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14. ABSTRACT In this project, the primary aim is to advance the science of controlling collectives of systems engaged in cooperative and adversarial behaviors, by focusing on: (a) robust dyadic (i.e. pair-wise) interaction laws between particles (modeling UAV/UGV/UUV interactions) to achieve a rich collection of spatio-temporal patterns of cooperative behavior; (b) robust scalable dyadic pursuit laws as building blocks of adversarial interactions; (c) optimality principles underlying interactions laws that gives rise to cooperative and adversarial behavior; (d) insight					
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Report Title

Gyroscopic Many Body Problems in Cooperative and Adversarial Control

ABSTRACT

In this project, the primary aim is to advance the science of controlling collectives of systems engaged in cooperative and adversarial behaviors, by focusing on: (a) robust dyadic (i.e. pair-wise) interaction laws between particles (modeling UAV/UGV/UUV interactions) to achieve a rich collection of spatio-temporal patterns of cooperative behavior; (b) robust scalable dyadic pursuit laws as building blocks of adversarial interactions; (c) optimality principles underlying interactions laws that gives rise to cooperative and adversarial behavior; (d) insight from biological processes such as prey-capture behavior or pursuit, in echolocating bats, and in visual insects such as dragonflies; (e) verification of the theoretical advances in a laboratory test-bed consisting of commercial robotic platforms equipped with distributed agent-based control software, and an indoor GPS system based on ultrasonic ranging. Methods used in this research include techniques from geometric mechanics and control on Lie groups, and optimal control and game theory. The project has contributed basic results in pursuit laws as building blocks for cooperative and adversarial control. In collaboration with biologists, methods developed in this project are enhancing our understanding of prey-capture behavior in single and multiple echolocating bats.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Number of Papers published in peer-reviewed journals: 0.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

Number of Papers published in non peer-reviewed journals: 0.00

(c) Presentations

GRASP Laboratory Colloquium, University of Pennsylvania: (March 26, 2010) "Geometry of Cyclic Pursuit"

ECE-GSA Seminar and TA Workshop, University of Maryland (April 2, 2010) "Geometric Phase: An Introduction"

CSCAMM Workshop on Nonlinear Dynamics of Networks 2010, University of Maryland, College Park: (April 6, 2010) "Pursuit and Collective Behavior"

Maryland Robotics Center, University of Maryland, College Park: (April 23, 2010) "Pursuit and Cohesion: Bio-inspiration for Collective Robotics"

Number of Presentations: 4.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

Peer-Reviewed Conference Proceeding publications (other than abstracts):

K. Galloway, Eric Justh and P. S. Krishnaprasad (2009), "Geometry of Cyclic Pursuit," Proceedings of the 48th IEEE Conference on Decision and Control, pp. 7485-7490, IEEE, New York.

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts): 1

(d) Manuscripts

M. Mischiati and P. S. Krishnaprasad (2009), "Motion camouflage for coverage," to appear, Proceedings of the American Control Conference, June 2010, American Automatic Control Council.

K. Galloway, Eric Justh and P. S. Krishnaprasad (2010), "Cyclic pursuit in 3 dimensions," submitted to 49th IEEE Conference on Decision and Control.

Eric Justh and P. S. Krishnaprasad (2010), "Extremal collective behavior," submitted to 49th IEEE Conference on Decision and Control.

Eric Justh and P. S. Krishnaprasad (2010), "Optimal natural frames," submitted to Communications in Information and Systems.

C. Chiu, P. V. Reddy, W. Xian, P. S. Krishnaprasad, and C. F. Moss (2010), "Effects of competitive prey capture on flight behavior and sonar beam pattern in paired big brown bats, Eptesicus fuscus," under revision, Journal of Experimental Biology.

Number of Manuscripts: 5.00

Patents Submitted

Patents Awarded

Graduate Students

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
FTE Equivalent:	
Total Number:	

Names of Post Doctorates

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	National Academy Member
P. S. Krishnaprasad	0.00	No
FTE Equivalent:	0.00	
Total Number:	1	

Names of Under Graduate students supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:..... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale): 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: 0.00

Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PhDs

NAME

Bijan Afsari

Total Number:

1

Names of other research staff

NAME

PERCENT SUPPORTED

FTE Equivalent:

Total Number:

Sub Contractors (DD882)

Inventions (DD882)

