

# Final Report: Case Number: AOARD - 104145

## Developing Optimal Strategies for Structural Health Monitoring: Stochastic Inverse Methodologies for Interrogating Dielectric Materials using Microwaves

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### 1 Background and Aim of Project

An insulation degradation of electrical cables of instruments and control facilities is one of the critical phenomena for aging management of large scale systems, such as in airplane, power plants, etc [1]. From the fact that dielectric properties of materials used in electrical cables can be degraded at the course of long-term service, use of microwaves includes potential applications in nondestructive test for cable degradation. The idea has been already applied to the in-service inspection in the variety of industrial applications. Time-Resolved Microwave Dielectric Absorption (TRMDA) enables to measure evanescent electrical field leaking from the probe into the cable insulation by using cavity resonator [2]. This electrical field changes with the dielectric properties of material, which in turn changes the resonant frequency. Figure 1 depicts the overall configuration of the current nondestructive testing. The nondestructive test is performed by evaluating a bunch width of reflected microwave. This is because the increasing the whole of cavity in cable insulation can be detected as variation of the bunch width. Although the method has been a conventional method for characterizing aging properties, there are several disadvantages on the practical implementation. One is that the detection and characterization of variation of bunch width are not feasible for on-site inspection. Also continuous surveillance of the variation is very crucial tasks. Taking into remind these, our concern in this paper is to develop a computational method for identifying dielectric properties of electrical cables based on microwave guide measurement system.

The quantitative nondestructive evaluation (QNDE) of material aging parameters continues to be a very challenging problem. In our approach, we formulate a forward problem arising in specific micro guided wave test. More specifically, it represents a mathematical model of specific nondestructive test using the parameter-to-output mapping with the appropriate admissible class of material parameters, To this end, the mathematical model is described by a two-dimensional Maxwell's equations. A numerical scheme of the forward problem is derived using the finite-difference time-domain method (FDTD) in two spatial dimensions. Thus, the inversion of material parameters is implemented with the aid of the forward problem. The non-linear least square identification (OLSI) is a conventional inverse methodology and there have been so many efforts in variety of industrial applications. For OLSI related to electromagnetic inversion, we refer [3][4][5][6][7]. However, it is well known that the problem mentioned above has many solutions due to the fact it is ill-posed. Recently, interest has grown in stochastic inversion using Markov Chain Monte Carlo (MCMC) methods [8][9]. Some previous efforts on

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the similar approach has been proposed in [10], [11]. The method has great advantages for the more practical aspects of inversions such as setting initial guesses and overcoming local minimums of the inverse solution. In this paper, a stochastic inversion technique based on the Metropolis-Hasting algorithm is applied to the problem presented here.

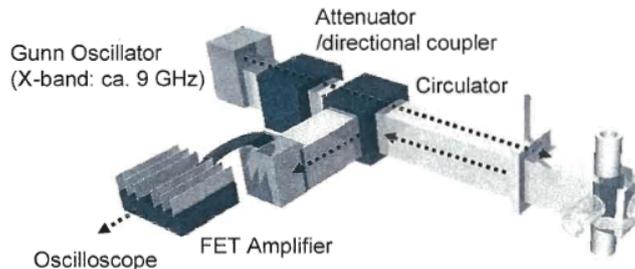


Figure 1: Nondestructive test using TRMDA

## 2 Methodical Procedures

### 2.1 Milestone 1: Formulation of nondestructive evaluation arising in microwave analysis

Guided microwaves analysis is considered for the detection and the characterization of corrosion under insulation. First, the forward problem is formulated by a Maxwell equation in two or three spatial dimensions. Secondly, the forward problem is considered by constructing deterministic parameter-to-output mappings based on the frequency dependent media with unknown material quantities as well as with the unknown geometries. Thirdly numerical scheme is developed by using FDTD method in order to solve the forward problem.

