ 1 & r_\tau \geq L \end{cases} \tag{5}$$

35. These equations become significantly more involved when uncertainty in helicopter launch times are added to the problem (see Annex A for details).

36. For very large values of r, the 1D model should do a good job of replicating the 2D model because the across-channel contribution to the range will be small compared to the along-channel contribution. However, if probability of interception within a short time limit is desired, it is necessary to identify and correct for the error related to the 1D approximation.

2.2.4 Error Correction

37. The main assumption of the 1D model is that a ship with a coverage range r can in fact cover a rectangular region that is $2r$ in length (Figure 8). This treatment makes the implicit assumption that the range is large compared to the across-channel distance, allowing the (curved) sides of the range circle to be treated as the (straight) sides of a rectangle. The error correction term identifies the actual area covered as a function of range, which permits the 1D model to be used in cases where the above-mentioned assumption breaks down. Here,

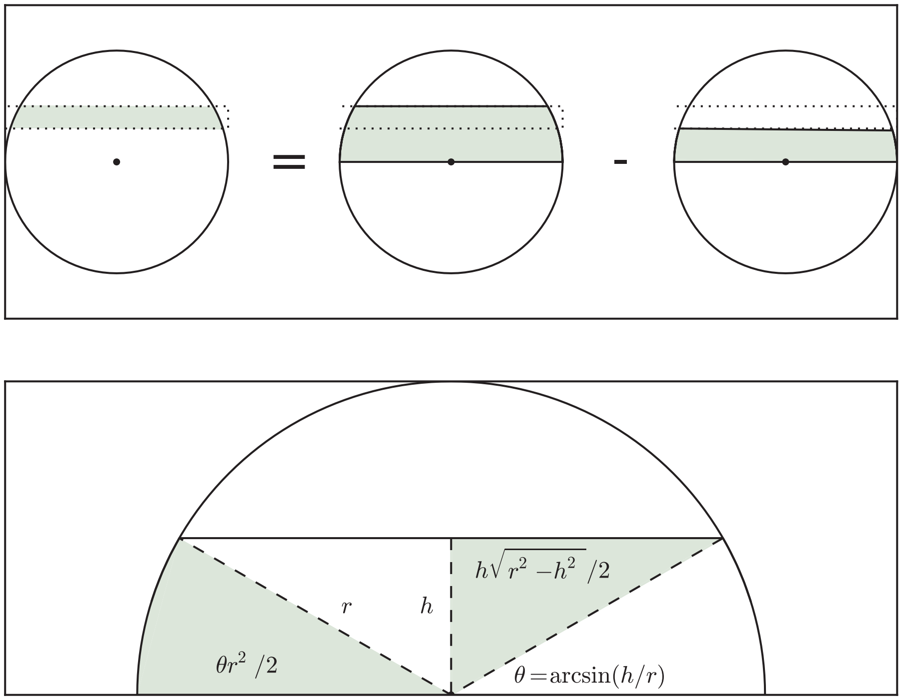

Figure 8: *Geometry used to identify overestimation of area covered by 1D model.*

the method used to quantify the degree of over-estimation in the 1D model is presented, but the mathematics are sequestered to Annex D.

38. The top panel of Figure 8 shows a series of semi-circles, with the dot at the circle centre representing the location of the ship and the dotted rectangle representing the portion of an IRTC lane assumed to be covered in the 1D model. In the figure, the ship is outside the IRTC lane, but the approach can be used if the ship is inside the IRTC lane as well [2]. In the left-most circle of the top panel, the filled region represents the region actually covered, so the overestimation can be taken as the area of the rectangle traced by the dotted line divided by the area of the filled section. The method for identifying the area of the filled section is shown graphically in upper panel, and calculated as difference between the two areas presented. The bottom panel shows how the areas are calculated by dividing the area into triangles and circular sections. Further details of the calculations are provided in Annex D.

2. When a ship is within the lane of interest, the area covered can be found by cutting the rectangular region into two sections along the axis of the IRTC. For each of the two sub-regions, the area is identified using the approach drawn out in the bottom panel of Figure 3, and the total area is simply the sum of the two.

39. Two possible patrol approaches are considered for the error estimates. In the first, the ship would remain stationed in the middle of the IRTC (Stationary). In the second, the ship travels back and forth across the region (Across). Overestimation of IRTC area covered as a function of range is shown in Figure 9. The results in the figure assume that the width of interest is the IRTC, which has a 2 nm buffer in between the two 5 nm-wide shipping lanes. Combining the two possible 1D patrol strategies (Random-1D and Centred-1D) with the across-channel error estimates (Stationary and Across) replicates each of the four patrol strategies considered by the 2D model (Table 2).

Table 2: *Linkage between 2D and 1D patrol coverage models*

2D Patrol	1D Patrol	Error Correction
Random	Random-1D	Across
Centred	Centred-1D	Stationary
Random Along (RAL)	Random-1D	Stationary
Random Across (RAC)	Centred-1D	Across

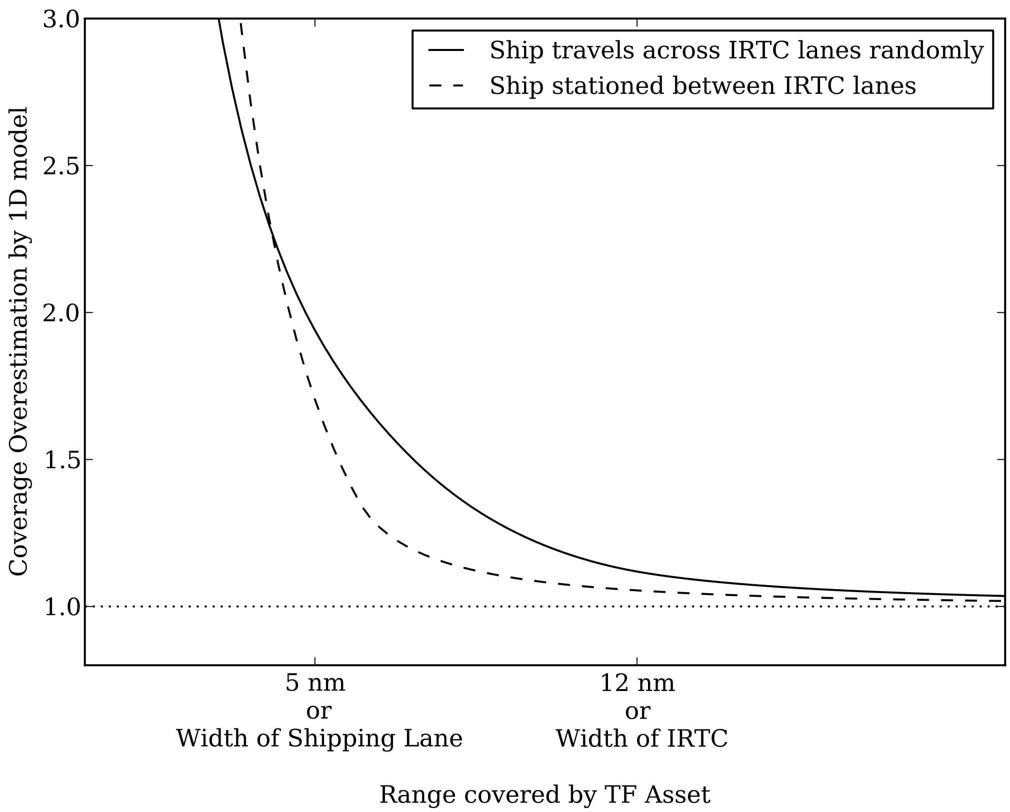

Figure 9: *Average area overestimation factor by 1D model.*

2.2.5 Limitations

40. The 1D model is an effective tool for identifying the ability of an asset to cover a long and thin region of length L, provided the measure of effectiveness is P_R or P_C. Unlike the 2D models, the 1D approach does not provide probability density functions of response distances (or response times). The correction term that would enable this capability would essentially reproduce the ripple propagation algorithm (Annex B), which as already stated is less efficient than the Marsaglia et al. approach. If detailed response probabilities are required, then a 2D model would be preferred.

3 Patrol and Positioning Strategies

41. This section considers the 2D and 1D models described above, and examines their ability to address questions regarding patrol strategies of single and multiple ships. It begins by examining how the four patrol strategies in Figure 3 affect response capabilities in the 2D problem, then compares the capabilities to those inferred from the 1D model. From there, focus is shifted to consider multiple-ship problems to identify how coordinating ship responses improves overall capabilities. Last, the effect of wind on helicopter response times is examined to determine how ship positioning can be altered to accommodate it.

42. For now, it is assumed that acts of piracy could happen at any location within the IRTC shipping lanes with equal probability. (This assumption would be relaxed in future work.) Under this assumption, the ships listed in Table 1 are used to compare the 1D and 2D models.

3.1 Comparison of Patrol Strategies for the One-Ship Problem

43. For the four patrol strategies, probability distributions of response distance calculated with the 2D models [3] are presented in Figure 10. For demonstration purposes, the PA in the figure has a fixed length of $L = 60$ nm. Each patrol strategy exhibits an initial steep increase in coverage ability as a function of range. This feature is due to these short ranges being smaller than the distance across the IRTC. There is a sharp peak at 6 nm for the Centred and RAL strategies when the range circle no longer fits within the width of the IRTC (see Figure 4). For the Random and RAC strategies, the peak is more diffuse and at a higher range-value [4]. The Centred and RAC patrol strategies show plateaus of the PDF with distance, dropping off dramatically when the range becomes half the length of the PA [5]. By contrast, the Random and RAL strategies tend towards a linear decrease with distance that reaches zero only when the range equals the length of the PA [6]. This difference is attributed to the effect of traveling along the length of the domain for the Random and RAL strategies.

44. The results shown for the range PDF demonstrate how remaining at the centre of the patrol box improves the probability of being close to a piracy incident. When response time

3. Figure 5 shows all 2D models give comparable results.

4. If one considers an expanding circle inside a rectangle, the PDF is similar to the length of circle's edge that is within the rectangle. For the Centred and RAL strategies, this peak is exactly at half the distance across the rectangle, giving the sharp peak. However, for the other two strategies where the ship position moves across the rectangular region, the peak gets blurred and shifted to a higher value.

5. Actually, it is $r = \sqrt{(L/2)^2 + (W/2)^2}$, where L and W are the length and width of the PA, respectively, but for the scales used in this example, $r \approx 1.02(L/2)$.

6. Similarly, the range is actually $r \approx 1.02L$.

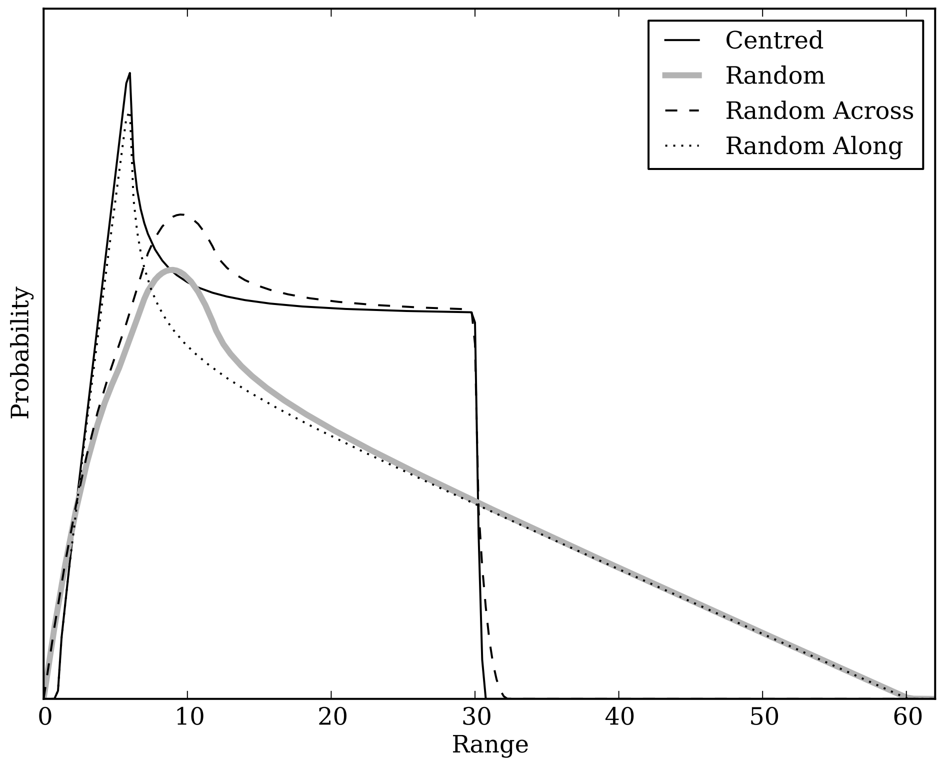

Figure 10: *PDFs of range to target for four Patrol Strategies in a 60 nm-long PA.*

is considered (Figure 11), the PDFs become more complicated as the TF asset capabilities come into effect. If the asset had a constant response time (e.g. constant ship transit speed with no helicopter), then the mapping from distance to time simply causes the distance PDF to be stretched by $(1/v_{ship})$. Because the launching of a helicopter increases the distance covered in a given time by a considerable amount, there is a step-like increase in the PDF when the helicopter is launched.