Problem Studied

The main problems of this project are (a) To determine robust scalable dyadic (i.e. pair-wise) interaction laws between particles (modeling UGV/UAV/UUV collectives) to achieve complex patterns of cooperative behavior; (b) To determine robust scalable dyadic pursuit laws that would serve as building blocks for adversarial interactions; (c) To uncover optimality principles underlying interaction laws that give rise to cooperative and adversarial behavior; (d) To draw insight from biological processes such as prey-capture behavior in individual echolocating bats, adversarial prey-capture encounters in multiple echolocating bats, and aggressive territorial maneuvers in visual insects such as dragonflies that employ stealth mechanisms in their flight patterns; (e) To exploit pursuit laws in cooperative behavior, such as cyclic pursuit; (f) To verify the theoretical advances made in attacking problems (a)-(c), and (e) by exploration of control laws in a laboratory test-bed.

Methods based on geometric mechanics and control, Lie group theory, and stochastic differential equations are currently used in the project to attack the stated problems. Methods from differential game theory and evolutionary game theory have also been introduced into our program of research. Evolutionary game-theoretic methods have been used to set up a competition between strategies and to uncover certain optimality properties of a particular strategy, the motion camouflage pursuit. Other pursuit strategies such as constant bearing pursuit and classical pursuit have been investigated and compared with motion camouflage pursuit. In collaboration with biologists, mathematical methods developed in this project are enhancing our understanding of the prey-capture behavior of echolocating bats. A mobile robot test-bed is used in carrying out a modest set of experimental verifications of the theoretical work. This test-bed includes a location sensing network employing ultrasound and RF transceivers for ranging, in a manner akin to how echolocation is accomplished in nature.

Scientific Barriers

At the start of this project, we faced a series of scientific barriers associated with the role of delay and stochasticity. In communicating cooperative collectives, communication delays as well as control delays contribute to possible instabilities. Additionally, stochasticity in the form of sensor noise as well as actuation noise in cooperative systems, and randomized strategies in adversarial systems are a source of complexity, and contribute to performance limitations. In adversarial situations such performance limitations may be exploited (for instance in evasive behavior). Delay and stochasticity are important issues in effective realization of feedback laws in neuronal and biomechanical substrates. They are also important issues in technological implementations. At the start of this project, we also were in the early stages of developing a 3-dimensional theory for pursuit laws in adversarial systems, with details remaining to be worked out. Additional barriers include: (a) limited understanding of the issues of sensory jamming when multiple members of a collective use active-sensing (e.g. bat vocalizations) in a swarm; (b) limited understanding of selective attentional mechanisms that would enable successful exploitation of dyadic interactions in cooperative and adversarial control; (c) limited understanding of the prospects for scalability of pursuit strategies to collectives of many agents.

Significance

The central theme of this project to advance the conceptual and analytical foundations of gyroscopic many body problems is an important one not only for its pure scientific interest but also for the applicability of results emerging from this project to the steering of collectives of engineered systems (such as unmanned aerial vehicles and unmanned ground vehicles) of interest to the Army. The informed use of biological inspiration through close collaboration with biologists, and extensive access and familiarity with the data arising from prey-capture behavior of echolocating bats (collected in the auditory neuroethology laboratory of Cynthia Moss) is a distinctive aspect of this project. It is appropriate and significant due to the strong functional similarities between the natural system (bat) and engineered systems (such as small unmanned aerial vehicles) in adversarial settings. The exploitation of commercial-grade distributed software environments for demonstrations of principle, and the introduction of ideas and techniques from evolutionary game theory for comparisons between alternative control strategies are recent advances in the project that promise greater relevance of this work for army missions. The approach in this project has the distinctive character of being strongly founded on principles of mechanics (second order dynamics), as opposed to pure kinematics (first order dynamics). During 2009, we initiated an international collaboration with a group of distinguished physicists from Italy engaged in observations of highly dynamic flocking behavior in large flocks of starlings (composed of many thousands of individuals). Using our techniques of pursuit, we worked on analyses of flock trajectory data, with the aim of developing control laws compatible with the topological interactions observed and investigated by the Italian researchers.

Accomplishments

Motion camouflage is a stealth strategy observed in nature in the aerial territorial battles of dragonflies which are highly visual insects capable of sensing optic flow, but are not very sensitive to looming cues. Early in this project it was observed that there is an equivalence between this stealth strategy (Justh and Krishnaprasad, 2006), and a prey-pursuit strategy in echolocating bats (Ghose, Horiuchi, Krishnaprasad and Moss, 2006). Motion camouflage is geometrically indistinguishable from the pursuit strategy known as constant absolute target direction (CATD) pursuit observed in bats engaged in prey-capture. The latter is familiar to missile guidance specialists under the name of parallel navigation. This understanding permitted the development of 3D pursuit control laws (Reddy, Justh and Krishnaprasad, 2006), that are biologically plausible.

