Physics-Based Modeling in Design & Development for U.S. Defense Conference

Laminated Composite Sandwich Plates with a Weak Compressible Core Impacted by Blast Loading

Terry Hause, Ph.D.,
Research Mechanical Engineer

And

Sudhakar Arepally
Deputy Associate Director
U.S. Army RDECOM-TARDEC, CASSI Analytics
Warren, MI 48397

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High bending stiffness and strength to weight ratio Excellent thermal and sound insulation Increased durability under a thermo-mechanical loading environment Tight thermal distortion tolerances Lightweight in structure

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OUTLINE

1. Motivation
2. Basic Assumptions and Preliminaries
3. Theoretical Developments
4. Solution Methodology
5. Blast Loading
6. Results
7. Concluding Remarks

MOTIVATION

- High bending stiffness and strength to weight ratio
- Excellent thermal and sound insulation
- Increased durability under a thermo-mechanical loading environment
- Tight thermal distortion tolerances
- Lightweight in structure

BASIC ASSUMPTIONS AND PRELIMINARIES

1. The face sheets fulfill the Love-Kirchoff assumptions and are thin compared with the core.
2. The bonding between the face sheets and the core is assumed to be perfect.
3. The kinematic boundary conditions at the interfaces between the core and the facings are satisfied.
4. The core is assumed to be a weak orthotropic transversely compressible core carrying only the transverse strains and the normal strain.
5. The shock wave pressure is uniformly distributed on the front face of the sandwich plate.

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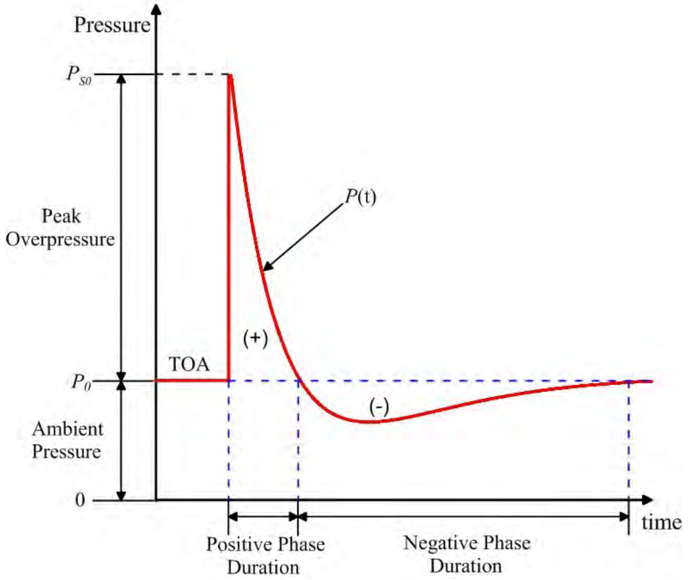

Fig 1a. Incident pressure profile

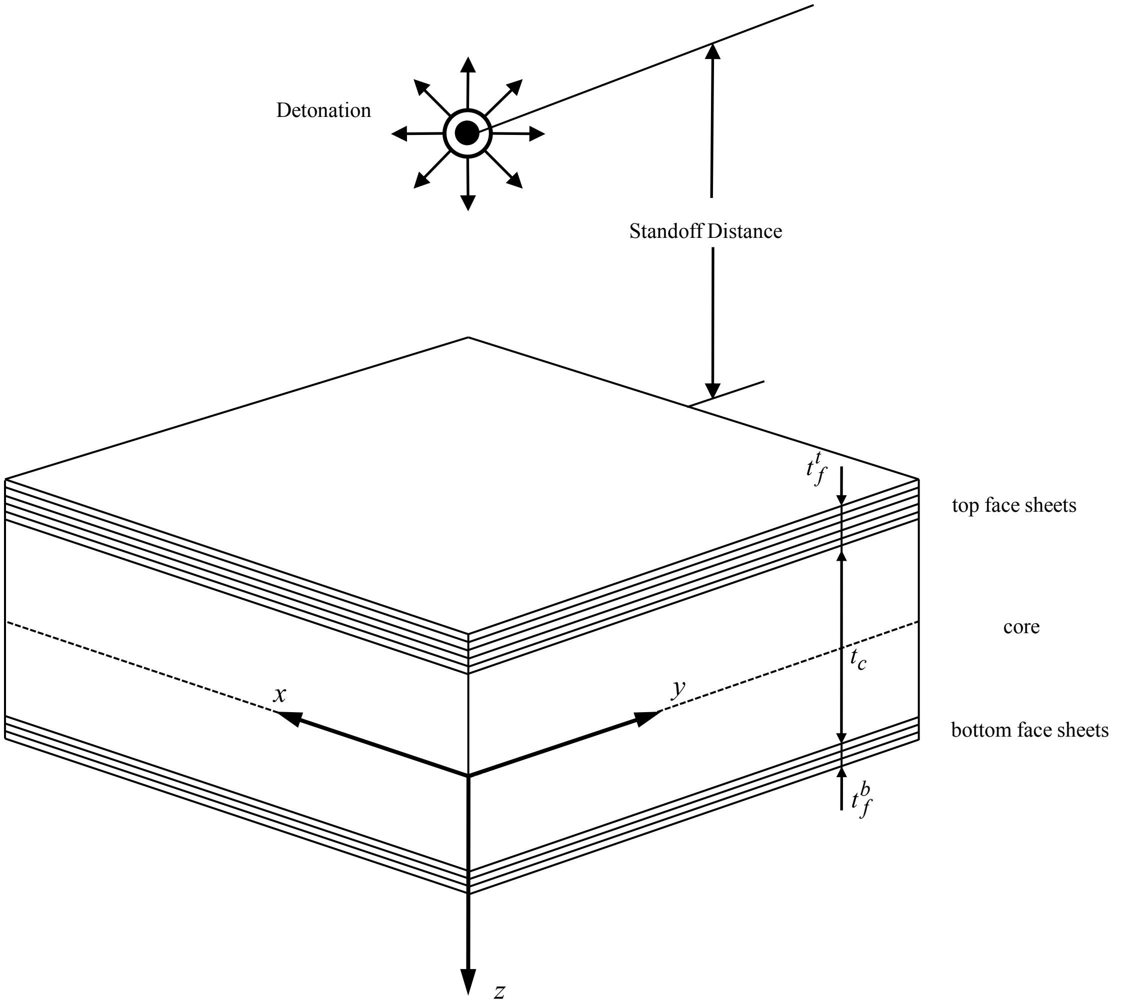

Fig 1b. An asymmetric sandwich plate under blast loading

THEORETICAL DEVELOPMENTS

Displacement Field

Top Face

$$v_\alpha^t = u_\alpha^a + u_\alpha^d - \left(x_3 + \frac{t_c + t_f^t}{2} \right) u_{3,\alpha}^a - \left(x_3 + \frac{t_c + t_f^t}{2} \right) u_{3,\alpha}^d$$

$$v_3^t = u_3^a + u_3^d$$

Bottom Face

$$v_\alpha^b = u_\alpha^a - u_\alpha^d - \left(x_3 - \frac{t_c + t_f^b}{2} \right) u_{3,\alpha}^a + \left(x_3 - \frac{t_c + t_f^b}{2} \right) u_{3,\alpha}^d$$

$$v_3^b = u_3^a - u_3^d$$

Core

$$v_\alpha^c = u_\alpha^a - \left(\frac{t_f^t - t_f^b}{4} \right) u_{3,\alpha}^a - \left(\frac{t_f^t + t_f^b}{4} \right) u_{3,\alpha}^d - \frac{2x_3}{t_c} u_\alpha^d + \left(\frac{t_f^t + t_f^b}{2t_c} \right) x_3 u_{3,\alpha}^a + \left(\frac{t_f^t - t_f^b}{2t_c} \right) x_3 u_{3,\alpha}^d + \left(\frac{4x_3^2}{t_c^2} - 1 \right) \Phi_\alpha^c$$

$$v_3^c(x,y,z,t) = u_3^a - \frac{2x_3}{t_c} u_3^d$$

Note:

the Greek indices have the range 1, 2, while the Latin indices have the range 1, 2, 3 and unless otherwise stated, Einstein's summation convention over the repeated indices is assumed. Also, denotes partial differentiation with respect to the coordinates , while superscripts *t* and *b* indicate the association with the top and bottom facings respectively.

