Detection and Estimation of Multi-Pulse LFMCW Radar Signals

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***Abstract*— The Wigner-Ville Hough transform (WVHT) has been applied to detect and estimate the parameters of linear frequency-modulated continuous-wave (LFMCW) low probability of intercept (LPI) radar waveforms. The WVHT, which is optimal for a single linear frequency modulated (LFM) signal, becomes sub-optimal when applied to LFMCW signals since the observed waveform is composed of concatenated LFM pulses. We formulate the detection and estimation problem to take into account the multiple pulses that are available in an observation interval at the intercept receiver. The new algorithm, called the periodic WVHT (PWVHT), is shown to significantly outperform the WVHT for LFMCW signals.**

I. Introduction

In electronic warfare, knowledge of enemies' electronic capabilities is desired. Electronic warfare includes the interception and analysis of radar signals (ELINT) to determine information about their capabilities [1]. The general process of ELINT includes a detection and estimation stage in which the sensor decides whether signals of interests (SOIs) are present and, if they are, estimates their parameters, such as the carrier frequencies and pulse durations. The main topic of this paper is the detection and parameter estimation of linear-frequency-modulated continuous wave (LFMCW) radar signals.

Various signal-processing techniques have been investigated for the detection and estimation of low probability of intercept (LPI) radar signals [2]–[6], including LFMCW signals. (For a detailed discussion of LPI radar signals, see [7]). In general, the detection and estimation of linear FM (LFM) signals has been well studied in the literature. One of the most prominent techniques is the Wigner-Ville Hough Transform [8], [9]. The Wigner-Ville Hough transform (WVHT), the Hough transform (HT) of the Wigner-Ville distribution (WVD), has been shown to be equivalent to the generalized likelihood ratio test (GLRT) and maximum likelihood estimator (MLE) (i.e., it is asymptotically optimal) in detection and estimation of an LFM signal [10]. A sawtooth LFMCW signal is a periodic extension of an LFM signals, i.e., the frequency within a modulation period of an LFMCW radar signal changes linearly, as illustrated in Figure 1. The WVHT is not optimal for the detection and estimation of LFMCW radar signals since they are not LFM when we consider the whole observation time.

In radar signal interception, the observation time can be controlled to the advantage of the intercept receiver: by having a longer observation time, more pulses are available within the observation signal and thus there is more energy available for detection. Intuitively, the detection and estimation performance ought to improve as the observation time is increased. Unfortunately, this is not the case with the WVHT, since it is tailored for LFM signals.

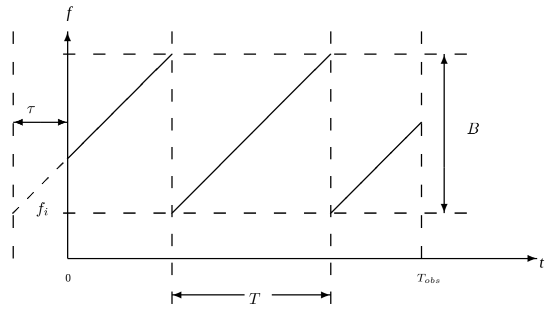

Fig. 1. Available LFMCW radar signal within an observation time T_{obs}

In this paper we present a detection and estimation technique named the periodic WVHT (PWVHT) that is optimal for LFMCW signals. The algorithm is similar to the WVHT except that it searches for patterns, such as a sawtooth waveform, in the time-frequency (TF) image. It is thus able to accumulate signal energy over the entire observation interval, unlike the traditional WVHT. The PWVHT is shown to outperform the WVHT applied to a LFMCW signal.

II. Interceptor for LFMCW Signals

We assume that the signal processing block in the interceptor is presented with a noisy discrete-time observation signal $r[n]$, $n = 0, 1, ..., N-1$, where N is the number of samples. If SOIs are present, these bandpass continuous LFMCW radar signals are denoted as $s'(t)$. Let $s(t)$ be the signal $s'(t)$ down-converted to a known intermediate frequency, which is then sampled with sampling period Δ (sec).

