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14. ABSTRACT

The project has been a success from a scientific point of view, producing 23 articles in 36 months, 17 of which have appeared in renowned academic journals and chapters of books. The PI's research team involved eight graduate students (four of whom also became direct beneficiaries of the ARO financial support). All eight successfully graduated between 2008 and 2010. The research turned out to be very compatible topically to the Army interests (in the area of warfare, economics, and anti-terrorism) and it has an unlimited potential for

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Report Title

RANDOM WALK ANALYSIS IN ANTAGONISTIC STOCHASTIC GAMES; FINAL REPORT

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List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

[1] Dshalalow, J.H., Random walk analysis in antagonistic stochastic games, Stochastic Analysis and Applications, 26 (2008), 738–783.
[2] Dshalalow, J.H. and Huang, W., On noncooperative hybrid stochastic games, Nonlinear Analysis: Special Issue Section: Analysis and Design of Hybrid Systems, 2:3, 2008, 803-811.
[3] Dshalalow, J.H. and Treerattrakoon, A., Set-theoretic inequalities in stochastic noncooperative games with coalition (jointly with Ailada Treerattrakoon), Journal of Inequalities and Applications, (14 pp), 2008.
[4] Dshalalow, J.H. and Huang, W., A stochastic games with a two-phase conflict, in Advances of Nonlinear Analysis, Cambridge Scientific Publishers, Chapter 18 (2009), 201-209.
[5] Dshalalow, J.H. and Ke, H-J., Layers of noncooperative games, Nonlinear Analysis, Series A, Theory and Methods, 71 (2009), 283-291.
[6] Dshalalow, J.H., On multivariate antagonistic marked point processes, Mathematics and Computer Modeling, 49 (2009) 432-452.
[7] Dshalalow, J.H. and Ke, H-J., Multilayers in a Modulated Stochastic Game, Journal of Mathematical Analysis and Applications, 353 (2009), 553-565.
[8] Dshalalow, J.H. and Treerattrakoon, A., Antagonistic games with an initial phase, Nonlinear Dynamics and System Theory, 9 (3) (2009) 277–286.
[9] Huang, W. and Dshalalow, J.H., Tandem antagonistic games, Nonlinear Analysis, Series A, Theory and Methods, 71 (2009), 259-270.
[10] Al-Matar, N. and Dshalalow, J.H., Maintenance in single-server queues. A game-theoretic approach, Mathematical Problems in Engineering, Vol. 2009, 23 pp.
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[12] Al-Matar, N. and Dshalalow, J.H., A game-theoretic approach in single-server queues with maintenance. Time sensitive analysis, Communications in Applied Nonlinear Analysis, 17, No. 1 (2010), 65-92.
[13] Dshalalow, J.H. and Treerattrakoon, A., Operational calculus in noncooperative stochastic games, Nonlinear Dynamics and System Theory, 10:1 (2010), 39-54.
[14] Dshalalow J.H. and Motir, R., Random walk processes in a bilevel (M-N)-policy queue with vacations, Quality Techn. And Quant. Manag., Special Issue, Queueing Systems. (2010) Currently in Press.
[15] Alghamdi, A. and Dshalalow, J.H., Multiphase fluctuation analysis in a queue with maintenance, Nonlinear Studies, 17:2 (2010), 163-176.
[16] Alghamdi, A. and Dshalalow, J.H., Multiphase fluctuation analysis in a queue with an enhanced maintenance. Continuous time parameter process, Nonlinear Studies, 17:3 (2010), 199-215.
[17] On fluctuations of a nonmonotone marked point process (jointly with Randy Robinson), Electronic Modeling, 32:3 (2010).

Number of Papers published in peer-reviewed journals: 17.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

Number of Papers published in non peer-reviewed journals: 0.00

(c) Presentations

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Dshalalow, J.H. and Ke, H-J., Layers of noncooperative games, Nonlinear Analysis, Series A, Theory and Methods, 71 (2009), 283-291.

Huang, W. and Dshalalow, J.H., Tandem antagonistic games, Nonlinear Analysis, Series A, Theory and Methods, 71 (2009), 259-270.

