

# REPORT DOCUMENTATION PAGE

*Form Approved*  
*OMB No. 0704-0188*

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. **PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.**

<b>1. REPORT DATE (DD-MM-YYYY)</b> 03-02-2009		<b>2. REPORT TYPE</b> Journal Article		<b>3. DATES COVERED (From - To)</b>	
<b>4. TITLE AND SUBTITLE</b>  Analysis of Accommodation Coefficients of Noble Gases on Aluminum Surface with an Experimental/Computational Method (Postprint)				<b>5a. CONTRACT NUMBER</b>	
				<b>5b. GRANT NUMBER</b>	
				<b>5c. PROGRAM ELEMENT NUMBER</b>	
<b>6. AUTHOR(S)</b> Nathaniel Selden (USC); Natalia E. Gimelshein & Sergey F. Gimelshein (ERC); Andrew Ketsdever (AFRL/RZSA)				<b>5d. PROJECT NUMBER</b>	
				<b>5f. WORK UNIT NUMBER</b> 50260568	
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b>  Air Force Research Laboratory (AFMC) AFRL/RZSA 10 E. Saturn Blvd. Edwards AFB CA 93524-7680				<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>  AFRL-RZ-ED-JA-2009-030	
<b>9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b>  Air Force Research Laboratory (AFMC) AFRL/RZS 5 Pollux Drive Edwards AFB CA 93524-7048				<b>10. SPONSOR/MONITOR'S ACRONYM(S)</b>	
				<b>11. SPONSOR/MONITOR'S NUMBER(S)</b> AFRL-RZ-ED-JA-2009-030	
<b>12. DISTRIBUTION / AVAILABILITY STATEMENT</b>  Approved for public release; distribution unlimited (PA #09081).					
<b>13. SUPPLEMENTARY NOTES</b> Published in Physics of Fluids <b>21</b> , 073101 (2009). © 2009 American Institute of Physics.					
<b>14. ABSTRACT</b>  A technique is proposed to assess gas-surface accommodation coefficients. The technique utilizes the fact that radiometric forces exerted on heated objects immersed in rarefied gases are governed by the interaction of gas molecules with the surface. In the present implementation, it connects measurements of radiometric forces on a heated vane in the transitional flow regime with the kinetic modeling of the flow, and derives the accommodation coefficients through the successive analysis of measured and computed results. A new combined ES-BGK / DSMC approach that allows accurate and time efficient analysis of radiometric forces on a vane in large vacuum chambers filled with rarefied gas is presented. Accommodation coefficients for the Maxwell model are estimated for argon, xenon, and helium on a machined aluminum surface, and found to be 0.81, 0.86, and 0.53, respectively.					
<b>15. SUBJECT TERMS</b>					
<b>16. SECURITY CLASSIFICATION OF:</b>			<b>17. LIMITATION OF ABSTRACT</b>	<b>18. NUMBER OF PAGES</b>	<b>19a. NAME OF RESPONSIBLE PERSON</b>
<b>a. REPORT</b>	<b>b. ABSTRACT</b>	<b>c. THIS PAGE</b>			Dr. Ingrid Wysong
Unclassified	Unclassified	Unclassified	SAR	9	<b>19b. TELEPHONE NUMBER</b> (include area code) N/A

# Analysis of accommodation coefficients of noble gases on aluminum surface with an experimental/computational method

Nathaniel Selden,<sup>1</sup> Natalia Gimelshein,<sup>2</sup> Sergey Gimelshein,<sup>2</sup> and Andrew Ketsdever<sup>3</sup>

<sup>1</sup>University of Southern California, Los Angeles, California 90089, USA

<sup>2</sup>ERC, Inc., Edwards AFB, California 93524, USA

<sup>3</sup>Propulsion Directorate, Edwards AFB, California 93524, USA

(Received 2 February 2009; accepted 12 June 2009; published online 22 July 2009)

A method that connects measurements of radiometric forces on a heated vane in the transitional flow regime with the kinetic modeling of the flow, and derives the accommodation coefficients through the successive analysis of measured and computed results, is proposed. The method utilizes the fact that radiometric forces exerted on heated objects immersed in rarefied gases are governed by the interaction of gas molecules with the surface. Experimental results on radiometric forces on a 0.11 m diameter circular vane are obtained on a nano-Newton thrust stand in a 3 m long vacuum chamber for pressures ranging from approximately 0.01 to 1 Pa. The vane was heated to 419 K on the hot side and 396 K on the cold side. The numerical modeling is conducted using a combined ellipsoidal statistical Bhatnagar–Gross–Krook/direct simulation Monte Carlo approach that allows accurate and time efficient analysis of radiometric forces on a vane in large vacuum chambers filled with rarefied gas. Accommodation coefficients for the Maxwell model are estimated for argon, xenon, and helium on a machined aluminum surface, and found to be 0.81, 0.86, and 0.53, respectively. © 2009 American Institute of Physics. [DOI: 10.1063/1.3187932]

## I. INTRODUCTION

The history of accommodation coefficients of energy and momentum of gas molecules colliding with solid surfaces spans well over a century.<sup>1</sup> Its beginning dates to the work of Kundt and Warburg<sup>2</sup> who studied the effect of gas density change in the damping of a vibrating disk. The viscosity appeared to decrease with density, which seemed unexplainable at the time. The authors suggested an incomplete interaction, or accommodation, of gas molecules at the surface, where a low density gas slips over a surface. Following that work, Maxwell<sup>3</sup> showed that the slip phenomenon has roots in kinetic theory, and he treated the solid wall as something intermediate between a perfectly reflecting and a perfectly absorbing surface. He proposed that “of every unit of area a portion  $\alpha$  absorbs all the incident molecules, and afterwards allows them to evaporate with velocities corresponding to those in still gas at the temperature of the solid, while a portion  $1-\alpha$  perfectly reflects all the molecules incident upon it.”<sup>3</sup>