#### 2.1.1 Problem Domain

(Term : From September 2010 to December 2010)

For the simplicity of discussions, we consider a two dimensional bounded domain  $x = (x_1, x_2) \in \Omega \subset R^2$  as an inspection area. Let  $\Omega_c$  be a cable domain defined on  $\Omega$ . The cable domain consists of a chemical material  $\Omega_d$  and a metal core  $\Omega_o$ . The chemical material corresponds to a polymer of which the dielectric constant  $\epsilon_r$  and conductivity  $\sigma_d$  often vary at different frequencies. Figure 2 depicts the two-dimensional problems space considered here. Our electromagnetic interrogation is then represented by electric and magnetic fields in  $\Omega$ . Let  $D_3(t, x)$ ,  $H_1(t, x)$ ,  $H_2(t, x)$  and  $E_3(t, x)$  be the component of electric flux density, magnetic and electric field at time  $t \in [0, T]$  and at location  $x \in \Omega$ . In order to use Gaussian units, we also use the normalizing terms,

$$\begin{aligned} \tilde{D}_3 &:= \sqrt{\frac{1}{\epsilon_0 \mu_0}} D_3 \\ \tilde{E}_3 &:= \sqrt{\frac{\epsilon_0}{\mu_0}} E_3 \end{aligned}$$

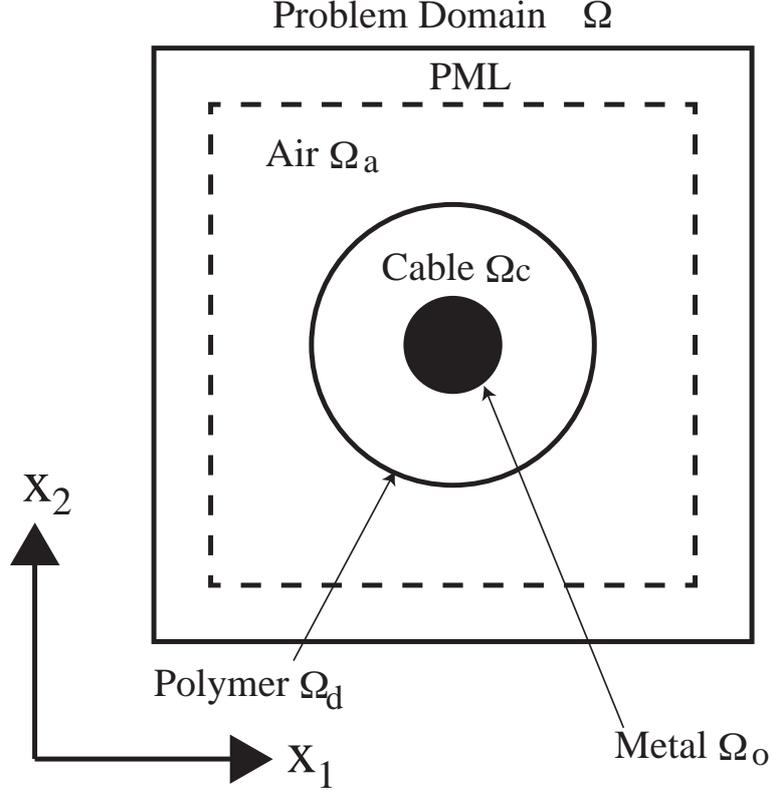


Figure 2: Schematic diagram of geometry

where  $\epsilon_0$  and  $\mu_0$  are permittivity and magnetic permeability of air. From Maxwell's equations, the electromagnetic propagation in the transverse magnetic (TM) mode is governed by

$$\frac{\partial \tilde{D}_3(t, x)}{\partial t} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \left( \frac{\partial H_2(t, x)}{\partial x_1} - \frac{\partial H_1(t, x)}{\partial x_2} \right) + J_s(t, x) \quad \text{in } [0, T] \times \Omega \quad (1)$$

$$\tilde{D}_3(\omega, x) = \epsilon_r^*(\omega) \tilde{E}_3(\omega, x) \quad (2)$$

