

## AOARD Grant 09-4013

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“A Fast and Robust Inverse Scattering Algorithm”

Period of performance reported: 23/06/2009 - 22/12/2010

### 1. Summary of the Progress

In implementing the project, five research topics in the area of inverse scattering problems have been conducted:

- (1) To propose an improved version of the Subspace-based Optimization Method (SOM) to solve general inverse scattering problems.
- (2) To apply the frequency hopping technique in one-sided transmitter/receiver inverse scattering problems.
- (3) To efficiently locate small scatterers that are embedded in inhomogeneous background, such as through-wall-imaging.
- (4) To reconstruct large scatterers that are embedded in inhomogeneous background.
- (5) To reconstruct scatterers using only the magnitude of wave information (i.e., the phase information is not available)

The above research works has been published as journal or conferences papers. In total, **six journal papers** have been published or accepted, and **five conference papers** have been published or presented.

### 2. Details of the Research Work

Safe, reliable, high-quality and low-cost imaging technologies are growing requirements for the modern world. In many imaging applications, the shapes, locations and composite materials of objects are reconstructed from the measurement of electromagnetic fields scattered by these objects. The problem of reconstructing such objects is basically an inverse scattering problem. In inverse scattering problems, transmitting antennas transmit electromagnetic waves to probe the targets, and the scattered fields are measured by receiving antennas.

It is well known that the inverse scattering problem is usually computationally demanding, characterized by (1) slow convergence rate; (2) possibility of convergence to a local optimum instead of the global minimum; (3) instability in the

## Report Documentation Page

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14. ABSTRACT <b>It is well known that the inverse scattering problem is usually computationally demanding, characterized by (1) slow convergence rate; (2) possibility of convergence to a local optimum instead of the global minimum; (3) instability in the presence of noise. The common way of solving an inverse scattering problem is to cast the problem into an optimization problem where the objective is to minimize the mismatch between the measured scattered field and the calculated scattered field. During the project, five important contributions have been documented: (1) Proposed improved version of the Subspace-based Optimization Method (SOM) to solve general inverse scattering problems. (2) Applied frequency hopping technique in one-sided transmitter/receiver inverse scattering problems. (3) Efficiently located small scatterers are embedded in inhomogeneous background, such as through-wall-imaging. (4) Reconstructed large scatterers embedded in inhomogeneous background. (5) Reconstructed scatterers using only the magnitude of wave information (i.e., the phase information is not available)</b>					
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presence of noise. The common way of solving an inverse scattering problem is to cast the problem into an optimization problem where the objective is to minimize the mismatch between the measured scattered field and the calculated scattered field.

During the one and half years of the project period, we have made five important contributions in solving inverse scattering problems.

(1). To propose an improved version of the Subspace-based Optimization Method (SOM) to solve general inverse scattering problems.

In Ref. [5], we propose an improved subspace-based optimization method by using a new construction method for the ambiguous part of the induced current. The new current construction method significantly reduces the computational complexity of the algorithm. Thus, the improved SOM is able to deal with the three dimensional (3-D) inverse-scattering problems. Numerical tests validate the algorithm. For example, the 3-D scatterer in Fig. 1 is successfully reconstructed, as shown in Fig. 2.

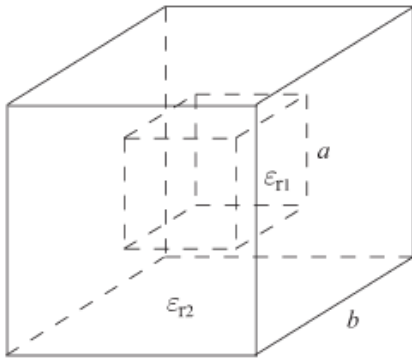


Fig. 1 A scatterer that consists of coated dielectric cubes.

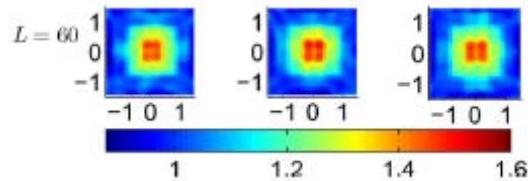


Fig. 2 Reconstructed scatterer that are displayed in three slices. It is clear to see coated cubes.

(2) To apply the frequency hopping technique in one-sided transmitter/receiver inverse scattering problems.

