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14. ABSTRACT
Over the five year period, this project has mainly been been concerned about developing a theory for statistical analysis of shapes of objects, both two and
three-dimensional. Focusing on the boundaries of these objects, our framework is for shape analysis of curves and surfaces. The main achievements were development of tools for: (1) Quantifying Shape Differences: Given any two objects, we can quantify differences between their shapes. (2) Achieve Desired Invariance: Our notion of shape is

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Report Title

A Statistical Theory for Shape Analysis of Curves and Surfaces With Applications in Image Analysis, Biometrics, Bioinformatics and Medical Diagnostics

ABSTRACT

Over the five year period, this project has mainly been been concerned about developing a theory for statistical analysis of shapes of objects, both two and
three-dimensional. Focusing on the boundaries of these objects, our framework is for shape analysis of curves and surfaces. The main achievements were development of tools for: (1) Quantifying Shape Differences: Given any two objects, we can quantify differences between their shapes. (2) Achieve Desired Invariance: Our notion of shape is invariant to certain transformations of curves – rigid motion, scaling and re-parameterization. (3) Compute Summary Statistics: Given a collection of shapes and shape classes we can generate summary statistics – mean, covariance, etc, to characterize a shape class. (4) Stochastic Modeling: We have developed probability models that capture observed variability in shape classes. These models form priors for Bayesian inferences. (5) Statistical Inferences: We have studied statistical evaluations, such as hypothesis testing, likelihood ratios, performance bounds, etc, for shape analysis.

The salient components of this differential geometric framework are following.
First, we define a space of curves or surfaces by choosing a mathematical representation for these objects and establish a Hilbert submanifold(s) for such representations. Then, we choose a Riemannian metric, usually an elastic metric for measuring distances on such manifolds.We arrive at a shape manifold by imposing remaining invariances in the representation. For these shape spaces, we have developed two numerical techniques for computing geodesic paths. Finally, we define and compute empirical statistics, and define probability models on tangent bundles.

The resulting statistical models are then used to characterize objects in images according to shapes, for using in object detection, tracking and recognition. We have demonstrated these tools in different application scenarios including general computer vision and image understanding, human biometrics, bioinformatics, and medical image analysis. More specially, we have studied the use of prior shape models in detecting targets in noisy/corrupted images (Bayesian active contours), finding shape models in point clouds derived from images, shape analysis of facial surfaces generated from laser scans, human and activity recognition in videos using shapes of silhouettes, classification of human subjects into control and diseased cases using shapes of their anatomical parts, and structural matching and classification of proteins using the shapes of their backbones.

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(a) Papers published in peer-reviewed journals (N/A for none)

1. W. Mio, A. Srivastava, and X. Liu, Contour Inferences for Image Understanding, International Journal of Computer Vision, vol. 69, no. 1, pages 137-144, August 2006.

2. C. Samir, A. Srivastava, and M. Daoudi, Automatic 3D Face Recognition Using Shapes of Facial Curves, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 28, no. 11, pages 1858-1863, November 2006.

3. W. Mio, A. Srivastava, and S. Joshi, On Shape of Plane Elastic Curves, International Journal of Computer Vision, vol. 73, no. 3, pages 307-324, July 2007.

4. U. Grenander, A. Srivastava and S. Saini, A Pattern-Theoretic Characterization of Biological Growth, IEEE Transactions on Medical Imaging, vol. 26, no. 5, pages 648-659, November 2007.

5. D. Kaziska and A. Srivastava, Classi?cation of Cyclostationary Processes on Nonlinear Shape Manifolds for Gait-Based Human Recognition, Journal of American Statistical Association, vol. 102, no. 480, pages 1114-1124, December 2007. Comments by K. Mardia, R. Chellappa and A. Veeraghavan. Published with a rejoinder page:1127, same issue.

6. D. Kaziska and A. Srivastava, The Karcher Mean of a Class of Symmetric Distributions on a Unit Circle, Statistics and Probability Letters, vol. 78, pages 1314-1316, 2008.

7. A. Srivastava, C. Samir, S. H. Joshi, and M. Daoudi, Elastic Shape Models for Face Analysis Using Curvilinear Coordinates, Journal of Mathematical Imaging and Vision, vol. 33, no. 2, pages 253-265, February 2009.

8. M.-C. Chiang, M. Barysheva, D. Shattuck, A. Lee, S. K. Madsen, C. Avedissian, A. D. Klunder, A.Toga, K. McMahon, G. De Zubicaray, M. Wright, A. Srivastava, N. Balov, and P. Thompson, Genetics of Brain Fiber Architecture and Intellectual Performance, Journal of Neuroscience, vol. 29, no. 7, 2212-2224, February 2009.

9. S. Joshi and A. Srivastava, Intrinsic Bayesian Active Contours for Extraction of Object Contours in Images, International Journal of Computer Vision, vol. 81, no. 3, pages 331-355, March 2009.

10. C. Samir, A. Srivastava, M. Daoudi, and E. Klassen, An Intrinsic Framework for Analysis of Facial Surfaces, International Journal of Computer Vision, vol. 82, no. 1, pages 80-95, April 2009.

11. A. Veeraraghavan. A. Srivastava, A. K. Roy-Chowdhury and R. Chellappa, Rate-invariant recognition of humans and their activities, IEEE Transactions on Image Processing, vol. 8, issue 6, pages 1326-1339, June 2009.

12. B. Ben Amor, H. Drira, L. Ballihi, A. Srivastava, and M. Daoudi, An Experimental Il lustration of 3D Facial Shape Analysis Under Facial Expressions, Annals of Telecommunications, vol. 64, no. 5, pages 369-379, June 2009.

13. A. Srivastava and I. H. Jermyn, Looking for Shapes in Two-Dimensional, Cluttered Point Cloud, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 31, no. 9, pages 1616-1629, September 2009.