$$\frac{\partial H_1(t, x)}{\partial t} = -\frac{1}{\sqrt{\epsilon_0 \mu_0}} \frac{\partial \tilde{E}_3(t, x)}{\partial x_2} \quad \text{in } [0, T] \times \Omega \quad (3)$$

$$\frac{\partial H_2(t, x)}{\partial t} = -\frac{1}{\sqrt{\epsilon_0 \mu_0}} \frac{\partial \tilde{E}_3(t, x)}{\partial x_1} \quad \text{in } [0, T] \times \Omega \quad (4)$$

with zero initial states and with the absorption boundary conditions. The complex relative dielectric constant  $\epsilon_r^*$  of the polymer  $\Omega_d$  can be represented by a lossy dielectric medium of the form;

$$\epsilon_r^* = \epsilon_r + \frac{\sigma_d}{j\omega\epsilon_0}. \quad (5)$$

On the other hand, the value of this parameter in  $\Omega_a = \Omega - \Omega_c$  becomes  $\epsilon_r^* = 1$ . The value of that in  $\Omega_o$  is also given by

$$\epsilon_r^* = 1 + \frac{\sigma_o}{j\omega\epsilon_0} \quad (6)$$

where  $\sigma_o$  denotes the conductivity of the metal core. The source current  $J_s$  is the test signal in our inspection process and is given by

$$J_s(t, x) = \delta(\Omega_s)g_s(t)I_{(0,t_s)}(t) \quad (7)$$

where  $\delta$  denote the array of actuator in  $\Omega$  such as  $\Omega_s \cap \Omega_c = \emptyset$ . In Eq. (7), the test signal is truncated at a finite time  $t_s$  by the indicator function  $I$ . The detecting signal can be made through the allocation of sensor arrays  $\Omega_o$  in  $\Omega$  by

$$Y(t, x; \mathbf{q}) = E_3(t, x; \mathbf{q}) \quad \text{in } \Omega_o \quad (8)$$

where the parameter vector  $\mathbf{q}$  to be identified is denoted by

$$\mathbf{q} = \{\epsilon_r, \sigma_d\}. \quad (9)$$

Thus the direct problem for our NDT is formulated as

$$\mathbf{q} \implies Y(t, x; \mathbf{q}). \quad (10)$$

### 2.1.2 Numerical Scheme

(Term : From January 2011 to March 2011)

The finite-difference time-domain method (FDTD) in two spatial dimensions can be implemented to construct the forward model [12]. Putting Eqs.(1), (3), and (4) into the finite differencing scheme results in the following difference equations:

$$\begin{aligned} \frac{D_3^{n+1/2}[i, j] - D_3^{n-1/2}[i, j]}{\Delta t} &= \frac{1}{\sqrt{\epsilon_0\mu_0}} \left\{ \frac{H_2^n[i + 1/2, j] - H_2^n[i - 1/2, j]}{\Delta x_1} \right. \\ &\quad \left. - \frac{H_1^n[i, j + 1/2] - H_1^n[i, j - 1/2]}{\Delta x_2} \right\} \\ \frac{H_1^{n+1}[i, j + 1/2] - H_1^n[i, j + 1/2]}{\Delta t} &= -\frac{1}{\sqrt{\epsilon_0\mu_0}} \left\{ \frac{E_3^{n+1/2}[i, j + 1] - E_3^{n+1/2}[i, j]}{\Delta x_2} \right\} \\ \frac{H_2^{n+1}[i, j + 1/2] - H_2^n[i, j + 1/2]}{\Delta t} &= -\frac{1}{\sqrt{\epsilon_0\mu_0}} \left\{ \frac{E_3^{n+1/2}[i + 1, j] - E_3^{n+1/2}[i, j]}{\Delta x_1} \right\} \end{aligned}$$

