Comparison of Response from Combined Axial and Blast Loads Calculated with SDOF and Finite Element Methods

By:

Charles J. Oswald, P.E., Ph.D.
Protection Engineering Consultants

Abstract

Single-degree-of-freedom (SDOF) analyses are used to design many blast-resistant structures, which is an important part of explosive safety. Typically, the SDOF analyses consider flexural component response, but they can also consider other response modes including compression and tension membrane, arching from axial loads, and component response that includes secondary moments from axial loads acting on deflections caused by flexure. This latter response is often called "P-delta" response, since the axial load (i.e. P) acting on the component with a midspan deflection (i.e. delta) causes additional bending moment that adds to the bending moment from the applied blast load. It is typically unconservative to neglect the additional bending moments from P-delta response for blast design of exterior wall components that support significant axial load from the roof or floors above.

This paper describes how P-delta response can be incorporated into the SDOF analyses in a simple manner and also summarizes a comparison study of P-delta response calculated using the SDOF methodology and dynamic finite element analyses for beam-columns and two-way spanning wall panels subjected to combined dynamic axial load and lateral blast loads. The comparison is based on maximum deflections calculated with each method since component blast damage is typically based on the maximum calculated deflections. Any eccentricity of the applied axial load, relative to the centroid of the component cross section, can also be incorporated into the SDOF analysis approach. This approach does not consider secondary moments from frame sway, or flexibility of the building lateral load system that allows the top of the axially loaded components to deflect relative to the bottom. Frame sway must be considered as part of an analysis of the lateral load resisting system of the whole building, rather than SDOF analyses of individual wall or column components with combined axial and lateral loads.

Introduction

Structural components are commonly designed to resist laterally applied blast loads assuming they respond as equivalent single-degree-of-freedom (SDOF) systems. This methodology, which combines design level simplicity and explicit consideration of dynamic response, has compared well to blast tests on a wide variety of structural components (PDC-TR 08-02, 2008). In some cases, blast-loaded wall and column components also resist an axial load applied by the dynamic reaction of a blast-loaded roof component and/or static gravity loads. The axial load generally acts to increase the dynamic component response to the blast load because of P-delta effects cause an additional secondary bending moment to act on the component. Therefore, in order to accurately determine maximum response, the P-delta effect should generally be included in the

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
JUL 2010	**N/A**	**-**

4. TITLE AND SUBTITLE	
Comparison of Response from Combined Axial and Blast Loads Calculated with SDOF and Finite Element Methods	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Protection Engineering Consultants**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES

See also ADM002313. Department of Defense Explosives Safety Board Seminar (34th) held in Portland, Oregon on 13-15 July 2010, The original document contains color images.

14. ABSTRACT

Single-degree-of-freedom (SDOF) analyses are used to design many blast-resistant structures, which is an important part of explosive safety. Typically, the SDOF analyses consider flexural component response, but they can also consider other response modes including compression and tension membrane, arching from axial loads, and component response that includes secondary moments from axial loads acting on deflections caused by flexure. This latter response is often called P-delta response, since the axial load (i.e. P) acting on the component with a midspan deflection (i.e. delta) causes additional bending moment that adds to the bending moment from the applied blast load. It is typically unconservative to neglect the additional bending moments from P-delta response for blast design of exterior wall components that support significant axial load from the roof or floors above. This paper describes how P-delta response can be incorporated into the SDOF analyses in a simple manner and also summarizes a comparison study of P-delta response calculated using the SDOF methodology and dynamic finite element analyses for beam-columns and two-way spanning wall panels subjected to combined dynamic axial load and lateral blast loads. The comparison is based on maximum deflections calculated with each method since component blast damage is typically based on the maximum calculated deflections. Any eccentricity of the applied axial load, relative to the centroid of the component cross section, can also be incorporated into the SDOF analysis approach. This approach does not consider secondary moments from frame sway, or flexibility of the building lateral load system that allows the top of the axially loaded components to deflect relative to the bottom. Frame sway must be considered as part of an analysis of the lateral load resisting system of the whole building, rather than SDOF analyses of individual wall or column components with combined axial and lateral loads.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**SAR**	**35**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

analysis of the dynamic response of components subject to simultaneous blast loads and axial loads.

This paper describes a method to include the P-delta effect into a SDOF analysis of a component subject to combined axial and laterally applied blast load and shows a comparison of this method to calculations with dynamic finite element analyses. The method uses a dynamic equivalent lateral load (ELL) to cause approximately the same secondary bending moment as the P-delta effect at each time step in the SDOF analysis. The ELL is added to the applied blast load at each step in the SDOF analysis. The comparison to the finite element calculations is based on maximum deflections calculated with each method since component blast design and damage assessment is typically based on the maximum calculated deflections. The SDOF calculations are performed with the SBEDS (Single-Degree-of-Freedom Blast Effects Design Spreadsheets) program (PDC-TR 06-01, 2008) that is distributed by the U.S. Army Corps of Engineers, Protective Design Center (PDC) and the finite element calculations are performed with the LS-DYNA computer program (LSTC, 2007). The ELL method for calculating P-delta effects is incorporated into the current version of SBEDS (Version 4.1).

Equivalent Lateral Load

The ELL is calculated at each time step in the SDOF analysis as shown in Equation 1. This equation also shows how the ELL is derived to cause a moment equal to the P-delta moment. Since the ELL is added to the applied blast load at each time step, it must have the same load distribution as the blast load. This is accounted for in Equation 1 with the constant C, which is a function of the load distribution (i.e. uniform or concentrated) and is also a function of the number of supported sides of the component. The constant C is *not* a function of degree of fixity at the supports (e.g. simple supports or fixed supports) because the degree of fixity does not directly affect the magnitude of the applied P-delta effect, even though fixity allows a component to resist the P-delta moment with lower stresses. Since the ELL representing the P-delta moment is applied over the same loaded width as the blast load (i.e. it is added to the blast load history), this approach assumes that the P-delta moment is distributed into the whole width of the blast-loaded component (i.e. there is no concentration of the P-delta moment only on one part of the component that is not distributed by the component over its width).

$$M(t) = P(t)\left[\Delta(t) + e\right] = \frac{p'(t)L^2}{C}$$

$$p'(t) = P(t)\left[\Delta(t) + e\right]\left(\frac{C}{L^2}\right)$$

Equation 1

where:

p'(t) = equivalent lateral load (ELL) causing same maximum moment in component as PΔ moment (psi)

P(t) = total axial load divided by supported width of blast-loaded component (lb/in) Note: P(t) can include dynamic and/or static axial loading

Δ(t) = maximum lateral deflection of component (in)

e = eccentricity of axial load relative to centroid of cross section in the direction of bending response of from the applied lateral load

M(t) = maximum moment applied by PΔ divided by blast-loaded width (lb-in/in)
L = span in direction of axial load (in)
C = constant corresponding to blast load distribution and locations of supports. (see Equation 2)

$$C = K_1 K_2$$

Equation 2

where:

K_1 = factor dependent on location of boundaries in direction of axial load for one-way spanning components (see Table 1)
K_2 = 1.0 for one-way spanning component
= 0.64 for two-way spanning component
Note: Only uniformly distributed blast loads can be considered for two-way spanning components.

Table 1. Values for Factor K_1

Case	Boundary Locations	Blast Load Distribution	K_1	Example
1	At both ends of component in direction of axial load	Uniform	8	Uniformly loaded column or one-way spanning wall with top and bottom supports. Supports may be fixed and/or simple.
2	At both ends of component in direction of axial load	Concentrated at midspan	4	Column with beam applying blast load as concentrated load at midspan. Supports may be fixed and/or simple.
3	At one end of component in direction of axial load	Uniform	2	Cantilevered column or wall that is not supported at top where axial load is applied (i.e. supported on two or three sides, not including top of wall). Uniform blast load in both cases.
4	At one end of component in direction of axial load (unloaded end)	Concentrated at free end	1	Cantilevered column with blast load applied by supported beam as concentrated load at free end.

The constant K_2 in Equation 2 is less than 1.0 for two-way spanning components because almost all of the axial load along the top support acts on a smaller deflection than the maximum component deflection that is calculated in the SDOF analysis. This is illustrated in Figure 1 for a four side supported wall panel, where Δ_{max} at the center of the panel is the deflection calculated by the SDOF analysis. The average deflection across the full width of the panel at mid-height is 64% of the center deflection, assuming the deflected shape for the wall panel is a double sine wave function. Therefore, the average P-delta moment across the full width of the wall, assuming the axial load is a uniformly distributed across the top of the wall, can be reduced to 64% for a two-way component compared to the same component spanning one-way in the direction of the axial load. The average deflection factor across the wall at mid-height in Figure 1 would actually be somewhat lower during plastic deflection assuming all the yielding is concentrated at yield lines, so the use of 64% is conservative overall. The case of any support fixity also decreases the average deflection factor across the wall and can be conservatively neglected.

