# **Multi-Resolution Estimation of Bottom Acoustic Properties**

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#### LONG-TERM GOAL

To develop an inversion procedure which will estimate the bottom acoustic properties with sufficient resolution to enable prediction of acoustic field over a range of frequencies.

#### **OBJECTIVES**

Investigate the usefulness of wavelets in determining the bottom compressional wave speed profile at different scales.

#### BACKGROUND

The wavelet  $\psi_{\lambda,u}(t)$  represents a family of functions  $\frac{1}{\sqrt{\lambda}}\psi\left(\frac{t-u}{\lambda}\right)$  where  $\lambda$  is the parameter that

controls the dilation or contraction of the function and u the translation of the function. In practical applications the scale parameter  $\lambda$  and translation parameter u are discretized. A discrete representation

of the wavelet is  $\psi_{m,n}(t) = \frac{1}{\sqrt{2^m}} \psi\left(\frac{t - n2^m}{2^m}\right)$ , where *m* is the scale parameter and *n* is the position

parameter. For certain class of wavelets the function  $\psi_{m,n}(t)$  are orthonormal to their dilates and translates. These wavelets can therefore form a complete set of basis functions for any function that has finite energy.

Let f(t) represent a function that has finite energy. The function f(t) can be approximated by a linear combination of the wavelets. It has been shown<sup>1</sup> that all the features of the function f(t) that are larger than the scale  $2^{m_0}$  can be approximated by a linear combination of the translates of the scaling function which is derived from the wavelet. Let this approximation be  $A_{m_0} f(t)$ . Then we have  $f(t) = A_{m_0} f(t) + d_{m_0}(t)$  where  $d_{m_0}(t)$  is called the detail. The approximation  $A_{m_0} f(t)$  can then be decomposed into its approximation at a higher scale and its detail. The approximations of the function at different scales can be obtained as outlined above.

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## APPROACH

We adopted two approaches for obtaining the estimates of the sediment compressional wave speed profile at different scales. In the first approach, the pressure field as a function of range obtained with a monochromatic source is used as data. Let  $p(r, \omega)$  be the measured pressure field. Using wavelets, approximation to the pressure field at different scales is first obtained. This approximated data can then be used to infer the sediment acoustic properties.

In the second approach, we use the scaling function at different scales to represent the unknown compressional wave speed profile. The unknown function c(z), which represents the compressional wave speed in the sediments, can then be approximated to scale *m* by the relation

$$A_m c(z) = \sum a_n^m \phi_{m,n}(z)$$

where  $\phi_{m,n}(z)$  is the scaling function at scale *m* and  $A_m c(z)$  is the approximation of c(z) to scale *m*. Inserting this expression into the inversion algorithm, we obtain the coefficients  $a_n^m$  which are then used in the equation above to obtain the compressional wave speed profile at scale *m*.

## WORK COMPLETED

The scheme was tested with synthetic data. We consider a range independent model of the ocean. The compressional wave speed profile in the sediment and the water column are shown in Figure 1. For this model and assuming a known constant density in the sediments, the pressure field as a function of range was obtained for a source frequency of 175 Hz. Approximations to the pressure field obtained using Daubechies wavelet for scales 7, 6, and 5 and the corresponding wavenumber spectrum are shown in Figure 2.



Figure 1: Sediment compressional wave speed profile and sound speed profile in water column. Depth of water column = 70 m.



Figure 2: True field and scale 7 and 5 approximations to the field and their spectrum.



Figure 3: Sediment compressional wave speed profile from different approximation to the field.

We note from Figure 2 that the number of propagating modes for the true field is 14. In the scale 7 approximation to the field there are only four propagating modes. However the scale 5 approximation of the field is in close agreement with the true field. Also the wavenumber spectrum is similar to that of the true field though the modal amplitudes of the higher order modes are less. The different approximations of the field, therefore, corresponds to spatially filtering the data with a band pass filter. The bottom compressional wave speed profile corresponding to these scales are presented in Figure 3. We note that at scale 7, the inversion results in estimation of the sediment compressional wave speed profile at very shallow depths. The inversion at scales 6 and 5 yield estimates of the compressional wave speed in the deeper layers. Similar analysis was performed using the data acquired during the Hudson Canyon experiment conducted in 1988. The results of this analysis are shown in Figure 4.

In the second approach, we obtained for the ocean model described earlier the eigenvalues of the propagating modes at 50 Hz, 100 Hz and 200 Hz using KRAKENC<sup>2</sup>. The use of modal eigenvalues for estimating the compressional wave speed profile in sediments has been well documented<sup>3</sup>. In our earlier work, boxcar functions were used to represent the compressional wave speed profile in the sediments. In this study we use scaling function derived from wavelet to represent the unknown compressional wave speed profile. The scaling function derived from Daubechies wavelet was used in the analysis. The results of reconstruction showing the results for different scales are presented in Figure 5.

## RESULTS

The approximation to the field using wavelets yields spatially filtered field in which at the highest scale only the low order modes exist. This provides an estimate of the acoustic properties close to the sediment water interface. As move to lower scales, the inversion yield properties at deeper depths. In the second method where we used multi-frequency data the maximum resolution attainable is related to the highest frequency of the source and is approximately half the wavelength at this frequency.

## **IMPACT/APPLICATIONS**

Multi-resolution method permits us to obtain the best possible estimate of the bottom properties that the data will support. It can be used to look selectively at different regions and obtain estimates with high resolution close to the water sediment interface and estimates with coarser resolution at deeper depths.

## **RELATED PROJECTS**

During the period of the contract, additional work on inversion for density profile from modal eigenvalues was completed. Also a comparison of local and global algorithms for inversion of bottom acoustic properties was performed.



Figure 4: Measured field (175 Hz) and field predicted by model from scale 5 approximation (Top). Model from scale 5 approximation of field and model obtained from multi-frequency inversion<sup>4</sup> (Bottom).



Figure 5: Sediment compressional wave speed profile at different scales from multifrequency data.

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