In order to prevent spurious reflections from the edge of the problem space, the perfectly matched layer (PML) is adopted to two dimensional domain ( See [13] for more details ). The principle of micro guided wave testing can be represented by the simulation of plane waves. Since the propagating plane wave must not interact with the absorbing boundary conditions like PML, the problem space can be divided up into two regions such as total fields and scattered field. Figure 3 illustrates how the incident plane wave behaves. To simulate a plane wave interacting with a cable object, the cable object according to its electromagnetic properties must be specified. The simulation of a plane wave pulse hitting a dielectric cylinder with  $\epsilon_r = 30$  and  $\sigma = 0.3[m/s]$  is demonstrated in Fig. 4. After the time step  $T = 50$ , the pulse is interacting with the cylinder. At time step  $T = 75$  some of it passes through the cylinder while the other part of it goes around that. It could be also recognized that, at  $T = 100$ , the main part of the propagation is being subtracted out the end of the total field.

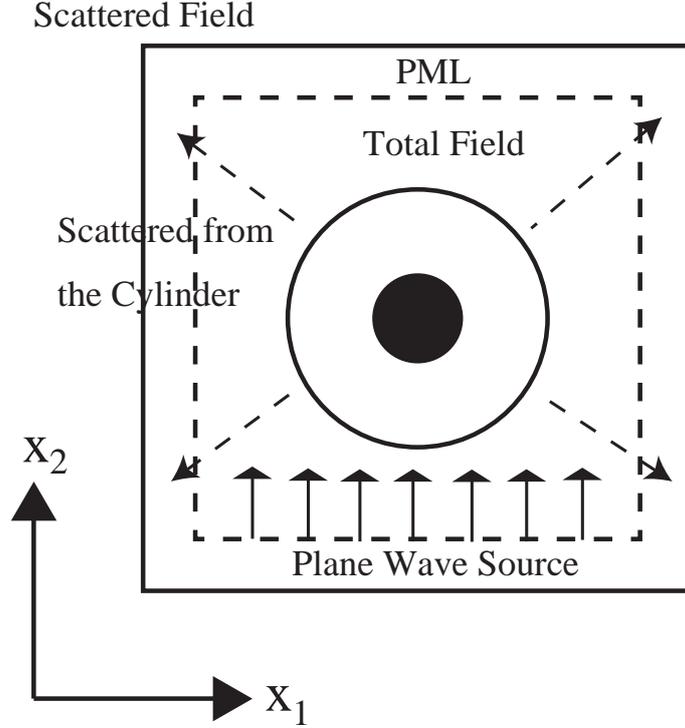


Figure 3: Diagram of a plane wave hitting a cable object

## 2.2 Milestone 2: Development of new stochastic inverse methodology

(Term : From April 2011 to June 2011)

With the background knowledge of Bayes formula, the full probability model is specified by the deterministic formula considered in Milestone 1 and the stochastic relationship for their measurement strategies. More specifically, the posteriori density function is provided with likelihood functional update based on iterative procedures of measurements and with a priori probability densities with respect to the unknown quantities. The application of extreme value statistics to corrosion analysis is also an important factor of our approach.

Our problem is to identify dielectric parameters related to cable insulation

$$\mathbf{q} = \{\epsilon_r, \sigma_d\}$$

using the information obtained through a scattering experiment for the forward problem stated in the previous section. The observations consist of sampling data associated with the electrical field. We suppose that the experimental observations consist of the values of electrical field at the measurement point  $x_p = \{x_p^1, x_p^2\}$  at the sampling time interval  $\Delta t$ . Let  $\mathbf{Y}_d^N = \{\hat{Y}_d[n]\}_{n=1}^N$  denote the collection of the data we seek to reconstruct and  $\mathbf{Y}^N(\mathbf{q}) = \{Y(n\Delta t, x_p; \mathbf{q})\}_{n=1}^N$  be the electrical field arising from the forward problem with dielectric parameter vector  $\mathbf{q}$ . Then the structural equation model is formulated as

$$\begin{aligned} \hat{Y}_d[n] &= Y(n\Delta t, x_p; \mathbf{q}) + \eta_n, \quad \eta_n \sim \mathcal{N}(0, \tau^2), \\ &(n = 1, 2, \dots, N). \end{aligned} \tag{11}$$