The model proposed by Maxwell is in fact the first theoretical model that describes gas-surface interaction, and it is still widely used today both in experiment and numerical simulation. According to the Maxwell model, the velocity distribution function of reflected molecules may be written as a function of the accommodation coefficient  $\alpha$  (see, for example, Ref. 4),

$$f_r(t, \mathbf{x}, \mathbf{v}_r) = (1 - \alpha)f_i(t, \mathbf{x}, \mathbf{v}_r - 2(\mathbf{v}_r \cdot \mathbf{n})\mathbf{n}) + \alpha \left( \frac{\beta_r^2}{\pi} \right)^{3/2} e^{-\beta_r^2 \mathbf{v}_r^2}, \quad (1)$$

where  $f$  is the distribution function,  $t$  is time,  $\mathbf{x}$  and  $\mathbf{v}$  are molecular position and velocity vectors, respectively, and  $\mathbf{n}$

denotes the surface normal. Subscripts  $i$  and  $r$  refer to incident and reflected molecules, respectively, and  $\beta = \sqrt{m/2kT_r}$ . The first term in Eq. (1) refers to specular reflection and the second term refers to diffuse reflection. The reflected temperature,  $T_r$ , is the wall temperature,  $T_w$ , according to the original Maxwell’s idea, but may generally be a free parameter of the model.

The tangential momentum transferred to the surface by the incident molecules may be written as<sup>4</sup>

$$\mathbf{P}_{i\tau} = -m \int_{\mathbf{v} \cdot \mathbf{n} < 0} f_i \mathbf{v}_{i\tau} (\mathbf{v}_i \cdot \mathbf{n}) d\mathbf{v}_i, \quad (2)$$

where the subscript  $\tau$  refers to the tangential to the surface components of molecular velocity, and the tangential momentum of reflected molecules is then

$$\mathbf{P}_{r\tau} = -m \int_{\mathbf{v} \cdot \mathbf{n} > 0} f_r \mathbf{v}_{r\tau} (\mathbf{v}_r \cdot \mathbf{n}) d\mathbf{v}_r = (1 - \alpha)\mathbf{P}_{i\tau}. \quad (3)$$

These equations show that the accommodation coefficient used in Eq. (1) may be considered as the coefficient of accommodation of the tangential momentum, and may be written as

$$\alpha \equiv \alpha_\tau = \frac{P_{i\tau} - P_{r\tau}}{P_{i\tau}}. \quad (4)$$

The accommodation coefficients for the normal momentum and energy may be introduced similar to Eq. (4) as

$$\alpha_n = \frac{P_{i\tau} - P_{r\tau}}{P_{i\tau} - P_w}, \quad \alpha_E = \frac{E_{i\tau} - E_{r\tau}}{E_{i\tau} - E_w}, \quad (5)$$

where the subscript  $w$  refers to the surface properties or the properties that would have had a gas at equilibrium with the wall.

Although the Maxwell model is still the most widely used model of gas-surface interaction, other models have also been proposed. Here we mention only a two-parameter Cercignani–Lampis model<sup>5</sup> that uses two accommodation coefficients,  $\alpha_r$  and  $\alpha_n$ , and a multiparametric Nocilla model<sup>6</sup> in which the velocity of reflected molecules is simulated by the function

$$f = n_r \pi^{-3/2} c_r^{-3} \exp\{-c_r^{-2} [\vec{\xi} - c_r \vec{S}_r]^2\},$$

$$c_c = \sqrt{2k/mT_r},$$

$$\vec{S}_r = \vec{\xi}/c_r.$$

The four parameters  $\vec{S}_r \equiv (S_{nr}, S_{\tau r})$ ,  $T_r$ ,  $n_r$  of this function are determined from experimental data.

The development and utilization of different gas-surface interaction models is related to various application areas where such interactions are important. One area of interest is the high altitude aerodynamics, and, in particular, free molecular aerodynamics of satellites (see, for example, Ref. 7). For the latter application, the Nocilla model is often used. The importance of the gas-surface interaction model in this case is obvious since the collisions of molecules with the spacecraft surface are the dominant process that influences drag, lift, and heat loads.

Another area where the gas-surface processes are important is gas flows in micro- and nanoscale devices. In such devices, the gas mean free path is comparable to characteristic flow dimensions, and the consideration of kinetic effects is essential for accurate prediction of device performance and peculiarities. The large surface-to-volume ratio further increases the influence of the wall. Note that for microscale flows, the preservation of the detailed balance in collisions of gas molecules with solid interfaces is critical. Therefore, the Nocilla model, which does not satisfy this requirement, is not a good choice, and the Maxwell and Cercignani–Lampis models are better suited for the description of low speed flows in microdevices. Beside these two areas, gas-surface interaction is important, if not determining, in many other applications. Near-continuum supersonic flows over sharp leading edges, contamination problems, and two-phase flows<sup>8</sup> are just a few examples of such applications.

Accurate prediction of the above flows requires the researcher not only to select an appropriate gas-surface interaction model, but also to specify the parameters of this model for each type of gas species-solid wall interface. Two principal approaches are used to determine parameters of the model, theoretical and experimental. The theoretical approach is usually based on the detailed studies of molecular interactions using classical or quasiclassical trajectory calculations in the framework of the molecular dynamics method.<sup>9</sup> In the experimental approach, parameters of the selected in-

teraction model are estimated directly from the measurement. The parameters for the Nocilla model, for example, are usually obtained from molecular beam experiments or flight experiments (see, for example, Refs. 10 and 11 and references therein). In Ref. 12, a connection between the exit velocity distribution described by the Nocilla model and the classical momentum and energy accommodation coefficients was given.