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

(d) Manuscripts

[1] Dshalalow, J.H., Huang, W., Ke, H-J., and Treerattrakoon, A., On antagonistic game with a constant initial condition, submitted for publication.
[2] Dshalalow, J.H. and Robinson, R., On one-sided stochastic games and their applications to finance, submitted for publication.
[3] Dshalalow, J.H. and Robinson, R., On fluctuations of a nonmonotone marked point process under a time constraint, submitted for publication.
[4] Alzahrani, M.S. and Dshalalow, J.H., Fluctuation analysis in a queue with (L-N)-policy and secondary maintenance. Discrete time parameter process, submitted for publication.
[5] Alzahrani, M.S. and Dshalalow, J.H., Fluctuation analysis in a queue with (L-N)-policy and secondary maintenance. Continuous time parameter process and numerics, submitted for publication.
[6] Al-Matar, N. and Dshalalow, J.H., Time sensitive functionals in classes of queues with sequential maintenance, submitted for publication.

Number of Manuscripts: 6.00

Patents Submitted

Patents Awarded

Graduate Students

NAME	PERCENT_SUPPORTED
Weijun Huang	0.35
Ailada Treerattrakoon	0.20
Hao-Jan Ke	0.20
Randy Robinson	0.25
FTE Equivalent:	**1.00**
Total Number:	**4**

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: ......

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:......

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:......

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale): ......

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering: ......

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense ......

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: ......

Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME
Weijun Huang
Ailada Treerattrakoon
Hao-Jan Ke
Randy Robinson
Najeeb Al-Matar
A. Alghamdi
M. Alzahrani
Ramy Motir

Total Number: **8**

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

RANDOM WALK ANALYSIS IN ANTAGONISTIC STOCHASTIC GAMES

Proposal No. 51468MA, Contract No. W911NF0710121,

PI Jewgeni H. Dshalalow, Department of Mathematical Sciences, Florida Institute of Technology, Melbourne, FL 32901, *eugene@FIT.edu*

1. A Brief Description of the Project

This project (awarded from April 1, 2007 and March 31, 2010) dealt with classes of conflicts in economics and warfare. By their nature, they fall into the category of antagonistic games between two mutually hostile parties referred to as players A and B.

The actions of the players are manifested by a series of strikes of random magnitudes at random times exerted by each player against his opponent. Each of the assaults is intended to inflict damage to vital areas of defense, infrastructure, utilities, economics, and industry (warfare), as well as drive down prices, cause the outsourcing of labor, render hostile advertisement, unfavorable trading of competitor's stock, lobbying of politicians (competition of enterprises), arrange terror acts that involve human casualties, poison utilities and spread infection through chemical or biological sabotage, hack into banks, country defense network, and stock exchange to cripple the economy (terrorism and cyber-terrorism).

In contrast with strictly antagonistic games best known in the literature, where a game ends with one single successful hit, in our setting, each player can endure multiple strikes before perishing (or sustaining severe damages bringing the conflict to a new phase). Therefore, we assign to each player a (hypothetical) threshold of endurance that represents how much damage he can sustain before succumbing. Each player will try to defeat his adversary at the earliest opportunity, and the time when one of them collapses is referred to as the *ruin time*. Normally, at this point the game is over. However, in many real-life situations a losing player may choose to continue fighting until the end of his resources (as in the case of fanatical Japanese resistance at the end of the World War II). Another reason to continue fighting is caused by the losing player's inability to figure out that he has lost; for example, Germany's inability to accept her defeat at the end of World War I. Even now, with the new and more advanced technology and media, there is a delay in processing the information and some lack of coordination. This brings us to a more realistic scenario of "delayed observations" during any conflict, whether it is economical or military.

- Our first and main objective was to model genuine and realistic combats and predict the loss of a campaign (or ruin time) of one of the combatants as well as the collateral damage to each combatant at game exit. As pointed out, a war can go on even after the losing player's resources are exhausted. The latter does not change the course of the war. However, continued war actions will surely amplify the casualties. In numerous infamous economic scandals, institutions like banks, companies, or corporations do not rush to publically announce a major failure either due to fear of consequences, resulting in a cover up, or simply living in denial, both of which cause even bigger losses. In a scandal that was a blockbuster prior to the recent economic crisis, a trader of future indexes apparently caused a $7.2 billion loss to the French Bank Société Générale making unauthorized trades, which by hacking into computers, he was able to hide for months. While the trader's supervisors missed several opportunities to stop him a year earlier, it took the bank officials six extra days to announce the loss after the fact. We can go on and on with hundreds of examples during the current economic crisis ranging from the recent infamous Ponzi schemes to massive foreclosures and frozen liquidities, delayed information about terror suspects and ambushes, just to name a few.