In many practical applications, transmitters and receivers can be placed on only one side of the scatterer. Therefore it is important to investigate the performance of the inverse scattering algorithm in such an experimental setup. In Ref. [4], we have investigated the performance of SOM in the case where transmitting/receiving antennas are located in a half circle around scatterers. In Ref. [2], we test the performance of SOM in a practical scenario, i.e., through-wall-imaging. The configuration of two-layer-wall is shown in Fig. 3.

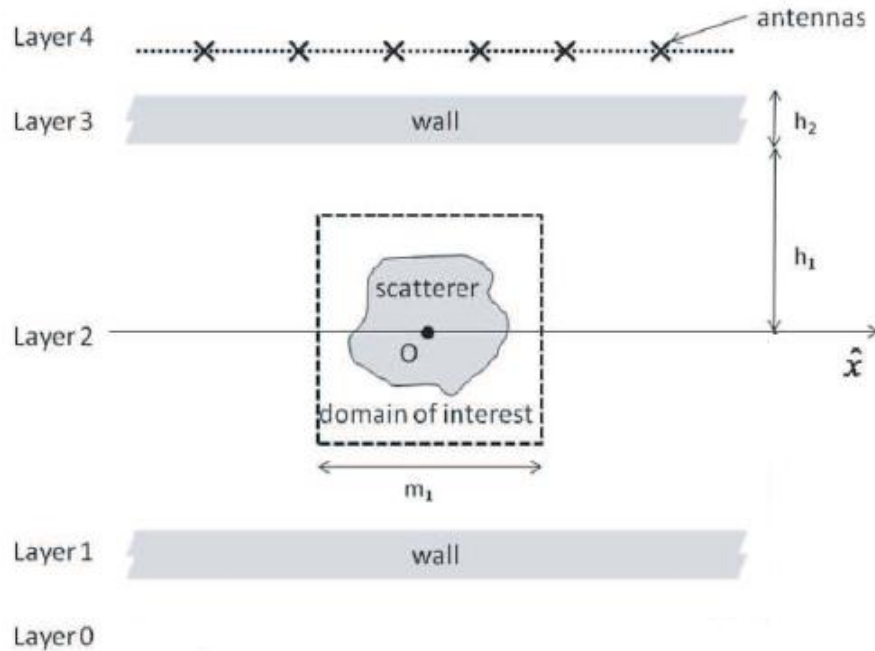


Fig. 3 Schematic configuration of the through-wall-imaging

The quality of reconstructed image is generally inferior to the case when full-aperture data is available. In order to improve the reconstruction, we use frequency hopping. For example, the relative permittivity reconstructed at 300MHz is used as the initial guess for performing optimization on the measurement data of 400MHz and the relative permittivity thus reconstructed at 400MHz is used as an initial guess for the optimization on the measurement data of 500 MHz. In Fig. 4, it is clearly seen that the frequency-hopping result at 500MHz is much better than the result at 300MHz.

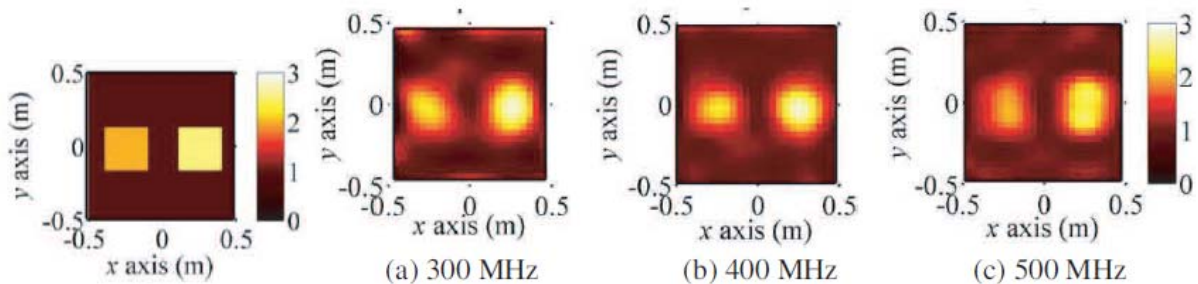


Fig. 4 Left: Exact scatterers (two squares). Right: reconstructed results using frequency hopping, where 300, 400, and 500 MHz frequencies are used.

(3) To efficiently locate small scatterers that are embedded in inhomogeneous background, such as through-wall-imaging.