Also,

$$u_i^a = \frac{1}{2}(u_i^t + u_i^b), \qquad u_i^d = \frac{1}{2}(u_i^t - u_i^b)$$

represent the average and the half difference of the face sheet mid-surface displacements while, the core displacements, Φ_α^c warping functions of the core.

Non-Linear Strain-Displacement Relationships

The strain-displacement relationships given by the Lagrangian Strain-Displacement Relationships used in conjunction with the Von-Karman assumptions is given in indicial notation as

$$\gamma_{11} = \upsilon_{1,1} + \frac{1}{2}(\upsilon_{3,1})^2$$

$$\gamma_{22} = \upsilon_{2,2} + \frac{1}{2}(\upsilon_{3,2})^2$$

$$\gamma_{33} = \upsilon_{3,3} + \frac{1}{2}(\upsilon_{3,3})^2$$

$$\gamma_{23} = \frac{1}{2}(\upsilon_{2,3} + \upsilon_{3,2}) + \frac{1}{2}\upsilon_{3,2}\upsilon_{3,3}$$

$$\gamma_{13} = \frac{1}{2}(\upsilon_{1,3} + \upsilon_{3,1}) + \frac{1}{2}\upsilon_{3,1}\upsilon_{3,3}$$

$$\gamma_{12} = \frac{1}{2}(\upsilon_{1,2} + \upsilon_{2,1}) + \frac{1}{2}\upsilon_{3,1}\upsilon_{3,2}$$

Substitution of the displacement relationships gives:

Top Layer

$$\gamma_{\alpha\beta}^{t} = \bar{\gamma}_{\alpha\beta}^{a} + \bar{\gamma}_{\alpha\beta}^{d} + \left(x_3 + \frac{t_c + t_f^t}{2} \right) \kappa_{\alpha\beta}^{a} + \left(x_3 + \frac{t_c + t_f^t}{2} \right) \kappa_{\alpha\beta}^{d}$$

Bottom Layer

$$\gamma_{\alpha\beta}^{b} = \bar{\gamma}_{\alpha\beta}^{a} - \bar{\gamma}_{\alpha\beta}^{d} + \left(x_3 - \frac{t_c + t_f^b}{2} \right) \kappa_{\alpha\beta}^{a} - \left(x_3 - \frac{t_c + t_f^b}{2} \right) \kappa_{\alpha\beta}^{d}$$

Where,

$$\bar{\gamma}_{\alpha\beta}^{a} = \frac{1}{2}(\bar{\gamma}_{\alpha\beta}^{t} + \bar{\gamma}_{\alpha\beta}^{b}), \quad \bar{\gamma}_{\alpha\beta}^{d} = \frac{1}{2}(\bar{\gamma}_{\alpha\beta}^{t} - \bar{\gamma}_{\alpha\beta}^{b})$$

$$\kappa_{\alpha\beta}^{a} = \frac{1}{2}(\kappa_{\alpha\beta}^{t} + \kappa_{\alpha\beta}^{b}), \quad \kappa_{\alpha\beta}^{d} = \frac{1}{2}(\kappa_{\alpha\beta}^{t} - \kappa_{\alpha\beta}^{b})$$

In the above expressions, $\bar{\gamma}_{\alpha\beta}^{(a,d)}$ are referred to as the average and half difference of tangential or membrane strains of the top and bottom facings; while, $\kappa_{\alpha\beta}^{(a,d)}$ are referred to as the average and half difference of the bending strains of the top and bottom facings. The expressions for the membrane and bending strains are not provided here.

For the core, the strain-displacement relationships take the form

$$\gamma_{i3}^{c} = \bar{\gamma}_{i3}^{c} + z\kappa_{i3}^{c}$$

In these expressions, $\bar{\gamma}_{i3}^{c}$ and κ_{i3}^{c} are the membrane and bending strains, respectively. These expressions are not provided here.

Constitutive Equations

Both the top and bottom face sheets are considered to be constructed from unidirectional fiber reinforced anisotropic laminated composites, the axes of orthotropy not necessarily being coincident with the geometrical axes. The stress-strain relationships for each lamina of the facings becomes

$$\begin{Bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ & \overline{Q}_{22} & \overline{Q}_{26} \\ \text{Sym} & & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \gamma_{11} \\ \gamma_{22} \\ 2\gamma_{12} \end{Bmatrix}$$

Where, $\overline{Q}_{ij}$ for *i, j* = (1, 2, 6) are the *Transformed plane-stress reduced stiffness measures*.

The stress-strain relationships for the orthotropic core with the geometrical and material axes coincident are expressed as

$$\tau_{33}^{c} = E^{c}\gamma_{33}^{c}, \quad \tau_{13}^{c} = G_{13}^{c}\gamma_{13}^{c}, \quad \tau_{23}^{c} = G_{23}^{c}\gamma_{23}^{c}$$

Hamilton's Variational Principle

$$\int_{t_0}^{t_1} (\delta U - \delta W - \delta T)dt = 0$$

U = strain energy,

W = represent the work done by external forces

T = represent the kinetic energy

$$\delta U = \int_A \left(\int_{-t_c/2-t_f^t}^{-t_c/2} \tau_{\alpha\beta}^t \delta\gamma_{\alpha\beta}^t dx_3 + \int_{-t_c/2}^{+t_c/2} \tau_{i3}^c \delta\gamma_{i3}^c dx_3 + \int_{t_c/2}^{t_c/2+t_f^b} \tau_{\alpha\beta}^b \delta\gamma_{\alpha\beta}^b dx_3 \right) dA$$

Where τ_{ij} are the tensorial components of the second Piola-Kirchoff stress tensor, while A is attributed to the area of the sandwich plate.

$$\delta W = \int_A \left(\hat{q}_3^t(x_1,x_2,t)\delta \upsilon_3^t + \hat{q}_3^b(x_1,x_2,t)\delta \upsilon_3^b - 2C^t \dot{\upsilon}_3^t \delta \upsilon_3^t - 2C^c \dot{\upsilon}_3^c \delta \upsilon_3^c - 2C^b \dot{\upsilon}_3^b \delta \upsilon_3^b \right) dA$$

Where $q^t(x_1,x_2,t)$ denotes the transverse pressure loading from a spherical air-blast and *C* is the structural damping coefficient per unit area of the plate.

$$\int_{t_0}^{t_1} \delta T dt = \int_{t_0}^{t_1} \int_A -\left(\int_{-t_c/2-t_f^t}^{-t_c/2} \rho_f^t \ddot{\upsilon}_3^t \delta \upsilon_3^t dx_3 + \int_{-t_c/2}^{t_c/2} \rho^c \ddot{\upsilon}_3^c \delta \upsilon_3^c dx_3 + \int_{t_c/2}^{t_c/2+t_f^b} \rho_f^b \ddot{\upsilon}_3^b \delta \upsilon_3^b dx_3 \right) dA dt$$

Where ρ^c and ρ_f^t, ρ_f^b are the mass densities of the core and the top and bottom face sheets, respectively, $\ddot{\upsilon}$ and denotes the transverse acceleration.