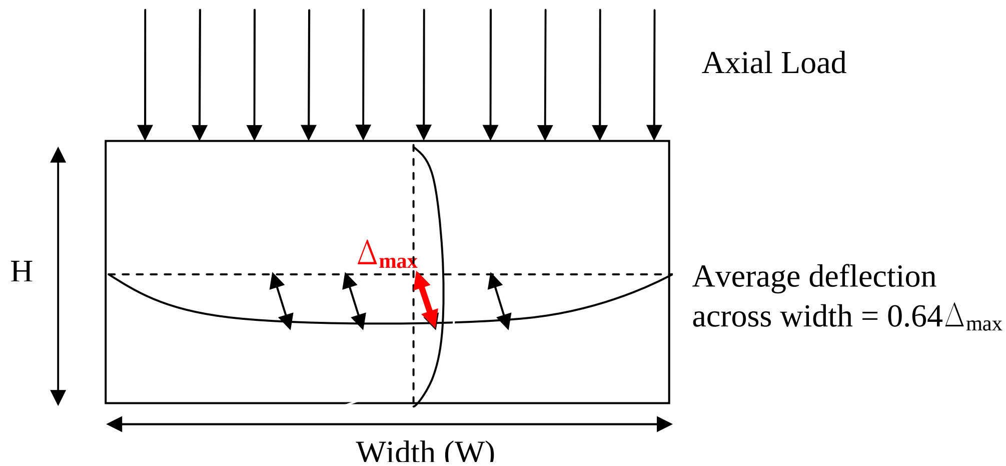


Figure 1. Two-Way Spanning Slab Supported on All Sides with Axial and Lateral Load

The 64% value is also applicable for a three-side supported wall that is free along the top axially-loaded edge, since the deflected shape along the top (where the maximum deflection occurs) is also a sine wave. The case of a vertically loaded wall supported on the top, bottom and one vertical edge is similar to that shown in Figure 1 over a width 0.5W, where the deflected shape along the mid-height is one-half of a sine wave, which has the same average deflection as the full sine wave shape (i.e. 64% also applies). The deflected shape of a wall supported only on two adjacent sides can be approximated with a double half sine wave function, in which case the average deflection would also be 64% of the maximum deflection at the unsupported corner.

Note that the maximum displacement (i.e. $\Delta(t)$ in Equation 1) in the ELL approach is the maximum component deflection calculated by the SDOF methodology relative to the component supports. Therefore, this approach does not account for the effects of any overall building sidesway and applies only for buildings without significant sidesway (i.e. adequately braced frames or shear wall construction), or buildings where the sideway is a relatively slow response that is out-of-phase with the lateral response of individual component to direct blast loading. In the latter case, the sideway response must be analyzed separately.

Comparison of Methodology to LS-DYNA for One-Way Spanning Components

The initial part of this study compared analyses of one-way spanning components. As a first step, the ELL methodology for SDOF analyses was compared to both theoretical and finite element calculations for a given column (W12x40 steel column) responding elastically to combined static axial and static lateral load. This column was analyzed in LS-DYNA with beam-column elements and an elastic-perfectly plastic material model. The theoretical analysis was based on the moment magnifier method (Timoshenko and Gere, 1961). The SDOF analysis was performed with SBEDS using Equation 1 to account for P-delta effects. To simulate static loading and elastic response, the lateral load was applied very slowly in the SDOF analysis and using an implicit load-stepping solution method in LS-DYNA. The effect of a constant eccentricity (i.e. “e” in Equation 1) was not included in this comparison since it is only a constant that is added to the calculated SDOF deflection at each time step and does not represent an additional level of complexity in the solution.

Table 2 shows a summary of this comparison analysis. The very good comparison on Table 2 between SBEDS and LS-DYNA, and between both methods and the theoretical deflection calculated with the moment magnifier method, indicates that the ELL method in Equation 1 accurately accounts for the P-delta moment under static loading for all three typical boundary condition cases.

Table 2. Comparison of Elastic Deflections from Static Lateral Load and Combined Lateral and Axial Load on W12x40 Column

Height (ft)	Boundary Condition	Lateral Load (W) (lb/ft)	Maximum Deflection From Lateral Load (in)			Axial Load (P) (lb)	Maximum Deflection From Combined Lateral and Axial Load (in)			
			SBEDS	DYNA	SBED/ DYNA		Theory[1]	SBEDS	DYNA	SBED/ DYNA
50	Fixed-fixed	587.5	1.83	1.87	0.98	186682	2.26	2.21	2.31	0.96
30	Fixed-fixed	1631	0.67	0.69	0.97	183245	0.72	0.70	0.73	0.96
50	Fixed-simple	392	2.53	2.59	0.98	121687	3.41	3.27	3.41	0.96
30	Fixed-simple	1088	0.92	0.94	0.98	167740	1.05	1.02	1.06	0.96
50	Simple-simple	391	6.09	6.18	0.99	54047	7.8	8.00	7.94	1.01
15	Simple-simple	4351	0.55	0.57	0.96	137436	0.58	0.58	0.60	0.97

Note 1: Theoretical deflection based on moment magnifier applied to SBEDS deflection from lateral load only.

In the next step of the comparison, yielding was allowed under dynamic lateral loading. Therefore, in these cases interaction between combined compressive and flexural stresses reduced the lateral load capacity of the beam-columns. The SDOF methodology (i.e. SBEDS) uses the column interaction formula in Equation 3 (AISC, 1989) to determine the available lateral moment capacity with axial load. The equation does not include the effects of secondary moments from the axial load (i.e. P-delta effects. The available moment capacity (M_{du}, in Equation 3) is used to determine the lateral load capacity (i.e. resistance) of the column in the SDOF analysis. The axial load capacity, P_A, in Equation 3 is a semi-empirical formula from AISC (2006) based on steel column tests, which account for the effect of small loading eccentricities and residual stresses (i.e. inelastic buckling) on axial load capacity (Salmon and Johnson, 1980). LS-DYNA, on the other hand, considers only theoretical combined stresses without any consideration of these additional effects.

$$M_{du} = M_m\left(1-\frac{P}{P_A}\right)$$

$$P_A = \left(0.658^{\lambda^2}\right)F_{dy} \quad if \quad \lambda \le 1.5$$

$$P_A = \left(\frac{0.877}{\lambda^2}\right)F_{dy} \quad if \quad \lambda > 1.5$$

$$where \quad \lambda = L'\sqrt{\frac{F_{dy}}{E}} \quad and \quad L' = \max\left(\frac{K_y L_y}{\pi r_y}, \frac{K_x L_x}{\pi r_x}\right)$$

Equation 3

where:

M_{du} = ultimate dynamic moment capacity accounting for axial compressive load (lb-in)
P = applied axial compression load (lb)
P_A = axial load capacity accounting for column slenderness (lb)
M_m = full plastic moment capacity (lb-in)
F_{dy} = dynamic yield strength (psi)
L = span length between supports about given bending axis (in)
r = radius of gyration about given bending axis (in)
E = modulus of elasticity (psi)
K = effective length factor about given bending axis

This difference initially confounded the intention of the comparison study, which was to focus on evaluating the accuracy of the simplified P-delta approach in Equation 1 using finite element analyses. Ideally, this would be resolved by accounting for the residual stress distribution in the steel column cross section in the LS-DYNA model so that it could match the inelastic buckling region of steel column response that is incorporated into Equation 3. However, this level of effort was outside the scope of this study. Instead, the SBEDS analyses were modified to make them more comparable to the theoretical consideration of axial load response in LS-DYNA, so that the lateral resistance of the columns under axial load would be similar in both methods and any differences would primarily be due to the simplification of using the ELL method in SBEDS.

In this approach, the LS-DYNA analyses were performed first and the ultimate moment capacities (i.e. yield moments) at midspan and at the supports (for fixed end conditions) were determined by inspection of the output dynamic moment histories. These yield moments were used in Equation 4 to calculate an effective lateral load resistance for the comparable SBEDS analyses (i.e. a LS-DYNA-based resistance). Equation 4 is a general form of the commonly used equations to calculate the ultimate resistance of beam components for SDOF analyses (UFC 3-340-02, 2008) (PDC PDC-TR 06-01, 2008). The interaction formula in SBEDS was disabled in the comparison analyses, and, instead, the effect of axial load on the available moment capacity was accounted for by inputting a reduced yield strength that caused the calculated ultimate resistance in SBEDS to match that calculated with Equation 4 based on yield moments from comparable LS-DYNA analysis.

$$R_u = \frac{\left(K'_1 M_{yn} + K'_2 M_{yp}\right)}{L^2}$$

Equation 4

where:

R_u = Resistance
M_{yn} = Negative yield moment at fixed support
M_{yp} = Positive yield moment at midspan
L = span length
K'_i = factors dependent on boundary conditions
$K'_1 = K'_2 = 8$ for a fixed-fixed beam-column
$K'_1 = 0$, $K'_2 = 8$ for a simply supported beam-column

Figure 2 has a comparison of the resistance histories calculated with LS-DYNA and SBEDS, showing how both methods calculated nearly the same component lateral resistance. The resistance for LS-DYNA in Figure 2 was calculated at each time step by using the moments at midspan and the support in the LS-DYNA output at that time step in Equation 4 in place of the yield moments. This comparison in Figure 2 is for a fixed-fixed, 20 ft, W12x40 column with the same applied static axial and lateral dynamic load applied in LS-DYNA and SBEDS.