In many real applications, the background medium is not homogeneous. In such case, it is numerically difficult to reconstruct targets since the background Green's function is not available. As mentioned earlier, the common way of solving an

inverse scattering problem is to cast the problem into an optimization problem where the objective is to minimize the mismatch between the measured scattered field and the calculated scattered field. It is usually computationally demanding to solve such optimization problems. However, when the sizes of scatterers are small compared with wavelength, we show that the problem can be solved without resorting to any optimization. In Ref. [3], we propose a multiple signal classification (MUSIC) method to locate small scatterers that are imbedded in an inhomogeneous background medium. Fig. 5 shows such an example.

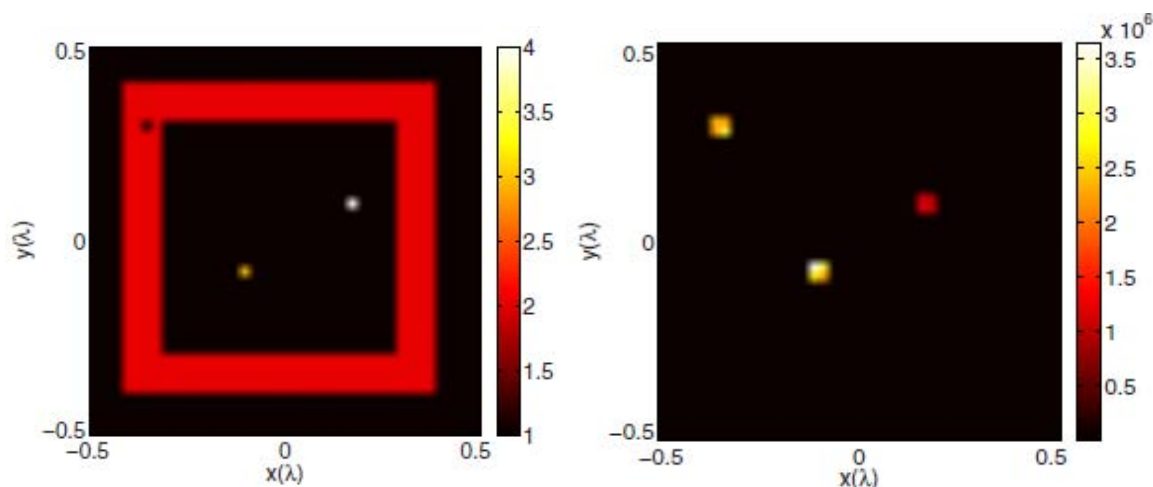


Fig. 5 Left: Three point-like scatterers are located in an inhomogeneous background medium, which is a square wall indicated in red color. Right: Reconstructed results, where three point scatterers are successfully located.

(4) To reconstruct large scatterers that are embedded in inhomogeneous background.

In Ref. [1], we propose a version of the subspace-based optimization method to solve the inverse scattering problem with an inhomogeneous background medium. The proposed method uses differential numerical method (such as the finite element method) instead of traditional integral equation approach. The SOM is found to be fast convergent and robust against noise. The proposed inhomogeneous-background SOM has a great potential in various applications, such as through-wall imaging and biomedical imaging. For example, Fig. 6 provides such an example.