Equations of Motion

$$\delta u_\alpha^a : \quad N^a_{\alpha\beta,\beta} = 0$$ Eqs. (1) , (2)

$$\delta u_\alpha^d : \quad N^d_{\alpha\beta,\beta} + \frac{N^c_{\alpha 3}}{t_c} = 0$$ Eqs. (3) , (4)

$$\delta \Phi_\alpha^c : \quad M^c_{\alpha 3} = 0$$ Eqs. (5) , (6)

$$\delta u_3^a : \quad u^a_{3,\alpha\beta} N^a_{\alpha\beta} + M^a_{\alpha\beta,\alpha\beta} + u^d_{3,\alpha\beta} N^d_{\alpha\beta} + \frac{1}{t_c}\left(\frac{2t_c + t_f^t + t_f^b}{4} - u_3^d\right) N^\alpha_{\alpha 3,\alpha} - \frac{2}{t_c} u^d_{3,\alpha} N^c_{\alpha 3}$$
$$- \left(\frac{t_f^t \rho^t + t_f^b \rho^b + t_c \rho^c}{2}\right)\ddot{u}_3^a - \left(\frac{t_f^t \rho^t - t_f^b \rho^b}{2}\right)\ddot{u}_3^d - \left(\frac{C^t + C^b}{2} + C^c\right)\dot{u}_3^a$$
$$- \left(\frac{C^t - C^b}{2}\right)\dot{u}_3^d + \frac{\hat{q}_3^t + \hat{q}_3^b}{2} = 0$$ Eq. (7)

$$\delta u_3^d : \quad u_{3,\alpha\beta}^a N_{\alpha\beta}^d + M_{\alpha\beta,\alpha\beta}^d + u_{3,\alpha\beta}^d N_{\alpha\beta}^a + \left(1 - \frac{2}{t_c} u_3^d\right) N_{33}^c + \left(\frac{t_f^t - t_f^b}{4t_c}\right) N_{\alpha 3,\alpha}^c$$

$$- \frac{1}{2}\left(t_f^t \rho^t + t_f^b \rho^b + \frac{t_c \rho^c}{3}\right)\ddot{u}_3^d - \left(\frac{t_f^t \rho^t - t_f^b \rho^b}{2}\right)\ddot{u}_3^a - \left(\frac{C^t + C^b}{2}\right)\dot{u}_3^d - \left(\frac{C^t - C^b}{2}\right)\dot{u}_3^a$$

$$+ \frac{\hat{q}_3^t - \hat{q}_3^b}{2} = 0 \qquad \text{Eq. (8)}$$

Where, the global stress resultants and stress couples are defined as

$$\left(N_{\alpha\beta}^a, M_{\alpha\beta}^a\right) = \frac{1}{2}\left\{\left(N_{\alpha\beta}^t + N_{\alpha\beta}^b\right), \left(M_{\alpha\beta}^t + M_{\alpha\beta}^b\right)\right\}$$

$$\left(N_{\alpha\beta}^d, M_{\alpha\beta}^d\right) = \frac{1}{2}\left\{\left(N_{\alpha\beta}^t - N_{\alpha\beta}^b\right), \left(M_{\alpha\beta}^t - M_{\alpha\beta}^b\right)\right\}$$

Where the local stress resultants and stress couples are given as:

$$\left\{N_{\alpha\beta}^{t}, M_{\alpha\beta}^{t}\right\} = \int_{-t_c/2-t_f^t}^{-t_c/2} \tau_{\alpha\beta}^{t}\left\{1, \left(x_3 + \frac{t_c + t_f^t}{2}\right)\right\} dx_3$$

$$\left\{N_{\alpha\beta}^{b}, M_{\alpha\beta}^{b}\right\} = \int_{t_c/2}^{t_c/2+t_f^b} \tau_{\alpha\beta}^{b}\left\{1, \left(x_3 - \frac{t_c + t_f^b}{2}\right)\right\} dx_3$$

$$\left\{N_{i3}^{c}, M_{i3}^{c}\right\} = \int_{-t_c/2}^{-t_c/2} \tau_{i3}^{c}(1, x_3)\, dx_3, \qquad (i = 1, 2, 3)$$

Boundary Conditions

For the case of *simply supported boundary conditions*, the boundary conditions become:
Along the edges $x_n = (0, L_n)$

$$N_{nn}^{a} = N_{nn}^{d} = N_{nt}^{a} = N_{nt}^{d} = M_{nn}^{a} = M_{nn}^{d} = u_3^{a} = u_3^{d} = 0$$

n and *t* are the normal and tangential directions to the boundary. When $n = 1,\ t = 2$ and when $n = 2,\ t = 1$

Solution Mthodology

Special Case: Symmetric orthotropic single layer facings

In fulfillment of the geometric boundary conditions, a suitable representation for u_3^a, and u_3^d is given by:

$$u_3^a = w_{mn}^a(t)\sin(\lambda_m x_1)\sin(\mu_n x_2)$$

$$u_3^d = w_{mn}^d(t)\sin(\lambda_m x_1)\sin(\mu_n x_2)$$

Where, $\lambda_m = m\pi/L_1$, $\mu_n = n\pi/L_2$

The transverse explosive loading is represented as

$$q_t(x_1, x_2, t) = q_{mn}(t)\sin(\lambda_m x_1)\sin(\mu_n x_2),$$

which implies through integration of both sides over the plate area that

$$q_{mn}(t) = \frac{4}{L_1 L_2} \int_0^{L_2} \int_0^{L_1} q_t(x_1, x_2, t) \sin(\lambda_m x_1) \sin(\mu_n x_2) dx_1 dx_2$$

Letting,

$$q_t(x_1, x_2, t) = q_t(t) = (q_{S0} - q_0)[1 - (t - t_a)/t_p] \exp[-\alpha(t - t_a)/t_p]$$

And integrating gives

$$q_{mn}(t) = \frac{16 q_t(t)}{mn\pi^2}$$

The first two Equations of Motion can be satisfied by a stress potential in conjunction with a compatibility equation not provided here. Equations of Motion (3) through (6) can be shown to be satisfied by expressing these equations of motion in terms of displacements and assuming appropriate functional forms in terms of unknown constant coefficients and the amplitudes as a function of time. The unknown constants are determined by substitution and comparing coefficients.

At this point the Extended-Galerkin Method is utilized by retaining the last two Equations of Motion within the energy functional and carring out the indicated integrations results in two nonlinear coupled second order ordinary differential equations in terms of the modal amplitudes. These are given as:

$$m_1\ddot{w}_{mn}^{a} + C\dot{w}_{mn}^{a} + C_{10}^{a}w_{mn}^{a} + C_{11}^{a}w_{mn}^{a}w_{mn}^{d} + C_{12}^{a}w_{mn}^{a}(w_{mn}^{d})^2 + C_{30}^{a}(w_{mn}^{a})^3 = \frac{q_{mn}}{2}$$

$$m_2\ddot{w}_{mn}^{d} + C\dot{w}_{mn}^{d} + C_{01}^{d}w_{mn}^{d} + C_{02}^{d}(w_{mn}^{d})^2 + C_{03}^{d}(w_{mn}^{d})^3 + C_{20}^{d}(w_{mn}^{a})^2 + C_{21}^{d}(w_{mn}^{a})^2 w_{mn}^{d} = \frac{q_{mn}}{2}$$

The coefficients $C_{10} - C_{12}, C_{30}, C_{01} - C_{03}, C_{20}, C_{21}$ are expressions which depend on the material and geometrical properties of the structure.