45. It is interesting to note that once the range is larger than the distance across the IRTC, there is only a small discrepancy between the Centred and the Random Across patrol strategies, and also only a small discrepancy between the Random and Random Along patrol strategies. This result suggests that, so long as the shape of the PA is rectangular, with the width across the region smaller than the length, the location of the TF ship across the PA is not important in determining response times. This finding is an important factor when applying the 1D model.

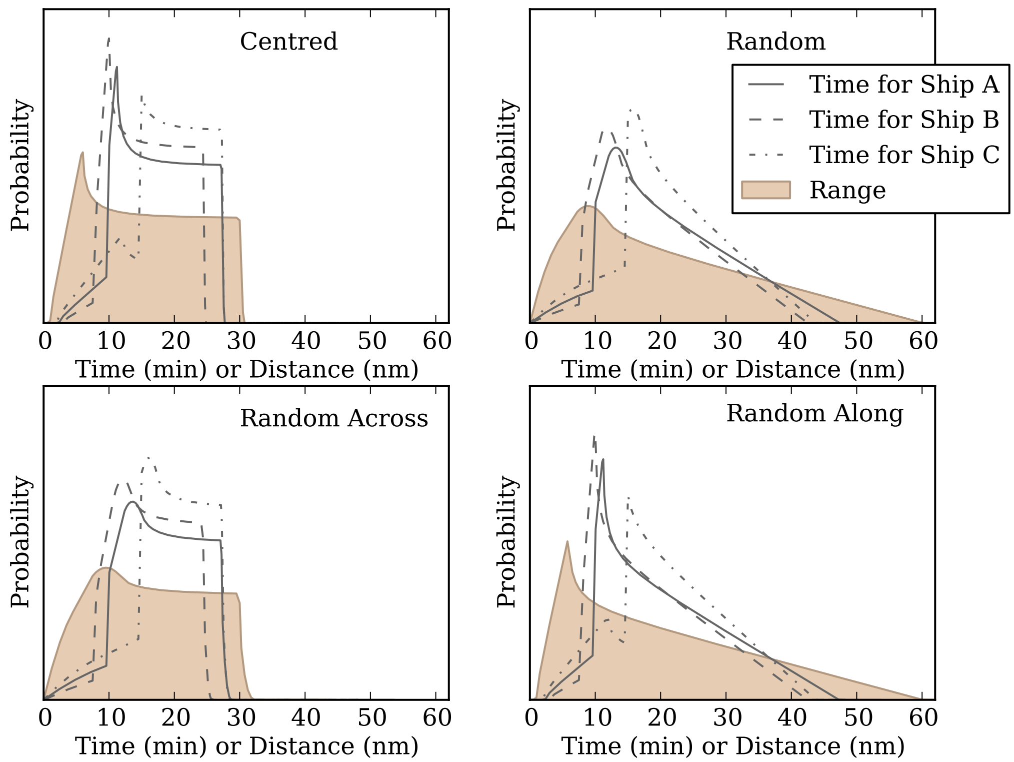

Figure 11: *PDFs of distance (fill) and times (lines) separating TF ship from site of pirate attack for the three ships profiles given in Table 1.*

3.2 Identifying the likelihood of a 30-minute response time

46. A key metric presently used to assess the effectiveness of patrols is the ability to respond to an event within 30 minutes. (Note that while $\tau = 30$ minutes is used here, for operational purposes related to CP in the GOA, the value could be set to an arbitrary value without altering the computational expense.) This section discusses how to answer the question, "Given a ship with some capability that is covering a patrol box of length L, what is the likelihood that an event will be intercepted within 30 minutes?"

47. To answer this question, the 2D models require integration of PDFs like those shown in Figure 11 to identify what portion of a region can be covered within the time limit. By comparison, the 1D model gives simple equations that can be solved quickly (see below), making them useful for optimization problems.

48. Figure 12 shows the probability of Ship A intercepting an incident within 30 minutes or less. Calculations based on the uncorrected 1D model are shown as dotted lines, and the results scaled by the error overestimation factors are shown as dashed lines. The symbols represent values calculated using the 2D models. The results for the same ship but with no helicopter are shown in Figure 13.

49. Consistent with Figure 9, Figures 12 and 13 indicate that the over-estimation factor is more important when short distances are being considered. In Figure 12, the error correction is larger for the RAC patrol than the RAL patrol. This result matches Figure 9, where ships traveling across the IRTC will require greater correction than those that do not.

50. Benchmarking performed on the effort required to generate Figures 12 and 13 show that the 1D model is well over four orders of magnitude faster than the 2D models. This factor, combined with the goodness of fit between the 1D and 2D models permits us to now move forward using the 1D model with greater confidence to consider more complex location-allocation problems.

3.3 Coordinating Multiple TF Assets

3.3.1 Assigning Patrol Boxes to Optimize Response

51. In the above section, it is shown that the 1D model can provide the likelihood of a TF ship responding to an incident in its PA within a given time limit provided that the ship characteristics, the patrol box length, and the patrol strategy are known. From here it becomes possible to optimize the response of multiple ships that must divide a given patrol area such as the IRTC.

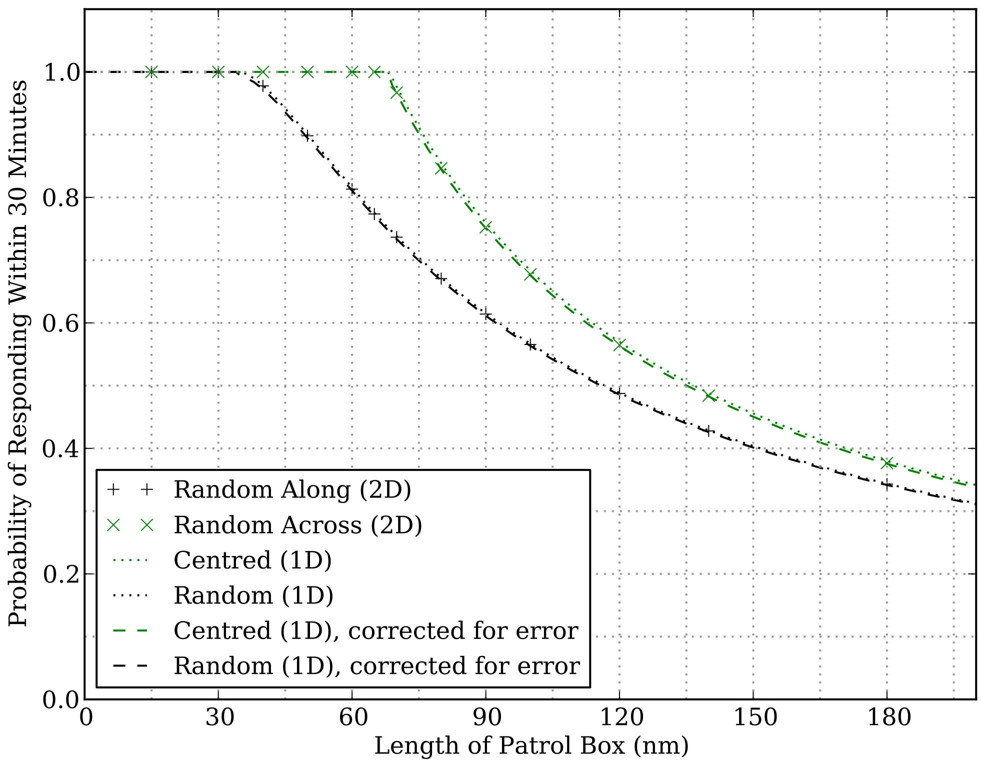

Figure 12: *Model comparison of response capabilities for Ship A, given a 30-minute response window.*

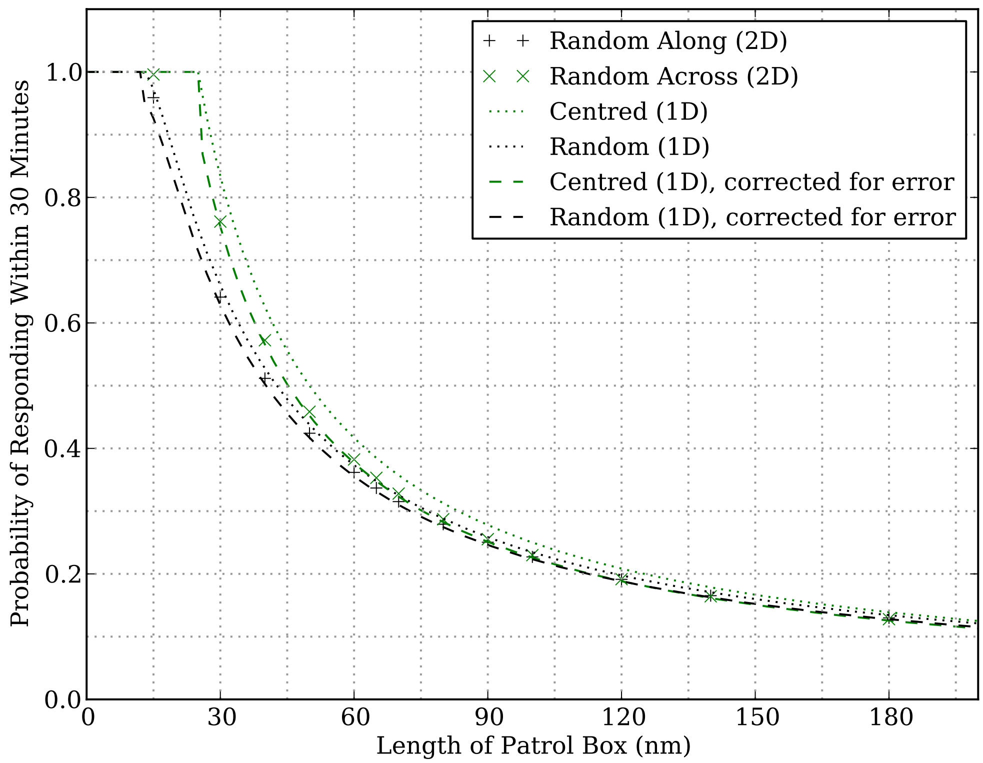

Figure 13: *Model comparison of response capabilities for Ship A without a helicopter, given a 30-minute response window.*

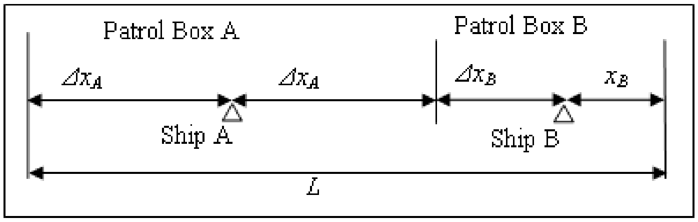

Figure 14: *Dynamic positioning of Ship A based on location of Ship B.*

52. For demonstration purposes, consider two ships that collectively patrol a 150 nm-long region of the IRTC with a length. A simplified allocation problem could be the following: for Ships A and B listed in Table 1, if Ship A has a centred patrol strategy and Ship B has a random patrol strategy, how will ship positioning affect probability of intercepting a pirate attack?