14. D. Kaziska and A. Srivastava, Joint Gait-Cadence Analysis for Human Identi?cation Using An Elastic Shape Framework, Communications in Statistics Theory and Methods, to appear in June 2010.

15. A. Srivastava, E. Klassen, S. Joshi, and I. Jermyn, Shape Analysis of Elastic Curves in Euclidean Spaces, in review at the IEEE Transactions on Pattern Analysis and Machine Intelligence, November 2009.

16. P. Turaga, A. Veeraraghavan, A. Srivastava, and R. Chellappa, Statistical Computations on Special Manifolds for Image and Video-Based Recognition, in review at the IEEE Transactions on Pattern Analysis and Machine Intelligence, October 2009.

17. C. Samir, P.-A. Absil, A. Srivastava and E. Klassen, A Gradient-Descent Method for Curve Fitting on Riemannian Manifolds, in review at the Foundations of Computational Mathematics, August 2009.

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(c) Presentations

1. ARO/AMCOM Workshop on Information Theoretic Imaging, US Army Missile Command Center, Huntsville, AL, June 2005.

2. ARO Workshop on Challenges and Opportunities in Image Understanding, University of Maryland, College Park Conference Center, College Park, MD, January 2007.

3. ARO Workshop on Challenges in Information Evaluation and Extraction in Distributed Sensing, MIT, Boston, MA, October 2009.

University Colloquium:

1. Department of Mathematics and Statistics, Air Force Institute of Technology, Dayton, OH, January 2006.

2. Statistical Engineering Division, National Institute of Standards and Technology, Gaithersburg, MD, February 2006.

3. Department of Biostatistics, Columbia University, New York, NY, March 2006.

4. INRIA Research Institute, Sophia-Antipolis, France, May 2006.

5. Department of Statistics, University of Chicago, Chicago, IL, November 2006.

6. Department of Electrical Engineering, North Carolina State University, Raleigh, NC, February 2007.

7. Department of Mathematics, University of Arizona, Tucson, AZ, February 2007.

8. CESAME (Applied Mathematics Group), Catholic University of Louvain, Louvain-la-Neuve, Belgium, May 2007.

9. Department of Computer Science (LIFL), University of Lille, France, June 2007.

10. Department of Statistics, The Ohio State University, Columbus, OH, September 2007.

11. The VISAGES group at IRISA, Rennes, France, December 2007.

12. Department of Statistics, FSU, January 2008.

13. Department of Electrical Engineering, University of Maryland, College Park, MD, September 2008.

14. Stochastic Systems Group, MIT, April 2009.

15. ARIANA Group, INRIA, Sophia Antipolis, France, June 2009.

16. Department of Computer Science and Engineering, Michigan State University, East Lansing, MI, March 2010.

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

1. IMA Workshop on Shape Spaces, Institute of Mathematics and its Applications, University of Minneapolis, Minneapolis, MN, April 2006.

2. SAMSI Workshop on Geometry and Statistics of Shape Space, SAMSI Institute, Research Triangle Park, NC, July 2007.

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

Peer-Reviewed Conference Proceeding publications (other than abstracts):

1. A. Jain, S. Joshi, A. Srivastava and D. Kaziska, Statistical Shape Models Using Elastic-String Representations, in Proceedings of Asian Conference on Computer Vision (ACCV), pages 612-621, LNCS 3851, P. J. Narayanan et al. (Eds.), Hyderabad, India, January, 2006.

2. D. Kaziska and A. Srivastava, Cyclostationary Processes on Shape Spaces for Gait-Based Recognition, in Proceedings of European Conference on Computer Vision (ECCV), pages 442-453, LNCS 3952, A. Leonardis, H. Bishof, and A. Prince (eds), Gratz, Austria, May 2006.

3. E. Klassen and A. Srivastava, Geodesics Between 3D Closed Curves Using Path-Straightening, in Proceedings of European Conference on Computer Vision (ECCV), pages, LNCS 3952, A. Leonardis, H. Bishof, and A. Prince (eds), Gratz, Austria, May 2006.

4. C. Samir, A. Srivastava and M. Daoudi, 3D Face Recognition Using Shapes of Facial Curves, in Proceedings of International Conferences on Acoustics, Speech, and Signal Processing (ICASSP), Toulouse, France, May 2006.

5. S. Joshi, E. Klassen, A. Srivastava and I. Jermyn, An Efficient Representation for Computing Geodesics Between n-Dimensional Elastic Curves, IEEE Conference on computer Vision and Pattern Recognition (CVPR), Minneapolis, MN, June 2007.

6. A. Srivastava, I. Jermyn and S. Joshi, Riemannian Analysis of Probability Density Functions with Applications in Vision, IEEE Conference on computer Vision and Pattern Recognition (CVPR), Minneapolis, MN, June 2007.

7. M. Kulikova, M. Mani, A. Srivastava and X. Descombes, Tree Species Classi¿cation Using Radiometry, Texture and Shape Based Features, European Conference on Signal Processing
(EUSIPCO), Poland, 2007.

8. N. Balov, A. Srivastava, C. Li, and Z. Ding, Shape Analysis of Open Curves in R3 with Applications to Study of Fiber Tracts in DT-MRI Data, in Proceedings of Sixth International Workshop on Energy Minimization Methods in Computer Vision and Pattern Recognition (EMMCVPR), pages 399-413, Hubei, China, August 2007.

9. A. Srivastava, S.H. Joshi, E. Klassen, and I.H. Jermyn, Removing Shape-Preserving Transformations in Square-Root Elastic (SRE) Framework for Shape Analysis of Curves, in Proceedings of Sixth International Workshop on Energy Minimization Methods in Computer Vision and Pattern Recognition (EMMCVPR), pages 387-398, Hubei, China, August 2007.