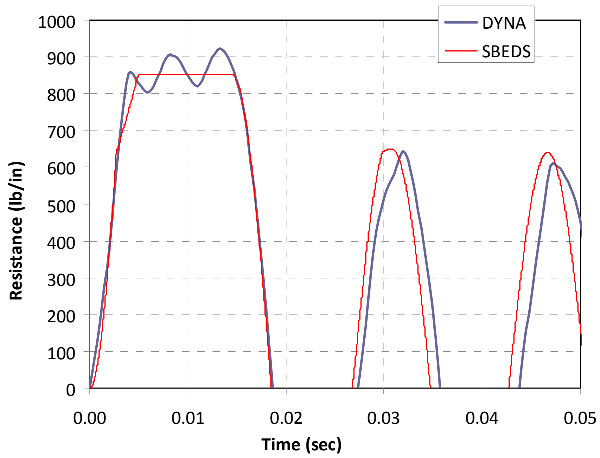


Figure 2. Comparison of Lateral Load Resistances Based on Dynamic Moments in LS-DYNA and SBEDS for 20 ft W12x40 Column with Fixed Supports

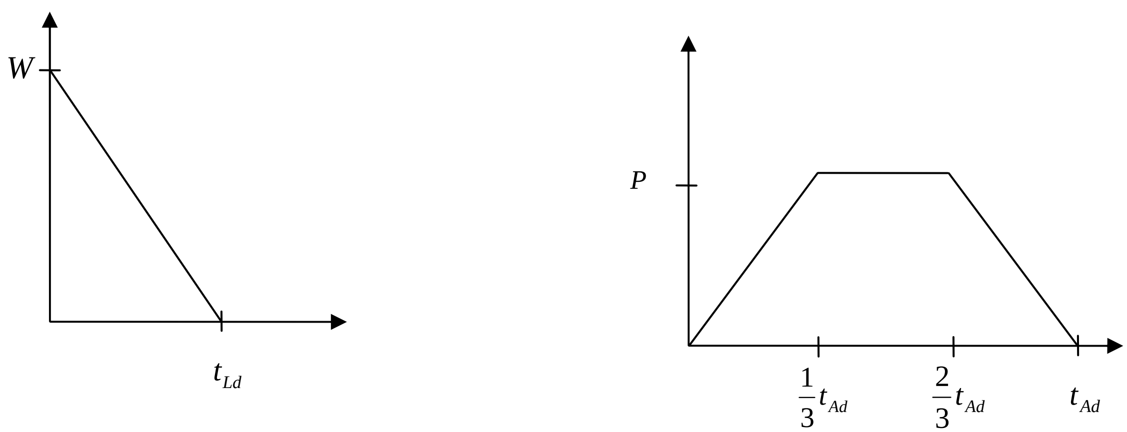
The results (i.e. maximum deflections) calculated from the comparisons cases for steel columns with combined dynamic axial and lateral loads that cause significant yielding are shown in Table 3. As described in the previous paragraphs, both methods used the nearly the same lateral load resistance. The reduced dynamic yield strengths input into SBEDS to achieve equivalent lateral load resistance as LS-DYNA are shown in Table 3. The applied dynamic loads used in the comparison cases have the shapes shown in Figure 3. The cases in Table 3 cover typical boundary conditions and column heights, and a range of natural periods. They include static and dynamic axial loading cases that cause response with ductility ratios between 1.2 and 5. This is a representative range for axially loaded components subject to lateral blast load (i.e. primary components). Specific details of the column cross section, span, and boundary conditions in the comparison cases are primarily important in that they create a relatively broad range of resistances, stiffnesses, and natural periods for comparison, which are the parameters that directly affect the dynamic responses.

Table 3 shows that the maximum deflections calculated with SBEDS are within 16% of those calculated with LS-DYNA for all comparisons. The LS-DYNA deflections are generally greater than those in SBEDS. This is not necessarily due to inaccuracy in Equation 1 since there are a number of differences in the dynamic response calculations between the two methods in spite of previously discussed efforts to minimize them. For example, comparisons of purely flexural response of wall slabs to lateral dynamic loads (i.e. no axial loading or P-delta effects) with LS-DYNA and SBEDS discussed in the next section show that LS-DYNA typically calculates 10% more lateral deflection than SBEDS. In these analyses SBEDS and LS-DYNA have nearly the mass, blast load, and resistance-deflection curves.

Table 3. Comparison of Maximum Deflections from Combined Dynamic Lateral and Axial Loads on W12x40 Column Causing Yielding

Height (ft)	Natural Period T_n (ms)	Boundary Condition	SBEDS Dynamic Yield Strength[3] (psi)	Yield Defl. (inch)[4]	Dynamic Lateral Load[1]		Static Axial Load and Dynamic Lateral Load				Dynamic Axial[2] Load and Dynamic Lateral Load				
					Load W (lb/ft)	Duration t_{Ld} (ms)	Axial Load, P_{AS} (lb)	SBEDS Max. Defl. (inch)	DYNA Max. Defl. (in)	SBEDS/ DYNA	Peak Axial Load, P_{AD} (lb)	Dur-ation t_{Ad} (ms)	SBEDS Max. Defl. (inch)	DYNA Max. Defl. (in)	SBEDS/ DYNA
30	80	SS	56554	4.9	4320	20	104253	7.23	8.58	**0.84**	104253	30	7.08	8.18	**0.87**
20	16	FF	53979	1.0	15600	16	144653	4.94	5.30	**0.93**	144653	15	4.95	5.11	**0.97**
50	144	FS	61069	10.5	1200	72.5	91265	11.87	12.16	**0.98**	91265	72	11.62	11.74	**0.99**

Note 1: See Figure 3 for shape of dynamic lateral load, W. No supported mass was included in any of the dynamic analyses.
Note 2: See Figure 3 for shape of dynamic axial load, P_{AD}.
Note 3: Dynamic yield strength available for flexural response (exclusive of axial load effects). This caused SBEDS moment capacities to match those in LS-DYNA that included axial load effects and had total yield strength of 62,500 psi.
Note 4: Yield deflection from equivalent SDOF system in SBEDS. Corresponding ductility ratios for based on SBEDS maximum deflections and yield deflection range from 1.2 to 5.0.



(a) Dynamic Lateral Load (b) Dynamic Axial Load

Figure 3. Dynamic Load-Time Histories for Beam Analyses

Furthermore, over 100 comparisons of the SDOF methodology in SBEDS to test data with only lateral blast loads showed that SBEDS was conservative compared to data by approximately 20% on the average (PDC-TR 08-02, 2008). Considering all of this, the relatively minor amount of apparent non-conservatism in SBEDS relative to LS-DYNA is not considered to be very significant. Therefore, Equation 1 is considered acceptable for SDOF-based calculations for one-way spanning components, even though LS-DYNA tends to predict somewhat more lateral deflection under combined lateral and axial loads in this study.

Comparison of SBEDS to LS-DYNA for Two-Way Spanning Components

The second part of the study focused on two-way spanning components. This comparison between the SDOF methodology with Equation 1 and LS-DYNA was based on a 7 inch thick concrete wall reinforced with 0.5 inch rebars at 10 inches on center, each face, each way. It includes a relatively wide range of boundary conditions and span lengths under a uniform lateral blast load and a uniform axial load along the top of the wall. A typical wall was initially input into SBEDS to determine its calculated moment capacity, mass density, and flexural stiffness (EI value) without axial load. LS-DYNA was then used to model the two-way spanning wall with each boundary condition using 4-noded shell elements and an elastic-perfectly plastic material model. The yield strength, modulus of elasticity, and mass density of the material were selected to cause the LS-DYNA model to match the mass density, flexural stiffness, and moment capacity calculated in SBEDS for the wall slab. Therefore, the SBEDS and LS-DYNA walls both had the same cross sectional properties under lateral loading.

The first step of this comparison was to apply only lateral blast loads to the same walls in SBEDS and LS-DYNA to check if both approaches were modeling the dynamic lateral response of the walls similarly. In order to make sure that both SBEDS and LS-DYNA would use the same resistance-deflection curves in the comparisons, static load-deflection analyses were initially performed with LS-DYNA and these curves were fit with piece-wise linear approximations to generate very nearly identical resistance-deflection curves that were input into the SBEDS analyses. Also, no dynamic increase factors were used in SBEDS or LS-DYNA. These steps helped to create a more equivalent comparison of dynamic response calculations in SBEDS and LS-DYNA. Table 4 shows a comparison of the calculated maximum deflections.