These two governing differential equations are then solved using the 4th Order runge-Kutta Method.

Blast Loading

For a free in-air spherical air burst, the pressure profile over time is given in figure 2 as

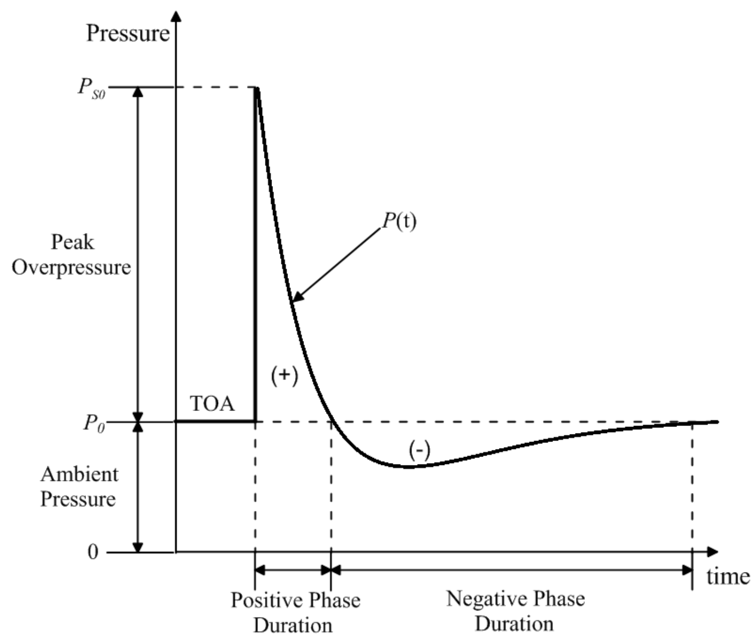

Fig 2. Incident Profile of a blast wave

The wave form shown in figure 4 is given by an expression known as The Friedlander equation and is give as

$$P_t(t) = (P_{so} - P_o)\left(1 - \frac{t - t_a}{t_p}\right) e^{-\alpha\left(\frac{t - t_a}{t_p}\right)}$$

Where,

$$P_{so} = \frac{1772}{Z^3} - \frac{114}{Z^2} + \frac{108}{Z} \Rightarrow \text{Peak Overpressure over ambient}$$

$$Z = R / W^{1/3} \Rightarrow \text{scaled distance} \begin{cases} R \text{ is the Standoff Distance} \\ W \text{ is the equivalent weight of charge} \\ \text{of TNT in terms of kilograms} \end{cases}$$

P_o is the ambient pressure

t_a is the time of arrival

t_p is the positive phase duration of the blast wave

t is the time

For conditions of STP at sea level, the time of arrival and the positive phase duration can be determined from

Arrival time or positive phase duration

$$\frac{t}{t_1} = \frac{R}{R_1} = \left(\frac{W}{W_1}\right)^{\frac{1}{3}} \Rightarrow \text{Cube root scaling}$$

Arrival time or positive phase duration for a reference explosion of charge weight, W_1

It should be noted that the standoff distances are themselves scaled
According to the cube root law

Results-*Validation*

To validate the present approach, the dynamic response of a simply supported plate impacted by a uniform pressure pulse was chosen from R.S. Alwar et Al.

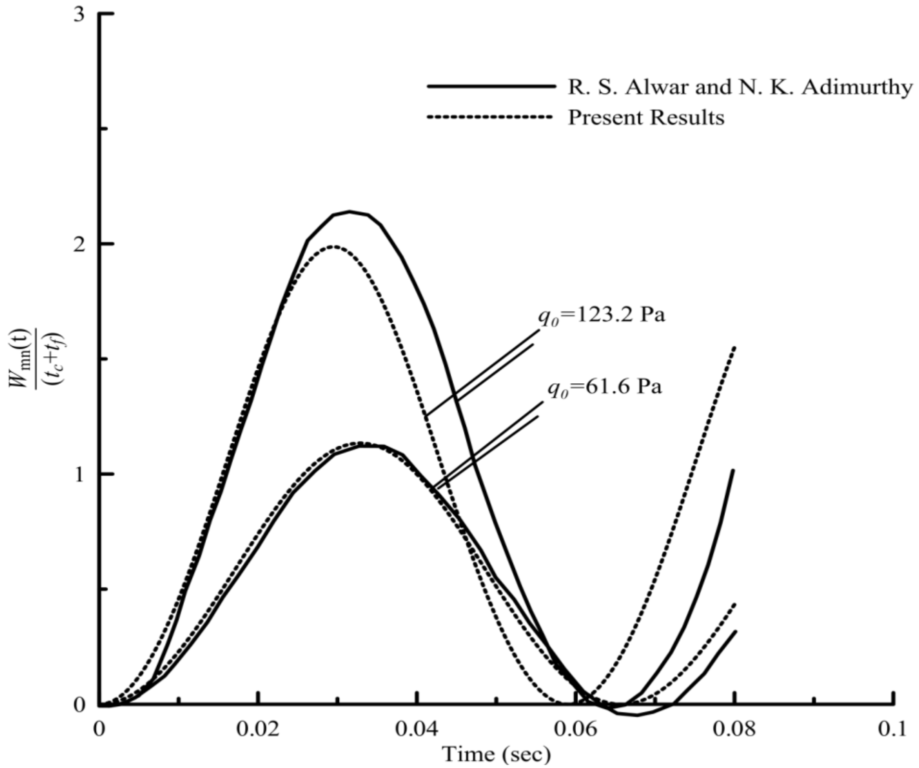

Fig 3. The nondimensional global deflection-time response of a simply supported sandwich plate impacted by a uniform pressure pulse

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Results-*Present*

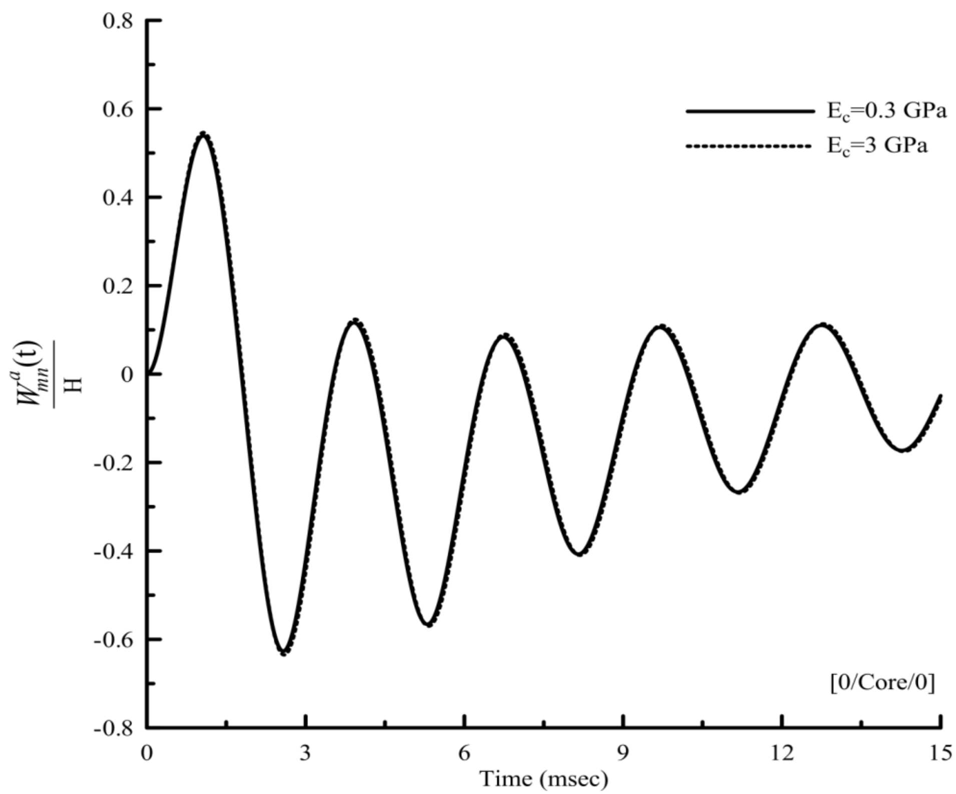

Fig. 4 The effect of the transverse modulus of the core on the global response of a sandwich plate with orthotropic facings.