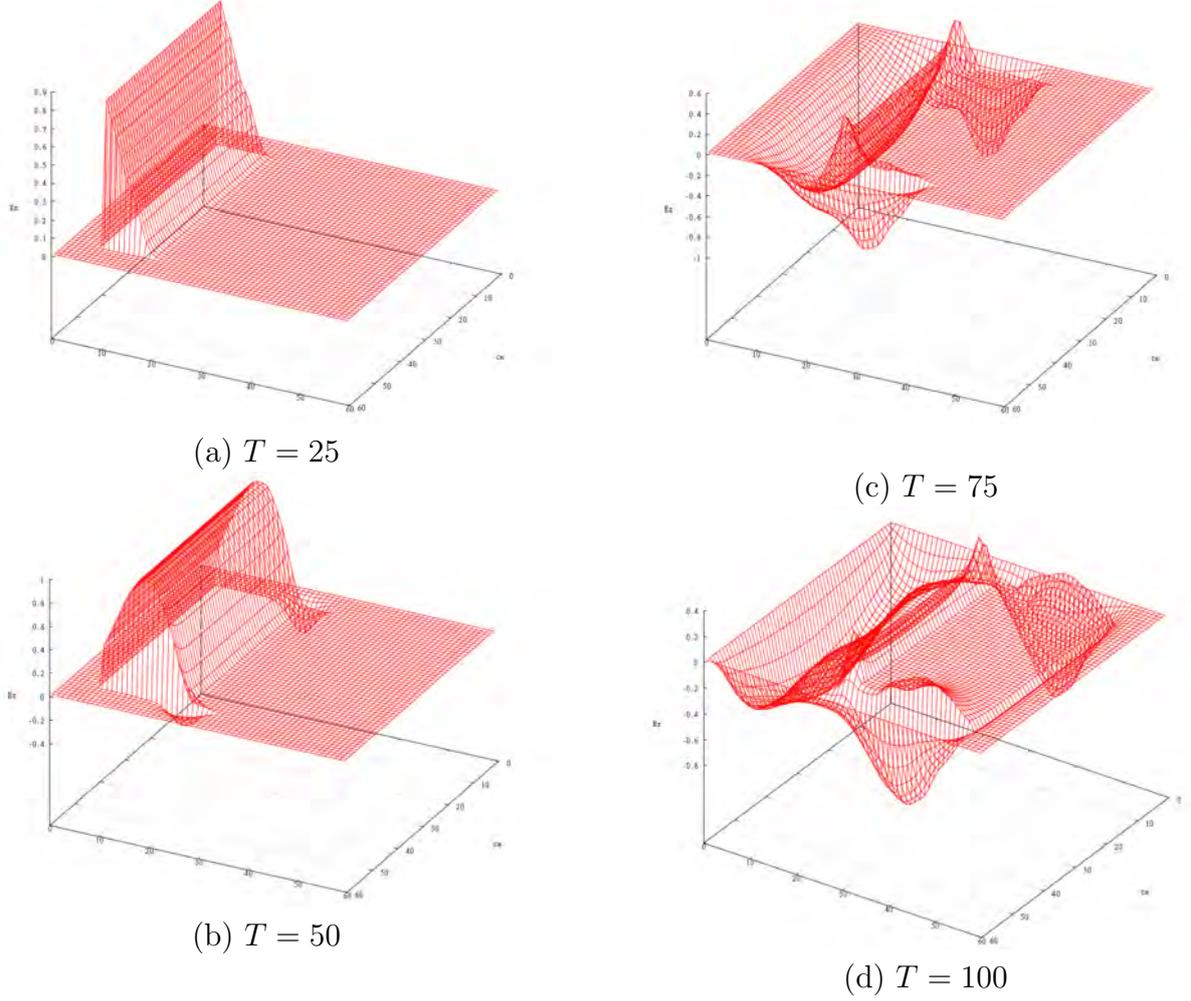


Figure 4: Simulation of a plane wave pulse

where  $\eta_n$  denotes the mutually independent sequence of additive measurement noise. With the background knowledge of Bayes formula, the posteriori density function with respect to the set of unknown parameter vector  $\mathbf{q}$  can be simply written by