As shown in Table 4, the maximum dynamic deflections calculated with both methods are close, although the deflections calculated with SBEDS are generally less than those calculated with LS-DYNA (10% less on average). The reasons for this relatively small difference are not known, but they may involve some differences in the lateral load resistance developed under dynamic response in LS-DYNA compared to static response (i.e. higher mode shapes that were excited in the LS-DYNA analyses that are not considered with the basic first-mode shape approach in SBEDS).

Table 4. Comparison of Maximum Dynamic Deflections on Two-Way Components from Lateral Load Only

Case	Height (ft)	Width (ft)	Boundary Condition[1]	Yield Defl. (inch)	Natural Period T_n (ms)	Dynamic Lateral Load[2]				
						Load P (psi)	Duration t_{Ld} (ms)	SBEDS Max. Deflection	DYNA Max. Deflection	SBEDS/ DYNA
1a	25	25	SSSS	0.62	108	3.5	125	5.25	5.73	**0.92**
1b	25	25	SSSS	0.62	108	4	125	7.21	7.60	**0.95**
2a	10	10	SSSS	0.11	17	20	40	1.95	2.05	**0.95**
2b	10	10	SSSS	0.11	17	23	40	3.27	3.34	**0.98**
3a	10	25	SSSS	0.18	32	14	40	2.55	3.15	**0.81**
3b	10	25	SSSS	0.18	32	16	40	3.70	4.42	**0.84**
4a	25	25	FFFF	0.69	60	10	30	2.5	2.82	**0.89**
4b	25	25	FFFF	0.69	60	13	30	4.55	4.79	**0.95**
5a	10	10	FFFF	0.11	10	50	10	0.8	0.83	**0.96**
5b	10	10	FFFF	0.11	10	61	10	1.30	1.37	**0.95**
6a	10	25	FFFF	0.18	15	14	40	0.33	0.42	**0.79**
6b	10	25	FFFF	0.18	15	16	40	0.55	0.68	**0.81**
7a	25	25	FSFS	0.69	60	3.5	125	2.2	2.24	**0.98**
7b	25	25	FSFS	0.69	60	4	125	3.30	3.41	**0.97**
8a	10	10	FSFS	0.11	10	21	40	0.56	0.56	**1.00**
8b	10	10	FSFS	0.11	10	30	40	2.97	2.84	**1.05**
9a	10	25	FSFS	0.10	15	14	40	0.44	0.49	**0.90**
9b	10	25	FSFS	0.10	15	16	40	0.72	0.80	**0.90**
10a	10	25	SSSFree	1.09	15	8	40	5.8	7.17	**0.81**
10b	10	25	SSSFree	1.09	15	10	40	8.80	10.67	**0.82**

Note 1: FSFS is fixed on both long sides and simple on both short sides. SSSFree is simple on three sides, with a long free edge along at the top of wall. SSSS is simple on all edges and FFFF is fixed on all edges.
Note 2: See Figure 3 for shape of dynamic lateral load.

The next step of the comparison was to apply combined axial and dynamic lateral loads causing yielding of the wall panels. This presented a problem similar to the column comparison analyses, since the axial load caused compression stress that reduced the flexural stress available to resist lateral load in the shell elements in LS-DYNA. This does not occur in SBEDS because under-reinforced reinforced concrete components, such as the wall considered in these comparisons, actually gain moment capacity at typical applied axial load levels such as those used in these comparisons (i.e. less than 25% of the axial capacity), rather than loose moment capacity. SBEDS conservatively ignores this gain in the moment resistance of wall slabs (although it is accounted for in analysis of reinforced concrete columns).

This difference was resolved by generating static load-deflection curves in LS-DYNA using the same axial load that was used in the SBEDS analyses and then inputting those curves as the resistance-deflection curves for the SBEDS analyses. Figure 4 shows a typical comparison of three static load-deflection curves for a typical wall; 1) the load-deflection curve from only static lateral load in LS-DYNA, 2) the load-deflection curve for combined static axial and lateral load in LS-DYNA with instability at yield (statically the P-delta effect causes instability at yielding), and 3) the lateral resistance in dynamic LS-DYNA analyses with combined axial and lateral loads. The LS-DYNA analysis with combined lateral and axial load (i.e. Case 2) shows the reduced flexural resistance caused by combined axial load. This reduction was applied to the static resistance-deflection curve for lateral load only to create a reduced resistance-deflection curve accounting for combined axial and lateral load that is assumed applicable during dynamic response in LS-DYNA (i.e. Case 3). The LS-DYNA analyses with combined axial and lateral load became unstable at the deflections shown in Figure 4, but dynamic LS-DYNA analyses with combined axial and lateral load deflected to much large deflections and rebounded, so that they were not unstable.

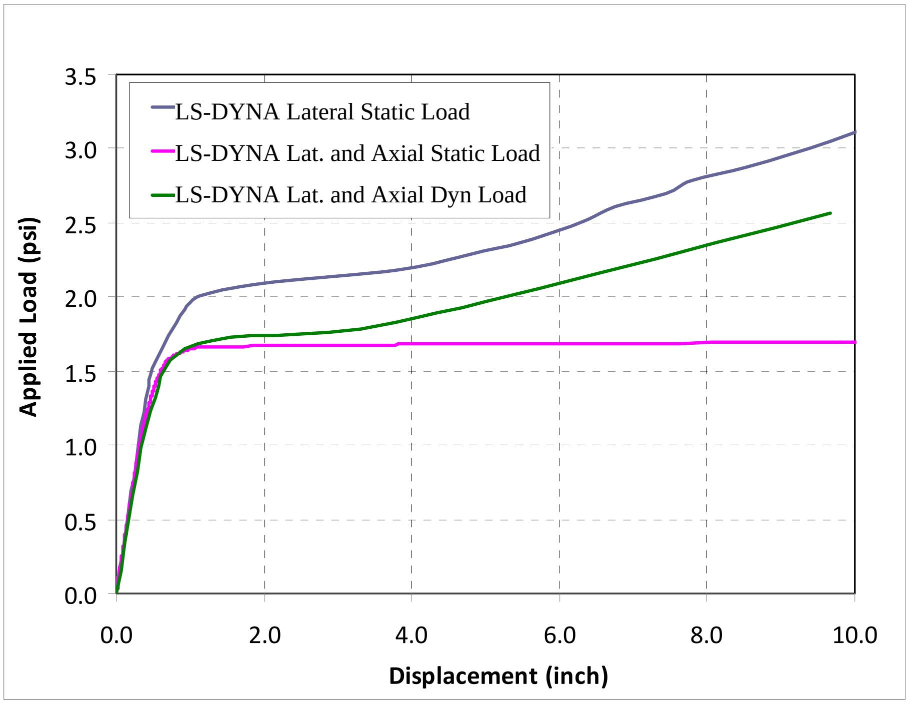


Figure 4. Comparison of Static Load-Deflection Curves for Case 1

The resistance-deflection curves represented by Case 3 in Figure 4 (i.e. green curve for Lateral and Axial Dynamic Load) were used in the SBEDS analyses for the comparison cases. These curves were calculated from the static load-deflection curves for lateral load only from LS-DYNA (i.e. blue curve in Figure 4) using Equation 5. The resistance-deflection curve representing Case 3 in Figure 4 shows how the resistance-deflection curves generated with Equation 5 typically fit between the static load-deflection curves from LS-DYNA with and without applied axial load. The K_{ab} factor in Equation 5 was determined by trial and error to cause the best match to the LS-DYNA curve with lateral and axial load prior to its instability.

$$p'(x) = p(x)\left[K_{ab}\left(1 - K_{rf}\,\frac{\sigma_c}{\sigma_y}\right)\right]$$

Equation 5

where:

p(x) = load from LS-DYNA load-deflection curve for lateral only load (psi)
p'(x) = corresponding modified load at each deflection in SBEDS resistance-deflection curve (i.e. SBEDS curve Figure 4) (psi)
K_{ab} = aspect ratio correction factor
= 1.1 for aspect ratio of 1.0 and 1.0 for aspect ratio of 0.4
σ_c = compression stress from applied axial load on wall
σ_y = input yield strength for wall in LS-DYNA analyses
K_{fr} = two-way action reduction factor since axial load applied in one direction and component resists load based on resisting moments in two directions
= 0.5 for all cases

Table 5 shows the comparisons between the maximum dynamic lateral displacements calculated with SBEDS and LS-DYNA for combined axial and lateral loads with load shape histories as shown in Figure 3. The axial loads were uniformly distributed along the top edge of the walls. The resistance deflection curves in the SBEDS analyses in Table 5 were very close point-wise linear representations of curves created with Equation 5 from static lateral load-deflection curves from LS-DYNA without axial load. These static curves were calculated with LS-DYNA prior to dynamic analyses for each case.