53. Ship A can position itself using one of two possible approaches. In the first, it assumes a static location, which is to say that if Ship B is assigned a PA with a length L_B, Ship A will position itself in the middle of a PA with length $L_A = L - L_B$. This approach is defined as static positioning. In the second, Ship A considers the current position of Ship B and positions itself in a manner that maximizes the overall response capabilities of both ships. This approach is defined as dynamic positioning.

54. A definition sketch for the dynamic positioning scenario is provided in Figure 14. In the figure, the distances Δx_A and Δx_B are defined so that both ships can reach their shared patrol-box edge in the same amount of time [7]. Mathematically, the position of Ship A is found by solving

$$\Delta x_B + 2\Delta x_A = L - x_B \tag{6}$$

where for Ships A and B, the Δx value is defined as

$$\Delta x = \begin{cases} v_s t & t \leq t_h \\ v_s t_h + v_h(t - t_h) & t > t_h \end{cases} \tag{7}$$

with v_s, v_h, and t_h defined based on the individual ships and t set to same value for both ships. Noting that Equation (7) is really two equations (one for Ship A and one for Ship B), Equations (6) and (7) provide a system of three equations to solve for the three unknowns Δx_A, Δx_B, and t.

55. If it is assumed that Ship A will shadow the movements of Ship B in a dynamic fashion so that the above equations remain satisfied, the average coverage capabilities can be found by averaging the ship response capabilities over the possible values of x_B (which range from 0 to L_B).

7. In cases where a there is uncertainty in the helicopter launch delay time, this shared edge would be based on the average response capabilities.

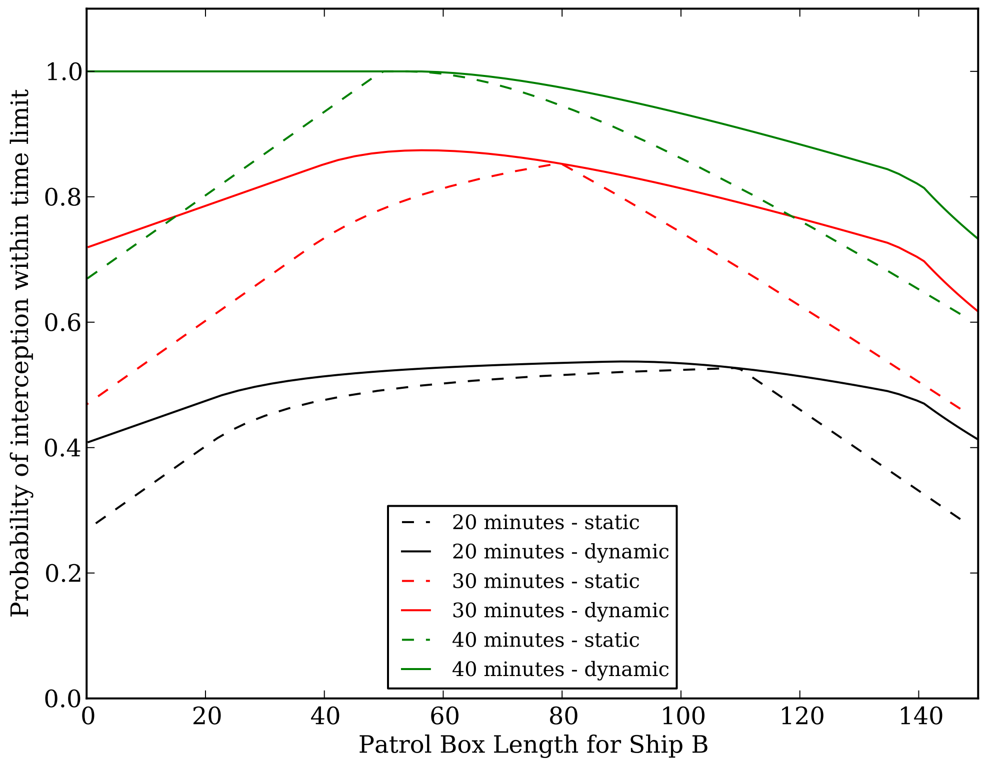

Figure 15: *Response capabilities for a two-ship problem.*

56. The effect of static or dynamic positioning on overall response effectiveness is shown in Figure 15 for response windows of 20, 30, and 40 minutes. For all patrol box sizes, the response capabilities are improved when the PA for Ship A is varied dynamically based on the location of Ship B (statistics are provided in Table 3). While dynamic positioning does not alter the outcome if the assets are positioned in an optimal manner to begin with, it mitigates the effects of poor positioning considerably. Hence, if this example is used to consider the problem of managing patrol strategies near a ship that does not coordinate with the major coalitions, this approach delivers a method to position coalition ships in a manner that utilizes position of the uncoordinated ships in an optimal manner.

3.3.2 Considering Helicopter Readiness

57. Helicopters waiting to launch will remain at high alert states for short periods of time, returning to lower readiness levels for most of the time. In this section, the effect of helicopter readiness on dynamic asset allocation is examined with the following model. A three-ship problem was considered, under the following assumptions:

Table 3: *Statistics for optimal patrol box sizes for Ships A and B covering a 150 nm section of the IRTC.*

	20 min. Response		30 min. Response		40 min. Response	
	Static	Dynamic	Static	Dynamic	Static	Dynamic
Ship A PA Length	40 nm	Variable	70 nm	Variable	95 nm	Variable
Ship B PA Length	110 nm	90 nm	80 nm	55 nm	55 nm	60 nm
Max(P Intercept)	52%	53%	85%	87%	100%	100%
Min(P Intercept)	27%	40%	46%	62%	61%	74%
Mean(P Intercept)	45%	49%	68%	77%	84%	92%

a. The helicopters on these ships can have one of two readiness levels, high (H) and low (L). At the high readiness level a helicopter can be launched within 5 minutes, while at the lower readiness level it will take 15 minutes to launch.

b. A helicopter may remain at a heightened alert level for one shift, after which it must be stood down to a lower level for at least two shifts. (The length of the shift does not matter, so long as all shifts are of equal length.)

c. The length of PAs can be coordinated (i.e. the assets with helicopters at high-alert cover a larger area) or uncoordinated (all assets cover an equal area).

d. The asset that can reach the location of the incident the fastest will respond.

e. The combined patrol area is set such that when two helicopters are at heightened alert and one at the lower alert level, it is possible to coordinate PAs so that the entire region can be covered in 30 minutes or less.

f. At any given time, the three ships can exhibit one of six possible readiness states that can fit into one of four categories [8]:

   (a) All helicopters at high alert level (HHH)

   (b) Two helicopters at high alert level

   Low-alert helicopter on end of patrol box (HHL or LHH)

   Low-alert helicopter in centre of patrol box (HLH)

   (c) One helicopter at high alert level

   High-alert helicopter on end of patrol box (HLL or LLH)

   High-alert helicopter in centre of patrol box (LHL)

   (d) All helicopters at low alert level (LLL)

58. Results for the various readiness levels are shown in Figure 16 and are grouped into two sets. On the left, the six permutations of helicopter readiness are shown. On the right, the average response for combinations of three successive "rotations" of readiness are presented. For the permutations of possible readiness levels among the three assets, the

8. Note that HHL and LHH are the same for modeling purposes, as are LLH and HLL.

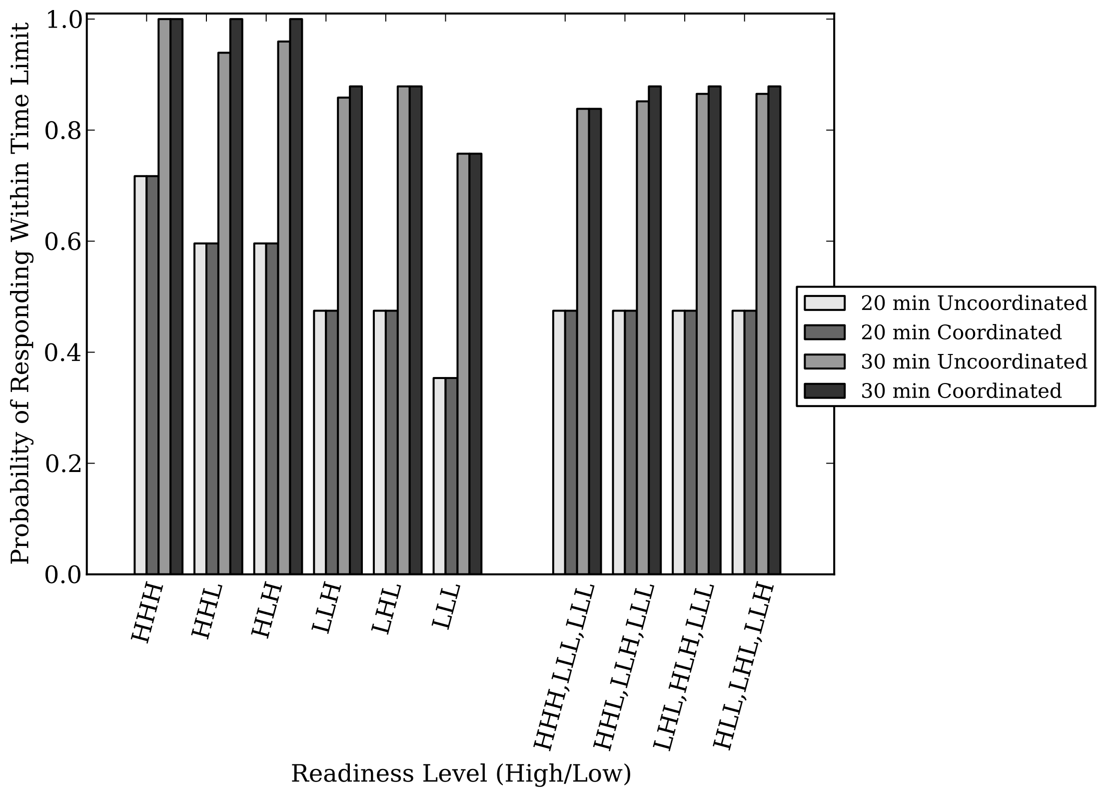

Figure 16: *Effect of helicopter readiness on response capabilities.*

configurations HHH, HHL and HLH all have the ability to cover 100% of the patrol area in 30 minutes or less if the assets coordinate their positioning. However, these combinations are not sustainable, and lower readiness combinations would need to be included in any cycle. For any set of cycles that maximizes the number of high-readiness rotations, 30-minute coverage capabilities are in the range of 84% to 88%. For a shorter time limit of 20 minutes, the patrolled domain is too big relative to the response distances of the assets for there to be benefit to asset coordination.

59. Based on these findings, it appears that it is likely that helicopter readiness levels should be based on times and locations of elevated pirate activities and higher commercial traffic density, rather than the readiness levels of nearby ships. To be certain, however, this result would need to be sought using data that reflected the actual positioning and capabilities of TF assets.

3.4 Altering Ship Position to Compensate for Wind

60. Discussions with analysts from fellow TTCP nations indicated that some operators have expressed concerns that asset allocation tools being prepared for these types of patrols should be able to compensate for wind. While wind can affect asset positioning within its PA, simple modeling efforts indicate that winds in the GOA should not greatly affect the size of a patrol box that may be covered by a TF asset. This finding was identified by calculating how wind affects the bearing a helicopter must follow to reach a target as well as the resulting helicopter velocity over ground. Using a sine law condition, it can easily be shown that the bearing of the helicopter needs to be adjusted by an amount

$$\theta_h = -\arcsin\left(\frac{v_{wind}}{v_{helo}}\sin\theta_w\right) \tag{8}$$

where v_{wind} is the wind speed and θ_w is the direction of the wind relative to the direction to the target. For example, if the wind is blowing at 25 kts in a direction 20 degrees to the right of the intended direction of travel, then a helicopter with a transit speed of 90 kts would need to follow a bearing 5.5 degrees to the left of the intended direction of travel to arrive at its desired destination. The effective speed of the helicopter is also changed to

$$\hat{v}_{helo} = v_{helo}\cos\theta_h + v_{wind}\cos\theta_w, \tag{9}$$

which permits the travel time along the adjusted vector to be calculated as well.