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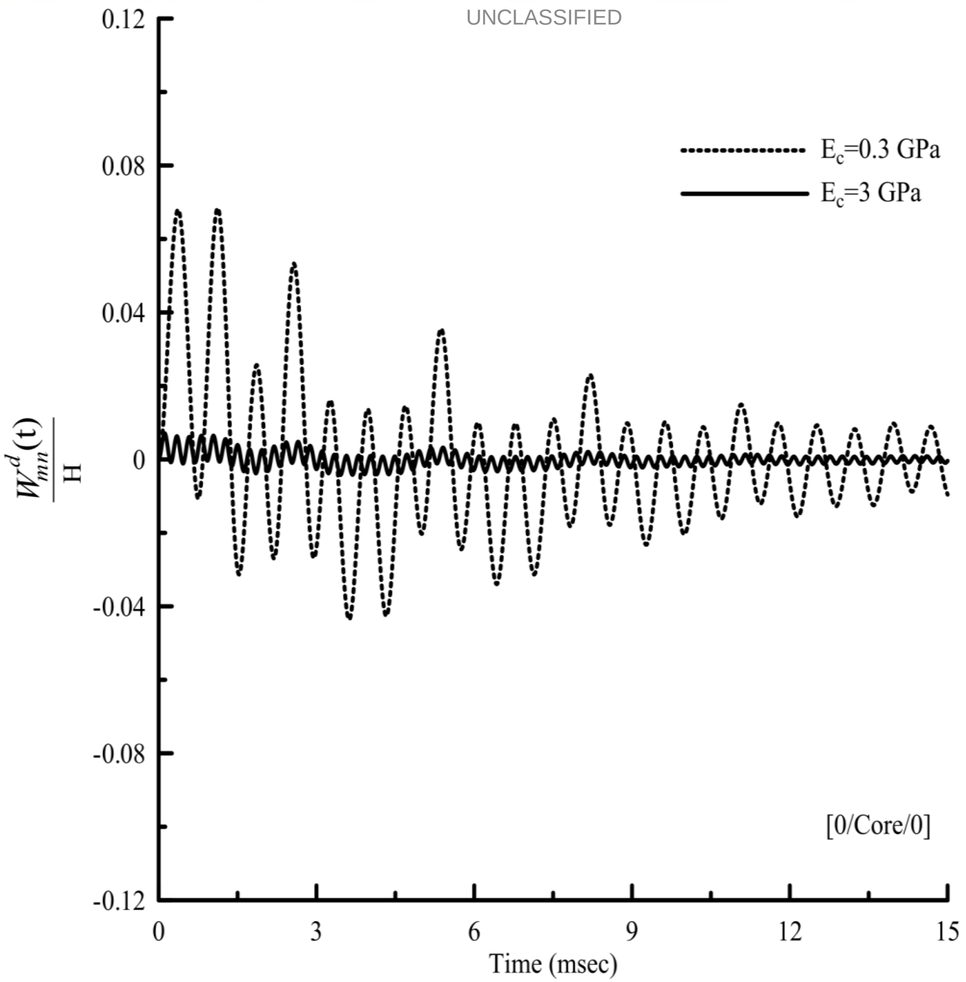

Fig. 5 The counterpart of Fig. 4 for the wrinkling response of a sandwich plate.

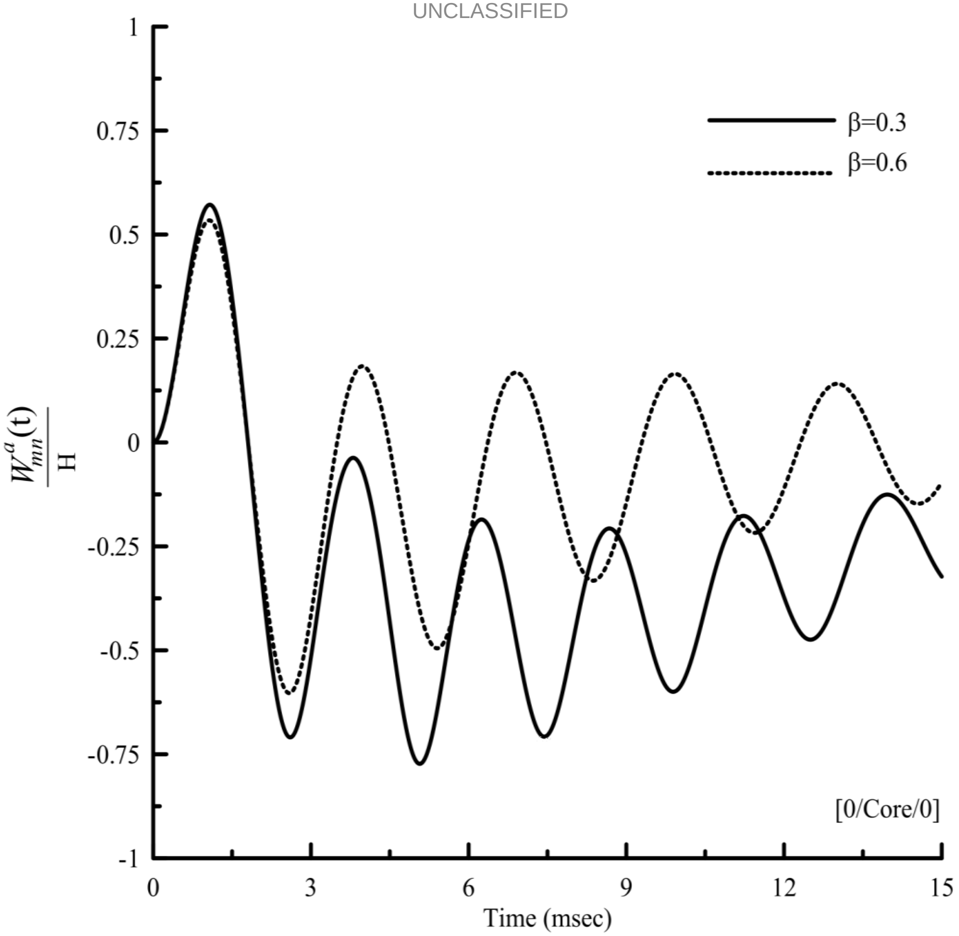

Fig. 6 The effect of the rate-of-decay parameter on the global response of a sandwich plate with orthotropic facings.