The comparisons in Table 5 show that the lateral deflections calculated with the ELL method in SBEDS were very nearly equal to those from LS-DYNA for comparison cases with static axial loads, on the average, and were conservative by 16% compared to LS-DYNA for comparison cases with dynamic axial loads, on the average. The comparisons in Table 5 include a wide range of response with some support rotations over 4 degrees and ductility ratios over 40. They also include a wide range of boundary conditions, ultimate resistances, and natural periods for the walls. The ultimate flexural resistances for the walls in Table 5 ranged from 2 psi to 20 psi. The actual conservatism in the SBEDS analyses is probably greater than that shown in the averages in Table 5 when the results from Table 4 are considered. The results in Table 4 show that LS-DYNA typically calculates more lateral deflection than determined by SBEDS when only lateral loads are considered.

Table 5. Comparison of Maximum Dynamic Deflections on Two-Way Components from Combined Axial and Lateral Load

Case	Height by Width (ft)	Natural Period (ms)	Boundary Condition[1]	Yield Defl. (inch)	Static Axial and Dynamic Lateral Load[2]				Dynamic Axial[3] and Dynamic Lateral Load[2]				
					Axial Load, P_s (lb)	SBEDS Max. Deflection	DYNA Max. Deflection	SBEDS/ DYNA	Axial Load,P_D (lb)	Duration t_{Ad} (ms)	SBEDS Max. Deflection	DYNA Max. Deflection	SBEDS/ DYNA
1a	25x25	108	SSSS	0.62	2000	9.35	9.36	**1.00**					
1b	25x25	108	SSSS	0.62					2000	125	11.5	10.4	**1.11**
2a	10x10	17	SSSS	0.11	2500	4.94	4.35	**1.14**					
2b	10x10	17	SSSS	0.11					2500	40	6.7	5.16	**1.30**
3a	10x25	32	SSSS	0.18	2500	Failure	Failure	**N/A**					
3b	10x25	32	SSSS	0.18					2500	40	7.65	7.44	**1.03**
4a	25x25	60	FFFF	0.69	2000	3.1	3.36	**0.92**					
4b	25x25	60	FFFF	0.69					2000	30	5.1	5.18	**0.98**
5a	10x10	10	FFFF	0.11	2500	1.21	1.16	**1.04**					
5b	10x10	10	FFFF	0.11					2500	10	1.93	1.64	**1.18**
6a	10x25	15	FFFF	0.18	2500	1.1	1.22	**0.91**					
6b	10x25	15	FFFF	0.18					2500	40	1.9	1.47	**1.29**
7a	25x25	80	FSFS	0.69	2000	3.85	3.87	**0.99**					
7b	25x25	80	FSFS	0.69					2000	125	5.86	5.29	**1.11**
8a	10x10	10	FSFS	0.11	2500	1.65	1.57	**1.05**					
8b	10x10	10	FSFS	0.11					2500	40	6.9	4.96	**1.39**
9a	10x25	15	FSFS	0.10	2500	1.44	1.58	**0.91**					
9b	10x25	15	FSFS	0.10					2500	40	2.44	1.87	**1.30**
10a	10x25	120	SSSFree	1.09	800	7.8	9.52	**0.82**					
10b	10x25	120	SSSFree	1.09					800	40	9.9	11.15	**0.89**
Average			**All**					**0.98**					**1.16**

Note 1: All FSFS (fixed on opposite sides, simple on opposite sides) cases have fixity across top support with axial load. SSSFree (Simple on three sides with free surface along long side at top of wall) has axial load along top free edge. In all cases, axial load is applied over short span of walls.
Note 2: See Figure 3 for shape of dynamic lateral load.
Note 3: Axial load uniformly distributed along top of wall.

Summary and Conclusions

The study described in this paper indicates that the ELL method can be used in SDOF-based methodologies to account for P-delta effects from combined axial and lateral dynamic loads. Based on comparisons to similar components, the ELL method predicts maximum dynamic deflections from combined lateral blast loading and static or dynamic axial load within 16%, on the average, of those calculated with dynamic finite element analyses (i.e. LS-DYNA) for one-way spanning and two-way spanning components. The comparison analyses ranged from causing elastic response to significant plastic response. Both the one-way spanning and two-way spanning components had a range of natural periods, boundary conditions, ultimate resistances, blast loads, and axial loads.

These comparisons indicate that the implementation of the ELL method in a SDOF methodology for one-way and two-way spanning components compares well to LS-DYNA since it is generally conservative by an acceptable margin for design purposes. Also, the ELL method is well-suited for blast resistant design because of its simplicity and its ease of implementation into the type of time-stepping SDOF analysis typically used for dynamic analyses of blast-loaded components in design. The ELL method does not consider secondary moments from frame sway, which is caused by flexibility of the building lateral load system that allows the top of the axially loaded components to deflect relative to the bottom. This type of response (i.e. frame sway) must be considered as part of a larger analysis of the lateral load resisting system of the whole building, rather than SDOF analyses of individual components with combined axial and lateral loads.

This study was conducted using modifications to the equations in the SDOF methodology (i.e. SBEDS) that would typically be used to account for axial load effects for steel columns and reinforced concrete slabs. These effects (i.e. residual stresses in steel column cross sections and moment increase due to low applied axial loads in under-reinforced concrete wall panels) are easier to account for in a SDOF approach, which uses empirically-based design equations, than in a first-principles finite element analysis and explicit modeling of them was outside the scope of this study. Therefore, the lateral resistance-deflection relationships for combined axial and lateral loading were determined for each LS-DYNA analysis and inputs were used in the comparable SDOF analyses to cause nearly the same lateral resistance-deflection relationships. The goal was to make the lateral response characteristics of the two analyses methods as equivalent as possible, so that any differences could be attributed to the different consideration of P-delta effects in the two methods (i.e. the simplified ELL approach in SBEDS and explicit considering the dynamic moment distributions applied to the component by the P-delta effect in the LS-DYNA analyses).

Acknowledgements

This study was funded by the U.S. Army Corps of Engineers, Protective Design Center at Protection Engineering Consultants (PEC) as part of the development effort for the SBEDS program. Dr. Daniel Williams performed the LS-DYNA calculations as a consultant to PEC. SBEDS has been developed at PEC and Baker Engineering and Risk Consultants.

References

American Institute of Steel Construction (AISC). (1989). *Steel Construction Manual for Allowable Stress Design (ASD)*. 9th Edition.

American Institute of Steel Construction (AISC). (2006). *Steel Construction Manual for Load and Resistance Factor Design (LRFD)*. 13th Edition.

Livermore Software Technology Corporation (LSTC). (2007). *LS-DYNA Keyword User's Manual, Version 971*.

PDC-TR 06-01 Rev 1, "Methodology Manual for the Single-Degree-of-Freedom Blast Effects Design Spreadsheets (SBEDS)", U.S. Army Corps of Engineers, Protective Design Center (PDC) Technical Report, 2008.

PDC-TR 08-02, "Comparison of Calculated Single-Degree-of-Freedom Response to Blast Test Data", U.S. Army Corps of Engineers, Protective Design Center (PDC) Technical Report, 2008.

UFC 3-340-02, "Structures to Resist the Effects of Accidental Explosions," Department of Defense, 2008.

Salmon, C.G. and Johnson, J.E., *Steel Structures*, Harper and Row, New York, NY, 1980.

Timoshenko, S.P. and Gere, J.M., *Theory of Elastic Stability*, McGraw Hill, New York, NY, 1961.

Comparison of Response from Combined Axial and Blast Loads Calculated with SDOF and Finite Element Methods

Chuck Oswald, Ph.D., P.E.