The problem of sensing-actuation feedback delay is important in nature and in engineered systems. Data from the echolocating bat showed that this delay is in the vicinity of a little over 100 msec. We extended our theoretical work on motion camouflage to analyze pursuit in the presence of sensorimotor delay. We have shown that motion camouflage (or pursuit to capture) is achievable under suitable hypotheses on delay and feedback gain (Reddy, Justh and Krishnaprasad, 2007). The structure of delay-gain feasible set for effective pursuit in 2D was elucidated through detailed analysis in the Master of Science Thesis of Viswanadha Reddy. In recent work, we have extracted actual curvature and speed signals associated with bat flight trajectories and compared these to our theoretical feedback laws for the setting of sensorimotor delay. The results show strong correlation for a delay of about 112 msec. We have developed an

optimization algorithm for extraction of curvature and speed signals from noisy flight data, based on a regularization approach that penalizes very rapid changes in curvature or speed. These results are being readied for publication (Reddy, Ghose, Justh, Horiuchi, Krishnaprasad and Moss, – in preparation). Work on further developments along these computational lines, using the maximum principle of Pontryagin from optimal control theory is being pursued.

In nature and in engineered systems involving adversarial encounters (e.g. missile guidance), noise in actuation is important. An evader (prey) may use noisy or unpredictable steering maneuvers to confound and possibly escape a pursuer (predator). Given the geometrical structure of our models, we asked and answered the question of what the appropriate noise models might be. We also allowed for jump processes in the noise. We further developed a result on weak accessibility of motion camouflage in the presence of noise (Galloway, Justh and Krishnaprasad, 2007).

In ongoing work, we have developed a differential game formulation of motion camouflage (with Kevin Galloway), which form a part of Galloway's Ph.D. work (Galloway, 2008). An evolutionary game-theoretic model for comparison of pursuit strategies (Wei, Justh and Krishnaprasad, 2009), has yielded a new way of looking at competition between different pursuit strategies. In a recent collaboration with Chen Chiu and others (Chiu et. al. 2010), new insights on competitive prey-capture behavior in echolocating bats have been obtained, using analysis methods developed in this project. Further progress has been made in cyclic pursuit in 3 dimensions (Galloway, Justh and Krishnaprasad, 2010), and optimal control in coupled systems (Justh and Krishnaprasad, 2010a and 2010b).

In collaboration with undergraduate researchers in the Intelligent Servosystems Laboratory, we have extended our test-bed for investigation of pursuit laws in mobile robotic systems based on Cricket an indoor localization system. The specific extensions include integration of odometry with INS, GPS, and Cricket by means of filtering and estimation algorithms. This work involved 3 undergraduates during summer 2006, fall 2006 and spring 2007. Three additional undergraduates joined the team in June 2007 for the summer and refined the filtering and estimation algorithms, and their implementation. They also carried out demonstrations of this work on mobile robots executing pursuit strategies. In summer 2008, two undergraduates, Scott Livingston (University of Tennessee) and Laura Freyman (University of Maryland), obtained extensive data on the interaction of a bat in flight with obstacles in a flight room during the course of pursuit of an insect. They then analyzed this data from the viewpoint of geometric models of the current project to examine hypotheses on correlations between geometric features of obstacles (nets) and bat trajectories. The work received the top prize for best project in the summer research program. During summer 2009 two undergraduates, Michael Kuhlman (from Rensselaer Polytechnic Institute) and Katherine McRoberts (from Grove City College), supported under the same NSF program for summer research, developed experimental implementations of control laws for collision avoidance using a sonar sensor signal processing system (due to T. K. Horiuchi) inspired by bat sonar. An undergraduate Physics student, Raffaele Tavarone from Rome University worked in the PI's lab during summer 2009 to analyze flocking trajectory data for starlings, using algorithms developed under the current ARO project. Results so far are very promising, yielding insight into the interaction structure of the flock.

Collaborations and Leveraged Funding

A key collaboration was initiated with colleagues Timothy Horiuchi (Associate Professor, Electrical and Computer Engineering and Institute for Systems Research), and Cynthia Moss (Professor of Psychology, affiliated with Biology, and member of the Institute for Systems Research), and postdoctoral fellow Kaushik Ghose. Further, the collaboration with Eric Justh continues (he is now on the staff of the Tactical Electronic Warfare Division of the Naval Research Laboratory).