The advantage of the molecular beam technique is that it may provide detailed information on the velocity distributions of reflected molecules. There are many situations, however, when such detailed information is not necessary, and the knowledge of accommodation coefficients, either momentum or energy, would suffice. Examples include the force estimate of spacecraft at high altitudes, or the evaluation of heat loads in microdevices. Over the past three decades, molecular beam experiments have been used extensively to determine both the normal and tangential momentum and energy accommodation coefficients<sup>13–15</sup> for various gas-surface pairs. For the energy (thermal) accommodation coefficient, parallel plates, coaxial cylinders, and hot-wire methods have been widely used. A comprehensive review of different approaches to the thermal accommodation coefficient measurements may be found in Ref. 16. A wide range of results for accommodation coefficients have been reported for the three gases studied here (argon, xenon, and helium). In view of this, the results of Ref. 17 are particularly important; these are state-of-the-art measurements of energy accommodation coefficients for argon and helium at temperatures similar to those for the present work, and thus provide the most useful comparison with the present results. Various experimental techniques used in the past to measure tangential momentum accommodation coefficient, such as the rotating cylinder method, the spinning rotor gage method, the flow through microchannel approach, as well as the molecular beam technique, are discussed in recent review article.<sup>18</sup>

In contrast to high-enthalpy flows around space vehicles, gas-driven flows in microscale devices are characterized by relatively low gradients in gas velocity and temperature, and the velocity distribution function in these flows is often close to Maxwellian. As a result, prediction of gas-driven flows in such devices typically requires knowledge of momentum and/or energy accommodation coefficients as a function of gas and surface temperature. The use of molecular beam technique may be quite difficult in this case since the after-collision velocities need to be analyzed for a large number of precollisional energies. On the other hand, standard techniques for the accommodation measurement may not be applicable when information on momentum accommodation, normal, or tangential is needed.

One major issue with the need of gas-surface interaction parameters to predict complicated flow interactions is the range of experimental data for similar flows. For example, various experiments can have different, and in some cases conflicting, results. Take, for instance, the measurement of the energy accommodation of helium on a platinum surface. References 19–21 present results which vary by more than 30%. Thus there is a need to reinvestigate these data sets in order to study the effects of gas temperature, surface tem-

perature, surface preparation, gas adsorption on surfaces, and gas pressure.

The objective of this work is to evaluate the feasibility of using a new method for measurements of momentum accommodation coefficients, based on the comparing experimental and computational results on radiometric forces on heated plates. Radiometric forces are typically exerted on nonuniformly heated objects immersed in rarefied gases, and tend to move these objects in the direction from the cold to the hot side. The authors of Ref. 22 have recently remarked that the measurement of radiometric forces may yield data on gas-surface interaction. However, there are two problems that make the direct use of such measurements to infer momentum accommodation coefficients extremely difficult. First, there are usually molecular collisions present in radiometric flows, and these collisions do not allow simple and accurate analytic evaluation of accommodation coefficients from force measurements beyond the free molecular regime. Second, while the availability of force measurements in free molecular regime would offer the benefit of accommodation coefficient evaluation, there is a physical limitation in the accuracy of such measurements. The fewer gas-surface collisions a radiometer vane experiences, the greater the experimental error. To avoid these difficulties, it is suggested in the present work to measure radiometric forces in the transitional regime, and then use kinetic modeling of radiometric flows to infer the momentum accommodation coefficients.

## II. RADIOMETRIC APPROACH TO MOMENTUM ACCOMMODATION STUDY

The radiometric forces on a heated plate may be described analytically only in a free molecular regime; the presence of even relatively small number of molecular collisions in the transition regime complicates the flow to the point where accurate analytical description is not possible, and a numerical approach has to be used to address the problem. Even for a free molecule flow, some model needs to be used for the gas-surface accommodation in order to make analytical treatment possible.

Generally, for a plate with its opposite sides heated uniformly to different temperatures  $T_h$  and  $T_c$ , the forces in the direction normal to the plate, created by molecules reflected from the hot and the cold sides of the plate, may be written as

$$F_h = n_h m \int_{\mathbf{v} \cdot \mathbf{n}} f_h \mathbf{v}_r (\mathbf{v}_r \cdot \mathbf{n}) d\mathbf{v}_r \quad \text{and} \quad (6)$$

$$F_c = n_c m \int_{\mathbf{v} \cdot \mathbf{n}} f_c \mathbf{v}_r (\mathbf{v}_r \cdot \mathbf{n}) d\mathbf{v}_r,$$

where subscripts  $h$  and  $c$  refer to the hot and cold sides, respectively. The number density that describes the flux of reflected molecules may be obtained from the assumption of the equality of the incident and reflected mass flux (i.e., no sticking on the surface),

$$n_{h,c} \int_{\mathbf{v} \cdot \mathbf{n}} f_{h,c} (\mathbf{v}_r \cdot \mathbf{n}) d\mathbf{v}_r = n_g \int_{\mathbf{v} \cdot \mathbf{n}} f_g (\mathbf{v}_r \cdot \mathbf{n}) d\mathbf{v}_r, \quad (7)$$

where subscript  $g$  refers to the incident gas molecules. In a free molecular flow, the Maxwellian distribution function of  $f_g$  may be reasonably assumed. For fully diffuse accommodation, the number density of reflected molecules is obtained by integrating Eq. (7) over equilibrium distribution functions to give

$$n_{h,c} = n_g \sqrt{\frac{T_g}{T_{h,c}}}, \quad (8)$$

where indices  $h$  and  $c$  refer to either hot or cold side of the plate. The force on the side of the plate will be

$$F_{h,c} = \frac{p_g}{2} + \frac{p_g}{2} \sqrt{\left(\frac{T_{h,c}}{T_g}\right)}, \quad (9)$$

where  $p_g$  is the gas pressure. The first term in Eq. (9) is for the incident molecules and the second term accounts for the contribution from the reflected molecules. If the accommodation coefficient is introduced according to the expression suggested by Knudsen,<sup>23</sup>

$$\alpha_K = \frac{T_g - T_r}{T_g - T_w}, \quad (10)$$

then assuming the same accommodation coefficient on the hot and cold sides of the plate (a small temperature difference between the plates), and using  $T_w$  from Eq. (10) instead of  $T_h$  and  $T_c$  in Eq. (9), one can obtain the expression for the total radiometric force on the plate,

$$F_K = \frac{p_g}{2} \left( \sqrt{\frac{(1 - \alpha_K)T_g + \alpha_K T_h}{T_h}} - \sqrt{\frac{(1 - \alpha_K)T_g + \alpha_K T_c}{T_c}} \right). \quad (11)$$

Note that the contributions from the incident molecules cancel out in the free molecular flow; the force is directed from the hot to the cold surface.