10. A. Srivastava, W. Liu and S. H. Joshi, Modeling Spatial Patterns of Shapes, International Conference on Image Processing (ICIP), San Diego, CA, October 2008.

11. A. Srivastava and I. H. Jermyn, Bayesian Classification of Shapes Hidden in Point Clouds, Proceedings of 13th Digital Signal Processing Workshop, Marco Island, FL, January 2009.

12. H. Drira, B. Ben Amor, M. Daoudi, and A. Srivastava., Nasal Region Contribution in 3D Face Biometrics Using Shape Analysis Framework, In Proceedings of 3rd IAPR/IEEE International
Conference on Biometrics, Sassari, Italy, June 2009.

13. C. Samir, A. Srivastava, M. Daoudi, and S. Kurtek, On Analyzing Symmetry of Objects Using Elastic Deformations, In Proceedings of International Conference on Computer Vision Theory and Applications (VISAPP), February 2009, Lisboa, Portugal.

14. C. Samir, P.-A. Absil, A. Srivastava, and E. Klassen, Fitting Curves on Riemannian Manifolds Using Energy Minimization, IAPR Conference on Machine Vision Applications, Keio University, Japan, 2009.

15. H. Drira, B. Ben-Amor, A. Srivastava and M. Daoudi, A Riemannian Analysis of 3D Nose Shapes for Partial Human Biometrics, International Conference of Computer Vision, Kyoto, Japan, October 2009.

16. M. Mani, S. Kurtek, C. Barillot, and A. Srivastava, A Comprehensive Riemannian Framework for Analysis of White Matter Fiber Tracts, International Symposium on Biomedical Imaging (ISBI), Rotterdam, The Netherlands, April 2010.

17. S. Kurtek, E. Klassen, Z. Ding, and A. Srivastava, A Novel Riemannian Framework for Shape Analysis of 3D Objects, IEEE Conference on computer Vision and Pattern Recognition (CVPR), San Francisco, CA, June 2010.

18. J. Su, Z. Zhu, F. Huffer, and A. Srivastava, Detecting Shapes in 2D Point Clouds Generated from Images, International Conference on Pattern Recognition (ICPR), Istanbul, Turkey, August 2010.

19. A. Malej, B. Ben Amor, M. Daoudi, A. Srivastava and S. Berretti, 3D Face Analysis for Facial Expression Recognition, International Conference on Pattern Recognition (ICPR), Istanbul, Turkey, August 2010.

20. J. Su, Z. Zhu, F. Hu¿er, and A. Srivastava, A Ful ly Statistical Framework for Detection of Shapes in Image Primitives, Seventh IEEE Workshop on Perceptual Organization in Computer Vision (POCV), in conjunction with CVPR, San Francisco, CA, June 2010.

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts): 20

(d) Manuscripts

Number of Manuscripts: 0.00

Patents Submitted

Patents Awarded

Graduate Students

NAME	PERCENT_SUPPORTED
Shantanu Joshi	1.00
Sanjay Saini	1.00
Nikolay Balov	1.00
Sebastian Kurtek	0.50
Wei Liu	0.50
FTE Equivalent:	**4.00**
Total Number:	**5**

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Anuj Srivastava	0.11	No
FTE Equivalent:	**0.11**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period:	0.00
The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:......	0.00
The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:......	0.00
Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):	0.00
Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:	0.00
The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense	0.00
The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:	0.00

Names of Personnel receiving masters degrees

NAME	
Sanjay Saini	
Sebastian Kurtek	
Total Number:	**2**

Names of personnel receiving PHDs

NAME	
Shantanu Joshi	
Nikolay Balov	
Total Number:	**2**

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

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Statistical Shape Analysis for Automated Target Recognition

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Contents

List of Figures

1 Executive Summary

Over the five year period, this project has mainly been been concerned about developing a theory for statistical analysis of shapes of objects, both two and three-dimensional. Focusing on the boundaries of these objects, our framework is for shape analysis of curves and surfaces. The main achievements were development of tools for: (1) Quantifying Shape Differences: Given any two objects, we can quantify differences between their shapes. (2) Achieve Desired Invariance: Our notion of shape is invariant to certain transformations of curves rigid motion, scaling and re-parameterization. (3) Compute Summary Statistics: Given a collection of shapes and shape classes we can generate summary statistics mean, covariance, etc, to characterize a shape class. (4) Stochastic Modeling: We have developed probability models that capture observed variability in shape classes. These models form priors for Bayesian inferences. (5) Statistical Inferences: We have studied statistical evaluations, such as hypothesis testing, likelihood ratios, performance bounds, etc, for shape analysis. The salient components of this differential geometric framework are following. First, we define a space of curves or surfaces by choosing a mathematical representation for these objects and establish a Hilbert submanifold(s) for such representations. Then, we choose a Riemannian metric, usually an elastic metric, for measuring distances on such manifolds. We arrive at a shape manifold by imposing remaining invariances in the representation. For these shape spaces, we have developed two numerical techniques for computing geodesic paths. Finally, we define and compute empirical statistics, and define probability models on tangent bundles.

The resulting statistical models are then used to characterize objects in images according to shapes, for using in object detection, tracking and recognition. We have demonstrated these tools in different application scenarios including general computer vision and image understanding, human biometrics, bioinformatics, and medical image analysis. More specially, we have studied the use of prior shape models in detecting targets in noisy/corrupted images (Bayesian active contours), finding shape models in point clouds derived from images, shape analysis of facial surfaces generated from laser scans, human and activity recognition in videos using shapes of silhouettes, classification of human subjects into control and diseased cases using shapes of their anatomical parts, and structural matching and classification of proteins using the shapes of their backbones.