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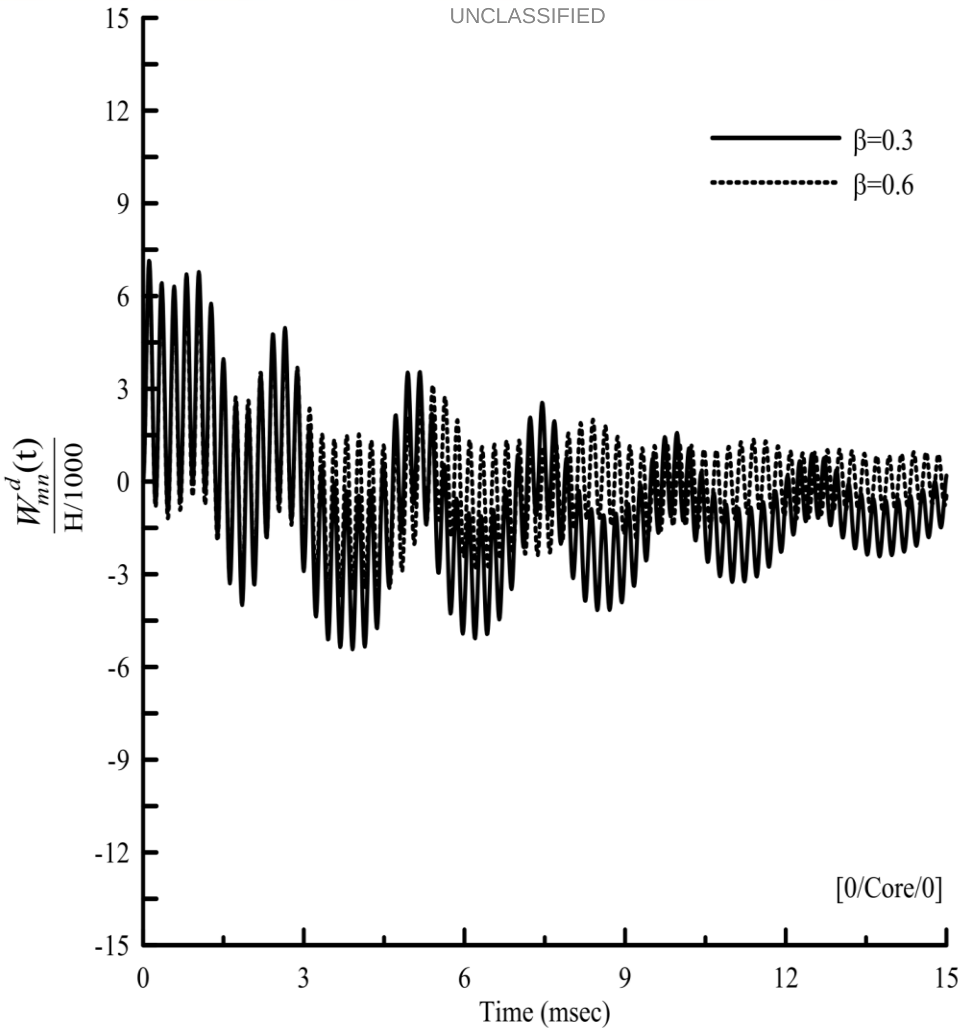

Fig. 7 The counterpart of Fig. 6 for the wrinkling response.

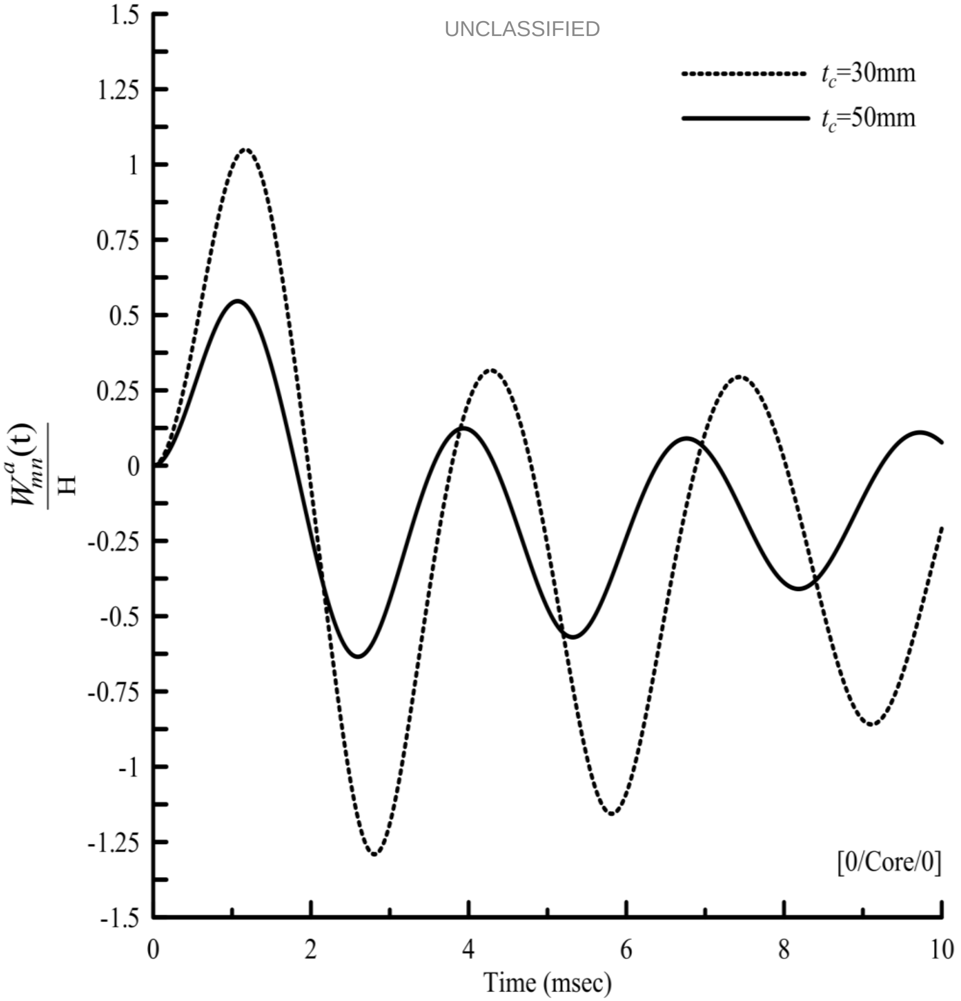

Fig. 8 The effect of the core thickness on the global deflection-time history of a sandwich plate with orthotropic facings.

