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Aerospace Applications of Adjoint Theory

Domenic Bucco

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Defence Science and Technology Organisation

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The adjoint simulation method is an efficient, computerised method for the performance analysis of linear time varying (LTV) systems excited by deterministic and/or stochastic inputs. It is based on the impulse response function (or weighting function) of the so-called adjoint system – an associated linear system derived from the original system by clever utilisation of the mathematical principle of duality. Once the adjoint system is obtained, the solution process using this method is quite straightforward regardless of the nature and number of inputs. In this report, we demonstrate the power of adjoint simulation in the context of guided missile homing loop studies and present a MATLAB tool that can be used for its speedy implementation and execution. The tool is demonstrated by using it to analyse the miss distance performance of a generic guided missile against an evading target.

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Aerospace Applications of Adjoint Theory

Executive Summary

Adjoint theory has widely been employed by various missile companies as a pivotal tool in support of their weapons programs. These companies have traditionally adopted this practical engineering approach due to its simplicity, accuracy and efficiency of use, particularly during the preliminary and conceptual stages of a new missile design. In this approach, design parameters for the missile system can be readily selected on the basis of sensitivity curves generated by the efficient simulation of an associated system adjoint to the missile system under study.

However, one of the difficulties with this approach is the determination of the associated adjoint system. Two methods exist for doing this. The adjoint system can be obtained via a practical engineering approach or via a more rigorous mathematical approach. Traditionally, defence engineers have used the engineering approach. This is based on a set of adjoint construction rules developed by Laning and Battin [7] while implementing their adjoint equations on an analogue computer. With this approach, given the original dynamic system in the form of a simulation block diagram with designated inputs and outputs, the adjoint system is constructed through manual manipulation of this block diagram in accordance with the Laning and Battin rules.

For very complex simulation diagrams with many feedback paths typically found in missile guidance loops, manual application of the adjoint construction rules may be extremely tedious, time consuming and prone to error. However, if the original simulation block diagram is implemented in a commercially available graphical simulation package such as MATLAB/Simulink, then the process required to construct the adjoint system can be conveniently automated to minimise or eliminate these shortcomings. A tool for doing this is outlined in this report. The tool can be used to support preliminary studies into new concepts such as hypersonic vehicles.

The objectives of this report are to review the classical theory of the adjoint method as applied to the guided missile problem, to present a pragmatic engineering implementation of the theory using block diagram representation and to outline the development of a specialised software package designed to automate the process of adjoint system construction within the MATLAB/Simulink environment. Several examples drawn from the open literature are used to illustrate the solution process for typical problems encountered in Aerospace/Defence science.

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Fellowship Publications and Presentations

1. Bucco, D. & Weiss, M., "Further Enhancements to COVAD: A MATLAB Tool Based on Covariance/Adjoint Methods," *AIAA M&ST Conference, Honolulu, Hawaii, Aug 2008.*
2. Weiss, M. & Bucco, D., "The State-Space Approach to the Method of Adjoint for Hybrid Guidance Loop Models," *AIAA GN&C Conference, Chicago, IL, USA, Aug 2009.*
3. Bucco, D. & Weiss, M., "Extending the COVAD Toolbox to Accommodate System Non-Linearities," *AIAA M&ST Conference, Chicago, IL, Aug 2009.*
4. Rajagopalan A. & Bucco D. "Applications of Adjoint Theory to Problems in Aerospace/Defence Science," *EMAC Conference, Adelaide, Dec 2009.*

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1. Introduction

Systems analysts and engineers are often confronted with understanding the behaviour and performance of complex, dynamic systems driven by various inputs predominantly of a stochastic nature. Some typical applications include miss distance studies of guided missiles, fire control problems, navigation problems and analysis of circuits due to random noise. For these systems, the most general method of tackling the problem and gaining critical statistical insight into the nature of the response is via the use of the Monte Carlo simulation technique [1]. However, when the system is linear or can be approximated by a linear system, the use of the Monte Carlo method would seem somewhat excessive and unwarranted in such cases. Instead, the stochastically-driven linear system may be more economically and rapidly studied using better matched linear techniques. One such technique is based on adjoint theory [2] and is traditionally known as the adjoint simulation method (or modified adjoint method).