If the Maxwell model of gas-surface interaction is used, then, substituting Eq. (1) into Eq. (6) and making use of Eq. (7), one can obtain for the free molecular force

$$F_M = \alpha \frac{p_g}{2} \left( \sqrt{\frac{T_h}{T_g}} - \sqrt{\frac{T_c}{T_g}} \right). \quad (12)$$

Therefore, the free molecular radiometric force calculated using the Maxwell model is linearly dependent on the tangential momentum accommodation coefficient. If the momentum accommodation coefficient in the Maxwell model,  $\alpha$ , is close to the energy accommodation coefficient in the Knudsen model,  $\alpha_K$ , then the force predictions obtained with Eqs. (11) and (12) are similar for small temperature differences. The difference between them becomes significant when the surface temperatures are not similar. Equation (12) allows one to easily calculate the accommodation coefficients when the radiometric force in the free molecular regime can be measured.

In reality, however, it is difficult to accurately measure the radiometric force in free molecular regime. Such measurements are possible for a transitional regime, for which the above analytic expressions are not applicable. Therefore, it is reasonable to infer the accommodation coefficients from a numerical simulation performed for a given gas-surface interaction model with varying parameters of the model. It is clear that the conventional continuum approaches of the computational fluid dynamics, such as those based on the solution of the full Navier–Stokes equations or boundary layer equations, cannot be used to compute radiometric forces in the transitional flow regime. In these approaches, developed for modeling gas flows close to equilibrium, the effects of rarefaction are typically accounted for through the boundary conditions of slip velocity and temperature jump on the surface. The assumption of small deviation from equilibrium makes them inapplicable for modeling radiometric flows and calculating radiometric forces. In this case, a kinetic approach based on the solution of the Boltzmann equation has to be used. For a kinetic approach, a kinetic model of gas-surface interaction needs to be used, such as the Maxwell model, and the approach naturally gives the velocity distribution functions for the incident and reflected molecules.

Thus, in order to obtain the accommodation coefficients for a given gas-surface interaction model, numerical results need to be obtained by comparing results from a kinetic approach for a computational setup that closely reproduces the experimental one. The details on the present experimental setup are given in Sec. III.

### III. EXPERIMENTAL SETUP

As radiometric phenomena occur in rarefied conditions, there are only two ways to study them experimentally. The first is to build extremely small devices on the order of nanometers and test them under atmospheric conditions. The second way is to build a larger device and modify the background pressure such that the local Knudsen number is large enough for the flow to be considered transitional (i.e.,  $Kn > 0.01$ ). In this work the latter method has been chosen, and all the experimental results that follow have been achieved under low pressure conditions in a large 3.0 m diameter vacuum chamber. The use of such a large chamber is critical to avoid the effect of chamber walls that was found to strongly impact the radiometric force in smaller chambers.<sup>24</sup>

To accurately measure the impact of various accommodation coefficients, and to be practical to model using an axisymmetric code, a circular radiometer vane with a diameter of 11.13 cm was used. The vane consisted of a Teflon insulator sandwiched between two aluminum plates with a resistive heater located between one of the plates and the insulator. The temperature of one side of the device was maintained by varying the power input to the heater, while the temperature of the opposite side was not actively maintained and was allowed to float. Each of the three pieces of the radiometer vane had a thickness of 0.32 cm, and when assembled yield a total device thickness of 0.96 cm.

One motivation for this particular configuration of radi-

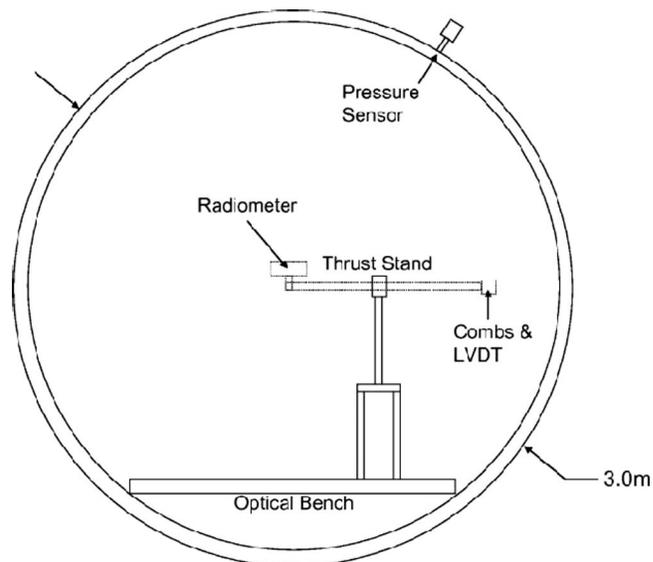


FIG. 1. Setup of the radiometric experiment.

ometer vane comes from historical work<sup>25</sup> where rudimentary temperature measurements of the vanes suggested that a significant temperature drop occurred at the outermost edges. This same work made it quite clear that to accurately deduce a theory for the operation of the radiometer, it would be necessary to discover exactly what effect the temperature variations at the edges had. For the sake of clarity it should be noted here that there are two gradients important to the flow: The first of these shall be referred to as the radial gradient and will refer to the temperature profile of a plate from the center to the periphery, while the second will be called the axial gradient and will refer to temperature profile along an axis normal to the face. In an ideal experiment, the axial gradient would be large and the radial gradient would be nonexistent such that the experiment and simulation share nearly identical temperature profiles. It is for these reasons that the particular aluminum “sandwich” design was chosen; not only does the high thermal conductivity maximize the surface temperature of the hot plate (and thus the axial temperature gradient), but it also minimizes the radial temperature gradients near the edges of the device.