The LFMCW radar signal can be written as

$$s(t) = Ae^{j(\varphi + 2\pi f_i t + \pi g \text{mod}(t+\tau,T)^2)}, \quad (1)$$

where A is the amplitude, f_i (Hz) is the initial frequency after down-conversion, and $g = \frac{B}{T}$ ($\frac{Hz}{sec}$) is the chirp-rate, where B (Hz) is the bandwidth, τ (sec) is an initial time-offset, and T (sec) is the modulation period. The symbol φ represents

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14. ABSTRACT
The Wigner-Ville Hough transform (WVHT) has been applied to detect and estimate the parameters of linear frequency-modulated continuous-wave (LFMCW) low probability of intercept (LPI) radar waveforms. The WVHT, which is optimal for a single linear frequency modulated (LFM) signal becomes sub-optimal when applied to LFMCW signals since the observed waveform is composed of concatenated LFM pulses. We formulate the detection and estimation problem to take into account the multiple pulses that are available in an observation interval at the intercept receiver. The new algorithm, called the periodic WVHT (PWVHT), is shown to significantly outperform the WVHT for LFMCW signals.

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the initial phase, which is distributed uniformly $U(-\pi, \pi)$, and $\mathrm{mod}(t, T)$ is a shorthand notation for (t modulo T). The observation time t (sec) at the intercept receiver is bounded by T_{obs}, i.e., $0 \leq t \leq T_{obs}$. A time vs. frequency (t, f) illustration of the LFMCW radar signal $s(t)$, along with its parameters is given in Figure 1.

The discrete-time LFMCW radar signals that may be present in $r[n]$ can be written as $s[n] = s(t) \mid_{t=n\Delta}$. Given a noisy observation signal $r[n]$, the decision problem is to decide whether hypothesis H_0 or H_1 is true, where

$$H_0 : r[n] = w[n] \quad (2)$$

$$H_1 : r[n] = s[n] + w[n] \quad (3)$$

The noise $w[n]$ is zero-mean, stationary, complex white Gaussian, i.e., $\mathcal{N}(0, \sigma_w^2)$. If the decision is that hypothesis H_1 is true, then an estimate of the true parameter vector $\Omega = (f_i, g, T, \tau)$ is desired. The amplitudes and phases of the signals are considered nuisance parameters.

III. Periodic Wigner-Ville Hough Transform

The crucial drawback of the WVHT when used for the detection and estimation of LFMCW radar signals at the intercept receiver is that there no longer is gain in performance when the observation time is increased. In this section we introduce the periodic Wigner-Ville Hough transform (PWVHT), which is based on the WVHT but tailored to LFMCW signals. The transformation from the time and frequency (t, f) domain to the initial frequency and chirp rate $(\tilde{f}, \tilde{g})$ domain in the WVHT is done by summing over straight lines, parameterized by $(\tilde{f}, \tilde{g})$, in the WVD. For an arbitrary signal $x[n]$ it can be written as

$$W_x[n, f] = 2 \sum_{k=-\infty}^{\infty} C_{xx}[n, k] F(f, k), \quad (4)$$

where $F(f, k) = e^{-j4\pi f k\Delta}$ is the matching function, $C_{xy}[n, k] = x[n+k]y^*[n-k]$ is the instantaneous discrete time cross-correlation, and the symbol "$*$" stands for the complex conjugate. The WVD is basically the Fourier transform of the instantaneous auto-correlation function with respect to k. For a single LFM signal $s[n] = Ae^{j(\varphi+2\pi f_i t+\pi g t^2)} \mid_{t=n\Delta}$, the instantaneous auto-correlation function is $C_{ss}[n, k] = A^2 e^{j4\pi k\Delta(f_i+gn\Delta)}$. Thus, the WVD of an LFM signal is

$$W_s[n, f] = \sum_{k=-\infty}^{\infty} A^2 e^{-j4\pi k\Delta(f-f_i-gn\Delta)}. \quad (5)$$

Notice that each term in the WVD is maximized only when it is evaluated at the signal parameters $f = f_i + gn\Delta$ at any given time n since the exponential terms cancel out.