The adjoint simulation method is a computerised method for the performance analysis of linear time varying (LTV) systems. Mathematically, it is derived from the formal theory of linear differential operators and their associated adjoints. Traditionally, the technique has been very popular in the study of guided missile homing loops and, in general, has been employed during the preliminary and conceptual definition stages of many new missile programs. For this work, the LTV system under study is typically represented in the form of a simulation block diagram. The associated adjoint system is also represented in block diagram form. The adjoint system is constructed by manual manipulation of the original system block diagram in accordance with a given set of rules. These rules were first proposed by Laning and Battin [7] as part of their adjoint simulation work on analogue computers.

In general, given an LTV system with n inputs and m outputs, the adjoint method can be used to determine the sensitivity of any of the outputs at a fixed time to each of the n inputs. If the inputs are stochastic in nature, then this approach can be easily formulated to compute the Root Mean Squared (RMS) value of the system states of interest without resorting to ensemble computations as generally required by the Monte Carlo method [1]. Hence, the technique provides the analyst with a simple but powerful alternative to the Monte Carlo approach in those cases when linearisation can be physically justified.

The purpose of this report is to review the theory of the adjoint simulation method as applied to LTV systems, to present a pragmatic implementation of the theory using block diagram representation and to outline the development of a software package designed to automate the adjoint system construction process within the Matlab/Simulink environment [50]. Several examples drawn from the open literature will be used to illustrate the solution process to typical problems encountered in Aerospace/Defence science.

2. Historical Perspective

According to Zarchan [2], the adjoint method can be traced back to the Italian mathematician Vito Volterra, circa 1870. However, the book by Ince [62] points to Lagrange as the discoverer of the method over a hundred years earlier. According to Ince, while exploring ways of solving the variable coefficient ordinary differential equation with non-zero right hand side, Lagrange was the first to mathematically formulate the adjoint differential equation, although he did not use the term “adjoint” to describe it. The term was later used by Fuchs in 1873 [62].

The first practical use of the adjoint method was subsequently developed by Bliss while serving as a mathematical expert at the Army Proving Ground at Aberdeen, Maryland, US, in 1918 [3]. Bliss used a property of the adjoint method to study the effects of perturbations on a system of equations associated with the flight of shells. Ballisticians then used the method in connection with their theoretical studies of artillery hit dispersions [4].

In the early 1950s, Bennett [5] employed the adjoint method for statistical performance analysis of linear systems on an analogue computer. Subsequently, Laning and Battin [6] described the use of the method for the statistical analysis of time variable networks. However, it seemed that the adjoint method was still relatively unknown until 1956 when Laning and Battin [7] included a discussion and proof of the method in their book on random processes. This book helped popularise adjoint simulation and is still a highly referenced resource on the method.

In the 1960s, many references appeared on the applications of the adjoint method to systems analysis using analogue computers. Books by Rogers & Connolly [8], Fifer [9] and Leondes [10] devoted chapters to the technique while Thorson [11] examined the method more closely while undertaking research as part of a Master of Science program. Peterson [12], and later Howe [13] and Tarrant [14], illustrated how

the method could be easily applied to the performance analysis of generic guided missile systems. In fact, with regards to guided missile studies, the Peterson book is referenced often. Several other references [15-17] discuss the adjoint simulation approach and its application on the analogue computer. In all these applications, the adopted methodology for the construction of the adjoint system followed that proposed by Laning and Battin in the form of their adjoint construction rules [7].

During the same time frame, a slightly different mathematical approach applying the adjoint operator to linear differential equations was being developed and presented by Sussman [18]. Similar expositions using the state space representation of the governing equations were published [19-20] relating the more mathematical approach to the so-called modified adjoint system. State space methodologies were also developed to handle discrete time systems [21-22]. Willems [23] used a combination of the state space approach with the more traditional block diagram approach to investigate the performance of a generic ground to ground missile guidance system.

The adjoint method had become a common analysis tool by the 1970s and 1980s and was being used on preliminary design studies by various companies in the missile industry. Advances to the basic adjoint methodology, such as the use of shaping filters to model target manoeuvres [24-26], adjoint representation of discrete systems [2] and applications to non-linear systems [27] extended the overall capability of the method. The extension to non-linear systems utilised the concept of statistical linearisation and is known as the Statistical Linearisation Adjoint Method (SLAM). An excellent report outlining the use of the adjoint method for sensitivity analysis and miss distance studies of generic guided missile homing loops is given by Bibel [28].