$$p(\epsilon_r | \sigma, \mathbf{Y}^N) \propto p(\epsilon_r) \prod_{n=1}^N \frac{1}{\sqrt{2\pi\tau}} \exp\left(-\frac{|Y(n\Delta t, x_p; \mathbf{q}) - \hat{Y}_d[n]|^2}{2\tau^2}\right) \quad (12)$$

$$p(\sigma_d | \epsilon_r, \mathbf{Y}^N) \propto p(\sigma) \prod_{n=1}^N \frac{1}{\sqrt{2\pi\tau}} \exp\left(-\frac{|Y(n\Delta t, x_p; \mathbf{q}) - \hat{Y}_d[n]|^2}{2\tau^2}\right)$$

where  $p(\epsilon_r)$  and  $p(\sigma_d)$  denote a priori density functions of  $\epsilon_r$  and  $\sigma_d$ . An estimation algorithm can be performed by sampling procedures for the posteriori distribution given by Eqs. (12) and (13) from which sample paths can be drawn using Markov chains. For the practical implementation of MCMC, Metropolis-Hasting algorithm is applied to the problem considered here.

### 2.3 Milestone 3: Feasibility studies based on computational experiments

(Term : From July 2011 to September 2011)

The validity and feasibility of our approach developed in Milestones -1 and -2 will be demonstrated through computational experiments for appropriate specific examples. The Gibbs sampler or the other appropriate MCMC samplers (e.g., Metropolis-Hasting algorithm, hierarchy method, etc) are tested for the above problems. The applicability of a real problem to our analysis is also considered.

To test the feasibility of the estimation approach, we produce synthetic data for the observations  $\mathbf{Y}^N = \{Y[n]\}_{n=1}^N$  by adding random noise to the results of the simulation with a known set of parameters  $\mathbf{q}^{true} = \{\epsilon_r^{true}, \sigma_d^{true}\}$ . The data to be identified are generated with the forward problem in the previous section using the following parameter values:

$$\begin{aligned}\epsilon_r^{true} &= 30.0 \\ \sigma_d^{true} &= 0.3 \quad [S/m]\end{aligned}$$

The carrier frequency of the plane wave is  $90GHz$  and the duration of sampling time is taken as  $\Delta t = [s]$ , respectively. The observation data consist of 512 measurements of the electric field taken at  $x_p = (0.5, 0.5)$ . Figure 5 depicts the model data with true parameters. *A priori* densities of  $\epsilon_r$  and  $\sigma$  are preassigned as normal distributions;

$$\begin{aligned}p(\epsilon_r) &= \frac{1}{\sqrt{2\pi}\tau_r} \exp\left(-\frac{|\epsilon_r - \epsilon_r^0|^2}{2\tau_r^2}\right) \\ p(\sigma_d) &= \frac{1}{\sqrt{2\pi}\tau_\sigma} \exp\left(-\frac{|\sigma - \sigma^0|^2}{2\tau_\sigma^2}\right)\end{aligned}$$

where  $\epsilon_r^0$  and  $\sigma^0$  denote the nominal values of dielectric parameters. The artificial noise term was provided using a standard Gaussian random generator  $N(0, 0.1)$ . After 10000 iterations and discarding the first 1000 iterations, the posterior means of  $\mathbf{q}$  were evaluated. Typical paths based on 9000 draws are presented in Fig. 6. The resulting marginal posteriors appear in Fig. 7. Estimation results are summarized in Table 1.