To measure the force produced by this device, it was mounted on a modified nano-Newton Thrust Stand<sup>26</sup> (nNTS) located inside the vacuum chamber. A schematic of the experimental setup is shown in Fig. 1. Here, every effort was made to minimize the impact of the thrust stand arm and attachment mechanism by using 6.35 mm tubing coupled with a  $2 \times 40$  mm<sup>2</sup> threaded rod. When calibrated using a pair of electrostatic combs,<sup>27</sup> the nNTS provides very accurate and repeatable data with typical force resolution of approximately 0.1  $\mu\text{N}$  and statistical scatter of about 1%. For the preliminary experiment, the experimental error based on standard deviation ranges from a few percent at the lowest pressures to less than 1% through most of the curve. However, due to the normalization by experimental temperature measurements and the small uncertainty of the calibration method, the total absolute experimental uncertainty is  $\approx 4\%$ .

Day-to-day variation of multiple data sets has been observed to be  $\approx 1\%$ .

The experimental data were obtained by evacuating the vacuum chamber to a base pressure below  $10^{-3}$  Pa. This low pressure was required to minimize the impact of the background gas to a level low enough as to be inconsequential to the measurements being made. While the evacuation of the chamber was taking place, a constant voltage was applied to the heater. This resulted in the main radiometer surfaces reaching temperatures of approximately 419 K (hot) and 394 K (cold), although the exact values fluctuated depending on both the species and pressure of the background gas. Force measurements were made by varying the background pressure of the gas in the chamber, where argon, helium, and xenon were all used. The highest background pressure achieved was approximately 1.6 Pa, but varied depending on the molecular weight of the background gas.

#### IV. NUMERICAL MODELING OF RADIOMETRIC FLOWS: A COMBINED KINETIC APPROACH

In this work, a combined ellipsoidal statistical Bhatnagar–Gross–Krook/direct simulation Monte Carlo (ES-BGK/DSMC) approach, where the final solution is obtained in two successive steps. First, an ES-BGK modeling is conducted in a large computational domain that includes both the radiometer vane and the chamber walls. The solution of this first step is used to set the boundary conditions for the second step. At the second step, the DSMC method is applied in a much smaller domain, with the subsonic boundary conditions taken from the first step. The use of such a new approach is based on the fact that DSMC modeling of a radiometric flow on a 10 cm vane in a 3 m chamber, where the accuracy of the radiometric force modeling needs to be on the order of 1%, is prohibitively expensive even for modern parallel computers. On the other hand, the ES-BGK method was found to be fairly accurate in predicting all gas macroparameters in the computational domain, but overpredicting the DSMC results on radiometric forces by  $\sim 10\%$  in the range of pressures where the force is near its maximum. This is related to the approximations inherent in the ES-BGK equation, and difficulty of modeling the radiometric force, that is typically less than 1% of the force on either cold or hot side of the vane.

In this work, the computational tool SMILE (Ref. 28) was used to obtain the solutions with the DSMC method. In DSMC runs, the variable soft sphere model with parameters listed in Ref. 29 was used for the molecular collisions, and the Maxwell model was used to calculate gas-surface collisions. A finite volume solver SMOKE (Ref. 30) has been used to deterministically solve the ES model kinetic equation. SMOKE is a parallel code based on conservative numerical schemes developed by Mieussens.<sup>31</sup> A second order spatial discretization was used. The solutions were typically obtained in two successive steps. First, an implicit time integration scheme was run until the result is converged. Second, a conservative explicit time integration scheme was used with the initial conditions from the first step. This two-step

approach allowed up to two orders of magnitude reduction in computational time compared to an explicit-only case.

The four macroparameters (density, temperature, and two velocities) from the ES-BGK solution were used at the external boundaries of the DSMC computational domain. That means that the velocities of molecules entering the DSMC computational domain are sampled from the Maxwellian distribution with parameters from the ES-BGK solution. It is important that the ES-BGK macroparameters used in the DSMC boundary conditions were computed from the incoming fluxes only. Good agreement between the full DSMC and the combined kinetic approach, obtained by the authors on a smaller, 0.2 m domain, allowed the application of the combined approach to analyze radiometric flows in a large vacuum chamber. A 3 m cylindrical chamber is simulated in this work, whose geometry with good accuracy reproduces the companion experimental setup. The radiometer size and location inside the chamber, as well as the temperature conditions, also correspond to those used in the experiment. Diffuse reflection with a complete energy and momentum accommodation was assumed on the chamber walls and the surface of the vane (with one exception explained below). Since the experimental setup closely approximates a flow with an axial symmetry, axisymmetric ES-BGK and DSMC codes were used in these computations. The subsonic boundaries of the DSMC computational domain were located 30 cm from the vane both in the axial and radial directions.

#### V. EVALUATION OF ACCOMMODATION COEFFICIENTS

Three gases were considered in this work, argon, xenon, and helium. The radiometric forces for these gases, obtained with the combined ES-BGK/DSMC approach as well as measured experimentally, are presented in Fig. 2. Generally, the radiometric force consists of two components, (i) the total radiometric force that includes the force resulting from the pressure difference between the hot and the cold sides of the vane, and (ii) the shear force on the lateral (circumferential) side of the vane. To show separate contribution of these forces, two sets of numerical results are shown, the total radiometric force that includes both component, and the radiometric force that is based on pressure alone. The results show that the shear force is a minor factor for pressures smaller than 0.6 Pa for argon, where the maximum force is observed. It becomes more significant for larger pressures, for which the contribution of the lateral side of the vane cannot be ignored. It may appear preferable to analyze the accommodation coefficients under conditions where the lateral side has a negligible effect, such as a much thinner vane, but it is difficult to realize in the experiment.