During the 1990s, the book by Zarchan [2] on "Tactical and Strategic Missile Guidance" helped promote the use of the adjoint method as a staple engineering tool for the performance analysis of homing missile guidance loops. Other practitioners used the method in conjunction with the SLAM concept to explore the effects of various noise sources on the miss distance of a radar homing missile [29-30]. The SLAM approach also featured in research undertaken as part of postgraduate degrees [31-32]. Also during this period, an interesting development saw the introduction of specialised software that automated the adjoint construction process using the Laning and Battin rules [33-34]. This provided faster turn around for analysis purposes while at the same time minimised potential errors during the adjoint construction phase.

Within the last decade, the application space for the adjoint simulation method has expanded considerably. A new state space framework to handle continuous and discrete system analysis by the adjoint method has been proposed by Weiss [35] and applied to mid-course guidance problems [36-37]. Raytheon has shown renewed interest in the traditional method with applications to the preliminary analysis and study of Ballistic Missile interceptors [38]. In addition, Raytheon researchers have extended the basic adjoint method to allow assessment of more complex homing guidance loops [39]. The method has also been used for comparison and evaluation of various missile guidance laws [40-41]. Further extensions of the method include the determination of appropriate shaping filters for estimating higher order statistics [42], performance analysis of guidance loops under model uncertainty [43], for robust guidance performance [44] and for conducting missile overload requirement analysis in terminal guidance using the L1 approach [45]. For non-linear homing loop analysis, a different approach to SLAM was proposed which made use of infinite dimensional linearisation to reduce the equations to linear form prior to applying the adjoint process [46]. There have also been investigations carried out into non-traditional uses of the method including applications to soft kill and survivability studies [47-48]. More recently, a new framework has been proposed for the analysis of hybrid systems consisting of multi-rate subsystems that may be characterised by non-periodic discrete events [49]. This facilitates the study of the effects of uplinks on the performance of a guidance system with the use of the adjoint method.

Following [33] which had been based on the MATRIXx environment, effort has continued on the software development front but with a major shift to the MATLAB environment [50-53]. The last decade has also seen much interest in the method as a topic for further research within the academic domain [54-59].

The adjoint technique has also received considerable attention in the teaching of aerospace guidance and navigation courses [60].

3. The Adjoint Simulation Method

3.1 Deterministic Inputs

Consider a linear time varying (LTV) system described by the following ordinary differential equation

$$\frac{d^n y}{dt^n} + a_{n-1}(t) \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_1(t) \frac{dy}{dt} + a_0(t) y = u \tag{1}$$

where y denotes the system output u , is the deterministic system input and the coefficients are functions of time.

According to linear systems theory [7], the solution to the above differential equation is given by

$$y(t) = \int_{t_i}^t u(\tau) w(t, \tau) d\tau \tag{2}$$

where t_i is the time at which the input is applied while $w(t, \tau)$ denotes the system weighting function or impulse response function. Physically, the function $w(t, \tau)$, represents the response of the system at time t to a unit impulse applied at the input at time τ . With the aid of this weighting function, the system described by equation (1) is conveniently represented in block diagram form as shown in Figure 1.

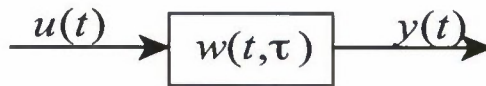


Figure 1. Linear time varying system

While equation (2) gives the formal solution to the problem posed in Figure 1, its practical usefulness is limited. The problem here lies in the fact that the variable of integration corresponds to the impulse application time. Thus, in order to generate $w(t, \tau)$ required for the above solution, it becomes necessary to simulate the system response for each impulse application time τ . Once the weighting function is obtained in this manner, it is then necessary to carry out the integration in equation (2) numerically.

Since every linear time-varying system has a corresponding adjoint system, a better and more efficient alternative is to reformulate equation (2) in terms of the weighting function of the associated adjoint system. To this end, utilising the so-called modified adjoint technique [7, 20] has the effect of requiring only one computer simulation run to yield the desired weighting function.