- Our second objective was to adapt to a larger variety of situations that range from rudimentary battles between two players to wars with coalitions and wars with distinct phases or periods, such as economic sanctions, ambushes, local conflicts, political pressure, eventually followed by full-scale wars.

- Our third objective was to extend the investigation of the games to yield a more detailed information as well as statistics derived from analytical models. This is achieved by focusing not only upon ruin times and ruin time casualties, but also on reviving the real time data (stochastic interpolation). Another improvement is to introduce auxiliary control levels and predict their crossings. Such information enables the course of the game to be modulated ahead of the time by adopting different tactics in defense.

- Our fourth objective was to include more complex modeling of games, in which not only are the players get involved in striking each other, but they work on rebuilding their damaged structures. We thus explored queuing modeling previously never utilized in gaming.

2. Problem Setting Related to the Project

A basic antagonistic game is comprised of two random walk processes recording damages to the players at random times with increments of random magnitudes. Each process (recording the collective damage) looks like a monotone step function, which upon one of the strikes crosses a threshold, meaning that the total casualties by this epoch of time essentially ruin the industry, infrastructure, and the defense system to the degree that a capitulation is the best outcome to prevent the losing player from a complete destruction. To make modeling more realistic, the real time data about the casualties is observed by a third party process and in particular, it is delayed. However, there is no restriction imposed on the observation process which can be reasonably controlled. In this basic model we predict the following key game components:

- the ruin time of the loser
- the amount of sustained casualties to both players at the ruin time
- the same information one observation period prior to the observed ruin time
- same information past ruin time in the event the game is still going on causing more casualties from both sides, but with no significant impact on the outcome of the game.

The next embellishment would be a game in which a conflict breaks into separate phases. Each of the phases represents a game on its own and thus each one of them takes a different course. What makes them common is that the same players are involved and that transition from one phase to the next one inherits some past (non-Markovian) information that needs to be taken care of.

We will start with the following illustration describing the W.W.II conflict between Japan and the United States. Phase 1 corresponds to the impact of the economic sanctions imposed on Japan by the US in 1940-1941 due to Japan's aggressive actions against China. Japan, as we know, tried to negotiate with the US (apparently until November 26 of 1941), but the concessions offered by the Japanese were not satisfactory to the US, and Japan not wishing to give up their ambitions had no other choice as to strike (on December 7). This corresponds to the beginning of phase 2. Undoubtedly, Japan would not be ruined economically, but it would be significantly crippled (being deprived of steel and oil, to name a few), and at the same time she would not want to stop her campaign in China (Manchuria), which the US did not want to tolerate, also fearing of Japan's further expansions. We may interpret the Pearl Harbor attack by the Japanese followed by their Pacific campaign as yet another intermediate phase prior to a full scale war, because Japan believed this will deter the US from further actions under the inflicted casualties and loss of key strategic territories in the Pacific.

Another variant of a conflict is a game between three players, A, B, and C, in which A and B form a coalition against C. The game is over whenever one of the two situations takes place: (a) the coalition of A and B jointly defeats player C, (b) player C defeats the coalition (by defeating first player A and then player B or defeating the coalition in the reverse order). In contrast with former models in which opposite parties could form coalitions who acted in unison, in this setting, the allies act on their own and consequently, C must wage the war on two fronts.