DDESB Explosive Safety Seminar
July, 2010

Overview

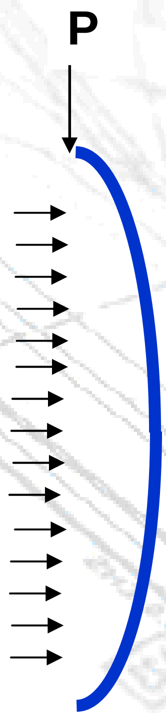
- Equivalent single-degree-of-freedom (SDOF) models usually used for blast design
- Load-bearing walls and exterior columns can have combined axial load and lateral blast load
- Axial load applies an additional "PΔ" moment to wall increasing dynamic response of component
- Equivalent lateral load (ELL) can be added to blast load to model the additional applied P-delta moment
  - ELL method can be incorporated into equivalent SDOF model

2

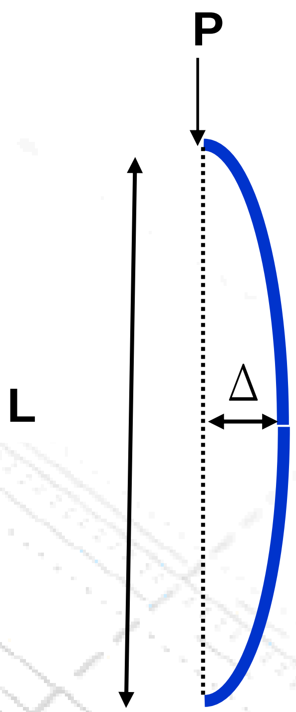
Overview (Cont'd)

- SDOF analysis with ELL method was compared to finite element analysis with LS-DYNA
- Purpose of comparison was to investigate accuracy of ELL method
  - ELL method is included in SBEDS (SDOF Blast Effects Design Spreadsheets)
  - Comparison study was funded by U.S. Army Corps of Engineers, Protective Design Center
- Other differences between blast analysis of components with SDOF and LS-DYNA methods were minimized
  - Focus of study was only to investigate ELL method

P-Delta Moment from Combined Blast and Axial Load



Component with Combined Blast and Axial Load



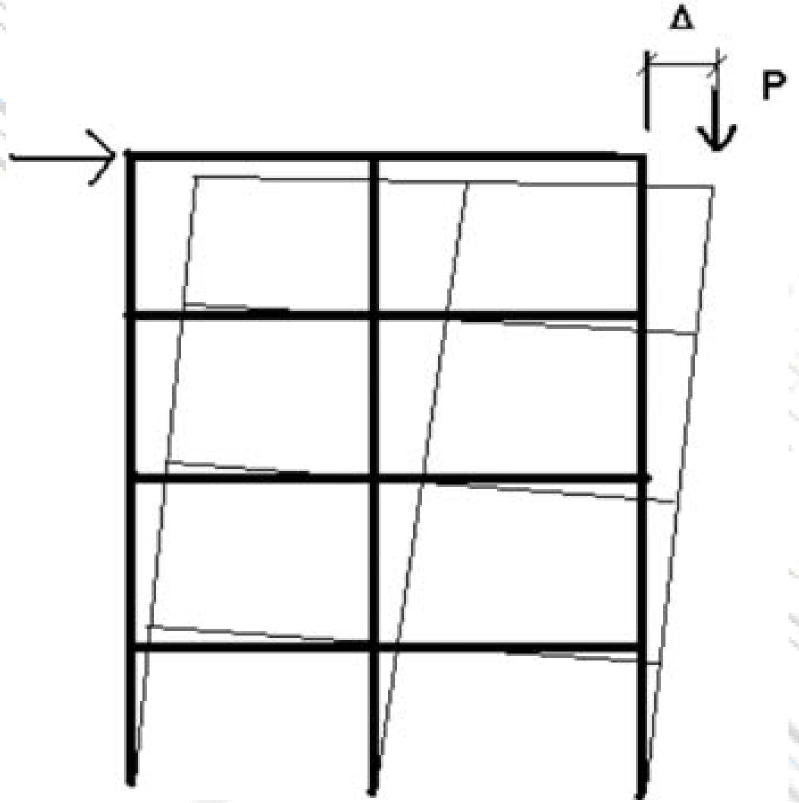
Secondary Bending Moment from PΔ

Note: P load can be from static gravity loads and/or dynamic reaction from supported roof component

Protection Engineering CONSULTANTS

P∆ Effect from Overall Frame Sway Not Considered in This Approach

- Additional secondary moments from frame sway are NOT considered in this approach
- This P∆ effect must be considered as part of larger lateral load resisting system analysis



Equivalent Lateral Load (ELL) Method

- Apply additional lateral load to cause same maximum moment as PΔ moment
- This "equivalent" lateral load is calculated at each time step in SDOF analysis based on maximum deflection ($\Delta(t)$) and axial load at previous time step
- ELL is added to blast load at each time step
  - ELL must have same spatial distribution as blast load
- SDOF response to both loads is calculated at each time step to determine dynamic response

6

Equivalent Lateral Load, p'(t)

Set Moments Equal

$$M(t) = P(t)\left[\Delta(t) + e\right] = \frac{p'(t)L^2}{C}$$

PΔ moment

Moment from ELL

Solve for ELL

$$p'(t) = P(t)\left[\Delta(t) + e\right]\left(\frac{C}{L^2}\right)$$

p'(t) = equivalent lateral load (ELL) causing same maximum moment in component as PΔ moment (psi)

P(t) = total axial load divided by supported width of blast-loaded component (lb/in) Note: P(t) can include dynamic and/or static axial loading

Δ(t) = maximum lateral deflection of component (in)

e = eccentricity of axial load relative to centroid of cross section in the direction of bending response of from the applied lateral load

M(t) = maximum moment applied by PΔ divided by blast-loaded width (lb-in/in)

L = span in direction of axial load (in)

C = constant corresponding to blast load distribution and locations of supports.

Engineering CONSULTANTS

Boundary Support Constant (C)

$$C = K_1 K_2$$

K_1 = factor dependent on location of boundaries in direction of axial load for one-way spanning components

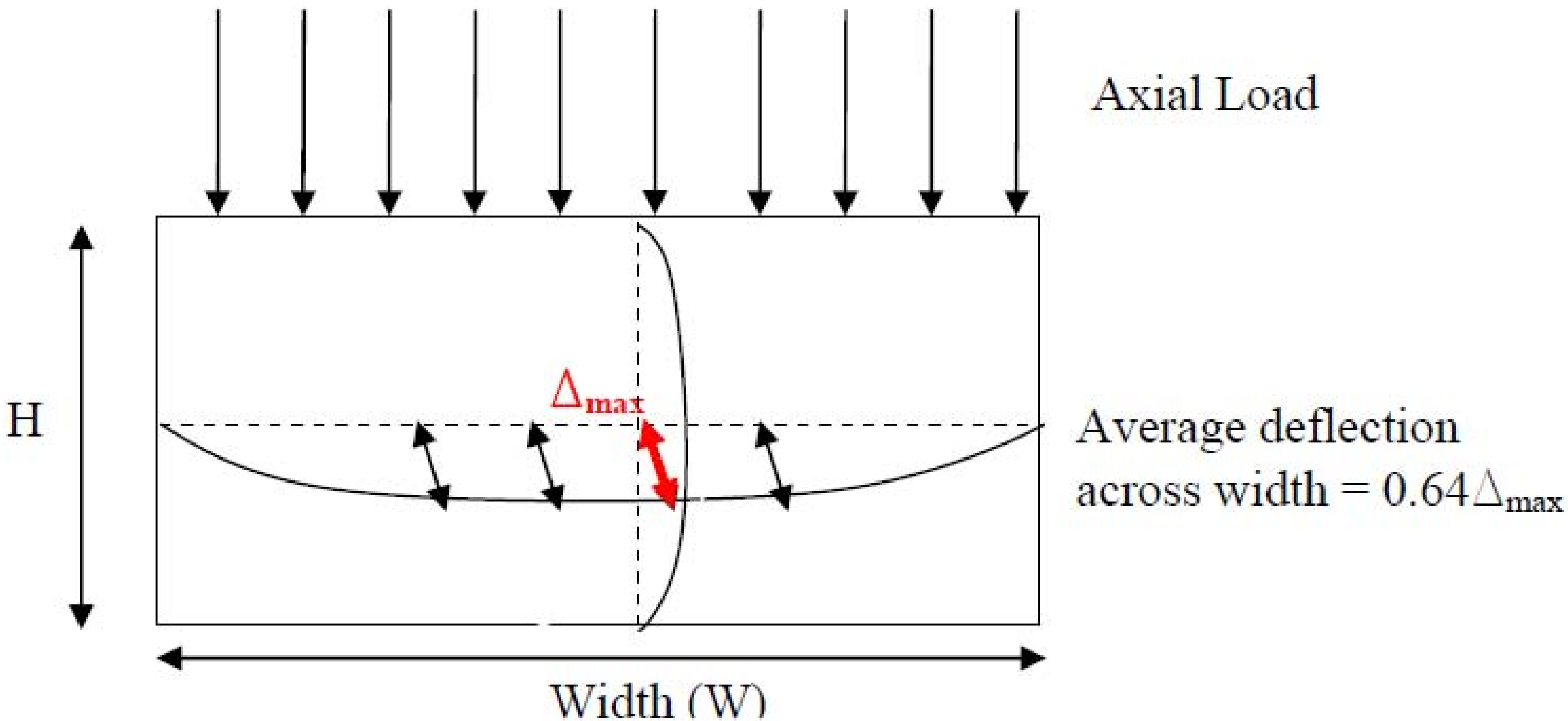
K_2 = 1.0 for one-way spanning component

= 0.64 for two-way spanning component

Case	Boundary Locations	Blast Load Distribution	K_1	Example
1	At both ends of component in direction of axial load	Uniform	8	Uniformly loaded column or one-way spanning wall with top and bottom supports. Supports may be fixed and/or simple.
2	At both ends of component in direction of axial load	Concentrated at midspan	4	Column with beam applying blast load as concentrated load at midspan. Supports may be fixed and/or simple.
3	At one end of component in direction of axial load	Uniform	2	Cantilevered column or wall that is not supported at top where axial load is applied (i.e. supported on two or three sides, not including top of wall). Uniform blast load in both cases.
4	At one end of component in direction of axial load (unloaded end)	Concentrated at free end	1	Cantilevered column with blast load applied by supported beam as concentrated load at free end.