2 Research Accomplishments

Over the five year period, this project has mainly been been concerned about developing a theory for statistical analysis of shapes of objects, both two and three-dimensional. Focusing on the boundaries of these objects, our framework is for shape analysis of curves and surfaces. The main achievements is this research are development of tools for the following important tasks:

1. **Quantifying Shape Differences**: Given any two objects, we can quantify differences between their shapes.

2. **Achieve Desired Invariance**: Our notion of shape is invariant to certain transformations of curves rigid motion, scaling and re-parameterization and the ensuing analysis is independent of these nuisance variables.

3. **Summary Statistics**: Given a collection of shapes and shape classes we can generate summary statistics mean, covariance, etc, to characterize a shape class.

4. **Stochastic Modeling**: We have developed probability models that capture observed variability in shape classes. These models form priors for Bayesian inferences.

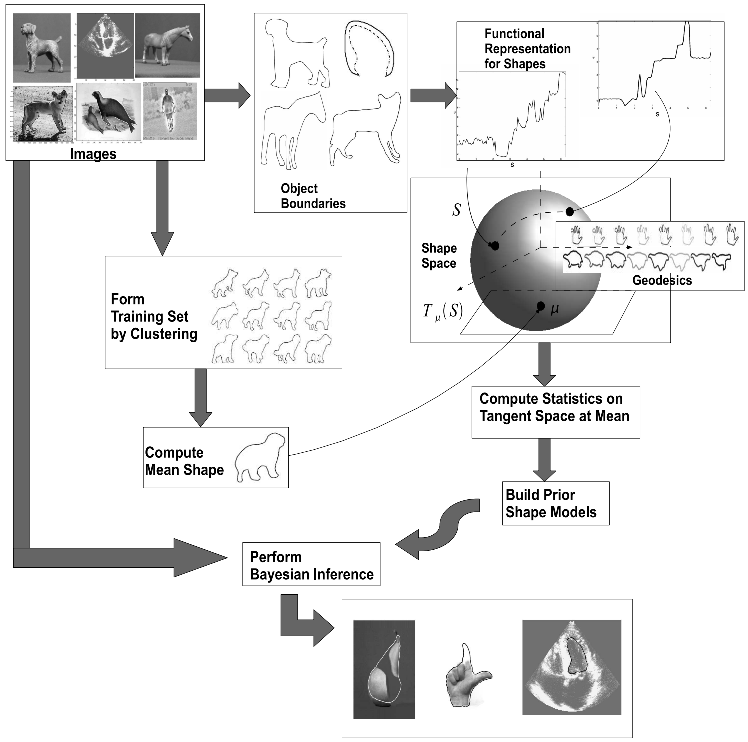


Figure 1: A system of statistical analysis of shapes in images.

5. **Statistical Inferences**: We have studied statistical evaluations, such as hypothesis testing, likelihood ratios, performance bounds, etc, for shape analysis.

In summary, our goal is to develop a statistical framework where we can treat shapes of curves and surfaces as random quantities, i.e. as variables taking values in well-defined spaces governed by underlying probability densities. An outline of such a comprehensive system for statistical shape analysis is laid out in Figure 1. In this system, the input images are processed to extract contours of interest, either by hand or automatically or both. These contours are mathematically represented as points on certain infinite-dimensional differentiable manifolds, denoted as the shape space in the figure. The dissimilarities between shapes of two contours are quantified using lengths of geodesic paths between the corresponding points on the shape space. Using the geometry of the shape space, tools for statistical analysis of shapes are derived. In particular, the concept of an average shape is developed on the shape space. Probability models, estimated from the training shapes in a shape class, are used for future Bayesian inferences on image data. A contour estimated in this Bayesian framework can then be used for classifying objects in images. Geodesic paths between a shape and its reflection are useful in symmetry analysis.

What makes statistical analysis of shapes difficult? It is quite easy for us, as human beings, to observe and to analyze shapes, and to perform many of the aforementioned tasks without much difficulty.

1. **Invariance**:
   One important challenge in shape analysis comes from the fact that our notion of shape is *invariant to certain transformations*, such as translations, rotations, and rescaling. Abstractly,

shape is described as a property that remains unchanged under these transformations and this aspect must be included in our mathematical formulations. Another aspect of invariance, of curves and surfaces, although not readily visible through pictures, is the issue of parametrization. A curve or a surface can be parameterized in many ways; a re-parametrization does not change its shape and, thus, any analysis of shape should be independent of the parameterization.

2. **Nonlinearity**:
   Although there are a variety of mathematical representations for studying shapes, they all share the following important property. The spaces formed by these representations are nonlinear. That is, they are not vector spaces and one can not use the classical vector calculus to perform operations on shapes. Operations such as addition, multiplication, and subtraction are not valid on these spaces.

3. **Infinite-Dimensionality**:
   Lastly, the study of shapes of (continuous) curves and surfaces introduces an additional challenge of high, often infinite, dimensionality. This is because curves and surfaces are formally represented by functions, and spaces of functions are usually infinite dimensional.

The salient components of this differential geometric framework are following whether we consider shape analysis of curves or surfaces. First, we define a space of curves or surfaces by choosing a mathematical representation for these objects and establish a Hilbert submanifold(s) for such representations. Since we are interested in continuous objects, the resulting spaces are infinite-dimensional; additionally, certain nonlinear constraints such as length (or size) constraint and the orientation constraints, result in the underlying spaces being nonlinear manifolds. Then, we choose a Riemannian metric, usually an elastic metric for measuring distances on such manifolds. We arrive at a shape manifold by imposing remaining invariances in the representation. This is done using the algebraic notion of forming quotient spaces by removing the actions of rotation and the re-parameterization groups (scale and translation are removed previously). For the resulting quotient space, termed shape spaces, we have developed two numerical techniques for computing geodesic paths: shooting method and path-straightening method. Using these methods we compute geodesics between arbitrary shapes on the shape manifold; these geodesics provide use with a tool for matching, comparing, deforming, and quantifying differences in shapes. Finally, we define and compute empirical statistics, and define probability models on tangent bundles.