While the above modeling has already emerged in our past work, we introduced a new class of models via queuing. Initially, queuing models have been used in operations research in connection with servicing processes, airlines, communications, computer networking, manufacturing processes to name a few. We have begun working on classes of queues with vacationing servers. When the queuing buffer gets exhausted the server commonly turns to render secondary jobs, which in the literature known as vacationing server. His (server's) duties during his absence are not specified, but it is assumed that some work is being done up until the queue gets replenished to some reasonable size to call the server back. We introduced a model with a real work in a secondary facility (such as a background work by the server during some ease of the primary duties in the area of computer networking, check-up for viruses, etc.). The server is usually programmed to perform this type of work in a limited fashion: restricted by time or by the amount of single tasks, which ever ends first. The latter forms a game which ends with server returning to the system, provided that the queue is filled up with enough units to warrant the return. The queuing process during the time when the server works on present and newly arriving units can be associated with new attacks and restoration of damages (departed units) until all of them are eventually finished. Then a (stochastically) similar cycle repeats again. The latter is understood in terms of cyclic games, when the outcome of the game for at least one of the players is positive in the sense that the player restored all of his structures by periodic repair of his damages. In this case, a new cycle of the game begins with mutual attacks. Players A and B have two thresholds L and N, but this time they are associated with two different phases. Phase I ends when damages to player A hit L and phase II ends when damages to player B hit N. Phase III starts thereafter, with subsequent restoration of damages to player B, while attacks on player B still continue.

These models are not fully explored yet but some work on them has already begun [10,12,14-16,20-22] (with [10,12,14-16] having been published and [20-22] submitted for publications), and it looks very optimistic.

3. Approach

The PI developed special **fluctuation techniques** extended to two or more random walk processes that model all possible paths of the games and evaluates their probabilities. These paths depict the casualties exerted to each player in real time as seen by an "observer process." The entering information is deliberately obscured to emulate more realistic situations than those modeled in real time. However, the observation process integrated into the game is by no way restricted and it can be arbitrarily refined to accommodate to a required frequency or intensity of data collection. Other ways of refinement of the information are achieved by **stochastic interpolation** (in time) or by **laying out the paths** of the involved processes, as per our discussion in section 4. The methods the PI used have not been previously known in the literatures except for stochastic interpolation (referred to as "**time sensitivity analysis**") some of which the PI developed in his earlier work.

All results are obtained in closed forms which can be further utilized. An advantage of a closed form is in part due to its ability to optimize an associated objective function in contrast with algorithms, for which optimization poses a separate problem. Secondly, closed form functionals enable us to calculate main statistical parameters, such as means, variances, and key reference intervals.

4. Scientific Challenges

- A main problem in the theory of fluctuations is to find the first moment of time when a stochastic process exits a fixed set and consequently, this instance is referred to as the ***exit time***. It is a rule of thumb that the complexity of this problem depends upon whether or not the underlying process is with continuous paths. If the process is piecewise linear (as it is in our case), it is more of a challenge, because the process is not continuously passing but rather jumps over the boundary. As a result, the location of the process immediately after the exit, and thus the exit time as well, are both less predictable. In addition, because every game in our setting represents a conglomerate of three or more dependent processes, the race of crossing the critical thresholds presents a further challenge. All of this is just about a basic game. Other types of games with sequential phases and coalitions are even more complex.
- With the first barrier of these three types of games (i.e. basic, phases, and coalition) we have set a goal to improve the results for each of these models in

two alternative ways. The first way is to investigate **continuous time parameter processes**, which deliver comprehensive information about the game continuously. The latter is generally more statistically refined, but a true difficulty is to attach the continuous time parameter process to the values of the process at key reference epochs, such as the **exit time of the game** or **transition times** when passing consecutive phases. Such analysis is referred to as **time sensitivity**. Two main articles [1,6] were produced on time sensitivity with very encouraging results. The method included stochastic interpolation around critical points thereby recovering the data from just stopping times to essentially continuous information without any additional prerequisites. This idea is also fruitful whenever is a need to expand the information about the game restricted to a crude observation. The results obtained are still explicit but somewhat more complex.

- Therefore, in the initial approach to expand the information on the course of the game, it would be of interest to get **continuous information** about the processes and at the same time not to lose the track on the **key reference points** such as exits from phases. A second type of detailing the game would be to draw multiple reference lines which the underlying processes are to cross in due time (in order to forecast bad losses, before bad losses turn very bad). In contrast with the first type of refinement, in the second case we set an almost inverse problem where we look into the crossings times of arbitrarily embedded auxiliary levels. We call this approach ***laying out*** **the paths** of the processes. A chief complexity takes place when combining these **layers** with the main thresholds a player is to cross at the exit. A very surprising result showed a local independence and an essential relaxation of analytical complexity. Two papers [5, 7] were recently published (one of which with more general layers) with one of PI's formed doctorate students (who graduated in the summer of 2009).