K_2 Value of 0.64 for Two-Way Spanning Components

- K_2 value accounts for difference between average deflection and maximum deflection calculated in SDOF analysis



Engineering CONSULTANTS

Comparison Analyses

- Comparisons were primarily between ELL method incorporated into SBEDS methodology for SDOF blast analysis and LS-DYNA
- Initial column comparison for elastic, static response
  - Included comparison to theory (i.e. moment magnifier method)
- Column comparisons with lateral blast load and static/dynamic axial load
- Two-way concrete wall panel comparisons with lateral blast load and static/dynamic axial load
- Comparisons focused on maximum deflection calculated with each method

LS-DYNA Model

- LS-DYNA models were kept as simple as possible
  - Beam-column and shell elements
  - Elastic-plastic material model
- SDOF inputs were modified as necessary to match LS-DYNA
  - Intent was to cause both approaches to model component lateral load response similarly
- Numerous LS-DYNA analyses were performed with components to verify that intended response was analyzed

11

Initial Static, Elastic Analysis Comparison

- Static combined lateral and axial load on W12x40 column
- Compared moment magnifier method with slowly applied SDOF and LS-DYNA loads

Height (ft)	Boundary Condition	Lateral Load (W) (lb/ft)	Maximum Deflection From Lateral Load (in)			Axial Load (P) (lb)	Maximum Deflection From Combined Lateral and Axial Load (in)			
			SBEDS	DYNA	SBED/ DYNA		Theory[1]	SBEDS	DYNA	SBED/ DYNA
50	Fixed-fixed	587.5	1.83	1.87	0.98	186682	2.26	2.21	2.31	0.96
30	Fixed-fixed	1631	0.67	0.69	0.97	183245	0.72	0.70	0.73	0.96
50	Fixed-simple	392	2.53	2.59	0.98	121687	3.41	3.27	3.41	0.96
30	Fixed-simple	1088	0.92	0.94	0.98	167740	1.05	1.02	1.06	0.96
50	Simple-simple	391	6.09	6.18	0.99	54047	7.8	8.00	7.94	1.01
15	Simple-simple	4351	0.55	0.57	0.96	137436	0.58	0.58	0.60	0.97

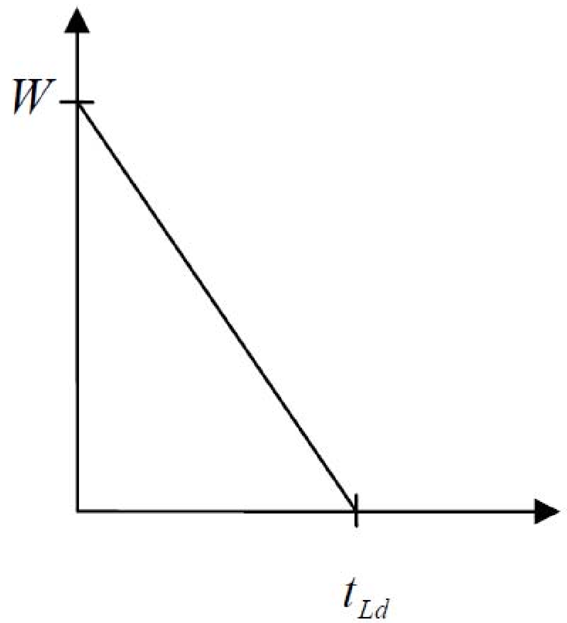
Note 1: Theoretical deflection based on moment magnifier applied to SBEDS deflection from lateral load only.

Protection Engineering CONSULTANTS

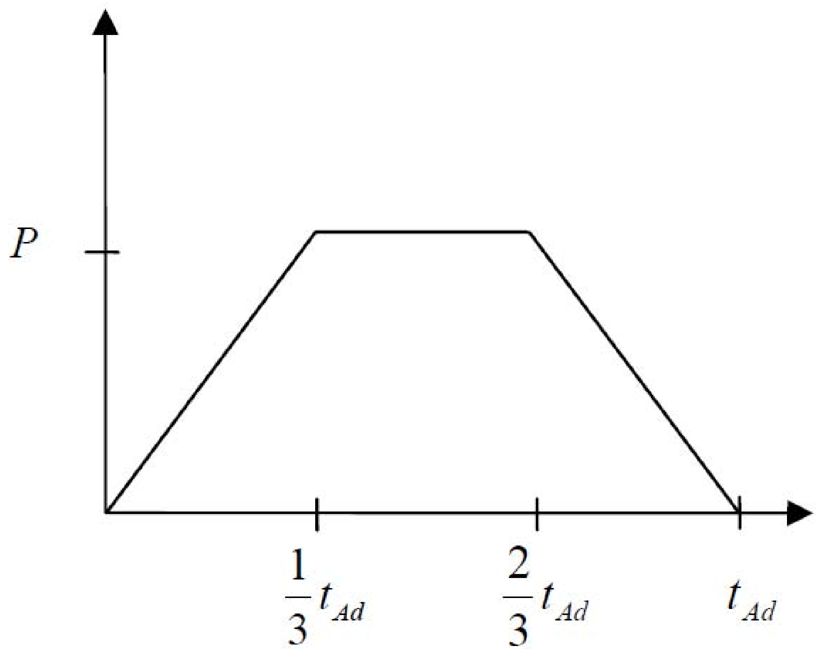
12

Column Comparison with Dynamic Loads

- High blast and axial loads causing plastic response
- Static and dynamic axial loads



(a) Dynamic Lateral Load



(b) Dynamic Axial Load

Column Comparison with Dynamic Loads

- W12x40 column with variety of boundary conditions, natural periods, loading combinations
- Comparisons within 16% where LS-DYNA calculated more deflection than SDOF
- Previous comparisons have shown SDOF generally conservatively calculates deflections vs. test data

Height (ft)	Natural Period T_n (ms)	Boundary Condition	Dynamic Lateral Load[1]		Static Axial Load and Dynamic Lateral Load				Dynamic Axial[2] Load and Dynamic Lateral Load				
			Load W (lb/ft)	Duration t_{Ld} (ms)	Axial Load, P_{AS} (lb)	SBEDS Max. Defl. (inch)	DYNA Max. Defl. (in)	SBEDS/ DYNA	Peak Axial Load, P_{AD} (lb)	Dur-ation t_{Ad} (ms)	SBEDS Max. Defl. (inch)	DYNA Max. Defl. (in)	SBEDS/ DYNA
30	80	SS	4320	20	104253	7.23	8.58	**0.84**	104253	30	7.08	8.18	**0.87**
20	16	FF	15600	16	144653	4.94	5.30	**0.93**	144653	15	4.95	5.11	**0.97**
50	144	FS	1200	72.5	91265	11.87	12.16	**0.98**	91265	72	11.62	11.74	**0.99**

Two-Way Wall Panel Comparisons with Dynamic Loads

- Similar blast and axial load shapes as column comparisons
- Initial comparison only considering lateral loads showed LS-DYNA typically calculated 10% to 20% more deflection than SDOF
- On average, ELL/SDOF and LS-DYNA were within 16%
  - ELL/SDOF generally calculated more deflection than LS-DYNA

Comparisons for Two-Way Wall Panels

Case	Height by Width (ft)	Natural Period (ms)	Boundary Condition[1]	Static Axial and Dynamic Lateral Load				Dynamic Axial and Dynamic Lateral Load				
				Axial Load, P_s (lb)	SBEDS Max. Deflection	DYNA Max. Deflection	SBEDS/ DYNA	Axial Load, P_D (lb)	Duration t_{Ad} (ms)	SBEDS Max. Deflection	DYNA Max. Deflection	SBEDS/ DYNA
1a	25x25	108	SSSS	2000	9.35	9.36	**1.00**					
1b	25x25	108	SSSS					2000	125	11.5	10.4	**1.11**
2a	10x10	17	SSSS	2500	4.94	4.35	**1.14**					
2b	10x10	17	SSSS					2500	40	6.7	5.16	**1.30**
3a	10x25	32	SSSS	2500	Failure	Failure	**N/A**					
3b	10x25	32	SSSS					2500	40	7.65	7.44	**1.03**
4a	25x25	60	FFFF	2000	3.1	3.36	**0.92**					
4b	25x25	60	FFFF					2000	30	5.1	5.18	**0.98**
5a	10x10	10	FFFF	2500	1.21	1.16	**1.04**					
5b	10x10	10	FFFF					2500	10	1.93	1.64	**1.18**
6a	10x25	15	FFFF	2500	1.1	1.22	**0.91**					
6b	10x25	15	FFFF					2500	40	1.9	1.47	**1.29**
7a	25x25	80	FSFS	2000	3.85	3.87	**0.99**					
7b	25x25	80	FSFS					2000	125	5.86	5.29	**1.11**
8a	10x10	10	FSFS	2500	1.65	1.57	**1.05**					
8b	10x10	10	FSFS					2500	40	6.9	4.96	**1.39**
9a	10x25	15	FSFS	2500	1.44	1.58	**0.91**					
9b	10x25	15	FSFS					2500	40	2.44	1.87	**1.30**
10a	10x25	120	SSSFree	800	7.8	9.52	**0.82**					
10b	10x25	120	SSSFree					800	40	9.9	11.15	**0.89**
Average			**All**				**0.98**					**1.16**