For all three gases, the experimental data lay lower than the numerical points, which is a clear indication of an incomplete surface accommodation. Beyond that, several other factors may play a role in this difference. First, there are numerical and experimental errors; they are not expected to cause a difference between the computation and the measurement larger than 5%. Then, there is a finite chamber size, with unknown accommodation on chamber walls. This has

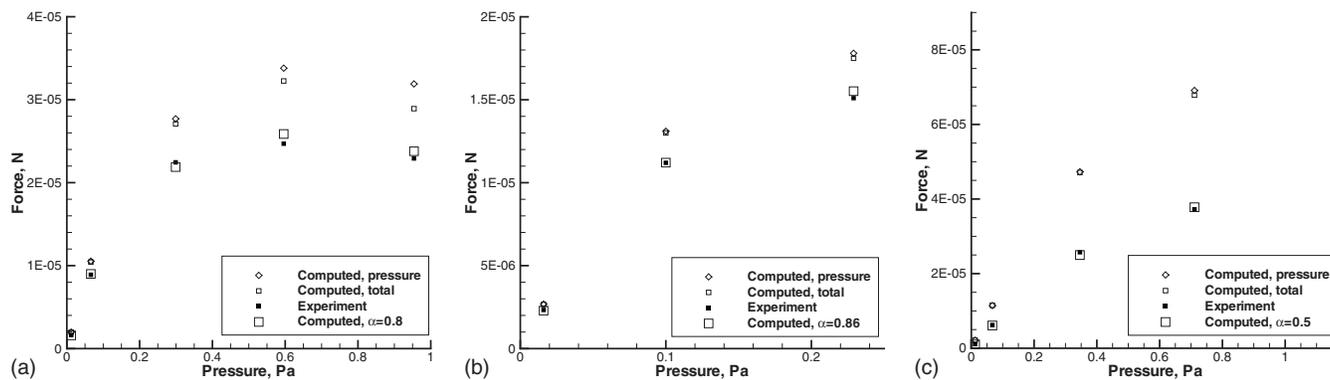


FIG. 2. Experimental and computed radiometric force for argon (left), xenon (center), and helium (right).

been found to be a minor issue in a series of ES-BGK calculations, where the chamber size larger than about 2 m was found to have a negligible effect on the radiometric force. Finally, there is some impact of intermolecular collision law, or, in other words, gas viscosity and heat conductivity. This factor is also believed to be minor, as the bulk gas properties correspond to well established experimental values for the temperature range under consideration. All these indicate that the gas accommodation on the vane surface is the main reason for the difference between the numerical and experimental values. For comparison, additional computations were conducted for the three gases using the Maxwell model with an accommodation coefficients of 0.8 for argon, 0.86 for xenon, and 0.5 for helium. It is clearly seen that the use of a lower accommodation coefficient allows one to obtain good agreement with experimental data for all pressures and gases considered.

The value of 0.5 for accommodation coefficient used for helium to reproduce the experimental data is in fact close to the experimental-to-computed ratio of 0.53 obtained after averaging over pressures. Remember that the Maxwell model is characterized by a linear dependence between the force and the accommodation coefficient; obviously, the dependence is close to linear in the transitional regime as well. The value of 0.5 may in fact be obtained if the Knudsen model of accommodation is assumed, and the functional dependence of Eq. (11) is applied. In this case, the unknown  $\alpha$  is calculated by equating the ratio of the right hand sides of Eq. (11) with  $\alpha_K = \alpha$  and  $\alpha_K = 1$  to the experimental-to-computed force ratio. Note that the value 0.5 is obtained when the Knudsen model of accommodation is assumed, and the functional dependence of Eq. (11) and equating the ratio of the right hand sides of Eq. (11) with an unknown  $\alpha_K$  and  $\alpha_K = 1$  to the experimental-to-computed force ratio.

Interestingly, the value of 0.5 also coincides with that of the Knudsen model of accommodation obtained assuming the functional dependence of Eq. (11) and equating the above ratio. As was mentioned earlier, the difference between the accommodation coefficients defined by the Maxwell model and the Knudsen expression is small for relatively small temperature differences examined in this work. It is therefore impossible to state which one is a better approximation for the transitional regime. For kinetic approaches, the authors believe that the use of Eq. (12) may be a better fit, with a

simple ratio between the experimental and numerical radiometric forces being an estimate of the accommodation coefficient in the Maxwell model. Such a ratio for different gases is presented in Fig. 3. The accommodation coefficients for the Maxwell model, obtained in this work, are 0.81 for argon, 0.86 for xenon, and 0.53 for helium, all of them on a machined aluminum surface. Note that the value of the accommodation coefficient increases with molecular mass, which is consistent with the experimental observation of Ref. 32 but contradicts to a hypothesis of Ref. 33.

## VI. COMPARISON WITH PREVIOUS MEASUREMENTS

Comparison of the above accommodation coefficients with those measured in the past is complicated by several factors in addition to their obvious dependence on particular gas and surface material. First, the coefficients obtained in this work are integral and not incident angle dependent. Therefore, it is difficult to compare them with molecular beam experiments. Second, the coefficients are generally sensitive to the wall and the surrounding gas temperature, and the results should be analyzed for the same temperature regime. Finally, the purity of the surface is very important, as the surface coverage and surface contamination change the accommodation coefficients. The last factor is related to the surface temperature and associated gas desorption, the sur-

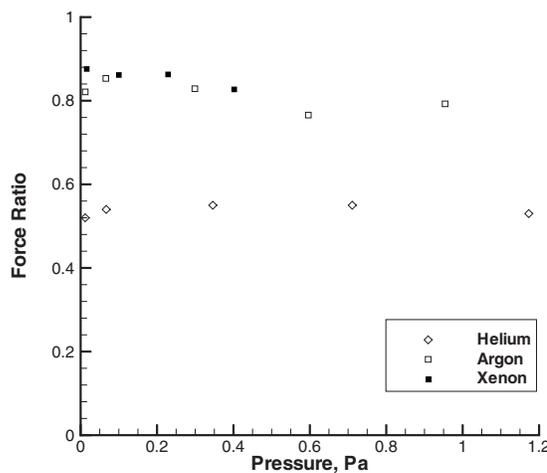


FIG. 3. Experimental-to-computed radiometric force ratio.

TABLE I. Comparison of the present accommodation coefficients to the previous measurements.