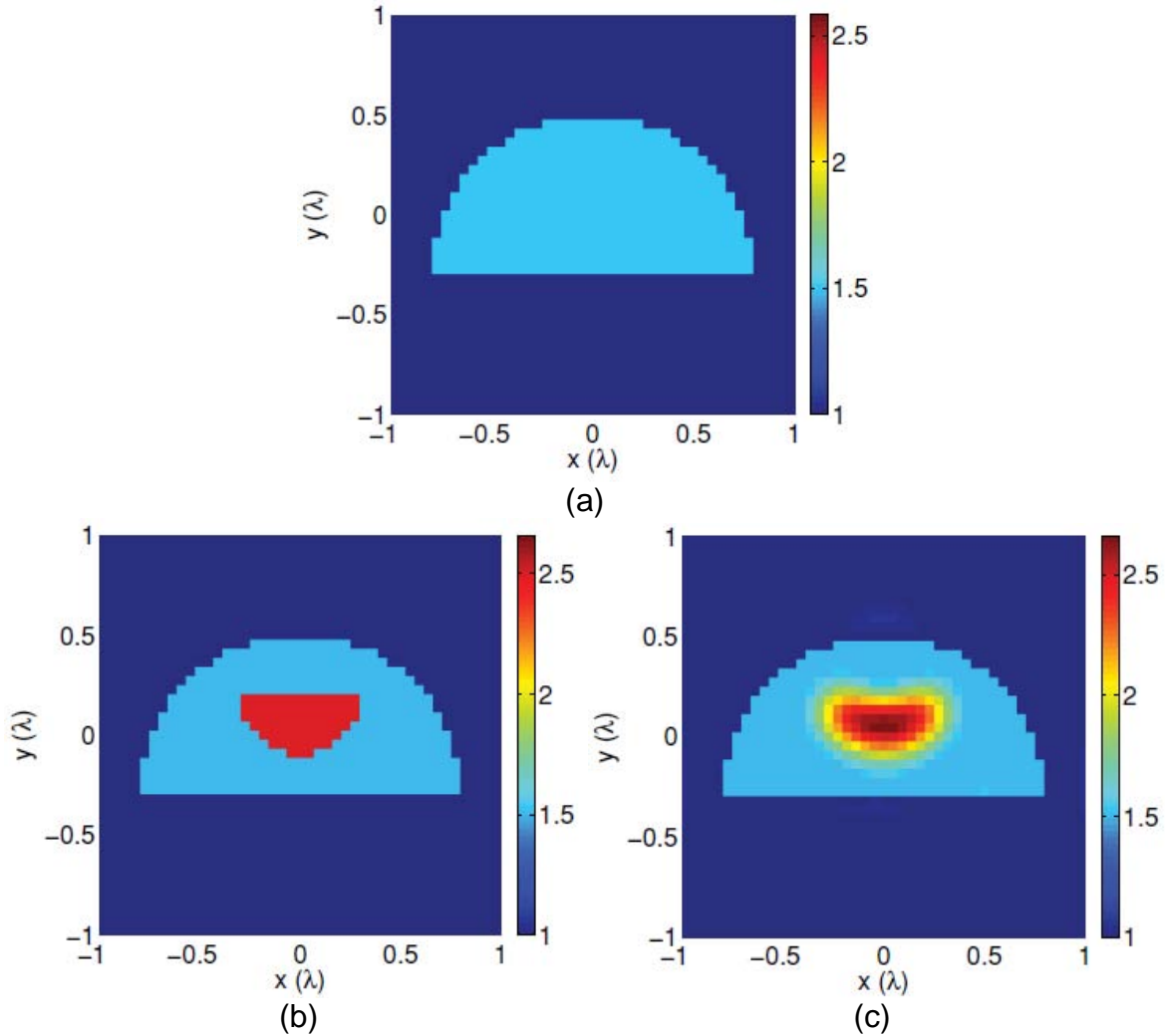


Fig. 6 (a): The inhomogeneous background is a half circular disk; (b) Exact profile, where the scatterer is a smaller half circular disk; (c) Reconstructed results.

(5) To reconstruct scatterers using only the magnitude of wave information (i.e., the phase information is not available)

In practice, when measuring a complex quantity, it is well-accepted that phase is generally more difficult to measure than amplitude. As a matter of fact, researchers have observed that the accuracy of phase measurements cannot be guaranteed for operating frequencies approaching the millimeter wave band and beyond. In [6], we present a novel variation of the subspace-based optimization method (SOM) to reconstruct the scatterer's permittivity profile by utilizing only phaseless measurements. The proposed method is capable of reconstructing complicated patterns with rapid rate of convergence and robust immunity to noise. Fig. 7 provides an example of reconstruction.