61. Winds blowing at 30 to 40 kts are likely to be associated with a Sea State of 5. Such conditions will include waves that are too energetic for pirates to engage in ship-boarding activities. If winds are this strong counter-piracy patrols are probably not necessary. Thus, to test whether winds can considerably alter coverage capabilities under relevant conditions, examining winds at this strength are sufficient.

62. Two base cases of winds blowing along or across the IRTC are examined, noting that any other direction can be considered a combination of these two. Figure 17 shows the results for the four pairings of wind speed (30 kts or 40 kts) and direction (along or across IRTC). In the figure, the black dot and black circle show ship location and the 30-minute coverage limit for Ship A in the absence of wind. The red dot and dashed red circle show adjusted ship position and resulting the patrol coverage capability under windy conditions, where the ship position is shifted upwind by a distance

$$v_{wind}(\tau - t_h), \tag{10}$$

where $\tau - t_h$ is the transit time for the helicopter. The distance upwind that a ship should be positioned is shown in Figure 18, which may be interpreted as follows. If a response time of $\tau = 30$ minutes is required and the helicopter launch delay is $t_h = 10$ minutes, then $\tau - t_h = 20$ minutes. Drawing a vertical line on the graph at this value shows how far the ship position should be shifted as a function of wind speed. For example if the wind

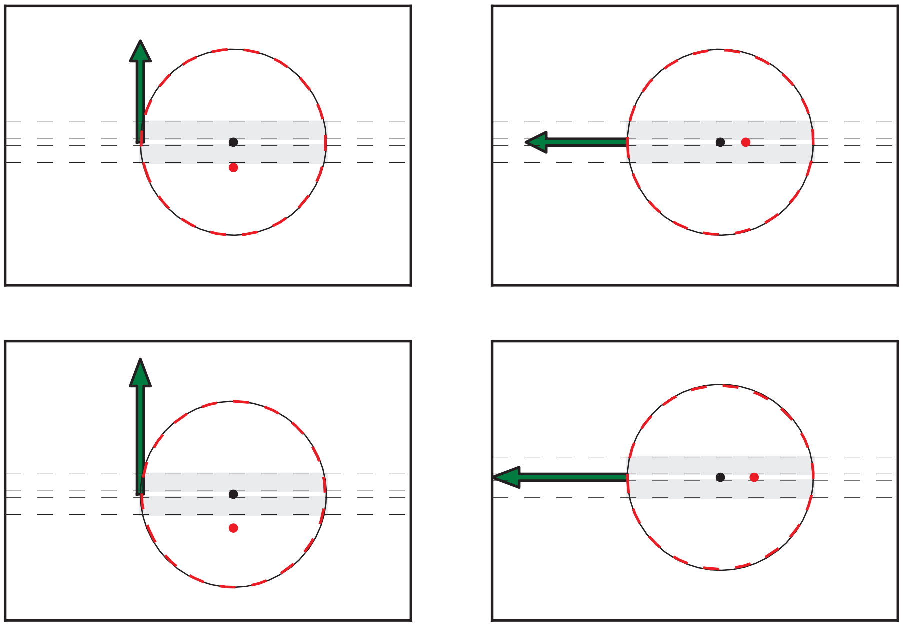

Figure 17: *Compensation of ship location to account for wind.*

speed is $v_{wind} = 15$ kts, then to cover the same region that it would cover under windless conditions, the ship should reposition itself 5 nm upwind.

63. Based on these results, the effects of wind can be expected to alter the positioning of a ship, but not necessarily its ability to effectively cover the region of interest.

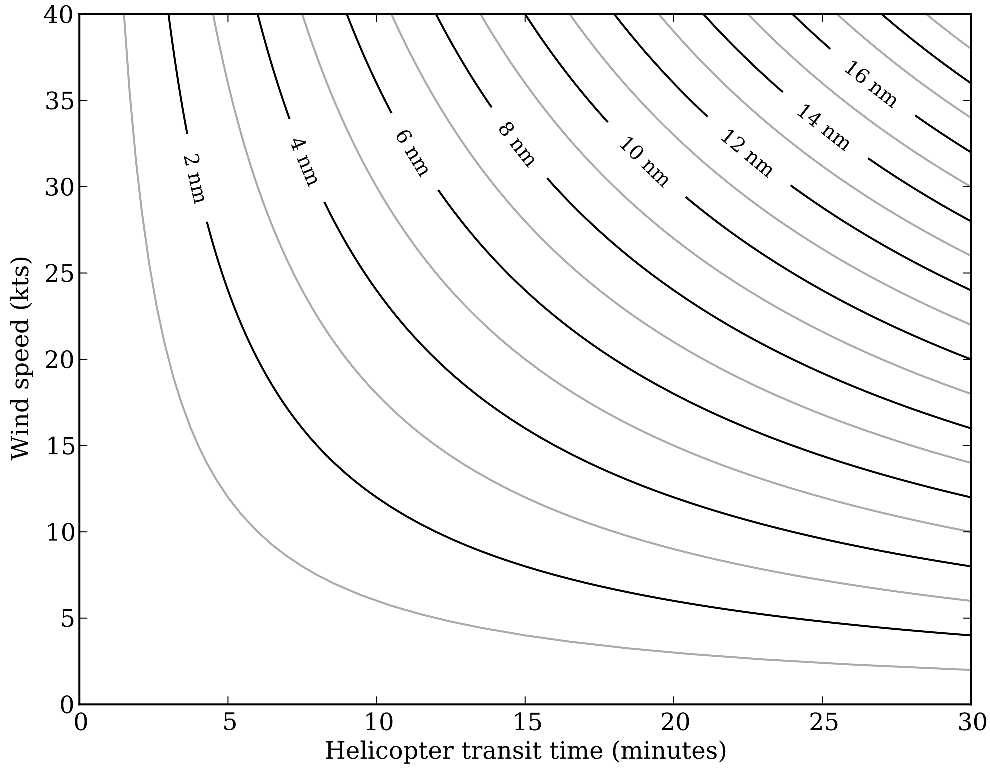

Figure 18: *Distance upwind that a ship position should be shifted to cover patrol region.*

4 Findings, Recommendations and Future Work

4.1 Findings

64. The modeling effort undertaken in this work revealed several key findings for assets patrolling long thin regions:

a. The 1D patrol coverage model can adequately identify the likelihood of a ship intercepting a target based on the size of the patrol box and the patrol strategy but it cannot give a probability distribution of response distances or response times. As a result the choice of patrol coverage model will depend on the desired measure of effectiveness.
b. In cases where 1D model is sufficient, it is the preferred option as it requires less than 0.01% of the time to perform its calculations (a few seconds rather than several minutes).
c. In responding to incidents that occur at random locations with their PA, assets centred within their PA consistently outperform assets that travel randomly through them.
d. The position of an asset across a long and thin PA has little effect on response capabilities compared to its position along its length.
e. A heterogeneous fleet (various assets with differing capabilities) will be more effective at covering a region if the length of PAs are customized to match the capabilities of the individual assets.
f. In cases where all asset taskings are not or cannot be optimized to match capabilities, dynamic positioning of the assets that can be tasked as needed can enhance response effectiveness considerably.
g. A fleet that places ship helicopters on heightened alert in a coordinated fashion *may* only mildly outperform a fleet that lets individual ships control their helicopter readiness levels, depending on the size of the patrol region.
h. While wind affects helicopter transit speeds over ground, a ship can position itself within its PA to accommodate the effect.

4.2 Recommendations

65. The modeling presented here lays forth several recommendations for allocating assets to patrol a long and thin region such as the IRTC in the GOA. These include:

a. PAs should be defined in a manner that considers the response capabilities of the individual assets. For example, a ship with a fast helicopter will be able to cover a larger area than a ship with a slower helicopter or no helicopter at all, and the size of PAs should be assigned accordingly based on the range an asset can cover in a given amount of time (see Equation (4)).

b. Assets should remain as close to the centre of their PA as possible to maximize response effectiveness. This strategy will minimize the number of assets required to patrol the entire corridor. If ship movement is desired to develop situational awareness, dynamic PAs should be used to keep assets centred in their PA. In addition, assets should patrol the regions in a coordinated fashion, keeping the along-channel distance between ships relatively constant.

c. In cases where the operations of one asset do not permit it to remain at the centre of its PA, adjacent assets should alter their position and PA in a dynamic fashion based on the real-time position of the other asset. This dynamic approach to positioning will improve patrol coverage effectiveness.

d. Following the findings in Section 3.3.2, the possibility that ships should place helicopters at heightened alert based on the locally-perceived risk of pirate activity (rather than in a coordinated effort that maintains an steady average readiness for the entire fleet) should be examined using a realistic example.

e. In windy conditions, ships should reposition themselves upwind by a distance that is based on helicopter speed, wind speed, time delay for helicopter launch, and desired response-time window. The desired distance can be extracted from Figure 18, following the example provided at the end of Section 3.4.

4.3 Future Work: Proposed Allocation Model

66. At present, executing these recommendations is a challenge as there is no readily available tool for analysts and operators who are setting the patrol schedules. Hence, the next step is to synthesize the findings presented in this report to produce an operationally useful asset position and allocation model. The model should include the ability to consider multiple ships with individual capabilities. Two modes of utilization would be beneficial. The first would be a planning mode that used anticipated asset availability to suggest where along the IRTC the assets should be positioned. The second would be a more responsive mode that suggested “fine-tuning” of asset positions as perceived risks and asset availability evolved.

67. The logical structure of such a model could be as follows. Consider a set of I ships assigned to patrol a section of a corridor of length L. Each ship will have the properties ship speed $(v_{ship})_i$, helicopter speed $(v_{helo})_i$, and an expected helicopter launch delay $(t_h)_i$. Combined, these features will provide a representative response curve $\rho_i(r)$ that describes how long it takes for a vessel with its helicopter to reach an incident some distance r away. This response curve can be calculated with the 1D model.

68. Further, these ships can be subdivided into two categories: ships are under direct coalition command (group M), and ships in the region that are performing counter-piracy duties outside of coalition control (group N). If ship i is within group M, then $M_i = 1$ and $N_i = 0$, otherwise $M_i = 0$ and $N_i = 1$. Since these groups are exclusive of each other, it is necessary that $M_i + N_i = 1$ and

$$\sum_{i \in I} M_i + N_i = I. \tag{11}$$

69. Each ship may be assigned to a location x_i, where $0 \leq x_i \leq L$. For ships in group N, this position is known, e.g. through observation, but cannot be changed since the ship is outside coalition command. For ships in group M, the values of x_i are found in a manner that minimizes

$$F = \int_0^L \mathscr{R} \rho_{\min} dx \tag{12}$$

where $\mathscr{R} = \mathscr{R}(x,t)$ is the risk function that defines the likelihood of an attack occurring at a particular location [9], and $\rho_{\min} = \rho_{\min}(x)$ is the shortest response time for any of the I assets to reach the location x, i.e.

$$\rho_{\min}(x) = \min(\rho_i(x)). \tag{13}$$

Note that the dependency of $\mathscr{R}$ on time implies that the value of F and therefore the optimal position of ships is likely to vary in time.

70. For scenarios where the ships are positioned and must adjust after one gets tasked or becomes otherwise occupied, readjustment of the positions would be performed in a manner that minimized repositioning the ships. For example, if the ship order is A, B, C, D, E and Ship D must remove itself from general patrol, the dynamic allocation model would give the best coverage for the remaining ships, ensuring that the order remains as A, B, C, E. If at some later time, Ship D were to become available once again but its location was between Ships B and C, then the dynamic allocation model would find the optimal positioning that kept the ship ordering as A, B, D, C, E.

71. It would also be beneficial to couple the proposed model with a reconnaissance tool that considered how changing ship patrols affected the quality of the regional maritime picture and whether making such changes would alter the requirements for other assets such as maritime patrol aircraft.