By definition, the differential equation that describes the modified adjoint system corresponding to the above system but with $u = 0$ is given by

$$\frac{d^n z}{dt^{*n}} + \frac{d^{n-1}[a_{n-1}(t^*)z]}{dt^{*n-1}} + \dots + \frac{d[a_1(t^*)z]}{dt^*} + a_0(t^*)z = 0 \quad (3)$$

where z is a function of adjoint time $t^* = t_F - t$ and t_F denotes final time. Note that the term “modified” adjoint system is used here to indicate a time reversal in the conventional adjoint system differential equation by definition of adjoint time t^* .

Now, if the adjoint system, described by the above differential equation, is subjected to an impulsive input at time τ^* , then the resulting system response, known as the adjoint weighting function, will be denoted by $w^*(t^*, \tau^*)$.

A fundamental relationship exists between a linear time varying system and its corresponding adjoint system [2], namely,

$$w^*(t_F - \tau, t_F - t) = w(t, \tau) \quad (4)$$

where τ , t and t_F are the impulse application, response observation and final time, respectively, of the original system. Thus, on substitution of equation (4) into equation (2) and, after a change of variable, $\eta = t_F - \tau$ we get

$$y(t) = \int_{t_F-t}^{t_F-t_i} u(t_F - \eta) w^*(\eta, t_F - t) d\eta \quad (5)$$

It is clear now that the variable of integration appears as the first argument in the adjoint weighting function and thus the solution for the output can be computed with just one computer simulation run. For example, if the input to the system is a step of magnitude K which is applied at time zero, and we seek the solution at final time t_F , then equation (5) becomes

$$y(t_f) = K \int_0^{t_f} w^*(\eta, 0) d\eta \quad (6)$$

Consequently, the original system response at final time can now be computed in one simulation run by integrating the weighting function of the corresponding adjoint system. This is represented in block diagram form in Figure 2, where $\delta(t^*)$ denotes the standard Dirac delta function or impulse function.



Figure 2. Adjoint solution for system with step input

This procedure can be extended to include many step input disturbances as well as other types of deterministic inputs such as ramp or sinusoidal inputs. Furthermore, since the system is linear, the superposition principle allows one adjoint computer simulation to yield the system response to a combination of disturbances, along with a detailed error budget showing how each disturbance influences total system performance.

3.2 Stochastic Inputs

Suppose the linear system is driven by some noise process $n(t)$. The system output is now random in nature and given by

$$y(t) = \int_{t_i}^t n(\tau) w(t, \tau) d\tau \quad (7)$$

Squaring both sides of equation (7) and taking expectations yields the mean square value of the output, namely,

$$E[y^2(t)] = \int_{t_i}^t \Phi_0(\tau) w^2(t, \tau) d\tau \quad (8)$$

where we have assumed the noise input to be non-stationary white noise with autocorrelation function $R(t, \tau) = \Phi_0(t)\delta(t - \tau)$. For stationary white noise input, the output mean square value reduces to

$$E[y^2(t)] = \Phi_0 \int_{t_i}^t w^2(t, \tau) d\tau \quad (9)$$

where the white noise power spectral density Φ_0 is double sided with dimension unit^2/Hz . Again, the problem with computation of the weighting function can be circumvented by considering the corresponding adjoint system. In this case, we have

$$E[y^2(t)] = \Phi_0 \int_{t_f-t}^{t_f-t_i} [w^*(\eta, t_f - t)]^2 d\eta \quad (10)$$

If the noise input begins at $t_i = 0$ and the observation time is taken as final time t_f , the mean square response simplifies to

$$E[y^2(t_f)] = \Phi_0 \int_0^{t_f} [w^*(\eta, 0)]^2 d\eta \quad (11)$$

Thus the mean square response of the original system due to a white noise input can be determined by computing the weighting function of the adjoint system in one computer simulation run. This is conveniently depicted in block diagram form in Figure 3.

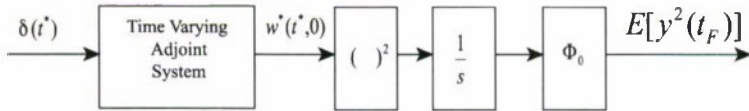


Figure 3. Adjoint solution for white noise input

Because of the superposition principle, the above procedure can be extended to the case of many white noise inputs. In this case, one adjoint simulation run yields an exact statistical solution of the noise-driven system including an error budget showing how each white noise error source contributes to the total system response. Coloured noise inputs can be treated in a similar fashion by the use of appropriate shaping filters.