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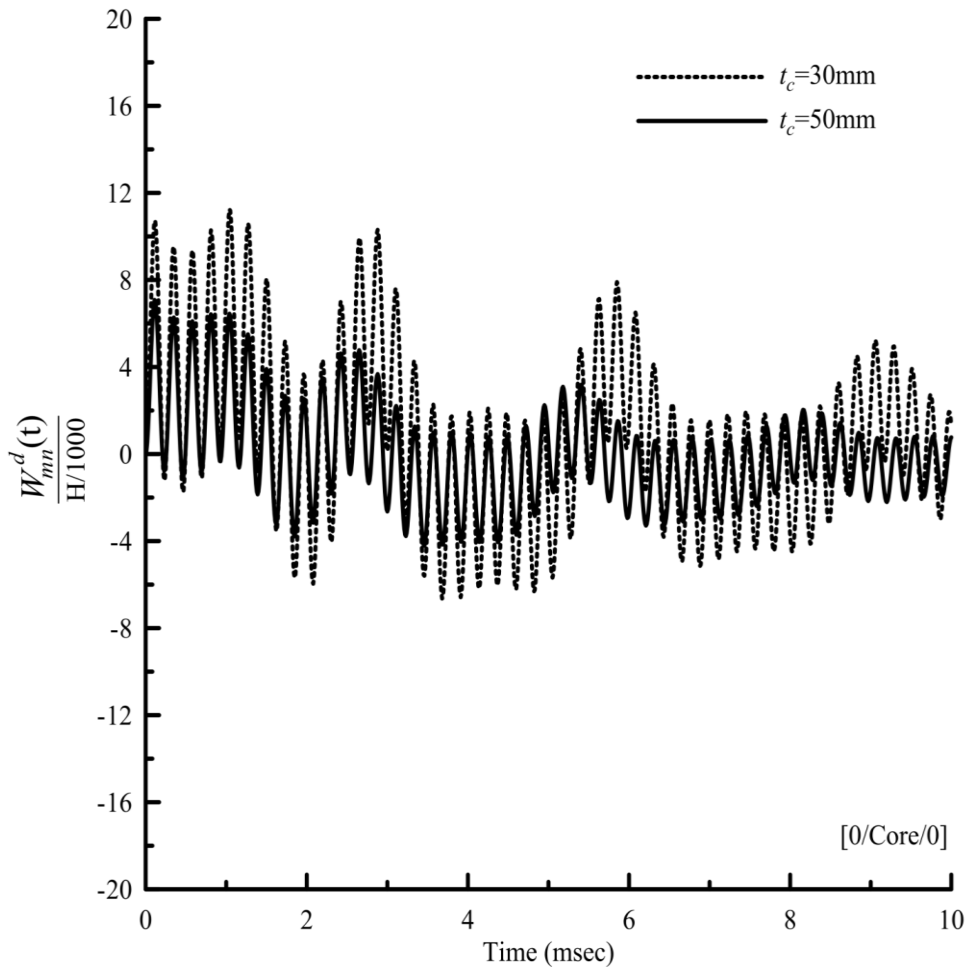

Fig. 9 The counterpart of Fig. 8 for the wrinkling response.

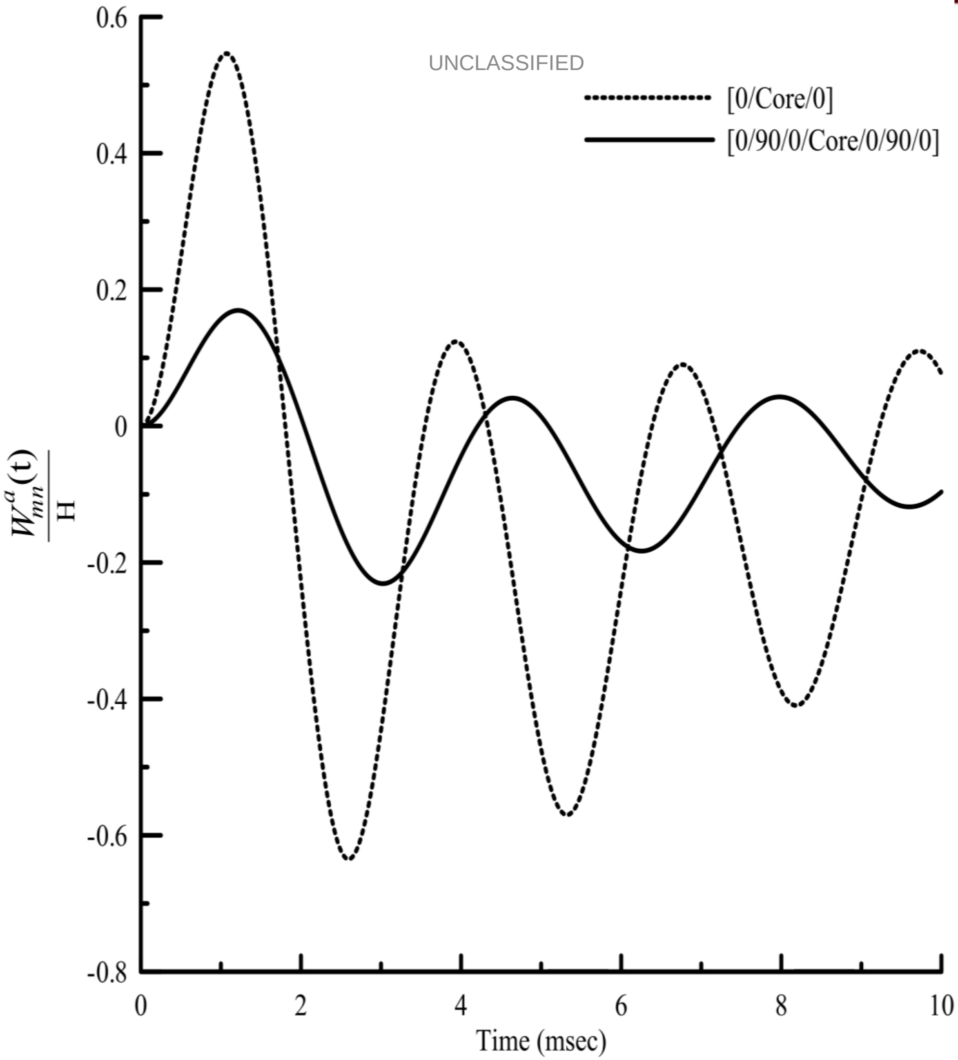

Fig. 10 The effect of the stacking sequence of the facings on the global response of a sandwich plate.

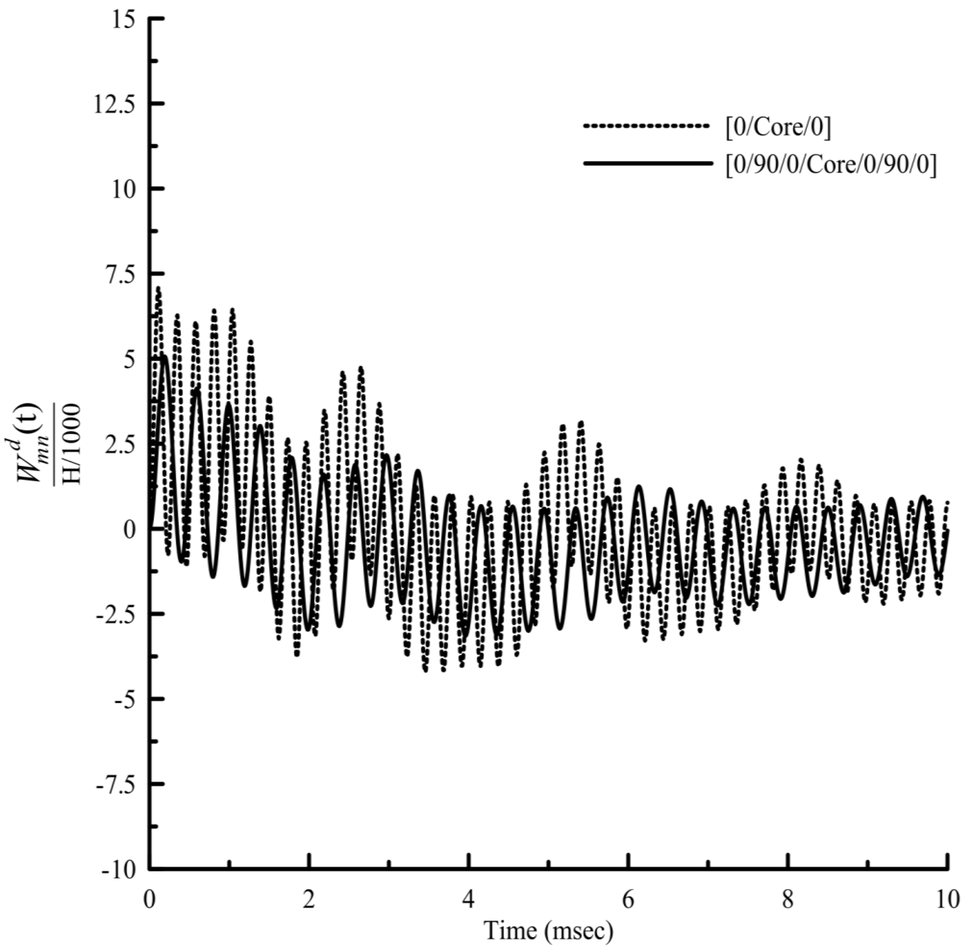

Fig. 11 The counterpart of Fig. 10 for the wrinkling response.