- In the context of **queuing**, a fully unexpected benefit of such modeling enabled the PI and his current doctoral students to explore far more advanced gaming, in which at least one player recovers during random restoration epochs. The latter, on occasion, led to a full recovery (regeneration), with subsequent resurrected conflicts of in a cyclic manner. Here an assumption of integer-valued attacks and restorations were essential, with virtually minor restrictions to the generality of models (in which associated integer-valued random quantities could be easily converted to any fractional quantities with appropriate multipliers). The PI and his students' efforts [10,12,14,15] are just starting but quite successful on the new path to more sophisticated game-theoretic modeling.

- In the overall complex machinery with rather theoretical results of [1-5,7,10-12] the PI and his former and current doctoral students became aware of direct applicability and numerics of the outcomes. This was not of our initial concern due to our deliberate plans to lay out mathematically sound foundation of our modeling and techniques. In due time, it has become the PI's strong drive of proving first that the results are analytically tractable. Consequently, it stood for reason to look into a conversion of our modeling and mathematical outcomes to readily available tools for engineers and scientists in the US Army who would find our accomplishments more appealing; not to mention opening new avenues for applications. Thus a new (and more applied) work has been produced [6,8,13,14,16-23], some of which [13,14,16,17,22] with wealth of numerical demonstrations.

5. Significance

The class of games we analyze can model economic conflicts between corporations as well as small and large scale wars waged between two opponents, each acting synchronously or coalitions that act on their own even if they have a common goal. A good example of the latter would be a coalition of Germany and Japan who were formally allies but acted asynchronously. Economically driven antagonism between corporations (such as buying or selling the stock of the opponent, political lobbying, outsourcing labor, driving artificially prices too low, and potentially hostile advertisements) can also offer opportunities for modeling via antagonistic random processes with fluctuations. A typical example is an antagonistic game model of the competition between two enterprises that manufacture the same homogeneous goods and operate on a given interval of time [4,6,9]. Hostile actions between nations in the modern world can also be manifested by economic sabotage and terrorism yielding very serious consequences.

The results obtained so far can be useful in predicting outcomes of various phases of conflicts: econo-political hostilities, small-scale wars, terror acts, and large-scale wars. Damages can be estimated in terms of physical casualties or economic losses. By prognosticating their time and quantities (in economy, industry, utilities, infrastructure, and defense) we can take various preventive measures.

One classical example is the 9/11 sinister terror act against the World Trade Center preceded by a heavy sell out of airline stocks traded about four weeks prior to the disaster. In this particular case, watching unusual activities in any particular stock market area evolving into a very aggressive trading (as oppose to benign sell out due to bad economic figures) can predict when the trade volume and stock price would cross critical thresholds before this actually takes place. The area of trading can also reveal a potential terror target. Similar examples also exist in the realm of cyber-terrorism, so that some initial activities can morph into security breach occurring later on, which can be predicted by watching some critical indicators. In the beginning, it is always difficult to tell a malignant activity from benign one, but a probabilistic prediction based on our current research is better than or compliments any deterministic logic and electronic defense.

While all kinds of terror attacks seem to be novel in contemporary warfare (since we deal with invisible enemies), in terms of our modeling and analysis they are no different from those preliminary phases of conflicts mentioned above that precede larger scale wars. One of the reasons is that no matter how serious the casualties of an initial strike can be, the enemy often reveals himself thereby giving us an opportunity to destroy him or at least take retaliatory measures.

6. Accomplishments

Between April 1, 2007 and March 31, 2010, the PI and a team of his eight doctoral students have managed to produce a research authenticated in 23 articles (with the total length of more than 420 pages) published, accepted, or submitted for publication. The PI wrote two of the 23 papers and co-authored the rest with his students. Of the 23 submissions, 17 by now have appeared in the *Journal of Inequalities and Applications, Nonlinear Analysis, Stochastic Analysis and Applications, an Honorary Volume of Cambridge Scientific Publishers, Journal of Mathematical Analysis and Applications, Mathematical and Computer Modeling, Mathematical Problems in Engineering, Communications in Applied Nonlinear Analysis, Nonlinear Dynamics and System Theory, Advances in Nonlinear Analysis, Quality Technology And Qualitative Management, Nonlinear Studies, Engineering Simulation* (*Electronic Modeling*).