Protection Engineering Consultants

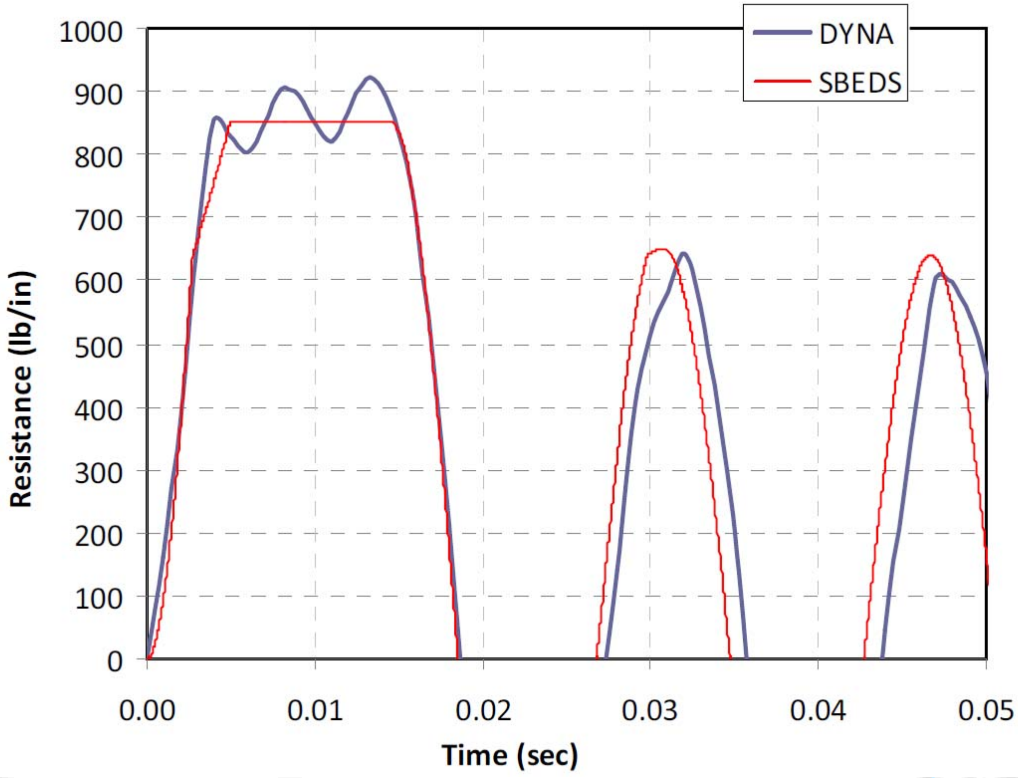
Difficulties Causing Equivalent Lateral Load Resisting Systems in SDOF and DYNA

- The accuracy of ELL method vs. finite element analysis can only be assessed to extent that two approaches are otherwise the same
- Numerous modeling differences in SBEDS and LS-DYNA
  - Consideration of combined axial and bending stress for reinforced concrete panels
  - Modeling of inelastic buckling
  - Compression ring effect allowing tension membrane in two-way components
- To focus on ELL investigation, lateral load resistance curves were input into SDOF to match LS-DYNA
  - Dynamic moment history output or static analyses used to determine lateral load resistance curves in LS-DYNA

17

Lateral Resistance-Deflection Curve for Columns

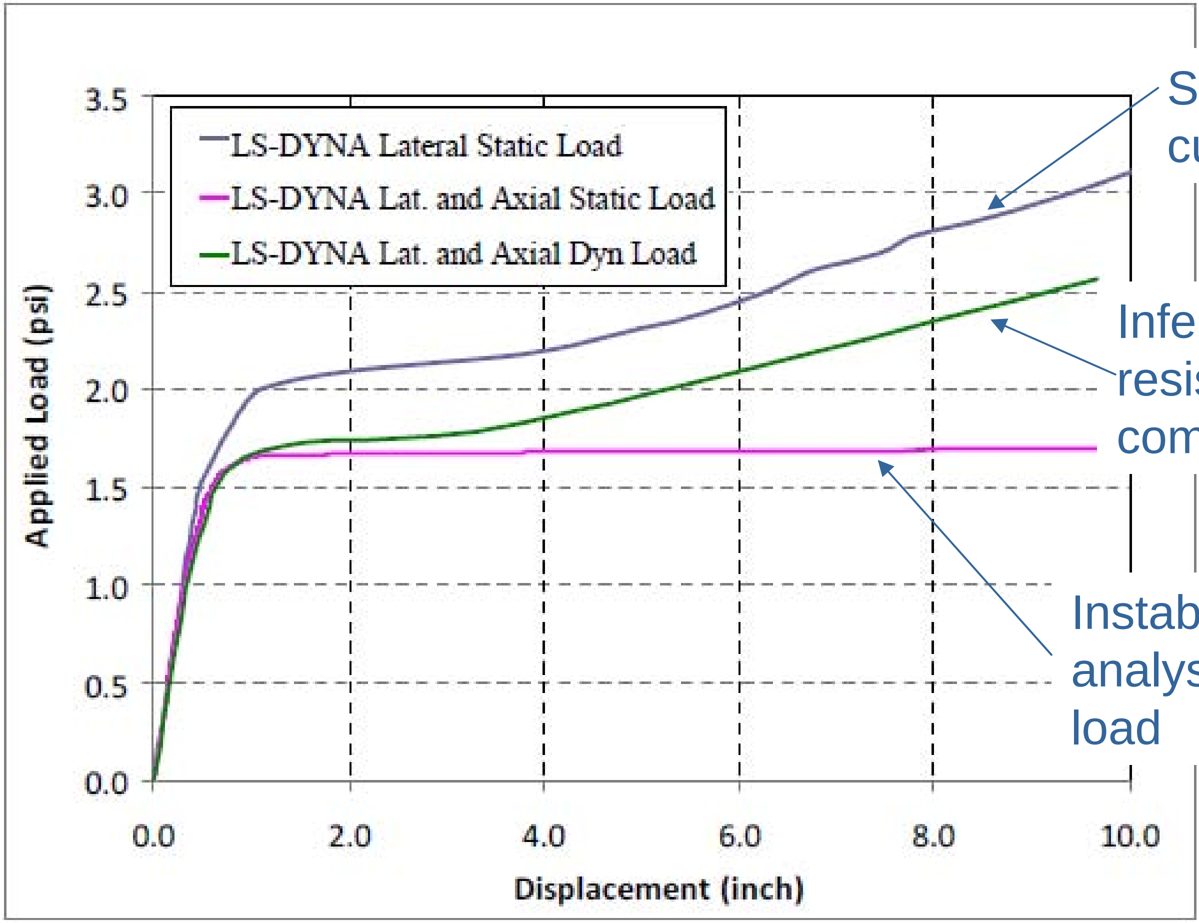
- Moment histories from LS-DYNA analyses with combined axial and blast loads used to "back-out" corresponding resistance-deflection curve
- This matched SDOF resistance-defl. curve well



Protection Engineering CONSULTANTS

Lateral Resistance-Deflection Curve for Two-Way Spanning Components

- Static analyses used to infer dynamic resistance-deflection curve in LS-DYNA
- These inferred curves input for SDOF analyses



Static resist.-defl. curve w/o axial load

Inferred dynamic resist.-defl. curve with combined load

Instability in static analysis with combined load

Protection Engineering CONSULTANTS

Summary and Conclusions

- ELL method acceptable for SDOF design
  - Maximum deflections within 16% of LS-DYNA analyses for one-way cases
  - Maximum deflections generally conservative compared to LS-DYNA analyses for two-way cases and within 16% on average
- More cases could be considered in larger comparison project
  - More accurate modeling with LS-DYNA also possible
- An inherent difficulty in this type of comparison is multiple differences between SDOF and finite element analysis
  - Must try to minimize these differences to assess ELL method

Protection Engineering CONSULTANTS

20