Gas	Present	Ref. 17	Ref. 32	Ref. 16	Ref. 18
Helium	0.53	0.38–0.47	0.65		
Argon	0.81	0.86		0.334–0.75	0.893
Xenon	0.86			0.4	0.95

face roughness and multiple gas-surface encounters, and the gas pressure. All these factors contribute to large differences often observed between accommodation coefficients measured by different authors. Some of the published results for the three gases considered in this work, along with the present data, are given in Table I.

For helium, the present accommodation coefficients, 0.53 for the Maxwell model and 0.5 for the Knudsen model, are close to that obtained in Ref. 17 for the thermal accommodation coefficient on machined aluminum kept at room temperature, for which the value of 0.47 was measured. The accommodation coefficient of helium on a plasma treated surface obtained in Ref. 17 is lower, 0.38. The normal momentum coefficients recommended<sup>32</sup> for helium on aluminum are somewhat higher, 0.65. It was also shown in Ref. 32 that the efficiency of the momentum transfer process increases with the mass of gas molecules, and relatively weakly depends on the surface material for temperatures ranging from 25 to 550 °C. The tangential momentum coefficients of helium on aluminum are not available, but for other materials were found to vary in a wide range depending on the experimental technique used, from 0.2 (Ref. 34) to about 0.9.<sup>35</sup>

The thermal accommodation coefficient of argon on aluminum, tabulated in Ref. 16, ranges from 0.334 to 0.75 for different experimental techniques and surface temperatures from 400 to 800 K. A larger value of 0.86 was measured<sup>17</sup> for argon atoms colliding with a machined aluminum surface. A tangential momentum accommodation coefficient of 0.893 was recommended in Ref. 18 based on the analysis of a large array of experimental data.

The accommodation of xenon on aluminum has not been extensively studied in the past. The thermal accommodation coefficient was reported for temperatures from 500 to 800 K as 0.4,<sup>16</sup> where a concentric-cylinder method was used. Among other materials, platinum was studied theoretically<sup>36</sup> and the energy and momentum accommodation coefficients were calculated for room temperature conditions to be 0.85 and 0.81, respectively. A mean value of 0.95 was recommended in Ref. 18 for the tangential momentum accommodation coefficient of xenon on commonly employed surface materials. In measurement,<sup>33</sup> this coefficient was estimated as 0.9 for xenon on bronze ribbon.

## VII. CONCLUSIONS

A method for estimation of gas-surface accommodation coefficients, based on comparing measured and computed radiometric forces on heated vanes in rarefied flows, is presented. The method applies a new combined ES-BGK kinetic approach to match accurately measured force on a circular

radiometer installed on a nNTS and mounted in a large vacuum chamber. Accommodation coefficients for the Maxwell model of gas-surface interaction may be deduced for a given pressure and gas-surface pair whether through the successive use of the combined approach with different values of the accommodation coefficient, or assuming a linear dependence of radiometric force on the accommodation coefficient.

Helium, argon, and xenon were considered in this work, for pressures ranging from approximately 0.01 to 1 Pa, and an aluminum vane with a diameter of 0.113 m was examined. The suggested values of the Maxwell model accommodation coefficients are 0.81 for argon, 0.86 for xenon, and 0.53 for helium, which reasonably agree with momentum and energy accommodation coefficients proposed in literature. The proposed experimental-computational method is general enough to be applied to a wide range of gases, surfaces, and temperature conditions.

## ACKNOWLEDGMENTS

This work was supported in part by the Propulsion Directorate of the Air Force Research Laboratory at Edwards Air Force Base, California. The authors thank Ingrid Wysong and Dean Wadsworth for many fruitful discussions.

- <sup>1</sup>H. Y. Wachman, "The thermal accommodation coefficient: A critical survey," *Am. Rocket Soc. J.* **32**, 2 (1962).
- <sup>2</sup>A. Kundt and E. Warburg, "Ueber Reibung und Wärmeleitung verdünnter Gase," *Ann. Phys.* **232**, 177 (1875).
- <sup>3</sup>J. C. Maxwell, "On stresses in rarified gases arising from inequalities of temperature," in *The Scientific Papers of James Clerk Maxwell*, edited by W. D. Niven (Dover, New York, 1965), Vol. 2, p. 707.
- <sup>4</sup>M. N. Kogan, *Rarefied Gas Dynamics* (Plenum, New York, 1969).
- <sup>5</sup>C. Cercignani and M. Lampis, "Kinetic models for gas-surface interactions," *Transp. Theory Stat. Phys.* **1**, 101 (1971).
- <sup>6</sup>S. Nocilla, "The surface re-emission law in free molecule flow," *Proceedings of the Third International Symposium on Rarefied Gas Dynamics*, edited by J. A. Laurmann (Academic, New York, 1963), Vol. 1, pp. 327–346.
- <sup>7</sup>G. Koppenwallner, D. Johannsmeier, H. Klinkrad, M. Ivanov, and A. Kashkovsky, "A rarefied aerodynamic modelling system for earth satellites (RAMSES)," *Proceedings of the 19th International Symposium on Rarefied Gas Dynamics*, edited by J. Harvey and G. Lord (Oxford University Press, Oxford, 1995), Vol. 2, pp. 1366–1372.
- <sup>8</sup>S. Gimelshein, G. Markelov, and J. Muylaert, "Numerical modeling of low thrust solid propellant nozzles at high altitudes," in *Proceedings of the 9th AIAA/ASME Joint Thermophysics and Heat Transfer Conference*, 5–8 June 2006, San Francisco, CA, AIAA Paper No. 2006-3273, 2006.
- <sup>9</sup>J. M. Haile, *Molecular Dynamics Simulation* (Wiley, New York, 1997).
- <sup>10</sup>S. R. Cook and M. A. Hoffbauer, "Nocilla model parameters obtained from forces exerted on surfaces by molecular beams," *J. Spacecr. Rockets* **34**, 379 (1997).
- <sup>11</sup>F. C. Hurlbut, "Gas surface interactions: Recent observations and interpretations," *Proceedings of the 20th International Symposium on Rarefied Gas Dynamics*, edited by C. Shen (Peking University Press, Beijing, 1997), pp. 355–367.