In the research conducted by the PI and his team they formalized and analyzed games of two players, games with coalition of players, and modulated games with several distinct phases (referred to as ***sequential games***). Sequential games included conflicts broken into separate periods ranging from political disagreement, economic sanctions, ambushes, to full scale wars. All results are obtained in closed forms, which among various other things, allow one to calculate pertinent predicted parameters including means, variances, and quantiles of losses as well as probability intervals of exit times.

The results of such modeling can influence the course of any war as it helps calibrating the default strategy after predicting critical times and casualties. The results obtained allowed us to refine the predicted losses (initially restricted only to key reference points such as exits from games and phases) in continuous time as well as by using the multilevel crossings method.

The entire studies have also become topics of doctoral dissertations to eight of the PI's graduate students. Three of them graduated in the Fall of 2008 and Spring of 2009, and the other five graduated in the Spring and Summer of 2010.

Some of our results the PI and his team had been working on were presented in the fourth World Congress of Nonlinear Analysts in July of 2008 (held in Orlando, FL). One of PI's students presented a 45-minutes talk in a stochastic session.

7. Conclusions

My project in stochastic analysis of antagonistic games has been a success from a scientific point of view, producing 23 articles in 36 months 17 of which have appeared in renowned academic journals and chapters of books. Our research team involved eight graduate students (four of whom also became direct beneficiaries of the ARO financial support) all of whom successfully graduated between 2008 and 2010. The research turned out to be very compatible topically to the Army interests (in the area of warfare, economics, and antiterrorism) and it has an unlimited potential for exploration. It also attracts more students (several more are about to join my program) in the years to come and if the ARO will choose to continue supporting us, we will surprise them with more accomplishments. The conducted research promises not only to attract followers from probability community, but also catch the attention of statisticians, engineers, financial analysts, computer scientists, and economists, to whom the results obtained (and those in progress) must be most appealing.

8. Produced References

[1] Dshalalow, J.H., Random walk analysis in antagonistic stochastic games, *Stochastic Analysis and Applications*, 26 (2008), 738–783.

[2] Dshalalow, J.H. and Huang, W., On noncooperative hybrid stochastic games, *Nonlinear Analysis*: Special Issue Section: Analysis and Design of Hybrid Systems, 2:3, 2008, 803-811.

[3] Dshalalow, J.H. and Treerattrakoon, A., Set-theoretic inequalities in stochastic noncooperative games with coalition (jointly with Ailada Treerattrakoon), *Journal of Inequalities and Applications*, (14 pp), 2008.

[4] Dshalalow, J.H. and Huang, W., A stochastic games with a two-phase conflict, in *Advances of Nonlinear* Analysis, *Cambridge Scientific Publishers*, Chapter 18 (2009), 201-209.

[5] Dshalalow, J.H. and Ke, H-J., Layers of noncooperative games, *Nonlinear Analysis*, Series A, Theory and Methods, 71 (2009), 283-291.

[6] Dshalalow, J.H., On multivariate antagonistic marked point processes, *Mathematics and Computer Modeling*, 49 (2009) 432-452.

[7] Dshalalow, J.H. and Ke, H-J., Multilayers in a Modulated Stochastic Game, *Journal of Mathematical Analysis and Applications*, 353 (2009), 553-565.

[8] Dshalalow, J.H. and Treerattrakoon, A., Antagonistic games with an initial phase, *Nonlinear Dynamics and System Theory*, 9 (3) (2009) 277–286.

[9] Huang, W. and Dshalalow, J.H., Tandem antagonistic games, *Nonlinear Analysis*, Series A, Theory and Methods, 71 (2009), 259-270.

[10] Al-Matar, N. and Dshalalow, J.H., Maintenance in single-server queues. A game-theoretic approach, *Mathematical Problems in Engineering*, Vol. 2009, 23 pp.

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