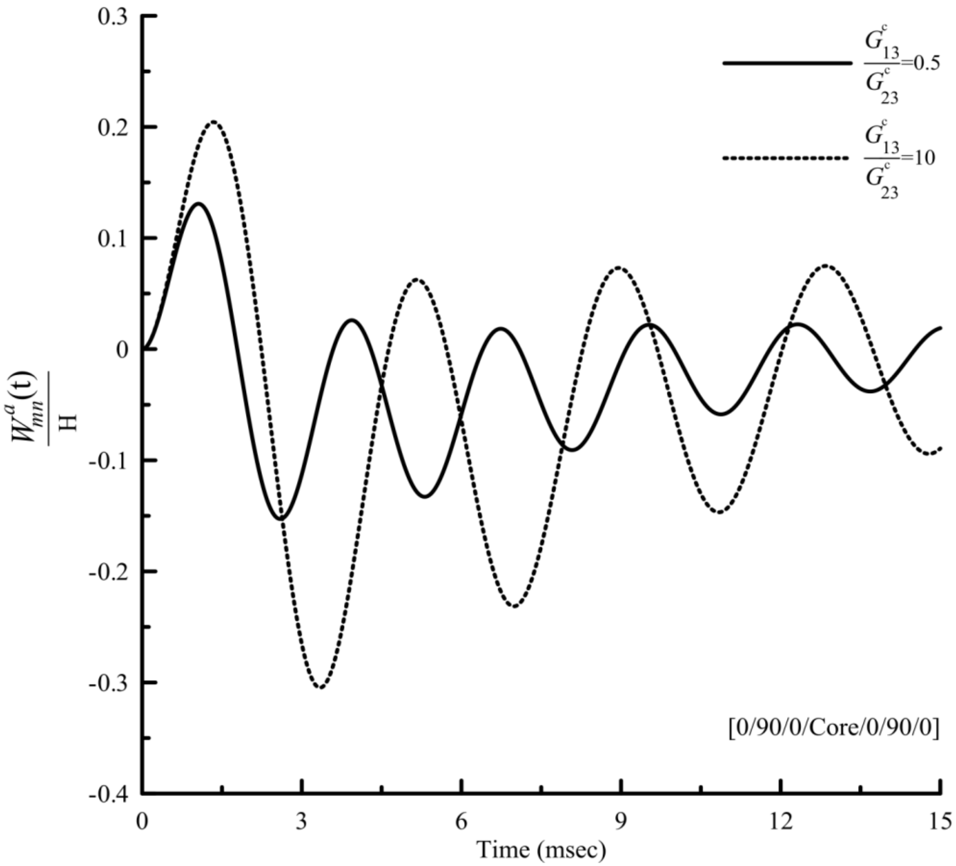

Fig. 12 The effect of the core shear modulus ratio on the deflection-time history of cross-ply laminated sandwich plate.

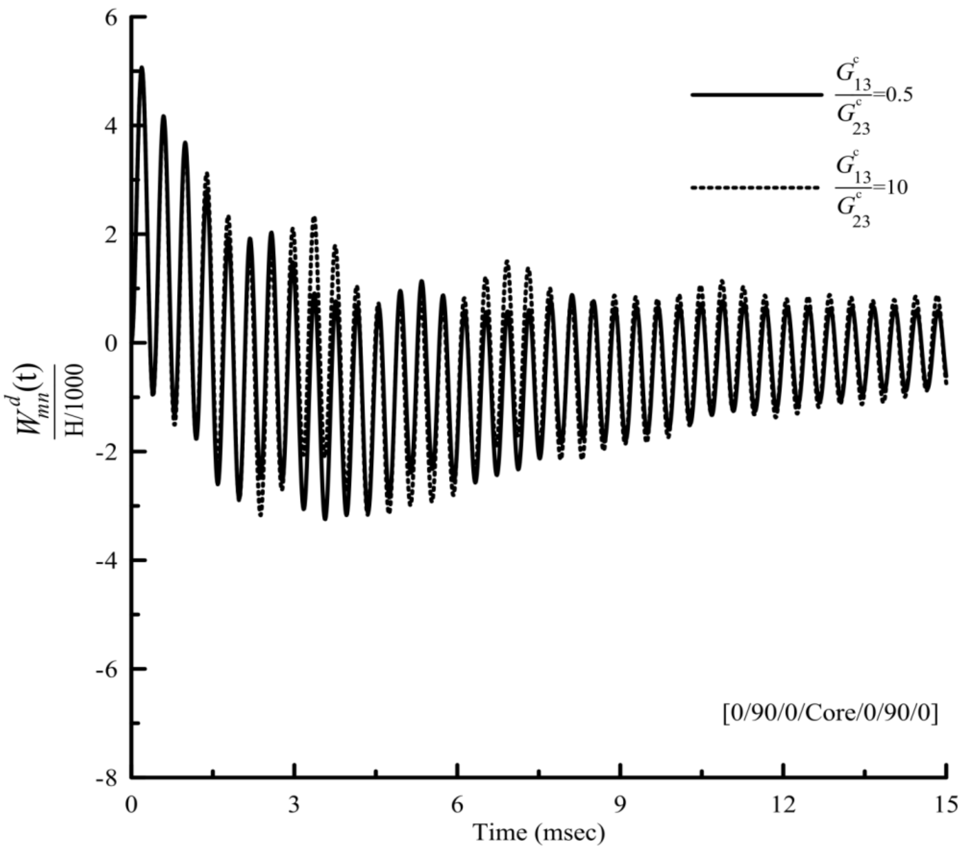

Fig. 13 The counterpart of Fig. 12 for the wrinkling response.

Concluding Remarks

The governing theory of asymmetric sandwich plates with a first-order compressible core impacted by a Friedlander-type of blast has been presented and simplified for the case of symmetric cross-ply and single-layered orthotropic facings. In all cases, it was mentioned that all four edges are simply supported and freely movable. Results were then presented for this simplified case and validated against results found in the literature from R. S. Alwar et al. It was found that for the incompressible core case that there was close agreement among the results. In regards to the compressible core case, no appropriate results have been found in the literature for the theory presented in this paper for the simply supported case with all edges freely movable. The effect of a number of important geometrical and material parameters were analyzed with conclusions drawn. Some of the important conclusions were that wrinkling response seems to be diminished as the young’s modulus of the core is increased. The same is the case for larger rates of decay. Also, for thicker cores, both the global and wrinkling responses are less severe. It was also revealed that the compressibility of the core has only a marginal effect upon the global response of the sandwich plate. Finally, the cross-ply type layup when compared with single-layered facings seemed to have a large effect on the global response and less effect on the wrinkling response.

One should keep in mind that both the stress and strain profiles should be determined to determine possible failure of the structure.