9. For the counter-piracy problem, the risk index value is likely to be determined based on the locations of recent incidents pirate activity, locations of suspected pirates, density of merchant ships that are at risk of high-jacking, and meteorological/oceanographic considerations such as wind and waves. This term is beyond the scope of the proposed modeling effort but could be based on ongoing and future analyses of coalition allies and TTCP partners.

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Annex A: Response-Time Uncertainties

A1. The models presented in Section 2 determine the probability distributions of response distance, then convert the results to response times based asset transit speed, which depends on the delay in helicopter launch t_h. This annex discusses how uncertainties in the helicopter launch delay can be incorporated to improve estimates of response delays.

A.1 Time Required to Cover a Known Distance

A2. Uncertainty in launch delay affects asset transit speed v. If the helicopter will launch after some known time t_h, the function for v will have a step function where

$$v(r) = \begin{cases} v_{ship} & r \leq v_{ship} t_h \\ v_{helo} & r > v_{ship} t_h \end{cases} \tag{A.1}$$

Also, the time necessary for an asset to reach a range r is

$$\tau_r = \begin{cases} \dfrac{r}{v_{ship}} & t_h \geq \dfrac{r}{v_{ship}} \\ \dfrac{r}{v_{helo}} + t_h \left(1 - \dfrac{v_{ship}}{v_{helo}}\right) & t_h < \dfrac{r}{v_{ship}} \end{cases} \tag{A.2}$$

A3. However, if the launch delay is not constant but instead fits a probability density function $t_h \sim P_h(t)$, then the mean transit speed as a function of distance will follow the more generalized form [10],

$$\bar{v}(r) = v_{helo} \int_0^r P_h\left(\frac{r'}{v_{ship}}\right) dr' + v_{ship} \int_r^\infty P_h\left(\frac{r'}{v_{ship}}\right) dr'. \tag{A.3}$$

and the mean time required to reach a target a distance r away is

$$\bar{\tau}_r = \int_0^r \frac{dr}{\bar{v}}. \tag{A.4}$$

Note that this is the mean response time: it does not give information about the distribution of response times. By inserting $P_h(t)$ into (A.2) and observing that to reach a target a distance r away in a time τ requires the launch delay to be

$$t_h^* = \frac{v_{helo}\tau - r}{v_{helo} - v_{ship}}, \tag{A.5}$$

10. Note that for normal distributions (like those used in GAPP), the integrals in this equation will have analytical solutions that make use of the error function and complementary error function (see, e.g., Abramowitz and Stegun [12] for more information on these functions).

the distribution of response times at a distance r away is found to fit

$$\tau_r \sim P_h^*(t_h^*) \tag{A.6}$$

where

$$P_h^*(t_h^*) = \begin{cases} 0 & t_h^* > \dfrac{r}{v_{ship}} \\ \displaystyle\int_{r/v_{ship}}^{\infty} P_h \, dt & t_h^* = \dfrac{r}{v_{ship}} \\ P_h(t_h^*) & t_h^* < \dfrac{r}{v_{ship}} \end{cases} \tag{A.7}$$

A4. Generic values of average transect speed and response time PDFs are shown in Figure A.1. The figure assumes helicopter launch delays fit a normal distribution and compares results from the theoretical curves defined above with results from a GAPP-style algorithm, showing good agreement. The top panel, which shows transit speed, follows the shape of a scaled error function, as is expected for a normal distribution. The bottom panel shows response-time curves for different four distances and is useful for understanding (A.7). In the first two response time panels, the ship commonly arrives on scene before the helicopter has had time to launch. For cases where the helicopter launches before arrival on scene, the response-time curve fits part of a bell-curve. However, any times where the launch would have been after the ship already arrives on scene get binned together, resulting in a spike of probability at a time that matches when the ship arrives on scene. In the last two panels, the helicopter always launches before the ship arrives on scene, so the response time fits a distribution like that of the helicopter launch delay.

A5. Examples of $v(r)$ for these two cases are shown in Figure A.2. The figure shows cases for the three ships listed in Table 1, with and without an uncertainty in launch time. For the solid line, there is no variability in the launch time, while for the dashed line the launch time is extracted from a normal distribution with a standard deviation of $\sigma_{t_h} = 3$ minutes.

A.2 Distance Covered in a Known Time

A6. The range covered in a time τ is

$$r_\tau = \begin{cases} v_{ship}\tau & \tau \leq t_h \\ v_{helo}\tau - \left(v_{helo} - v_{ship}\right) t_h & \tau > t_h \end{cases} \tag{A.8}$$

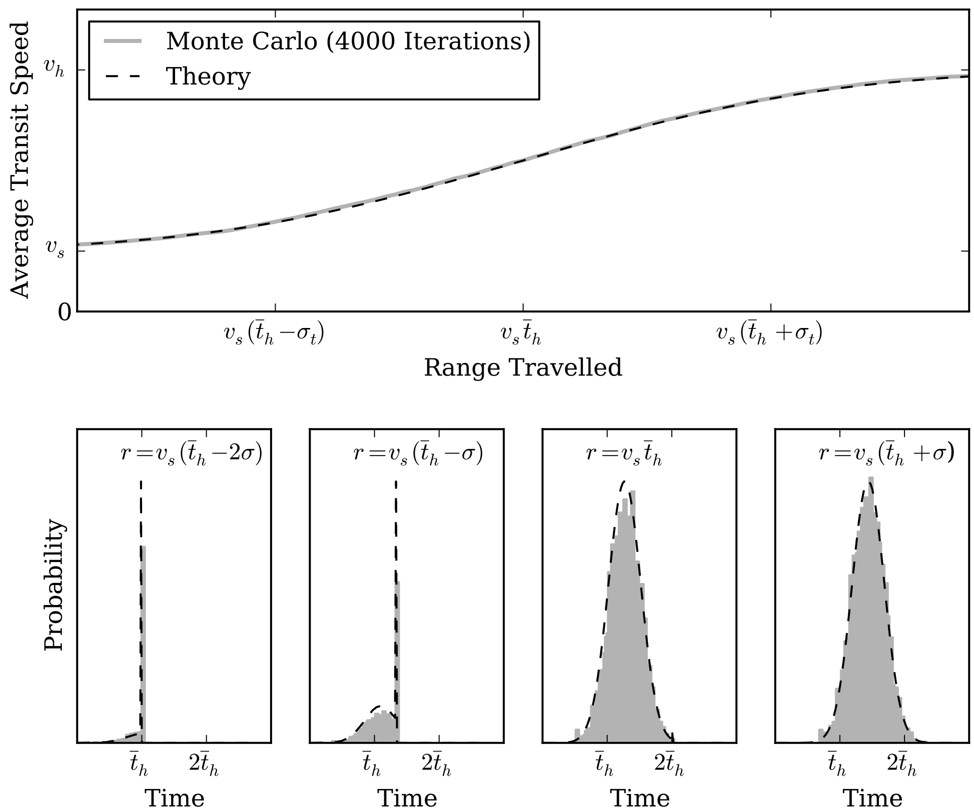

Figure A.1: *Average transit speed for a helicopter with normally distributed launch delay (top panel), and representative response times (bottom panels).*

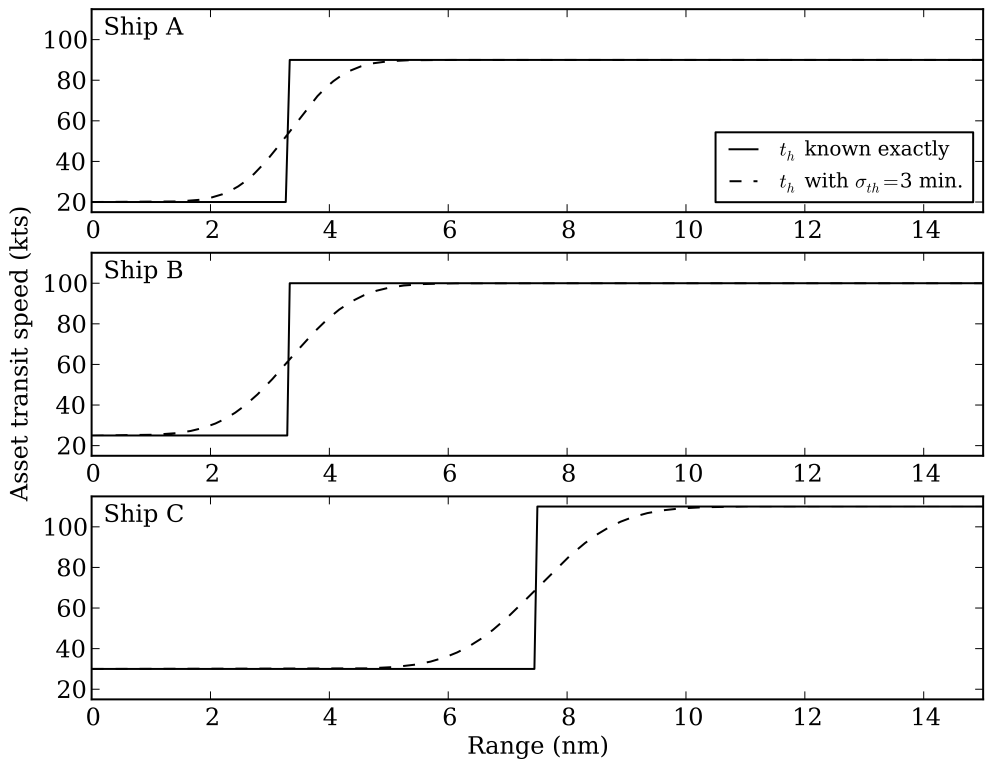

Figure A.2: *Velocity profiles for 2D model using exact and probabilistic treatments to helicopter launch time.*

Thus, when the launch delay follows $t_h \sim P_h(t)$, the range covered follows

$$r_\tau \sim P_\tau(r) = \begin{cases} 0 & \tau > \dfrac{r}{v_{ship}} \\[2ex] \displaystyle\int_{\frac{r}{v_{ship}}}^{\infty} P_h dt & \tau = \frac{r}{v_{ship}} \\[2ex] P_h\left(t_h^*\right) & \tau < \dfrac{r}{v_{ship}} \end{cases} \tag{A.9}$$

A.3 Covering a Known Distance in a Known Time

A7. The likelihood that an asset can reach a range r within a time τ is

$$P(r,\tau) = \begin{cases} 1 & r \leq v_{ship}\tau \\[2ex] \displaystyle\int_0^{t_h^*} P_h(t)\, dt & v_{ship}\tau < r \leq v_{helo}\tau \\[2ex] 0 & r > v_{helo}\tau \end{cases} \tag{A.10}$$

Figure A.3 shows results when $\tau = 30$ minutes for the three ships in Table 1.