Some specific items of this framework are elaborated further in the next sections.

2.1 Theory of Shape of Analysis

1. **Shape Analysis of Planar Elastic Curves**: We have studied shapes of planar arcs and closed contours modeled on elastic curves obtained by bending, stretching or compressing line segments non-uniformly along their extensions. Shapes are represented as elements of a quotient space of curves obtained by identifying those that differ by shape-preserving transformations. The elastic properties of the curves are encoded in Riemannian metrics on these spaces. Geodesics in shape spaces are used to quantify shape divergence and to develop morphing techniques. The shape spaces and metrics constructed are novel and offer an environment for the study of shape statistics. Elasticity leads to shape correspondences and deformations that are more natural and intuitive than those obtained in several existing models. Applications of shape geodesics to the definition and calculation of mean shapes and to the development of shape clustering techniques were also investigated. For further details, please refer to the article [14].

2. **Shape Analysis of Elastic Curves in Euclidean Spaces**: This work introduces a square-root velocity (SRV) representation for analyzing shapes of curves in Euclidean spaces using an elastic metric. This SRV representation has several advantages: the well-known elastic metric simplifies to the $\mathbb{L}^2$ metric, the re-parameterization group acts by isometries, and the space of unit length curves becomes the familiar unit sphere. The shape space of closed curves is quotient space of (a submanifold of) the unit sphere, modulo rotation and re-parameterization groups, and one finds geodesics in that space using a path-straightening approach. These geodesics and geodesic distances provide a framework for optimally matching, deforming and comparing shapes. Several experiments are presented to demonstrate these ideas: (i) Shape analysis of cylindrical helices for studying structures of protein backbones, (ii) Shape analysis of facial curves for use in recognition, (iii) A wrapped probability distribution to capture shapes of planar closed curves, and (iv) Parallel transport of deformations from one object to another for predicting shapes from novel poses. For further details, please refer to the conference articles [3, 4] and the journal article [18].

3. **Path-Straightening for Computing Geodesic Paths**: In order to analyze shapes of continuous curves in $\mathbb{R}^n$, we have developed a numerical technique for computing geodesics between them in the shape space. To compute geodesics between any two curves, we connect them with an arbitrary path, and then iteratively straighten this path using the gradient of an energy associated with this path. The limiting path of this path-straightening approach is a geodesic. Next, we consider the shape space of these curves by removing shape-preserving transformations such as rotation and re-parametrization. To construct a geodesic in this shape space, we construct the shortest geodesic between the all possible transformations of the two end shapes; this is accomplished using an iterative procedure. We provide step-by-step descriptions of all the procedures, and demonstrate them with simple examples in [9].

4. **Parameterization-Invariant of Shape Analysis of 3D Objects**: In this work, we have introduced a novel Riemannian framework for shape analysis of 3D objects. Focusing on the boundaries of these objects, i.e. surfaces, we derive a distance function between any two surfaces that is invariant to rigid motion, global scaling, and re-parameterizations. It is the last part that presents the main difficulty. Our solution to this problem is twofold: (1) define a special representation, called q-function, to represent each surface, and (2) develop a gradient-based algorithm to optimize over different re-parameterizations of a surface. The second step is akin to moving the mesh on a surface to optimize its placement. (This is different from the current methods that work with fixed meshes of 3D objects.) Under the chosen representation with the $\mathbb{L}^2$ metric, the action of the re-parameterization group is by isometries This results in, to our knowledge, the first Riemannian distance between surfaces to have all the desired invariances. We demonstrate this framework with several examples using toy shapes, anatomical objects, and facial surfaces. For further details, please refer to the paper [11].

2.2 Applications of Shape Analysis in Image Understanding

2.2.1 General Computer Vision and Image Understanding

1. **Intrinsic Bayesian Active Contours**: We have developed a framework for incorporating prior information about high-probability shapes in the process of contour extraction and object recognition in images. Here one studies shapes as elements of an infinite-dimensional,

non-linear quotient space, and statistics of shapes are defined and computed intrinsically using differential geometry of this shape space. Prior models on shapes are constructed using probability distributions on tangent bundles of shape spaces. Similar to the past work on active contours, where curves are driven by vector fields based on image gradients and roughness penalties, we incorporate the prior shape knowledge in the form of vector fields on curves. Through experimental results, we demonstrate the use of prior shape models in the estimation of object boundaries, and their success in handling partial obscuration and missing data. Furthermore, we describe the use of this framework in shapebased object recognition or classification. For further details, please refer to the article [5].