## 3 Summary of Achievement

Guided microwaves analysis was considered for the detection and the characterization of cable insulation. The forward problem in two spatial problem domain was considered by constructing deterministic parameter-to-output mappings based on the frequency dependent media with

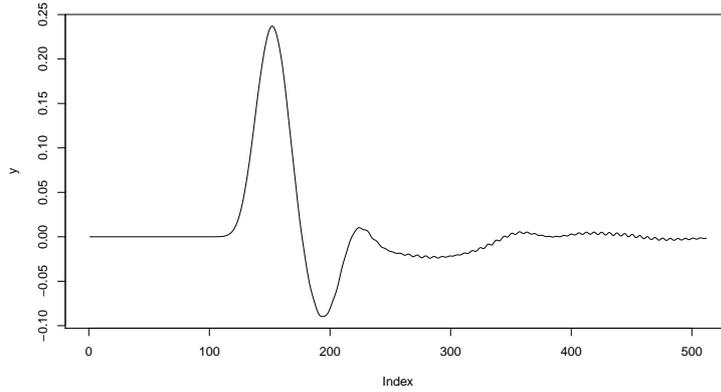


Figure 5: Synthetic data with true parameter values (noise free)

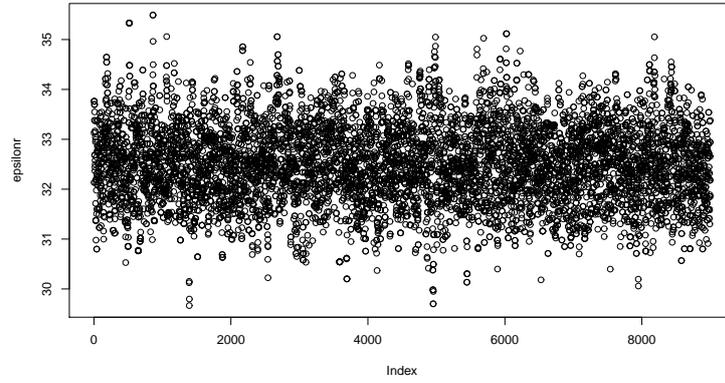
Table 1: Estimation summary

Quantities	$\epsilon_r$	$\sigma_d[S/m]$
True	30.00	0.3
Min	29.67	0.2179
1st Quater	31.98	0.2838
Median	32.49	0.2986
Mean	32.51	0.3000
3rd Quarter	33.04	0.3154
Max	35.49	0.4157

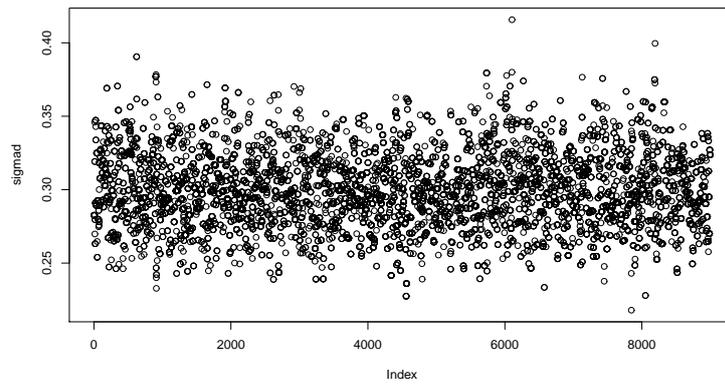
material quantities of cable insulation. Numerical scheme is developed by using FDTD method in order to solve the forward problem. Secondly, stochastic inverse methodology was considered with the background knowledge of Bayes formula. The full probability model was specified by the deterministic formula treated in the forward problem and by the stochastic relationship for their measurement strategies. The posteriori density function was provided with likelihood functional update based on iterative procedures of measurements and with a priori probability densities with respect to the unknown quantities. The validity and feasibility of our proposed method were demonstrated through computational experiments for appropriate specific examples. The MCMC samplers using Metropolis-Hasting algorithm was tested for our inverse problem.

## 4 Refereed Publications

Fumio Kojima and J.S. Knopp, “Quantitative Evaluation for Two-dimensional Electromagnetic Propagation in a Dielectric Medium using Markov Chain Monte Carlo Method”, Proceedings of the 43th ISCIE International Symposium on Stochastic System Theory (SSS’ 11), Otsu, Japan, October 27-28, 2011 (Submitted).

