ADVANCED SIGNALING STRATEGIES FOR THE HYBRID MIMO PHASED-ARRAY RADAR

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ABSTRACT

The Hybrid MIMO Phased Array Radar, or HMPAR, is a notional concept for a multisensor radar architecture that combines elements of traditional phased-array radar with the emerging technology of Multiple-Input Multiple Output (MIMO) radar. A HMPAR comprises a large number, MP, of T/R elements, organized into M subarrays of P elements each. Within each subarray, passive elementlevel phase shifting is used to steer transmit and receive beams in some desired fashion. Each of the M subarrays are in turn driven by independently amplified phase-coded signals. This paper proposes new transmit signal selection strategies based on the observation that some MIMO signal sets, such as those proposed by us previously, cause a very rapid sequential or raster scan across some field of view. Exploiting this property allows one to create and process multiple beams simultaneously. Furthermore, there exists a range-angle coupling in the transmit and receive signals that may lead to high-resolution target localization.

1. INTRODUCTION

Multiple-Input Multiple Output, or MIMO, radar systems are next-generation radar systems with multiple transmit and receive apertures, equipped with the capability of transmitting arbitrary and differing signals at each transmit aperture. MIMO radar systems are often contrasted with phased-array radars, that transmit the same signal at each aperture, shifted by an arbitary phase using analog electronics. The added flexibility of individual signal selection at each aperture brings with it the promise of enormous performance improvements, and the challenge of finding solutions to extremely high-dimensional optimization problems associated with choosing the right signals. See [1] and the many references therein. ²U.S. Air Force Research Laboratory Sensors Directorate, Radar Signal Processing Branch Wright-Patterson AFB, OH USA

In this paper we present new results for a variation of the MIMO radar concept for colocated sensor assets that we term the Hybrid MIMO Phased Array Radar, or HMPAR, first proposed by Browning *et al.* [2]. The HMPAR concept brings together elements of both MIMO and phased-array radar. There are a large number MP of T/R elements, organized into M subarrays of P elements each. This is illustrated in Figure 1, which depicts the notational concept for the HMPAR with elements arranged in a rectangular array.



Figure 1. HMPAR notional concept.

Within each subarray, passive element-level phase shifting is used to steer transmit and receive beams in some desired fashion. Each of the *M* subarrays are in turn driven by independently amplified phase-coded signals which could be quasi-orthogonal, phase-coherent, or partially correlated. Such a radar system could be an electronically steered planar array deployed in an airborne platform, *e.g.* in the radome of a fighter aircraft, for concurrent search, detect, and track missions. The objective of the ongoing research described in part here is to identify transmit signaling strategies and adaptive receive signal processing algorithms consistent with these requirements.

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In a previous paper [3] we described the general HMPAR concept, transmit signaling strategies that could be employed to achieve arbitrary spatial transmit beampatterns, and the MIMO ambiguity functions that are achievable with this approach. Here we summarize the main points of this previous work, then propose new signaling strategies that extend that work to the case of multiple independent transmit beams. The new approach results from an interpretation of our signal sets as a rapid sequential or raster scan across some field of view. This approach also reveals an interesting range-angle coupling in the transmit and receive signals that could be used to advantage.

2. SUMMARY OF HMPAR OPERATION

2.1. Signal Selection Methods

Throughout we assume that the phased array comprises MP elements, arranged as M subarrays of P elements each. We envision two modes of operation for the HMPAR: 1) Fanned subarrays and quasi-orthogonal signals, and 2) focused subarrays and correlated signals. In Mode 1, each of the subarrays forms a transmit beam in a different direction, and the M different signals chosen to drive the subarrays are quasi-orthogonal so that there is little cross-talk between channels. This mode could be used in a search operation where the radar energy is spread over a large angular region. In Mode 2, all the subarrays form a transmit beam in the same direction. Additional directivity and beampattern control is introduced by the selection of correlated signals. In this paper we consider Mode 2 operation only.

We now review the signal selection methodology proposed in [3]. This is most easily explained using onedimensional uniform linear arrays; the 2-D transmit beampatterns can later be defined as Kronecker products of 1-D beampatterns.

Our signal model assumes coded waveforms and thus the signals can be represented as discrete sequences, with the sampling rate approximately equal to the radar bandwidth *B*. The time-bandwidth product *BT* is then approximately equal to the number of symbols *N* in one pulse. With *M* subarrays, there are *M* such signals $s_i(n)$, $i = 1 \cdots M$, $n = 0 \cdots N - 1$ which for analysis purposes could be put into a single $M \times N$ matrix **S**.

For the case of a uniform linear array with a single angular parameter (the electrical angle ϕ) we define a set of signals in which the beamwidth is controlled by a single scalar parameter α . Let $s_i(n)$, $i = 1 \cdots M$, $n = 0 \cdots N - 1$ be given by the expression

$$s_i(n) = e^{\frac{j2\pi\alpha(i-1)n}{N}} e^{-j\psi(i-1)}$$
 (2.1)

where

$$\Psi = \pi \alpha \left(\frac{N-1}{N} \right) \,. \tag{2.2}$$

This particular value of ψ will steer the center of the beam toward array broadside; other choices will cause the beam be steered in other directions.