A.4 Application to 2D models

A8. In the absence of any uncertainty in the helicopter launch delay, conversion of the probability density function $f(r)$ to response time $g(t)$ is calculated as [11]

$$g(t) = f(r)\left|\frac{dr}{dt}\right| = v\, f(r) \tag{A.11}$$

where $v = v(r)$ is the asset speed at range r. When the launch delay t_h is known exactly, v is known exactly at all times/distances, which makes (A.11) easy to solve. However, incorporation of variability in helicopter launch time will affect the PDF and the probability function becomes

$$g(\tau) = \int_0^{\infty} f(r) P_h^*\left(t_h^*\right) dr. \tag{A.12}$$

A9. An example of this transformation is shown in Figure A.4. The figure applies the helicopter launch profile of Ship A (with a standard deviation of $\sigma = 3$ minutes) to the distance PDF of the Random patrol strategy shown in Figure 10. In Figure A.4, the bottom panel reproduces the distance PDF shown in Figure 10. The centre panel shows a contour

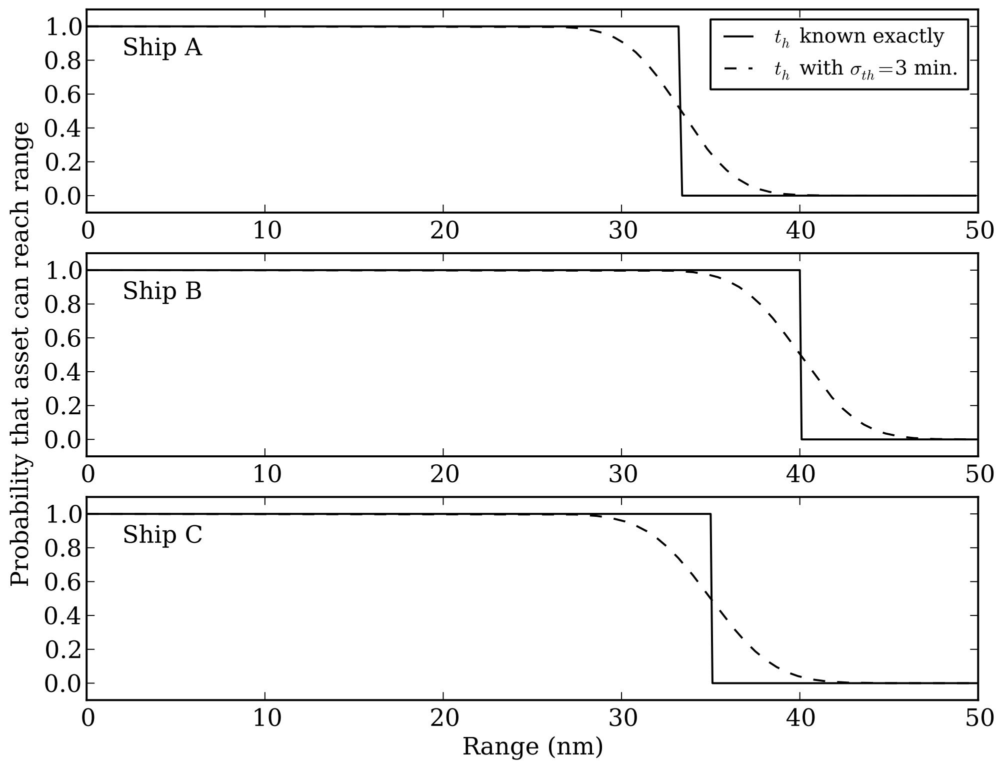

Figure A.3: *30-minute range of various assets.*

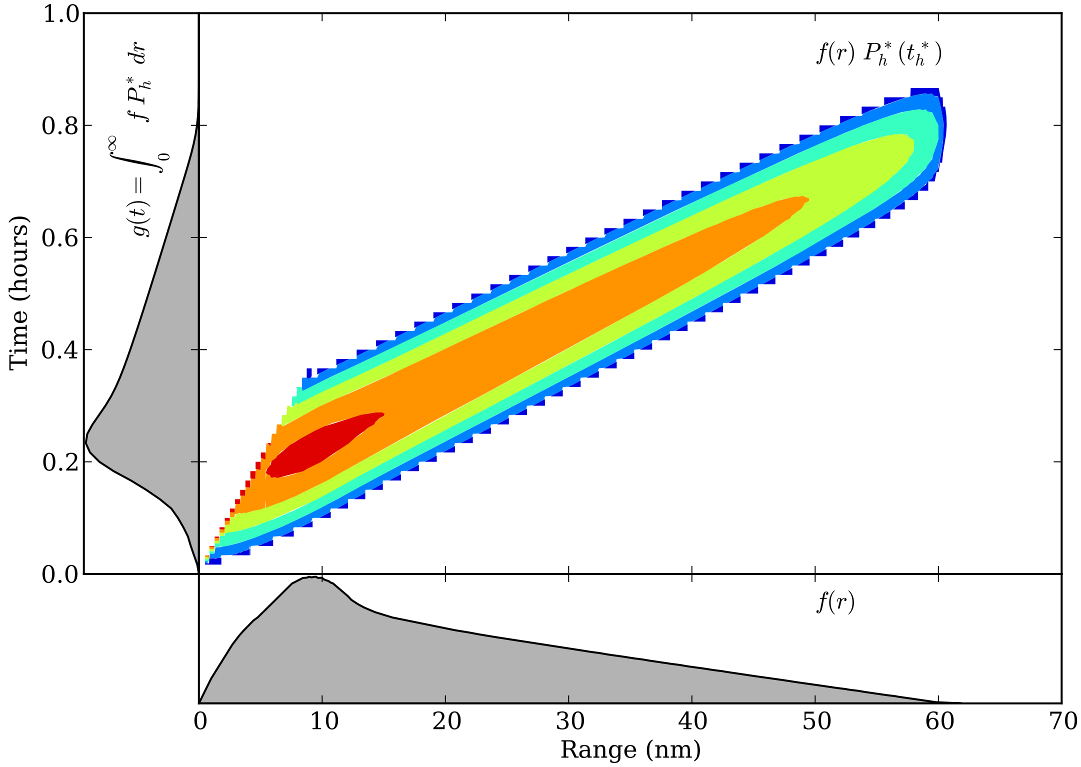

Figure A.4: *Deriving PDF of response times from PDF of response distance with uncertainty in helicopter launch.*

plot $f(r)P_h^*$ on a log scale (with values less than 1 in a million being masked). The left panel shows the result of (A.12), i.e. integrating the contour plot along lines of constant t.

A.5 Application to 1D Model

A10. When the exact launch time is not known, the probability of reaching a target in a domain of length L in a time τ can be described by

$$P_c(L,\tau) = \int_0^{L/2} \frac{2r}{L} P(r,\tau)dr + \int_{L/2}^{\infty} P(r,\tau)dr \tag{A.13}$$

for the Centred-1D approach and

$$P_R(L,\tau) = \int_0^{L/2} \frac{2r}{L}\left(L - \frac{r}{2}\right) P(r,\tau)dr + \int_{L/2}^{L} \frac{r}{L}\left(2 - \frac{r}{L}\right) P(r,\tau)dr + \int_{L}^{\infty} P(r,\tau)dr \tag{A.14}$$

for the Random-1D approach, where $P(r,\tau)$ is defined in (A.10).

A.6 Other Time Delays

A11. Although not discussed in the demonstrations used here, other delays may also be incorporated in identifying the total response time. For example, GAPP also permits one to consider the time required for commercial vessels to broadcast that they are under attack (which we will assume has the PDF $R(t)$) as well as the time required a ship to begin its response from when it learns of the attack (which we will assume has the PDF $S(t)$). Incorporating these effects gives a total response time PDF

$$\hat{g}(t) = g(t) * R(t) * S(t) \tag{A.15}$$

where $*$ is the convolution operator.

Annex B: Radial Propagation Algorithm

B1. This annex describes the radial propagation algorithm that provides an alternative to the Marsaglia et al. [10] approach. The premise is that the proportion of a rectangular PA of area A that is some range r from an asset is

$$f(r) = \frac{\Delta\theta \; r \; \delta r}{A} \tag{B.1}$$

where $\Delta\theta$ is the arc of a circle of radius r that passes through the patrol region (Figure 4) and δr is an infinitesimally small width.

B2. In presenting the approach, it is assumed that the asset is positioned at one edge of the patrol area and is either outside the patrol area or bordering it. To consider cases where the asset is somewhere along the length of the patrol area, or inside it, it is necessary to break the problem into components. The left panel of Figure B.1 shows a situation where the asset is within the PA, making it necessary to divide the PA into four sections with the asset at a point where the four sections meet.

B.1 Calculation of PDF

B3. The PDF itself is calculated as $f = f(r, h_1, h_2, \ell)$ where h_1 and h_2 are the lower and upper bounds of the region and ℓ is the length (see the right panel of Figure B.1 for a definition sketch). The region of interest can have one of two possible configurations. In the first (Case 1), the rectangle is long and thin, such that $h_2^2 \leq \ell^2 + h_1^2$. In the second (Case 2), the rectangle is short and wide, such that $h_2^2 > \ell^2 + h_1^2$.

B4. For Case 1, the PDF is described by inserting the following definition of $\Delta\theta$ into (B.1):

$$\Delta\theta(h_1, h_2, r, \ell) = \begin{cases} 0 & r < h_1 \\ \dfrac{\pi}{2} - \arcsin\left(\dfrac{h_1}{r}\right) & h_1 \leq r < h_2 \\ \arcsin\left(\dfrac{h_2}{r}\right) - \arcsin\left(\dfrac{h_1}{r}\right) & h_2 \leq r < \sqrt{\ell^2 + h_1^2} \\ \arcsin\left(\dfrac{h_2}{r}\right) - \arccos\left(\dfrac{\ell}{r}\right) & \sqrt{\ell^2 + h_1^2} \leq r \leq \sqrt{\ell^2 + h_2^2} \\ 0 & r > \sqrt{\ell^2 + h_2^2} \end{cases} \tag{B.2}$$

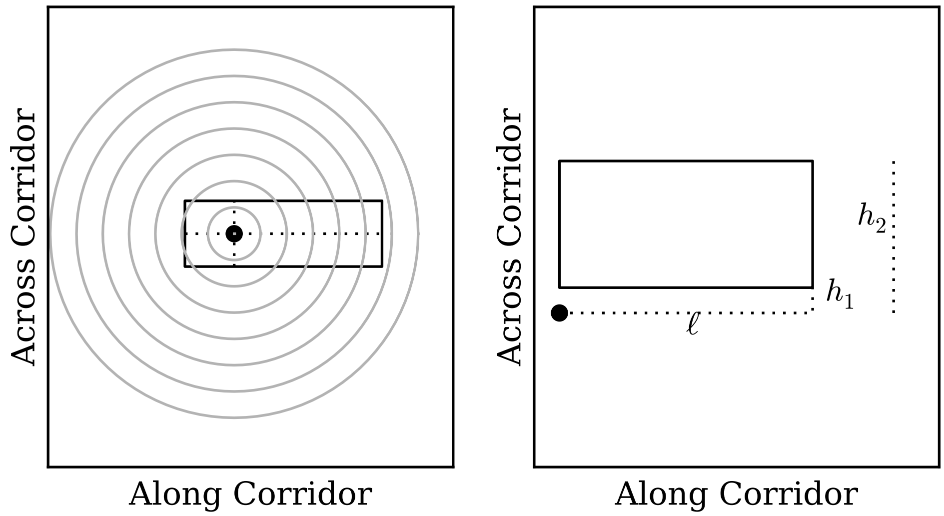

Figure B.1: *(Left) Division of Patrol Area based on asset position. (Right) Dimensions used in calculations.*

For Case 2, the definition of $\Delta\theta$ is

$$\Delta\theta(h_1,h_2,r,\ell)=\begin{cases} 0 & r<h_1 \\ \frac{\pi}{2}-\arcsin\left(\frac{h_1}{r}\right) & h_1\le r<\sqrt{\ell^2+h_1^2} \\ \frac{\pi}{2}-\arccos\left(\frac{\ell}{r}\right) & \sqrt{\ell^2+h_1^2}\le r<h_2 \\ \arcsin\left(\frac{h_2}{r}\right)-\arccos\left(\frac{\ell}{r}\right) & h_2\le r\le\sqrt{\ell^2+h_2^2} \\ 0 & r>\sqrt{\ell^2+h_2^2} \end{cases} \tag{B.3}$$

B.2 Application to Patrol Strategies

B5. The symmetry embedded in the patrol strategies and IRTC layout are used to reduce number of calculations required to identify f. For example, for the centred patrol strategy the region is bisected along the minor axis of the rectangle, so it is only necessary to calculate the distribution for one half of the region. Similarly, the symmetry of the IRTC requires that only one of the two lanes requires consideration. The results calculated with this approach are identical to those calculated by the Marsaglia et al. approach in Figure 10.