The WVHT can be written as

$$WH_x[\tilde{f}, \tilde{g}] = \sum_{n=0}^{N/2-1} \sum_{k=-n}^{n} C_{xx}[n, k] F(\tilde{f} + \tilde{g}n\Delta, k) + \sum_{n=N/2}^{N-1} \sum_{k=-(N-1-n)}^{N-1-n} C_{xx}[n, k] F(\tilde{f} + \tilde{g}n\Delta, k). \quad (6)$$

Therefore, the discrete WVHT of a discrete LFM signal is

$$WH_s[\tilde{f}, \tilde{g}] = \sum_{n} \sum_{k} A^2 e^{-j4\pi k\Delta\left((\tilde{f}-f_i)+(\tilde{g}-g)n\Delta\right)} \quad (7)$$

and is maximized only when evaluated at $\tilde{f} = f_i$ and $\tilde{g} = g$, which results in

$$WH_s[f_i, g] = \frac{N^2 A^2}{2}. \quad (8)$$

It can be seen that the canceling of complex exponentials in (5) and (7) when evaluated at the signal parameters is a key in detection and estimation of an LFM signal by the WVHT.

In the case of an LFM signal, $F(f, k)$ and $C_{xx}[n, k]$ are a good match. The Hough transformation involves integrating the WVD along the frequency law pattern of the signal in the TF plane for all the possible values for the parameters, and only at the correct signal parameters is the sum maximized, which is equivalent to when $F(f, k)$ completely cancels out the exponential term in $C_{xx}[n, k]$.

The WVHT is capable of detecting and estimating LFM signals even when more than one signal is present, as seen in Figure 2 where three LFM signals are processed, yet it is not optimal for more than one LFM signal. The main reason is that the matching function cannot completely remove the exponential term in $C_{xx}[n, k]$. For instance, the discrete instantaneous auto-correlation of a noise-free observation $r[n]$ consisting of two LFM signals $s_1[n]$ and $s_2[n]$ is

$$C_{rr}[n, k] = C_{s_1 s_1}[n, k] + C_{s_2 s_2}[n, k] + C_{s_1 s_2}[n, k] + C_{s_2 s_1}[n, k] \quad (9)$$

and it is easy to see that $F(f, k)|_{f=\tilde{f}+\tilde{g}n}$ cannot completely eliminate the exponential terms in (9). When the value of f in $F(f, k)$ matches one of the signal parameters, it cancels out one of the exponential term, either in $C_{s_1 s_1}[n, k]$ or $C_{s_2 s_2}[n, k]$, yielding prominent peaks in (6). It also shows why it is sub-optimal since $F(f, k)$ can only cancel one exponential term at a time. Thus, the WVHT of M LFM signals yields M peaks, which need to be searched by a peak finding algorithm.

The WVHT applied to LFMCW signals is also sub-optimal because the matching function $F(f, k)$ cannot completely cancel out the exponential terms in (6) when multiplied with the instantaneous auto-correlation function $C_{xx}[n, k]$ of an LFMCW radar signal, i.e., they are not a good match. For an LFMCW radar signal

$$s[n] = Ae^{j(\varphi+2\pi f_i(n\Delta)+\pi g \mathrm{mod}(n\Delta+\tau, T)^2)}. \quad (10)$$

The instantaneous auto-correlation function is

$$C_{ss}[n, k] = A^2 \exp\{j\pi(4f_i k\Delta + g[\mathrm{mod}((n+k)\Delta+\tau, T)^2 - \mathrm{mod}((n-k)\Delta+\tau, T)^2])\}. \quad (11)$$

Thus, the WVHT in (6) results in

$$WH_s[\tilde{f}, \tilde{g}] = \sum_{n} \sum_{k} A^2 \exp\{j\pi(4(f_i - \tilde{f} - \tilde{g}n)k\Delta + g[\mathrm{mod}((n+k)\Delta+\tau, T)^2 - \mathrm{mod}((n-k)\Delta+\tau, T)^2])\},$$