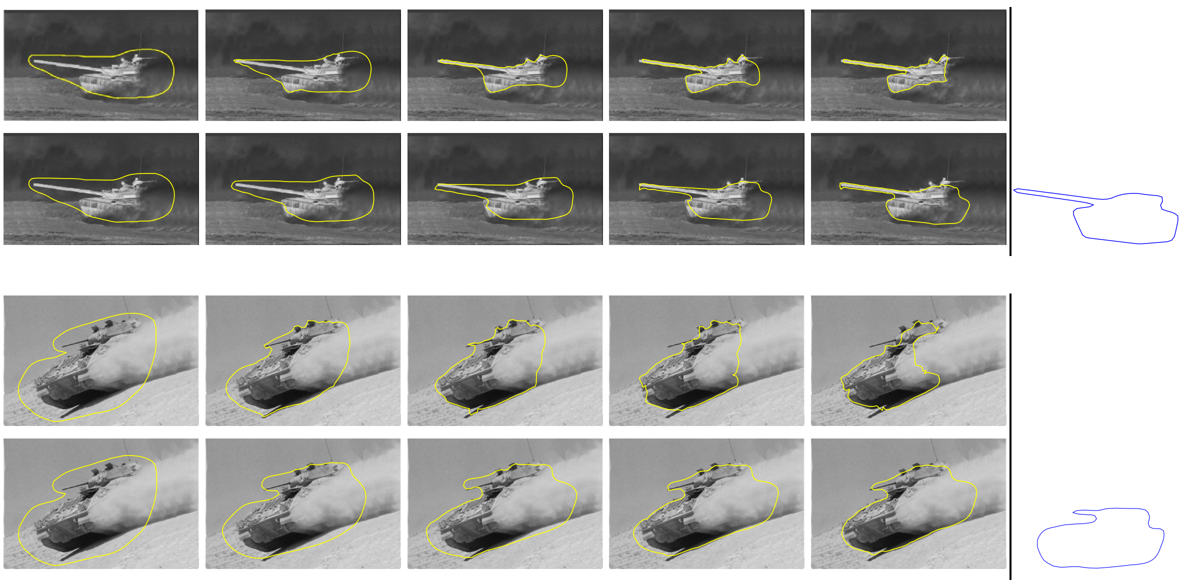
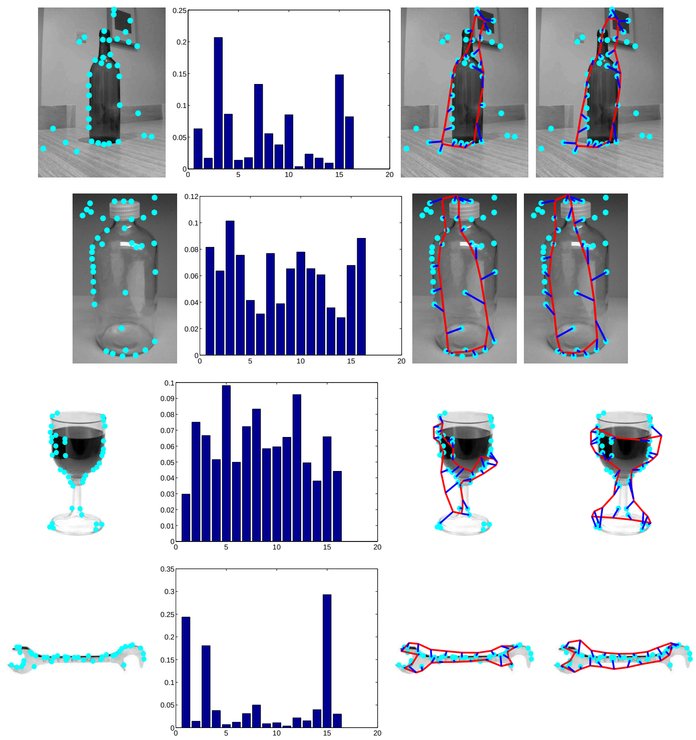


Figure 2: Two examples of shape extraction from corrupted images in the absence (top) and the presence (bottom) of the prior shape model. The last panel shows the prior mean. Example taken from [5].

2. **Shape Detection in Point Clouds**: We study the problem of classifying shapes in point clouds that are made of sampled contours corrupted by clutter and observation noise. Taking an analysis-by-synthesis approach, we simulate high-probability configurations of sampled contours using models learnt from the training data to evaluate the given test data. To facilitate simulations, we develop statistical models for sources of (nuisance) variability: (i) shape variations within classes, (ii) variability in sampling continuous curves, (iii) pose and scale variability, (iv) observation noise, and (v) points introduced by clutter. The variability in sampling closed curves into finite points is represented by positive diffeomorphisms of a unit circle and we derive probability models on these functions using their square-root forms and the Fisher-Rao metric. Using a Monte Carlo approach, we simulate configurations using a joint prior on the shape-sample space and compare them to the data using a likelihood function. Average likelihoods of simulated configurations lead to estimates of posterior probabilities of different classes and, hence, Bayesian classification. Further details can be found in the manuscript Srivastava and Jermyn [17].



(a) (b) (c) (d)

Figure 3: (a) Original images with points clouds extracted from them, (b) Estimated posterior of different shape classes in point clouds, (c) and (d) Some high probability shapes in given clouds.

2.2.2 Human Biometrics

1. **3D Face Recognition Using Shapes of Facial Surfaces**: This research focused on the problem of analyzing variability in shapes of facial surfaces using a Riemannian framework, a fundamental approach that allows for joint matchings, comparisons, and deformations of faces under a chosen metric. The starting point is to impose a curvilinear coordinate system, named the Darcyan coordinate system, on facial surfaces; it is based on the level curves of the surface distance function measured from the tip of the nose. Each facial surface is now represented as an indexed collection of these level curves. The task of finding optimal deformations, or geodesic paths, between facial surfaces reduces to that of finding geodesics between level curves, which is accomplished using the theory of elastic shape analysis of 3D curves. Elastic framework allows for nonlinear matching between curves and between points across curves. The resulting geodesics provide optimal elastic deformations between faces and an elastic metric for comparing facial shapes. We have demonstrates this idea using several public databases. The results and discussions have been presented in a number of papers [15, 19, 16, 1].