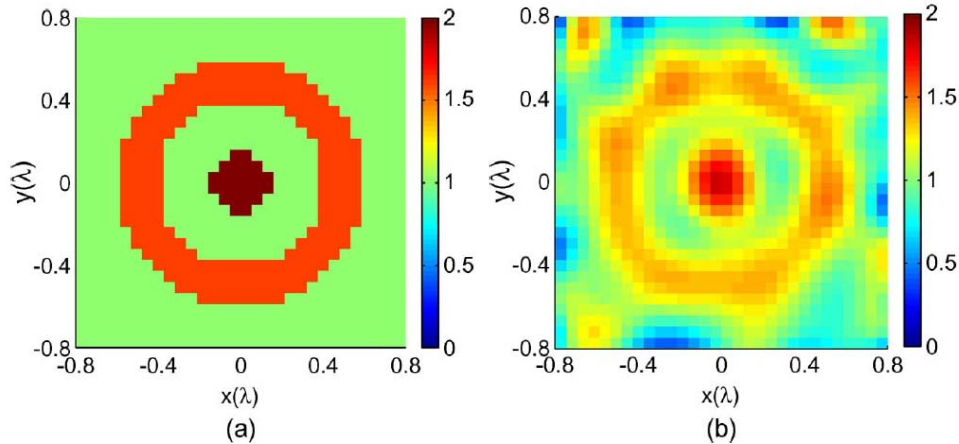


Fig. 7 Reconstruction using phaseless data: (a) Exact profile (b) Reconstructed results

### 3. Impact of the Research Work

It is well known that inverse scattering problems are difficult to solve, which is usually computationally demanding and characterized by (1) slow convergence rate; (2) possibility of convergence to a local optimum instead of the global minimum; (3) instability in the presence of noise. The four research works mentioned in Section 2 are important contributions to inverse scattering community. In particular, the works (1) (3) and (4) are, to the best of our knowledge, pioneering work in this area. Here are some comparisons of the proposed Subspace-based Optimization Method (SOM) with existing algorithms. In the literatures, many existing algorithms take hundreds of iterations of optimization to converge, whereas the SOM takes only 10 to 20 iterations. Many existing algorithms can bear up to 10% white Gaussian noise, whereas the SOM can bear up to 30% white Gaussian noise. The presented research work in reconstruction using only phaseless data is also pioneering in the community.

#### Publications:

##### Journal Papers published

1. X. Chen, "Subspace-based optimization method for inverse scattering problems with an inhomogeneous background medium", *Inverse Problems*, Vol. 26, ID: 074007, 2010
2. T. Lu, K. Agarwal, Y. Zhong, and X. Chen, "Through-wall imaging: application of subspace-based optimization method", *Progress in Electromagnetic Research*, Vol. 102, pp. 351-366, 2010
3. X. Chen, "Multiple signal classification method for detecting point-like scatterers embedded in an inhomogeneous background medium", *Journal of the Acoustical Society of America*, Vol. 127, pp. 2392-2397, 2010.



4. L. Pan, X. Chen, S. P. Yeo, "Nondestructive evaluation of nano-scale structures: inverse scattering approach", Applied Physics, A, Vol. 101, pp. 143-146, 2010
5. Y. Zhong, X. Chen, and K. Agarwal, "An Improved Subspace-based Optimization Method and its Implementation in Solving Three-dimensional Inverse Problems", IEEE Transactions on Geoscience and Remote Sensing, Vol. 48 pp. 3763-3768, 2010
6. L. Pan, Y. Zhong, X. Chen, and S. P. Yeo, "Subspace-based optimization method for inverse scattering problems utilizing phaseless data", IEEE Transactions on Geoscience and Remote Sensing, Vol. 49. pp. 981-987, 2011

Conference Papers published/presented:

6. Y. Zhong, K. Agarwal, and X. Chen, "Subspace-based optimization method with initial guess through a multipole-expansion based linear sampling method", Progress in Electromagnetics Research Symposium, Cambridge, MA, USA, 5-8, July, 2010
7. X. Chen, "Electromagnetic Inverse Scattering Problems Involving Small Scatterers", 5th Pacific Rim Conference on Mathematics, Stanford University, CA, USA, 28 Jun.- 2 Jul., 2010 (Invited)
8. X. Chen, "Subspace-based Optimization Method for Solving Inverse Scattering and EIT problems", International Workshop on Inverse Problems, Hong Kong, 23-24, Apr. 2010 (Invited)
9. L. Pan, X. Chen, S. P. Yeo, "Application of the Subspace-based Optimization Method in the Framework of the Method of Moments: Transverse Electric Case", Asia-Pacific Microwave Conference, Singapore, 7-10, Dec. 2009
10. L. Pan, X. Chen, S. P. Yeo, "Application of the Subspace-based Optimization Method in the Framework of the Method of Moments: Transverse Electric Case", Asia-Pacific Microwave Conference, Singapore, 7-10, Dec. 2009 (After peer review, the paper has been accepted for publication in Applied Physics, A, 2010. See Reference 4)