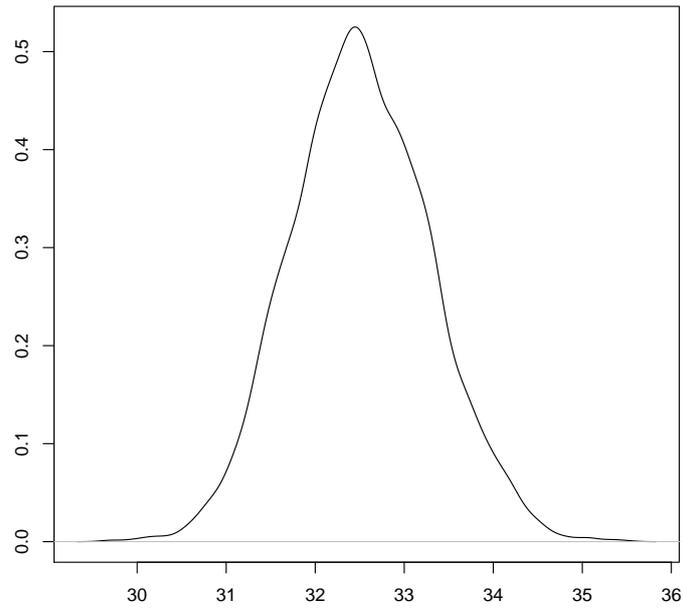


(a)  $\epsilon_r$

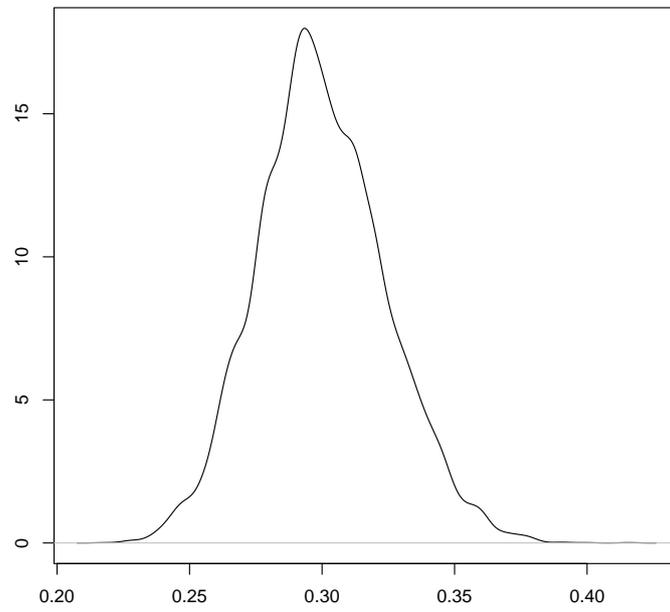


(b)  $\sigma_d$

Figure 6: Trajectory of the chains



(a)  $\epsilon_r$



(b)  $\sigma_d$

Figure 7: Marginal posteriori density estimates

## 5 Collaborations related in AOARD Work

[1] Professor H. T. Banks, Director, Center for Research in Scientific Computation, North Carolina State University, Raleigh NC USA

Research collaboration has been performed during April 17-21, 2011. Project on material damage parameter estimation for electromagnetic acoustic interaction has been started. We will accomplish the convergence properties of inverse solution of nondestructive test using electromagnetic acoustic transducer.

[2] Mr. Jeremy S. Knopp, Researcher, AFRL/RXLP, Nondestructive Evaluation Branch, Materials and Manufacturing Directorate, Air Force Research Laboratory, Dayton OH USA

Research collaboration has been performed during April 22-25, 2011 in Dayton. Also he visited Kobe University during January 26- 28, 2011. We discussed new inverse methodologies for structural health monitoring which includes how statistical approach by merging modeling of material failure process and how to design inverse methodologies.

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