   We have also explored the use of shapes of noses for partial human biometrics by looking at human nose. The basic idea is to represent nasal surfaces using indexed collections of iso-curves, and to analyze shapes of noses by comparing their corresponding curves. We extend past work in Riemannian analysis of shapes of closed curves in $\mathbb{R}^3$ to obtain a similar Riemannian analysis for nasal surfaces. In particular, we obtain algorithms for computing

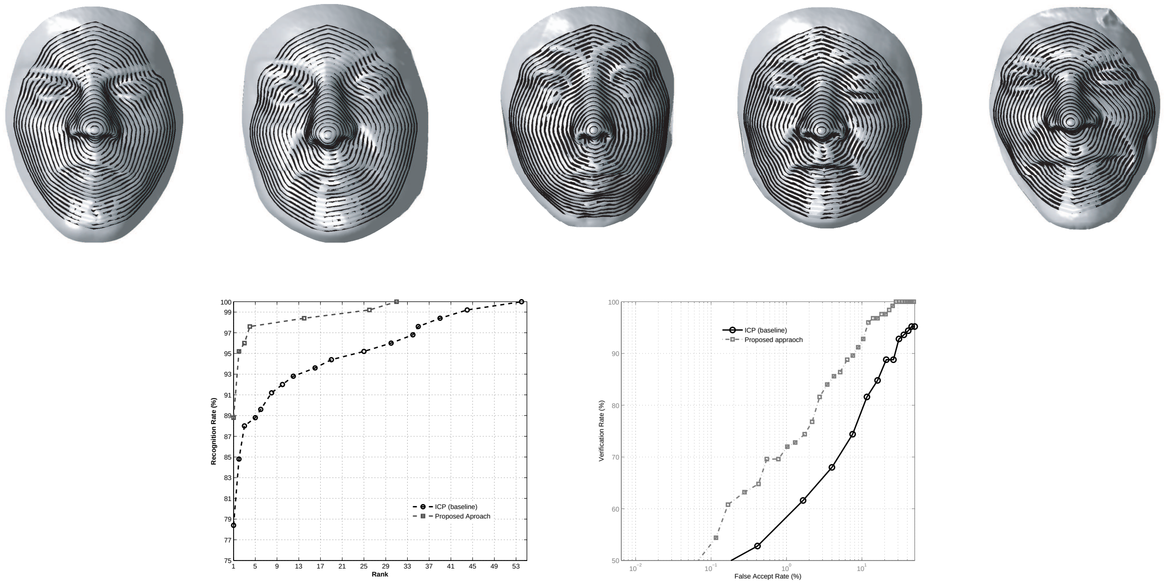


Figure 4: Top: Examples of facial surfaces represented by a collection of level curves. Performance curves (left: CMC and right: ROC) using elastic shape analysis and a commonly used ICP algorithm for FRGC face database.

geodesics, computing statistical means, and stochastic clustering. We demonstrate these ideas in two application contexts: evaluate authentication and identification performances using nasal shapes on a large database involving 2000 scans, and hierarchical organization of nose databases to allow for efficient searches. Please refer to the article [2] for more details.

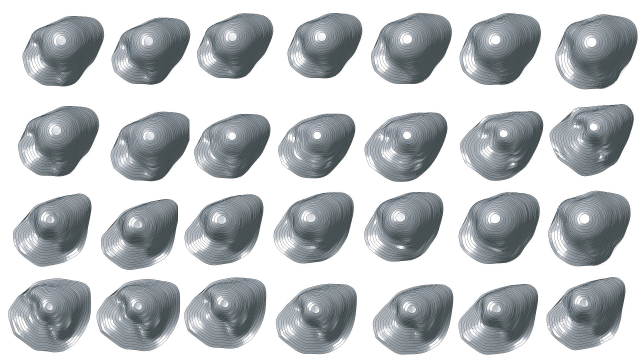


Figure 5: Geodesic paths between source and target noses (a) First row: intra-class path, source and target with different expressions (b) Three last rows: inter-class path

More recently, we have explored the use of shape analysis to estimate the facial expression and, thus, the state of mind of a person. Some promising preliminary results are presented in [13].

2. **Gait-Based Human Recognition**: In this work we have studied human identification by

gait recognition where subjects' gaits are represented by time series of silhouettes. Each silhouette is a simple, closed, planar curve that is represented by a Square-Root Velocity (SRV) function; this function enables comparisons of shapes of silhouettes in a manner that is invariant to rotations, translations, scalings, and re-parameterizations. We identify the nonlinear space (manifold) of allowable SRV functions and endow it with a Riemannian metric that compares shapes under optimal elastic deformations. A gait cycle becomes a cyclic stochastic process on this manifold and cadence relates to the rate of execution of this cycle. Using the differential geometry of the underlying shape manifold, we compute a mean gait cycle for each subject as a template. An observation model, that a test sequence is a result of a random perturbation of a subject's template, gives rise to a likelihood function for classification. Evaluation of likelihood involves temporal registration – linear and nonlinear – of a test cycle with the template, a process that may remove the effect of cadence. We study the effect of cadence and its removal on gait-based recognition of human subjects. It is observed that the linear registration, which preserves cadence, performs better than the nonlinear registration, which removes cadence. For further discussion, please refer to the papers [6, 8, 7]

3. **Activity Recognition Using 2D Shapes in Vidoes**: This research was performed in collaboration with Ashok Veeraraghavan and Prof. Rama Chellappa of University of Maryland, College Park. In this work we provide a systematic model-based approach to learn the nature of such temporal variations (time warps) while simultaneously allowing for the spatial variations in the descriptors. We illustrate our approach for the problem of action recognition and provide experimental justification for the importance of accounting for rate variations in action recognition. The model is composed of a nominal activity trajectory and a function space capturing the probability distribution of activity-specific time warping transformations.We use the square-root parameterization of time warps to derive geodesics, distance measures, and probability distributions on the space of time warping functions. We then design a Bayesian algorithm which treats the execution rate function as a nuisance variable and integrates it out using Monte Carlo sampling, to generate estimates of class posteriors. This approach allows us to learn the space of time warps for each activity while simultaneously capturing other intra- and interclass variations. For further details, please refer to [20].