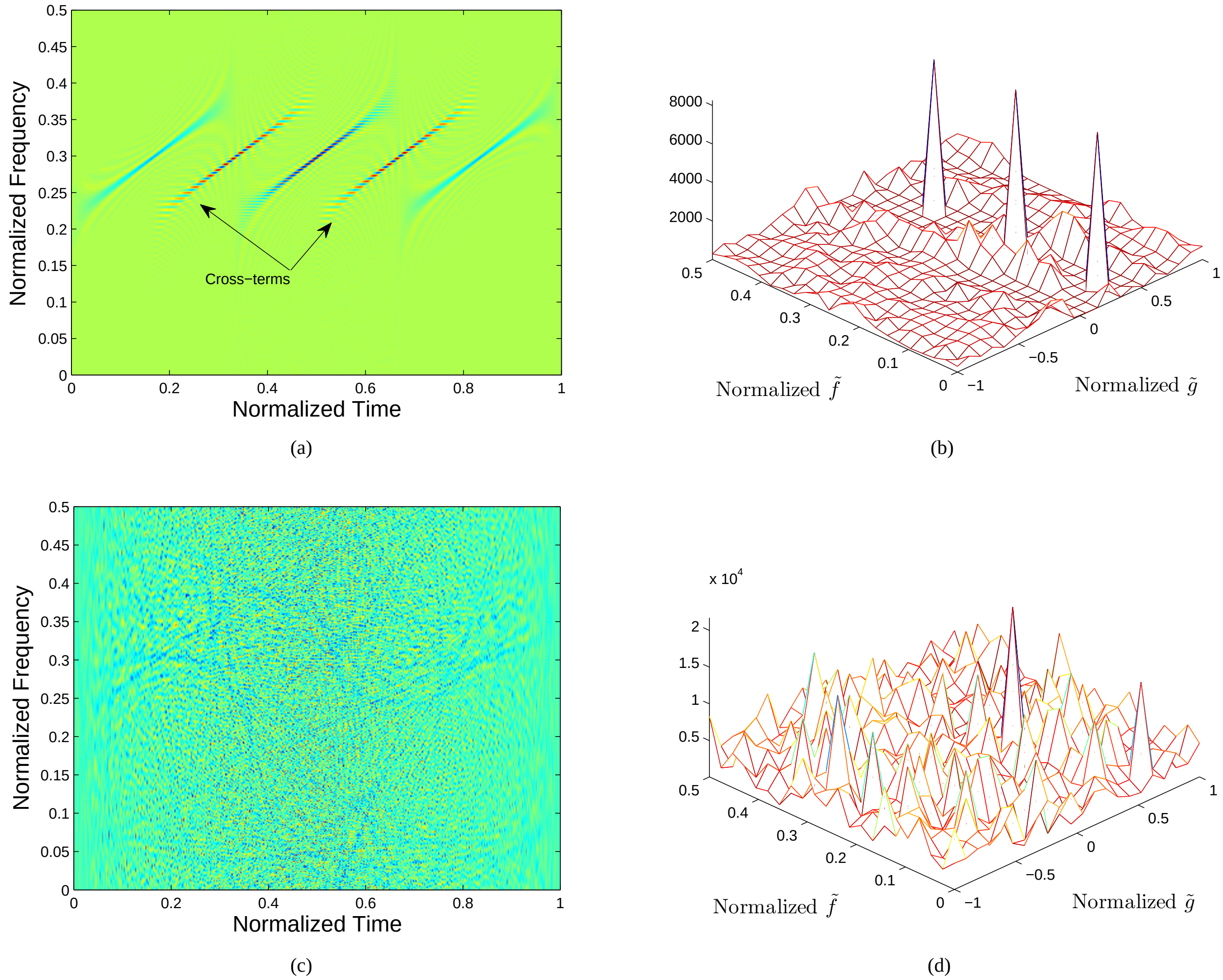

Fig. 2. WVD and WVHT parameterized by $(\tilde{f}, \tilde{g})$ of a single LFMCW signal formed by three LFM pulses. Noise-free: (a) WVD, (b) WVHT. Noisy with $\text{SNR}_i = -10$ dB: (c) WVD, (d) WVHT. In this and subsequent figures, the frequency is normalized by $1/\Delta$, the modulation period and time offset by T_{obs}, and the chirp-rate by the maximum chirp-rate searched.

and the exponentials do not cancel out for any values of $\tilde{f}$ and $\tilde{g}$. Thus, the summation results in a smaller peak.