- <sup>12</sup>F. C. Hurlbut, "Two contrasting models for the description of wall-gas interactions," *Progress in Astronautics and Aeronautics*, Proceedings of the 18th International Symposium on Rarefied Gas Dynamics, edited by B. D. Shizgal and D. P. Weaver (AIAA, Washington, DC, 1994), Vol. 158, pp. 494–506.
- <sup>13</sup>L. B. Thomas and R. G. Lord, "Comparative measurement of tangential momentum and thermal accommodation on polished and roughened steel spheres," *Proceedings of the Eighth International Symposium on Rarefied Gas Dynamics*, edited by R. Karamcheti (Academic, New York, 1974), pp. 405–412.
- <sup>14</sup>S. R. Cook, "Molecular beam measurements of absolute momentum accommodation on spacecraft surfaces using a specialized torsion balance," Ph.D. thesis, The University of Texas at Austin, 1995.
- <sup>15</sup>A. P. Nikiforov, "Measuring the momentum flux of molecules reflected from a surface of given roughness in free molecule flow," *Fluid Dyn.* **20**, 630 (1985).
- <sup>16</sup>S. C. Saxena and R. K. Joshi, "Thermal accommodation and adsorption coefficients of gases," *Thermal Accommodation and Adsorption Coefficients of Gases* (Hemisphere, New York, 1989).
- <sup>17</sup>W. M. Trott, D. J. Rader, J. N. Castañeda, J. R. Torczynski, and M. A. Gallis, "Measurement of gas-surface accommodation," Proceedings of the 26th International Symposium on Rarefied Gas Dynamics, Kyoto, Japan, July 2008.
- <sup>18</sup>A. Agrawal and S. V. Prabhu, "Survey on measurements of tangential momentum accommodation coefficient," *J. Vac. Sci. Technol. A* **26**, 634 (2008).
- <sup>19</sup>W. B. Mann, "The exchange of energy between a platinum surface and gas molecules," *Proc. R. Soc. London, Ser. A* **146**, 776 (1934).
- <sup>20</sup>P. Rolf, "The accommodation coefficient of helium on platinum," *Phys. Rev.* **65**, 185 (1944).
- <sup>21</sup>L. B. Thomas and F. Olmer, "The accommodation coefficients of He, Ne, A, H<sub>2</sub>, D<sub>2</sub>, O<sub>2</sub>, CO<sub>2</sub>, and Hg on platinum as a function of temperature," *J. Am. Chem. Soc.* **65**, 1036 (1943).
- <sup>22</sup>A. Passian, R. J. Warmack, T. L. Ferrell, and T. Thundat, "Thermal transpiration at the microscale: A Crookes cantilever," *Phys. Rev. Lett.* **90**, 124503 (2003).
- <sup>23</sup>M. Knudsen, "Die molekulare Wärmeleitung der Gase und der Akkommodationskoeffizient," *Ann. Phys.* **339**, 593 (1911).
- <sup>24</sup>N. P. Selden, S. F. Gimelshein, N. E. Gimelshein, and A. D. Ketsdever, "Effect of chamber wall proximity on radiometer force production," 26th International Symposium on Rarefied Gas Dynamics, Kyoto, Japan, 21–25 July 2008.
- <sup>25</sup>H. E. Marsh, "Further experiments on the theory of the vane radiometer," *J. Opt. Soc. Am.* **12**, 135 (1926).
- <sup>26</sup>A. J. Jamison, A. D. Ketsdever, and E. P. Muntz, "Gas dynamic calibration of a nano-Newton thrust stand," *Rev. Sci. Instrum.* **73**, 3629 (2002).
- <sup>27</sup>N. P. Selden and A. D. Ketsdever, "Comparison of force balance calibration techniques for the nano-Newton range," *Rev. Sci. Instrum.* **74**, 5249 (2003).
- <sup>28</sup>M. S. Ivanov, G. N. Markelov, and S. F. Gimelshein, "Statistical simulation of reactive rarefied flows: Numerical approach and applications," in Proceedings of the 31st AIAA Thermophysics Conference, Albuquerque, NM, June 1998, AIAA Paper No. 98-2669.
- <sup>29</sup>G. A. Bird, *Molecular Gas Dynamics and the Direct Simulation of Gas Flows* (Clarendon, Oxford, 1994).
- <sup>30</sup>D. C. Wadsworth, N. E. Gimelshein, S. F. Gimelshein, and I. J. Wysong, "Assessment of translational anisotropy in rarefied flows using kinetic approaches," Proceedings of the 26th International Symposium on Rarefied Gas Dynamics, Kyoto, Japan, July 2008.
- <sup>31</sup>L. Mieussens, "Discrete-velocity models and numerical schemes for the Boltzmann-BGK equation in plane and axisymmetric geometries," *J. Comput. Phys.* **162**, 429 (2000).
- <sup>32</sup>R. Stickney, "Momentum transfer between gas molecules and metallic surfaces in free molecule flow," *Phys. Fluids* **5**, 1617 (1962).
- <sup>33</sup>T. Gronych, R. Ulman, L. Peksa, and P. Repa, "Measurements of the relative momentum accommodation coefficient for different gases with a viscosity vacuum gauge," *Vacuum* **73**, 275 (2004).
- <sup>34</sup>R. G. Lord, "Tangential momentum coefficients of rare gases on polycrystalline surfaces," *Proceedings of the Tenth International Symposium on Rarefied Gas Dynamics* (American Institute of Astronautics and Aeronautics, New York, 1977), pp. 531–538.
- <sup>35</sup>B. T. Porodnov, P. E. Suetin, S. F. Borisov, and V. D. Akinshin, "Experimental investigation of rarefied gas flow in different channels," *J. Fluid Mech.* **64**, 417 (1974).
- <sup>36</sup>K. Yamamoto, "Slip flow over a smooth platinum surface," *JSME Int. J., Ser. B* **45**, 788 (2002).