2.2.3 Medical Image Analysis

We have applied the framework for shape analysis of surfaces of 3D brain structures to classify human subjects into control and disease cases using the shapes of their brain substructures. While past comparisons of surfaces optimized over rigid motions they mostly utilize the given parameterizations of surfaces. In our mathematical representation of surfaces, called q-maps, the $\mathbb{L}^2$ distances between such maps are invariant to re-parameterizations. This allows for removing the parameterization variability by optimizing over the re-parameterization group. This results in a proper parameterization-invariant distance shapes of surfaces. We demonstrate this method in shape analysis of eleven brain structures. Specifically, we show that the joint shape analysis of multiple brain structures, for 34 subjects in Detroit Fetal Drug and Alcohol Exposure Cohort study, results in approx. 92% classification rate for ADHD cases and controls [10].

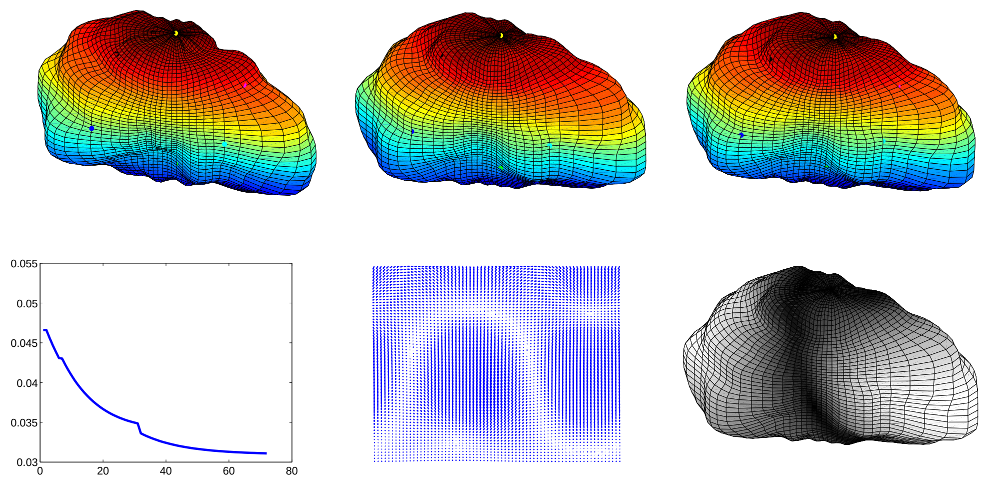


Figure 6: Top: Initial and final matching between two anatomical surfaces. Bottom: Cost function and deformation fields for surface 2.

2.2.4 Bioinformatics: Protein Structure Analysis

Structure comparison of proteins is an important tool for understanding the evolutionary relationships between proteins, predicting protein structures and predicting protein functions. There are two types of protein structure comparison problems, comparison of backbone structures (structure alignment) and comparison of the binding or active sites of proteins (surface matching). Proteins are flexible molecules and rigid matching of either backbones or surfaces of proteins, as used by most current methods, has the difficulty of recognizing relatively distant, functionally important similarities. Another well known issue in structure comparison is the lack of rigorous distance metric and comprehensive statistical framework for assessing the statistical significance of similarities between individual protein structures and classes of protein structures. Despite many past studies, protein structure alignment is still a challenging problem, especially for cases where structures undergo significant conformational changes or have large insertion or deletion of unrelated structural fragments. In this worked, we focused on the comparisons of backbone structures and develop methods based on elastic shape analysis. As a result, a formal distance can be calculated and geodesic paths, showing optimal deformations between conformations/structures, can be computed for any two backbone structures. It can also be used to average shapes of conformations associated with similar proteins. Using examples of protein backbones we demonstrate the matching and clustering of proteins using the backbone geometries, the secondary labels and the primary sequences. For details, please refer to the paper [12].

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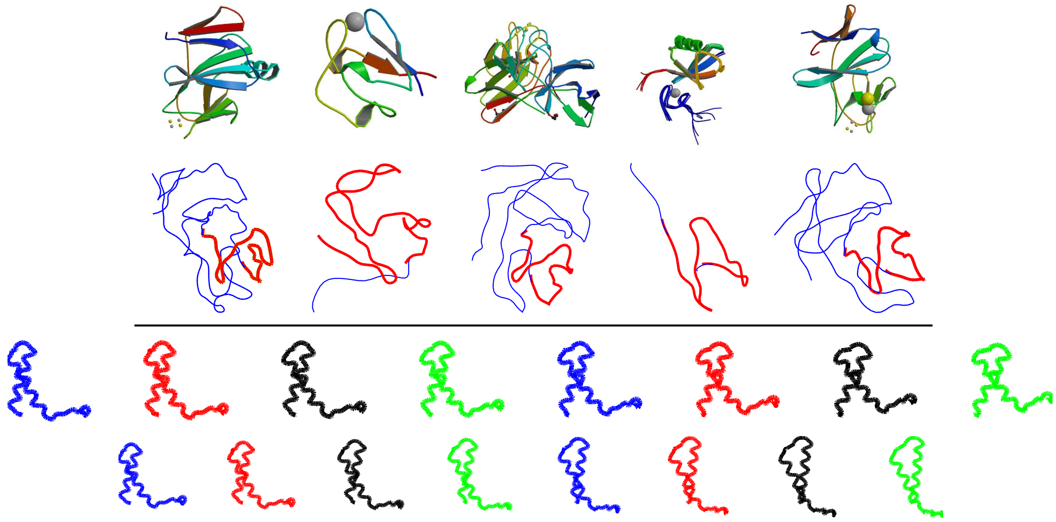

Figure 7: Top two rows: Some proteins and their backbones depicted as curves in $\mathbb{R}^3$. Bottom two rows: Examples of geodesic paths between some 2JVD conformations.

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