The cross-correlation between signal i and signal k is

$$r_{ik} = \frac{\sin[\pi\alpha(i-k)]}{\sin[\pi\alpha(i-k)/N]} \quad . \tag{2.3}$$

The full cross-correlation matrix R(0), with *ik* element r_{ik} as given above, is a Toeplitz matrix (constant along the diagonals) with the *l*th diagonal given by the Dirichlet function in (2.3). For $\alpha = 1$, R(0) is *N* times the identity matrix, and when $\alpha = 0$, R(0) is *N* times the rankone all-ones matrix. These correspond to the two extremes of quasi-orthogonal and phased-array signaling. For values of α between 0 and 1, the resulting transmit beampattern is approximately rectangular, where the beamwidth in electrical angle space is proportional to α . More precisely, the transmit beampattern is the convolution of a ideal rectangular pattern with a sinc-squared function. This is identical to the frequency response of an FIR filter designed using the window method with a triangular window.

In the 2-dimensional case, there are two parameters α_h and α_v , where the subscripts connote "horizontal" and "vertical", respectively. Signals are computed as Kronecker products of the same sort of complex exponential sequences described above. The parameters α_h and α_v determine the horizontal and vertical beamwidths of a rectangular beampattern in *u*-*v* space, which can be described as a Kronecker product of two one-dimensional beampatterns.

As an example, consider the display shown in Figure 2. We have chosen as our model a 30×30 array, partitioned in 25 subarrays of 36 elements each, as shown in the upper-left panel. The upper right panel shows the transmit beampattern for a single subarray, and it is assumed that all subarrays have this same pattern. The lower-left panel shows the transmit beampattern of the "meta-array", defined as the hypothetical array of omnidirectional transmitters located at the phase centers of the subarrays. By a straightforward pattern multiplication argument, it can be shown that the overall transmit beampattern is found as the product of these two, with the result shown in the lower-right panel.



Figure 2. Example HMPAR transmit beampatterns

Key to understanding the HMPAR operation is that the meta-array response is not the result of passive phaseshifting, but rather the selection of M different signals driving the M different subarrays in a MIMO mode.

2.2. MIMO Ambiguity Function

The MIMO radar ambiguity function is defined as the inner product of two normalized received signals under two different sets of target parameters. For a fully coherent radar with spatially distributed assets, the target parameters could be a 3-D position vector and a 3-D velocity vector, with a resulting ambiguity function a function of 12 parameters. The HMPAR has colocated assets and thus the ambiguity function can be expressed as a function of delay, Doppler, and one or two angles.

The key equations for the HMPAR ambiguity function, as derived in [3] are follows. There exists a subarray pattern $\mathbf{b}(\theta)$ and a meta-array pattern $\mathbf{a}(\theta)$, derived purely from spatial considerations. $\mathbf{b}(\theta)$ is the subarray pattern defined relative to the subarray local phase center, and $a_i(\theta)$ is the free-space phase of the *i*th phase center relative to the global phase center. The parameter θ could represent one angle or two (e.g. azimuth and elevation) depending on whether the array is 1-D or 2-D. The product pattern is

$$\mathbf{c}(\theta) = \mathbf{a}(\theta) \circ \mathbf{b}(\theta) \quad . \tag{2.4}$$

Delay and Doppler properties are determined by the selection of the signal matrix \mathbf{S} , which in turn defines the signal matrix cross-ambiguity function:

$$\chi_M(\tau, f) = \int_{-\infty}^{\infty} \mathbf{s}(t) \mathbf{s}^{\mathrm{H}}(t+\tau) e^{j2\pi f t} dt \qquad (2.5)$$

where $\tau = \tau_1 - \tau_2$ and $f = f_1 - f_2$.

The overall HMPAR ambiguity function is given by

$$\chi_H(\tau, f, \theta_1, \theta_2) = \mathbf{c}^{\mathrm{H}}(\theta_2)\mathbf{c}(\theta_1) \cdot \mathbf{c}^{\mathrm{T}}(\theta_1)\chi_M(\tau, f)\mathbf{c}^*(\theta_2)(2.6)$$

This expression can be viewed as the product of a transmit factor and a receive factor, where the transmit factor is a function of both spatial and signal parameters, and the receive factor is a function of the receive array only.

3. ADVANCED SIGNALING STRATEGIES

It has recently come to our attention [4] that the signals described in [3] and summarized above have the property that, at any instant in time, the symbol values taken across the subarrays form a vector that is the steering vector $\mathbf{a}(\theta)$ for a particular spatial angle θ . This tells us that for the short period of time associated with one symbol, the HMPAR is a behaving like a fully coherent phased array pointing a transmit beam in the direction θ . Over time, within one pulse, the angle varies, and as a result the radar is rapidly scanning a pencil beam across the field of view.

