Scattering of Internal Gravity Waves at Finite Topography

Peter Muller
University of Hawaii
Department of Oceanography
1000 Pope Road, MSB 429
Honolulu, HI 96822
phone: (808)956-8081 fax: (808)956-9164 email: pmuller@soest.hawaii.edu

Grant Number: N00014-96-1-0489

LONG-TERM GOAL

Description and modeling of the kinematic structure and the dynamical processes of oceanic internal gravity waves.

Understanding the role of internal gravity waves in the redistribution of momentum, potential vorticity, heat, and salt.

Development of a numerical model that predicts the internal wave field and the internal wave induced transports globally and regionally.

OBJECTIVES

The current research project investigates the interaction of internal gravity waves with topography. Its specific objectives are:

- to assess the role of reflection and scattering at topography in the redistribution of energy within the internal wave band and in providing energy for boundary mixing.
- to determine the modifications of the deep ocean internal wave field when it propagates up the continental slope onto the shelf and to compare this remotely forced wave field with the locally generated field.

APPROACH

The scientific approach combines theoretical analysis with numerical modeling. The specific approach differs for scattering in two and three dimensions.

In two dimensions the method of characteristics can be employed. A general theoretical framework was developed by Baines (1971 a, b) and by Sandstrom (1976). The major challenges are the implementation of the radiation condition and the singularities at points where the topographic slope equals the wave slope.

In three dimensions, no general approach has been developed as yet. The governing equation is a hyperbolic equation which is analogous to the ordinary wave equation with the vertical coordinate replacing the time coordinate. This difference causes difficulties since the vertical space coordinate does not increase uniformly along the ray path in the bottom scattering problem.

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **1998**	2. REPORT TYPE	3. DATES COVERED **00-00-1998 to 00-00-1998**
4. TITLE AND SUBTITLE **Scattering of Internal Gravity Waves at Finite Topography**		5a. CONTRACT NUMBER
		5b. GRANT NUMBER
		5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)		5d. PROJECT NUMBER
		5e. TASK NUMBER
		5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **University of Hawaii,Department of Oceanography,1000 Pope Rd,Honolulu,HI,96822**		8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR'S ACRONYM(S)
		11. SPONSOR/MONITOR'S REPORT NUMBER(S)
12. DISTRIBUTION/AVAILABILITY STATEMENT **Approved for public release; distribution unlimited**		
13. SUPPLEMENTARY NOTES **See also ADM002252.**		
14. ABSTRACT		
15. SUBJECT TERMS		

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**4**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

WORK COMPLETED

In the two-dimensional case we generalized Baines' (1971 a, b) theoretical results for an ocean without an upper surface to a finite depth ocean.

For applications a numerical code was developed for two types of topographic configurations: (a) slope-shelf configurations where a flat shallow shelf is connected to a flat deep ocean by various slope profiles and (b) ridge configurations which are obtained by mirroring the slope profiles. Two suites of numerical experiments were conducted. In the first suite a single plane wave is incident on the topography. Both the transmitted or forward-scattered wave field and the reflected or back-scattered wave field are calculated. The aim is to understand how the scattered wave fields depend on the parameters of the incident plane wave, (its frequency and modenumber), and on the parameters of the topography (its height, its slope and its higher derivatives). In the second suite of experiments a random wave field with a Garrett and Munk spectrum is used as the incident wave field. The transmitted and reflected energy flux, energy and inverse Richardson number spectra are calculated. A critical modenumber is calculated such that the inverse Richardson number from all waves with modenumbers less than the critical modenumber is one. The energy flux to modenumbers larger than this critical modenumber excites waves that are likely to break and cause mixing. The energy flux past this critical modenumber is thus interpreted as being available for internal wave induced boundary mixing.

In three dimensions, we have developed an approach based on the Green's function of the governing wave equation. Following Hurley (1972) this Green's function is obtained by analytic continuation of the Green's function of the elliptic problem into the hyperbolic range. With this Green's function the scattering problem can be cast into an integral equation for the bottom pressure from which the scattered wave field can be calculated. The approach correctly reproduces the scattering at a straight slope and at infinitesimal topography.

RESULTS

The numerical model runs with a single incident plane wave show that the results depend in a complex manner on the parameters of the problem. The most important parameter is the ratio of bottom to wave slope. Scattering at supercritical topography, where the bottom slope exceeds the wave slope, is very different from scattering at subcritical topography. For subcritical topography nearly all of the incident wave energy flux is transmitted. For supercritical topography it is partly reflected and partly transmitted. The partition depends on the incident modenumber and the depth ratio of the topography. The distribution of the scattered energy flux in modenumber space also depends on slope, depth ratio, and incident modenumber. Most of the reflected energy is scattered to modenumbers higher than the incident modenumber.

Higher order derivatives of the topography are also important. A convex slope is much more efficient in scattering the incoming energy flux to higher modenumbers than a concave slope, and a linear slope is more efficient than a curved slope. Abrupt changes in the topography, such as sharp corners, also affect the scattering, especially for subcritical topography.

Reflection theory works well for incident waves whose wavelengths are smaller than the radius of curvature of the topography. In these cases, reflection theory correctly predicts the locations of the peaks in modenumber space and the scattering to higher modenumbers for near critical slopes. However, as the incident modenumber decreases, predictions from reflection theory become less and less accurate, they especially violate the radiation conditions, and must be replaced by solving the full scattering problem.

Because of the complex parameter dependence of the scattering problem, care must be taken when replacing rugged topography by smooth topography, curved topography by linear slopes, and linear slopes by a series of steps.

The numerical model runs with an incident Garrett and Munk spectrum show:

> At frequencies less than the critical frequency most of the incident flux is reflected back. At frequency larger than the critical frequency most of the incident flux is transmitted onto the shelf or across the ridge. The topography thus acts like a filter in frequency space. Since typical topographic slopes imply a critical frequency close to the Coriolis frequency, this filtering leads to an elimination of the inertial peak in slope spectra.
>
> Both the reflected and transmitted modenumber spectra are flatter than the incident spectrum. This implies a redistribution of the incident flux from low to high modenumbers. This is partly a statistical tendency, an approach towards the equilibrium solution.
>
> The transfer to high modenumbers is accompanied by an amplification of the energy level and, even more so, of the shear level.
>
> The energy flux to modenumbers higher than the critical modenumber (to waves that are likely to break) is the energy flux available for internal wave induced boundary mixing. This flux depends on the topographic profile. Convex profiles are more efficient than linear and concave profiles.

Overall, our calculations indicate that the scattering at topography causes a significant distortion to the incident Garrett and Munk spectrum, and these distortions will induce a variety of dynamical adjustment processes. Topography is thus a dynamically active area for internal waves.

IMPACT/APPLICATION

The research will help to clarify the role of open ocean internal waves for (a) boundary mixing, and (b) for processes on the continental slope and shelf.

TRANSITIONS

None

RELATED PROJECTS

None

REFERENCES

Baines, P. G., 1971: The reflexion of internal/inertial waves from bumpy surfaces. *J. Fluid Mech.*, **46**, 273-291.

Baines, P. G., 1971: The reflexion of internal/inertial waves from bumpy surfaces. Part 2. Split reflexion and diffraction. *J. Fluid Mech.*, **49**, 113-131.

Hurley, D. G., 1972: A general method for solving steady-state internal gravity wave problems. *J. Fluid Mech.*, **56**, 721-740.

Sandstrom, H., 1976: On topographic generation and coupling of internal waves. *Geophys. Fluid Dyn.*, **7**, 231-270.

PUBLICATIONS

None