The collaboration with Moss and Horiuchi was supported by leveraged funding from NIH-NIBIB under the Collaborative Research in Computational Neuroscience program (CRCNS: Innovative technologies inspired by biosonar). Separately, there is an AFOSR 3 year project “Pursuit, Avoidance, and Cohesion in Flight: Multipurpose Control Laws and Neuromorphic VLSI Implementation,” under the auspices of the Directorate of Life Sciences, Information and Mathematics, that started on June 1 2007. This project enables collaboration on fundamental questions pertaining to biologically plausible implementation, and in-silico implementation. The investigators on this project are Horiuchi (PI) and Krishnaprasad (Co-PI). A new collaboration with a group of faculty from the Physics and Mathematics Departments in Maryland has been initiated and funded under the ONR MURI Program (Exploiting Nonlinear Dynamics for Novel Sensor Networks), with funding begun May 1, 2007. Under this project, Krishnaprasad has also completed work on stochastic control of free space optical communication links with his Ph.D. student Arash Komaee and colleague Prakash Narayan. This work on closed-loop control using point process detection models and nonlinear filters is expected to be useful in applications involving line-of-sight optical links for coordination of groups of ground and aerial vehicles. This line of work was initiated during a final year of funding under an ARO-MURI program begun in May 2001, on Communicating Networked Control Systems (lead institution: Boston University). Additional leveraged funding provided through a University of Maryland initiative supports some of the work with Cricket localization system.

Support for undergraduate researchers was also obtained from NSF through summer research programs during summer 2006-2009. A group of summer undergraduate researchers worked on demonstrating the use of Cricket location sensing system to guide a pair of robots to rendezvous using our 2D pursuit law in a cooperative manner. We continue to involve two undergraduates in the Intelligent Servosystems Laboratory on problems of pursuit and collision avoidance. Ermin Wei obtained new data on collision avoidance behavior in echolocating bats and Philip Twu studied cyclic pursuit in which pursuit laws are used to achieve cohesion (rendezvous). Both have graduated in May 2008 and moved on to conduct doctoral studies (Wei at MIT, and Twu at Georgia Tech).

A new collaboration with a group of physicists in Rome, Italy studying collective behavior in starlings has been initiated, including joint analyses of empirical data. This group is engaged in fundamental experimental and analytical studies of collective behavior under funding from the European Union.

Conclusions

Significant progress has been made in the project, on both theoretical aspects and collaborative experimental efforts. The success in investigating delay and stochasticity has been encouraging. These are key issues of broad interest to the field. The funding from ARO on this project has been crucial in providing the PI time to develop the results.

Technology Transfer

Graduate student Kevin Galloway (reserve officer in the US Navy) has played an active part in Technology Transfer, by serving as a summer 2007 intern at the Naval Research Laboratory (NRL), working on issues of implementation of pursuit and cohesion laws in unmanned aerial vehicles, working with a distributed hardware-in-the-loop simulation test-bed. (From January 2009, to February 2010 he was on leave, following call-up for active duty, in Iraq.). Additional technology transfer is being made via internships for students at Intelligent Automation Inc. (IAI), a company located in Rockville, MD, with significant involvement in DOD research on swarming and distributed systems. The internships have been set up as a formal collaboration and technology transfer mechanism between the PI's lab and IAI. Graduate student Matteo Mischiati (advisee of the PI) and undergraduate student Philip Twu both spent summer 2007 at IAI. Mischiati worked on path planning algorithms for a team of mobile robots to demonstrate mixed-initiative intruder stand-off strategies. Philip Twu was engaged in bringing to completion a new distributed agent-based implementation of MDLe for open source release. MDLe was invented in the PI's lab and was given a first implementation there. The new industrial-strength implementation at IAI is built on a distributed agent framework CybelePro. Graham Alldredge was intern at IAI during summer 2008, working on robotics projects using the new implementation of MDLe. He taught the undergraduates in the PI's lab during summer 2009 the basics of this technology as needed for their experiments.

Future Plans

The PI plans to extend the dyadic pursuit laws that have been developed so far into building blocks for cohesion in collectives of systems (UAVs, UGVs). Further work on delay and stochasticity as well as new efforts on the side of optimal control and game theory will be carried out. The collaboration with Professor Moss will be extended to include multiple bat behavior in the laboratory. The PI plans to investigate additional opportunities for technology transfer through Army labs. The PI will also continue collaborations with the Italian group on flocking.

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