This observation requires us to examine more closely the fine-scale temporal properties of the transmitted pulse. All of our previous work was based on the distribution of the total transmitted energy across space. The correlation matrix is

$$\mathbf{R} = \mathbf{S}\mathbf{S}^{\mathsf{H}} \tag{3.1}$$

and the transmit beampattern is proportional to

$$S(\theta) = \mathbf{a}^{\mathrm{T}}(\theta)\mathbf{R}\mathbf{a}^{*}(\theta) . \qquad (3.2)$$

Another way to write the correlation matrix in (3.1) is

$$\mathbf{R} = \sum_{n=1}^{N} \mathbf{s}(n) \mathbf{s}^{\mathrm{H}}(n)$$
(3.3)

where $\mathbf{s}(n)$ is the n^{th} column of **S**. Then

$$S(\boldsymbol{\theta}) = \sum_{n=1}^{N} |\mathbf{a}^{\mathrm{T}}(\boldsymbol{\theta})\mathbf{s}(n)|^{2} , \qquad (3.4)$$

that is, $S(\theta)$ is the sum of the instantaneous beampatterns across the entire sequence of transmitted symbols.

This suggests an alternative, indeed simpler, approach to signal design. Define Θ to be some fixed field of view, such as the rectangular region to be covered by the transmit beampattern. Then define a sequence of angles $\theta_1 \cdots \theta_N$ that covers this region uniformly, such as a sweep in one dimension or a raster scan in two. Then, choose

$$\mathbf{s}(n) = \mathbf{a}^*(\boldsymbol{\theta}_n) \ . \tag{3.5}$$

and set

$$\mathbf{S} = [\mathbf{s}(1)\cdots\mathbf{s}(N)] . \tag{3.6}$$

The transmitted pulse would rapidly scan the field of view.

This approach could be applied to totally arbitrary Θ , such as the union of disjoint regions. This would allow for the maintenance of multiple simultaneous beams. An example is shown in Figure 3. Here we simulate a 32×32 phased array with elements at half-wavelength spacing, with the signal matrix **S** chosen so that the scan points

cover a region in u-v which is the union of two rectangles. Each column of the matrix **S** is a direction vector pointed toward one of N scan points in u-v, shown in left panel of Figure 3. The right panel shows the resulting transmit beampattern. This is computed as the convolution of the high-gain transmit beampattern of the array with a set of impulses uniformly distributed over the regions of interest.

There is nothing in Figure 3 that reveals the time history of the scan points, for $n = 0 \cdots N - 1$. The sequence could be a simple raster scan over the first rectangle, followed by a raster scan over the second. Alternatively, the scan points could jump back and forth from one region to another, or they could represent a random permutation of an orderly sequence. What is shown in the figure is the distribution of *energy* over space, which is independent of the time history.

As a further extension, the set of scan points could be nonuniform, allowing the radar to "dwell" or devote more energy to some regions than others. The selection of such patterns would be driven by higher-level system considerations and the specific radar mission. As an example, consider the example shown in Figure 4. Here the scan points cover a tilted rectangular region of space, but in place of a uniform covering of the region, the scan points are spaced apart using the inverse cumulative distribution function for the Gaussian distribution. The resulting transmit beampattern has a two-dimensional Gaussian profile. Figure 5 shows the horizonal and vertical cuts through this beampattern. In [5] we show how such a pattern could be adapted to the prior knowledge on the target location, to maximize the performance of a target position estimator and target tracker.

Mathematically, there is no difference between what is proposed here and what one might obtain by using the phased array to scan some search volume using the analog electronics that drive the transmit elements. From a radar system viewpoint, however, the difference comes from the fact that the scanning is a consequence of the selected signal set \mathbf{S} , and not the phase shifts applied to the individual elements or subarrays. This allows for tremendous flexibility in the choice of the transmit beampatterns, which can be selected "on-the-fly" as part of a feedback loop in a fully adaptive radar.

This interpretation of the transmit signal sets also reveals an interesting range-angle coupling in the received data. Within one pulse epoch, if there exists a target at location θ_n then the reflected signal will be associated primarily with the transmitted symbol s(n) and its nearest neighbors. Thus, in addition to the usual phase-based spatial array processing, the timing of the received compressed pulse, after matched filtering, will carry information about the spatial location of the target.



Figure 3. Scan points and transmit beampattern for arbitrary region of *u*-*v* space.



Figure 4. Nonuniform scan points and Gaussian transmit beampattern



Figure 5. Cuts through Gaussian transmit beampattern

4. CONCLUSION

The basic operating principles of the notional Hybrid MIMO Phased Array Radar, or HMPAR, have been summarized. The selection of complex exponential symbol sequences to achieve desired transmit beampatterns, and the space-time ambiguity functions for the resulting signal sets, were described. Based on the observation that these signals will result in a rapid scan of the field of view within one pulse, a new signal selection strategy was described that is simple, intuitive, and flexible. The analysis reveals a range-angle coupling in the received data.

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