B.2.1 Centred

B6. For the Centred Patrol Strategy, $h_1 = B/2$, $h_2 = h_1 + W$, $\ell = L/2$, and $A = WL$, where B is the width of the buffer zone, W is the width of an IRTC shipping lane, and L is the length of the PA. The PDF is found by inserting these values into (B.1), and multiplying by two since only half the domain is tracked by $\Delta\theta$.

$$f(r) = 2\frac{\Delta\theta\ r\ \delta r}{A} \tag{B.4}$$

B.2.2 Random Along

B7. For the RAL Patrol Strategy, $h_1 = B/2$, $h_2 = h_1 + W$, and $A = WL$ but ℓ varies. As a result the PDF is found by integrating along the path, i.e.

$$f(r) = \frac{r\delta r}{A}\frac{2}{L}\int_0^L \Delta\theta d\ell \tag{B.5}$$

B.2.3 Random Across

B8. For the RAC Patrol Strategy, $\ell = L/2$ and $A = WL$ but h_1 and h_2 depend on where across the transect the asset is located. As a result

$$f(r) = \frac{r\delta r}{A}\frac{1}{2W+B}\left[\int_0^{W+B}\Delta\theta(h, 2W+B-h, r, L/2)dh + 2\int_0^W \Delta\theta(0, W-h, r, L/2)dh\right] \tag{B.6}$$

B.2.4 Random

B9. The Random Patrol strategy is similar to the RAL and RAC constructs, except that it is necessary to integrate both horizontally and vertically, i.e.

$$f(r) = \frac{r\delta r}{A}\frac{2}{L(2W+B)}\int_0^L\left[\int_0^{W+B}\Delta\theta(h, 2W+B-h, r, \ell)dh + 2\int_0^W \Delta\theta(0, W-h, r, \ell)dh\right]d\ell \tag{B.7}$$

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Annex C: Theory of Marsaglia et al. (1990)

C1. This annex provides an overview of the theoretical results presented by Marsaglia et al. [10] that are used to develop the 2D response model. Their work considers the probability distribution of the square of the distance between two points in two random rectangles of arbitrary size.

C2. The two rectangles (Rectangle A and Rectangle B) are described by their lengths in the x and y directions, i.e. a_x and a_y for Rectangle A and b_x and b_y for Rectangle B. Note that in their analysis the sides of the rectangles are parallel to the x and y axes. The spatial relationship between the two rectangles is identified by $\mathbf{c} = (c_x, c_y)$ that defines the translation vector required to get from the top-right corner of Rectangle A to the bottom-left corner of Rectangle B (Figure C.1). Note that in the figure, both c_x and c_y are positive, however they can be negative. By contrast, a_x, a_y, b_x, and b_y are always positive. For example, if one wished to identify the two random points in a square with sides of length ℓ with this model, then $a_x = a_y = b_x = b_y = \ell$ and $c_x = c_y = -\ell$.

C3. If the squared distance between two random points in the two rectangles is expressed as r^2, and the density of the square of the distances is $f(r^2)$, then

$$f(r^2) = \frac{1}{4a_xb_xa_yb_y} \sum_{i,j,k,l,m,n\in\{0,1\}} \int_0^{r^2} (-1)^{i+j+k+l} \left[(-1)^m - \frac{q}{\sqrt{r^2-x}}\right]_+ \left[(-1)^n - \frac{s}{\sqrt{x}}\right]_+ dx \tag{C.1}$$

where $q = c_x + ia_x + jb_x$ and $s = c_y + ka_y + lb_y$, and the "plus" subscript indicates a spline notation, i.e. $Q_+ = \max(0, Q)$.

C4. Solving (C.1) requires the solution to four possible integrals:

$$\begin{aligned}
\mathbf{I1}: &\quad \int_0^{r^2} \left(1 - \frac{q}{\sqrt{r^2-x}}\right)_+ \left(1 - \frac{s}{\sqrt{x}}\right)_+ dx = \mathscr{F}(r^2 - q_+^2, s_+^2, r^2, q, s) \\
\mathbf{I2}: &\quad \int_0^{r^2} \left(1 - \frac{q}{\sqrt{r^2-x}}\right)_+ \left(-1 - \frac{s}{\sqrt{x}}\right)_+ dx = -\mathscr{F}(0, \min((r^2 - q_+^2)_+, s^2), r^2, q, -s) \\
\mathbf{I3}: &\quad \int_0^{r^2} \left(-1 - \frac{q}{\sqrt{r^2-x}}\right)_+ \left(1 - \frac{s}{\sqrt{x}}\right)_+ dx = -\mathscr{F}(0, \min((r^2 - s_+^2)_+, q^2), r^2, -q, s) \\
\mathbf{I4}: &\quad \int_0^{r^2} \left(-1 - \frac{q}{\sqrt{r^2-x}}\right)_+ \left(-1 - \frac{s}{\sqrt{x}}\right)_+ dx = \mathscr{F}(\min(r^2, s^2), (r^2 - q^2)_+, r^2, -q, -s)
\end{aligned} \tag{C.2}$$

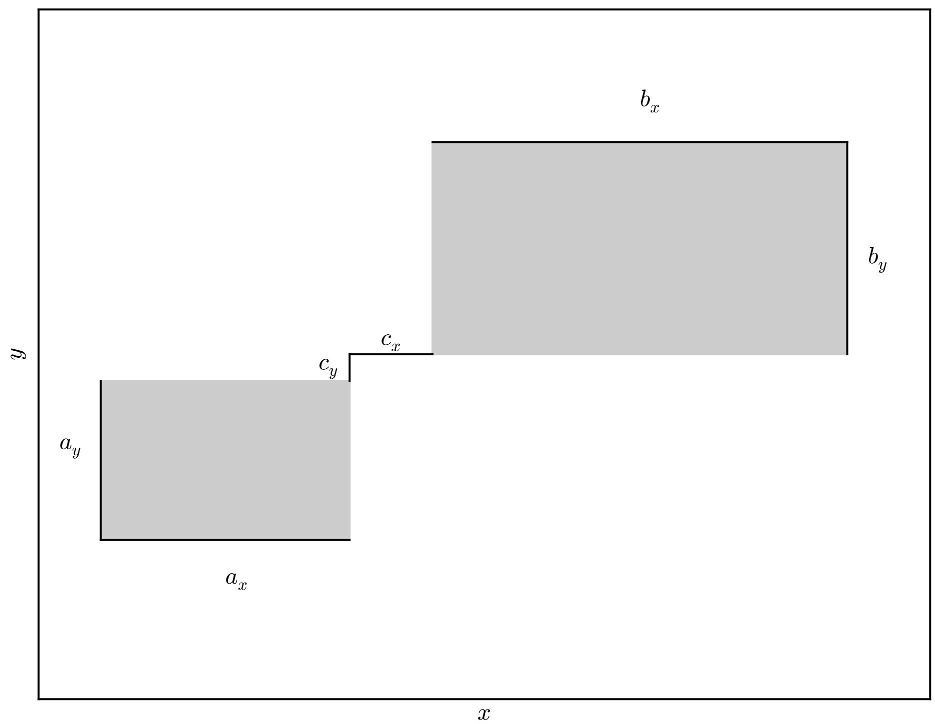

Figure C.1: *Spatial parameters used to describe rectangles.*

where

$$\mathscr{F}(x_1,x_2,x_3,x_4,x_5) = \begin{cases} 0 & x_1 \geq x_2 \\ G(x_2,x_3,x_4,x_5) - G(x_1,x_3,x_4,x_5) & \text{otherwise} \end{cases} \tag{C.3}$$

and

$$G(y_1,y_2,y_3,y_4) = y_1 + 2y_3\sqrt{y_2 - y_1} - 2y_4\sqrt{y_1} + y_3 y_4 \arcsin\left(\frac{2y_1}{y_2} - 1\right). \tag{C.4}$$

C5. It is noteworthy that both (C.2) and (C.4) have errors as presented in Marsaglia et al. They also provide FORTRAN code that has a few small errors. In preparing the equations given here, the Marsaglia model was tested heavily against the Monte Carlo and Ripple Propagation approaches to ensure correctness. If one is to turn to the original paper, take care when cross-referencing their equations with their code and with the equations provided here.

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Annex D: Error Calculations for 1D model

D1. The 1D model presented in this document estimates the part of the channel that a TF asset can reach within a certain amount of time by identifying the range that the asset can reach in that time, and then assuming that the total area is equal to the width of the channel times the range. This approach ignores the circular nature of the range, a problem that is illustrated in Figure 8.

D2. In Figure 8, the top panel shows a ship that is located just below a part of the channel it is patrolling. The part of the channel assumed to be within its reach is traced out by a rectangle (dotted line) that is a $2r$ long. Clearly from the figure, the model is overestimating the response capabilities since there are parts of the rectangle that are outside the range that can be reached by the asset. This section presents the theoretical analysis that allows the degree of overestimation, shown in Figure 9, to be quantified.

D3. The results rely heavily on the calculation of the area of half circles (representing half the ship range) cut by a horizontal line (representing the edge of a lane in corridor). The two areas are shown as a function of the range r and the height of the line h in Figure D.1 and defined as A_o and A_i.

D.1 Stationary Mode: Asset Remains Centred Between Shipping Lanes

D4. For cases where the strategy is for the patrol to remain between the shipping lanes (Stationary error correction), the area of a particular shipping lane that is within a range r depends on the size of the circle (Figure D.2). The figure shows an asset (red dot) and two IRTC lanes. Three circular regions are also depicted. In Region A, $r < B/2$. In Region B, $B/2 < r < W + B/2$. In Region C, $r > W + B/2$. Thus, the actual area covered when the ship remains between the two shipping lanes is

$$A = \begin{cases} 0 & r < \frac{B}{2} \\ A_o\left(r, \frac{B}{2}\right) & \frac{B}{2} \leq r < W + \frac{B}{2} \\ A_i\left(r, W + \frac{B}{2}\right) - A_i\left(r, \frac{B}{2}\right) & r \geq W + \frac{B}{2} \end{cases}, \quad \text{(D.1)}$$

and the correction factor for the actual area covered is

$$C = \frac{2rW}{A}. \quad \text{(D.2)}$$

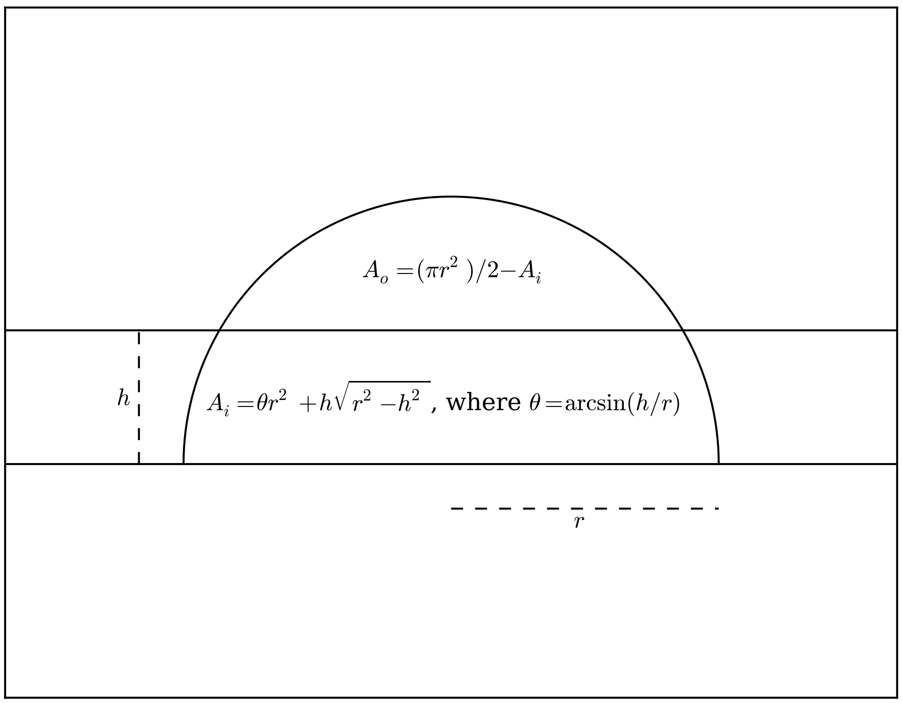

Figure D.1: *Area calculations used to determine overestimation correction for 1D model.*