The optimal algorithm for one LFMCW signal uses a matching function that can completely cancel out the exponential terms when multiplied with (11). We define the PWVHT of an arbitrary complex sequence $x[n]$, $n = 0, 1, ..., N-1$ to be

$$Z_x[\tilde{\Omega}] = \sum_{n=0}^{N/2-1} \sum_{k=-n}^{n} C_{xx}[n,k] F_{n,k}[\tilde{\Omega}] + \sum_{n=N/2}^{N-1} \sum_{k=-(N-1-n)}^{N-1-n} C_{xx}[n,k] F_{n,k}[\tilde{\Omega}], \quad (12)$$

where $\tilde{\Omega} = (\tilde{f}, \tilde{g}, \tilde{T}, \tilde{\tau})$. We assume that N is even and the matching function is

$$F_{n,k}[\tilde{\Omega}] = \exp\{-j\pi(4\tilde{f}k\Delta + \tilde{g}[\text{mod}((n+k)\Delta + \tilde{\tau}, \tilde{T})^2 - \text{mod}((n-k)\Delta + \tilde{\tau}, \tilde{T})^2])\}, \quad (13)$$

which, when evaluated at the true signal parameters $\tilde{\Omega} = \Omega = (f_i, g, T, \tau)$, is the complex conjugate of the exponential term of (11). The PWVHT is optimal as it is equivalent to the GLRT and the estimates it yields are MLE.

For an LFMCW radar signal at the intercept receiver, the peak of the PWVHT is achieved at the signal parameters Ω. For instance, if we substitute the LFMCW signal in (10) into (12), and evaluating it at the true signal parameters, it yields $Z_s[\Omega] = \frac{N^2 A^2}{2}$, which is the maximum and equal to (8). For other values, i.e., $\tilde{\Omega} \neq \Omega$, the matching function does not cancel out the exponential terms, resulting in a smaller value.

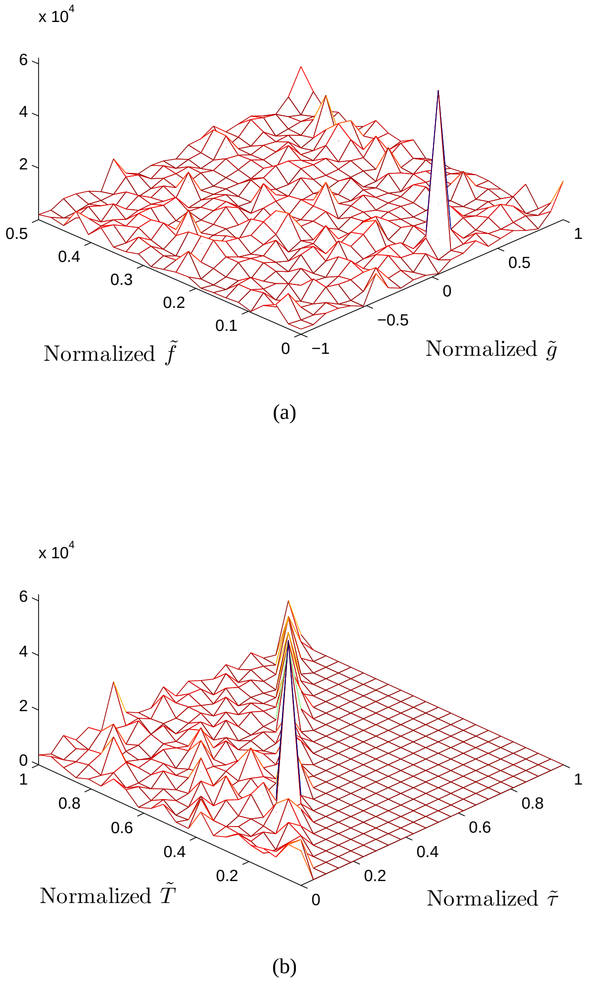

Fig. 3. Slices of $Z_r[\tilde{\Omega}]$ of a noisy LFMCW signal in Figure 2: (a) slice at the true modulation period and time offset of the signal, (b) slice at the true initial frequency and chirp rate of the signal. $\text{SNR}_i = -10$ dB.