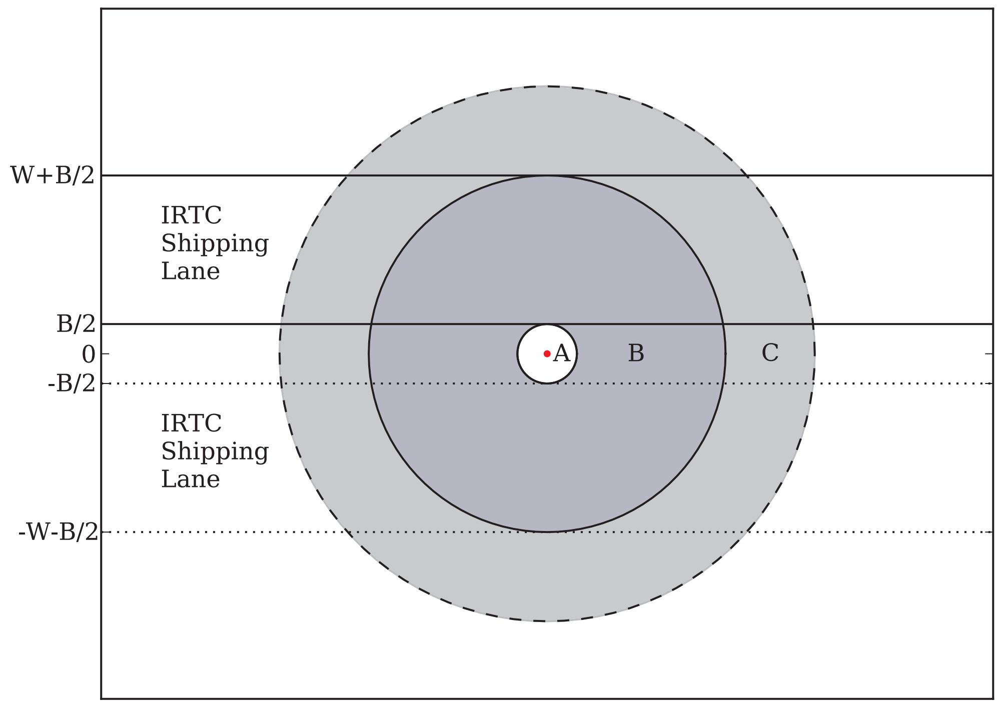

Figure D.2: *Calculating error correction when ship travels remains centred in corridor.*

Note that, due to the symmetry of the problem, only one lane needs to be considered to calculate the correction factor.

D.2 Across Mode: Asset Travels Across Shipping Lanes

D5. When the patrol travels across the shipping lanes (Across error correction), the area correction factor becomes more complex. It does remain symmetric, which simplifies calculations since only one shipping lane needs to be considered. The geometrical construct of the problem is shown in Figure D.3. Rather than having two areas as in the above problem, there are now two integrals that need to be considered:

$$I_i(r,h_1,h_2) = \int_{h_1}^{h_2} A_i(r,h)dh = \left\{ r^2 \left[h \arcsin\left(\frac{h}{r}\right) + \sqrt{r^2-h^2} \right] - \frac{(r^2-h^2)^{1.5}}{3} \right\}_{h_1}^{h_2} \quad \text{(D.3)}$$

and

$$I_o(r,h_1,h_2) = \int_{h_1}^{h_2} A_o(r,h)dh = \frac{\pi}{2}r^2(h_2-h_1) - I_i(r,h_1,h_2). \tag{D.4}$$

Tracing the path of the ship across the region shows that when the ship is traveling across the corridor, the average area becomes $A = A^*/(2WB)$, where

$$A^* = \begin{cases} 2I_i(r,0,r) + I_o(r,0,r) + \pi r^2(W-r) & r \leq W \\ 2I_i(r,0,r) + I_i(r,W,r) - I_i(r,0,r-W) + I_o(r,r-W,r) & W < r \leq L-W \\ 2I_i(r,0,W) + I_i(r,W,r) - 2I_i(r,0,r-W) + 2I_o(r,r-W,L-W) & L-W < r \leq L \\ 2I_i(r,0,W) + I_i(r,W,L) - I_i(r,0,L-W) & r > L \end{cases} \tag{D.5}$$

so the correction term is once again

$$C = \frac{2rW}{A}. \tag{D.6}$$

D.3 Range Coverage

D6. As previously discussed, the 1D model treats the area covered as a rectangular region with a width equal to the width of the corridor and a length equal to twice the range (assuming the ship is in the middle). This assumption neglects the radial spreading that takes place. As a result, while this model is useful for identifying the likelihood of intercepting an incident in a patrol region in a given amount of time, it does not produce reliable PDFs. If PDFs are desired, a 2D model is preferable.

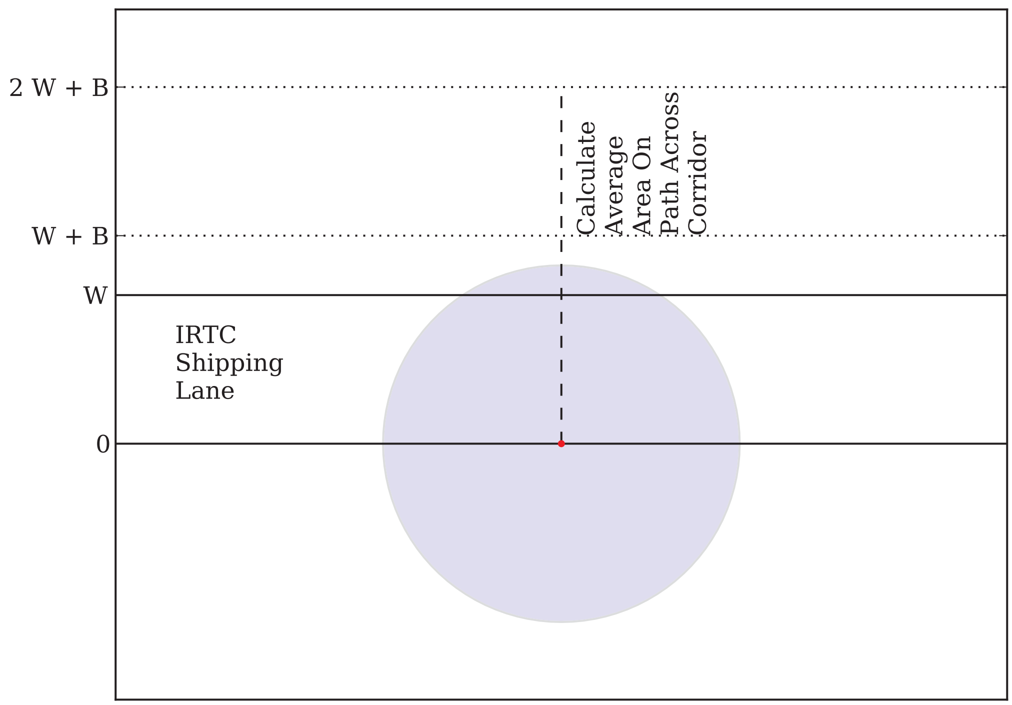

Figure D.3: *Calculating error correction when ship travels across corridor.*

List of Abbreviations Used

Term	Description
1D	One Dimensional
2D	Two Dimensional
AG	Action Group
CMF	Combined Maritime Force
CP	Counter-Piracy
CTF	Combined Task Force
EU NAVFOR	European Union Naval Force, Somalia
GAPP	Gulf of Aden Anti-Piracy Planning Tool
GOA	Gulf of Aden
H	High-readiness state
IMB	International Maritime Bureau
IRTC	International Recommended Transit Corridor
kts	knots (nautical miles per hour)
L	Low-readiness state
MAR	Maritime Systems Group
min	minutes
NATO	North Atlantic Treaty Organization
nm	nautical mile
PA	Patrol Area
PDF	Probability Density Function
RAC	Random Across Patrol Strategy
RAL	Random Along Patrol Strategy
TF	Task Force
TTCP	The Technical Cooperation Program

List of Mathematical Symbols Used

Symbol	Description
a_x	Width of rectangle a in x-direction
a_y	Width of rectangle a in y-direction
A	Area
A^*	Average area
A_i	Inner area of semi-circle cut by chord
A_o	Outer area of semi-circle cut by chord
b_x	width of rectangle b in x-direction
b_y	Width of rectangle b in y-direction
B	Width of IRTC buffer zone
c_x	x-translation from upper-right corner of rectangle a to lower-left corner of rectangle b
c_y	x-translation from upper-right corner of rectangle a to lower-left corner of rectangle b
C	Correction factor for 1D patrol models
f	Probability density function
$\mathscr{F}$	Distribution function in Marsaglia et al. derivation
F	Response function
g	Probability density function
$\hat{g}$	Convolved probability density function
G	Spline function in Marsaglia et al. derivation
h	Shortest distance from circle-centre to chord crossing circle
i, j, k, l, m, n	counters
I	Number of ships considered in proposed asset allocation model
I_i	Integral of A_i
I_o	Integral of A_o
ℓ	Partial length of a patrol area
L	Length of patrol area
M	Set of ships under control of command
N	Set of ships not under control of command
$P(r, \tau)$	Probability that an asset will be able to reach a range r in a time τ
$P_C(r, \tau)$	Probability of responding to an incident a distance r away in a time τ using a centred patrol strategy
P_h	Probability that a helicopter will have launched by time t.
P_h^*	Probability distribution for τ_r
$P_R(r, \tau)$	Probability of responding to an incident a distance r away in a time τ using a random patrol strategy
q	Distance variable in Marsaglia et al. derivation

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Symbol	Description
r	Distance or range covered by an asset
r_τ	Range covered by an asset in a time τ
R	Distribution of time delay for a commercial vessel to announce attack
$\mathscr{R}$	risk function
s	Distance variable in Marsaglia et al. derivation
S	Distribution of time delay for an asset to begin response
t	Time
t_h	Time delay for helicopter launch
t_h^*	Time delay required for an asset to exactly cover a range r in a time τ
$(t_h)_i$	Time delay for helicopter launch for asset i in proposed allocation model
v	Transit speed
$\overline{v}$	Average transit speed
v_{helo}	Helicopter transit speed
$\hat{v}_{helo}$	Helicopter transit speed corrected for wind
v_{ship}	Ship transit speed
v_{wind}	Wind speed
W	Width of IRTC shipping lane
x_i	Position of ship i in proposed asset allocation model
x_B	Fixed distance to edge of patrol area for ship B
x_1, x_2, x_3, x_4, x_5	Dummy variables in Marsaglia et al. derivation
y_1, y_2, y_3, y_4	Dummy variables in Marsaglia et al. derivation
z	Distance variable in Marsaglia et al. derivation
δr	Infinitesimal change in r
Δx_A, Δx_B	Range from asset A or B to edge of patrol area shared by assets A and B
$\Delta\theta$	Arc
ρ_i	response-time for asset i
ρ_{min}	minimum response-time
σ_{th}	standard deviation of helicopter launch delay t_h
θ	angle
θ_H	Change in helicopter azimuth
θ_w	Wind direction relative to desired direction of travel
τ	Response time
τ_r	Time required to cover a range r
$\overline{\tau}_r$	Mean time required to cover a range r

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This work examines the capability-models of ships performing counter-piracy patrols in the International Recommended Transit Corridor, located in Gulf of Aden (GOA). Specifically, it considers possible approaches to predict the response time of assets patrolling long and thin regions and to facilitate coordination between multiple assets.

For situations where the pirate attacks occur randomly, the across-channel location of the ship prior to the attack has only a limited impact on the response-time probability distribution, supporting the notion that the problem can be examined in a one-dimensional (1D) context. It is demonstrated that the 1D approach is extremely effective at reproducing the cumulative distributions of higher fidelity models. The 1D approach is then used to demonstrate that coordinating patrol ship positions is a key factor, while coordinating the rotation of helicopter crews between ships may be less important. It is also shown that for the types of wind fields in which pirates will operate in the GOA, a ship can reposition itself in a manner such that the winds will not heavily affect response capabilities.

Finally, a description of how the results from this work can be applied in the development of an asset positioning model are presented, and a description of the way forward is provided.

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Counter-piracy operations, modeling, response time, asset allocation

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