IV. Simulation Results and Analysis

To illustrate the performance of the PWVHT, $Z[\tilde{\Omega}]$, we apply it to the simulated LFMCW radar signal shown in Figure 2. When the PWVHT of the noisy signal is evaluated, a prominent peak appears, as seen in Figure 3, where slices of the $Z[\tilde{\Omega}]$ are shown. The right half of Figure 3(b) is smooth because only time offsets less than the modulation periods need to be searched. Each plot indicates a peak, revealing the correct number of signals (one instead of three). In addition, the location of the peak gives an estimate of the signal parameters. The detection and parameter estimation of the LFMCW signals can then be achieved by searching for peaks in the $\tilde{\Omega}$ domain.

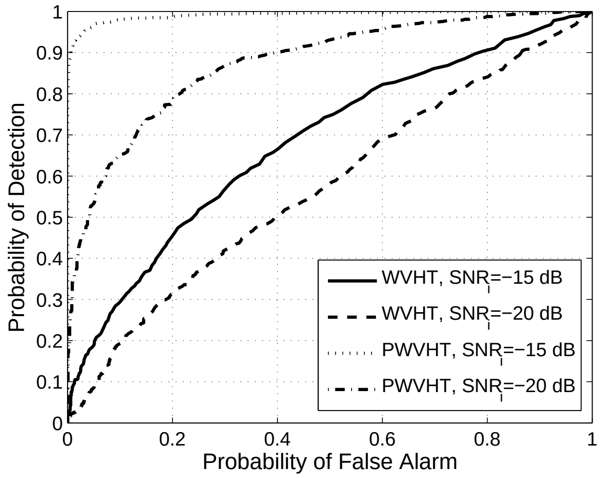

Fig. 4. ROC plot of the PWVHT and the WVHT for a single LFMCW signal, using $N = 330$ and $T_m = 110\Delta$ (3 concatenated LFW signals)

The output SNR can be canculated to be

$$\text{SNR}_{PWVHT} = \frac{\frac{N^2}{2}\left(\frac{A^2}{\sigma_w^2}\right)^2}{N\frac{A^2}{\sigma_w^2}+1} = \frac{\frac{N^2}{2}\text{SNR}_i^2}{N\text{SNR}_i+1}. \tag{14}$$

This is a remarkable result because when the input SNR is high, ($\text{SNR}_i = \frac{A^2}{\sigma_w^2} >> 1$), the output SNR is approximately $\text{SNR}_{PWVHT} = \frac{N\text{SNR}_i}{2}$, which indicates that it is almost as good as a matched filter's output SNR ($\text{SNR}_{MF} = N\text{SNR}_i$), but with approximately 3 dB loss due to quadratic detection. Most importantly, when the input SNR is very low ($\text{SNR}_i << 1$), detection can still be accomplished by increasing number of samples N and benefiting from processing gain. Not surprisingly, the PWVHT is equivalent to the GLRT and the MLE for an LFMCW radar signal at the intercept receiver. The detection performance of the PWVHT is shown using a received operating characteristic (ROC) plot in Figure 4. The probability of detecting a simulated LFMCW signal using the PWVHT is compared to the same metric using a standard WVHT for two input SNR values, as the probability of false alarm varies. The LFMCW algorithm is clearly superior to the WVHT, as predicted since the signal is LFMCW and not purely LFM.

Although the PWVHT is superior to the WVHT for detection and estimation of a signal that is composed of multiple LFM signals, the improved performance comes at a cost in computational complexity. However, for LPI application, the PWVHT is realizable. For detection and estimation of LFMCW radar signal, the search is usually bounded, i.e., there is *a priori* knowledge about the range of possible initial frequencies, chirp-rates, and modulation periods, which allows one to restrict the search in the parameter space and reduce the amount of computations.

V. Conclusion

In this paper we present a new signal processing technique, the PWVHT, for detection and estimation of a LFMCW radar signal. Within an observation session at an intercept receiver, the received signal is composed of multiple LFM pulses if an SOI is present. Currently available techniques are suboptimal since they are tailored for the detection and estimation of a single pulse. The PWVHT technique is equivalent to the GLRT and MLE for an LFMCW radar signal, and thus provides an optimal solution. The improvement in performance comes at a cost in computational complexity.

The idea in the PWVHT can be extended to other signal types (i.e., other signals composed of multiple LFM pulses) by designing a matching function that can completely remove the exponential term when multiplied with its instantaneous auto-correlation function. This warrants further research.

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