3.3 Adjoint Construction Rules

From the above theory, it is clear that, for linear time varying systems, the adjoint simulation method offers tremendous efficiencies over other methods as a performance projection tool. This fact had been realised early on by many weapons analysts and cleverly utilised in the preliminary design stages of many current missile systems.

One of the difficulties with this approach is the construction of the adjoint system from which the desired weighting function must be computed. Although several methods for the construction process have appeared in the literature, by far the most popular method has been that proposed by Laning and Battin [7]. This method works directly on the original simulation block diagram by application of a set of rules.

The adjoint construction rules are:

1. Convert all system inputs into equivalent systems driven by impulses or white noise processes,
2. Reverse all signal flow directions,
3. Switch all system inputs to adjoint outputs and all system outputs to adjoint inputs,
4. Change all summing points to branch points and all branch points to summing points,
5. Replace time in all time varying elements with adjoint time t' where $t' = t_f - t$,
6. Apply an impulse signal to the selected output of the original system,
7. At all the adjoint system stochastic output points, add the adjoint solution sequence as illustrated in Figure 3 to yield the mean square value of the system response.

3.4 Example of Adjoint System Construction

Traditionally, the construction of an adjoint system has been carried out manually by following the above set of rules. As an example consider the following simple system described by

$$\frac{dy}{dt} + (t_f - t)^2 y = u(t) \quad , \quad y(0) = 2 \tag{12}$$

where the input is given by $u(t) = \sin(t)$ and t_f denotes final time. This simple example has two inputs and one output. Suppose we are interested in the sensitivity of $y(t_f)$ due to each of the inputs. The problem is then ideally suited to adjoint analysis.

Firstly, we re-cast the system into block diagram form. The block diagram representation is given in Figure 4

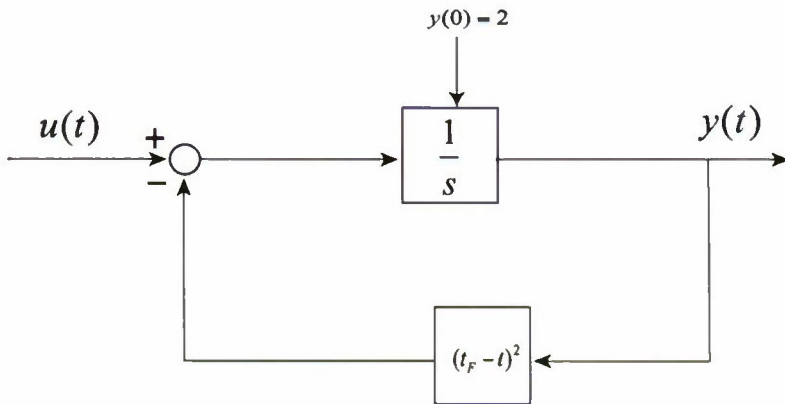


Figure 4. Block diagram representation of the system

After application of rule 1 in the adjoint construction process, the above block diagram is converted to that shown in Figure 5. Here, the input $\delta(t)$ represents the impulse function.

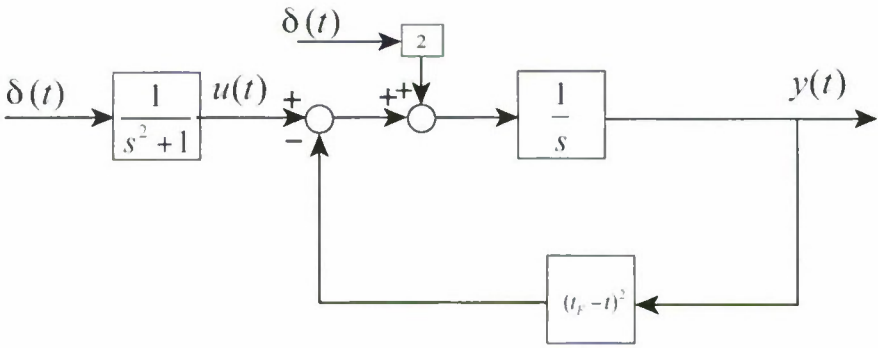


Figure 5 Modified block diagram following application of rule 1.

After applying the rest of the adjoint construction rules to the system block diagram shown in Figure 5, we obtain the adjoint system associated with the original system. This is given in block diagram form in Figure 6.

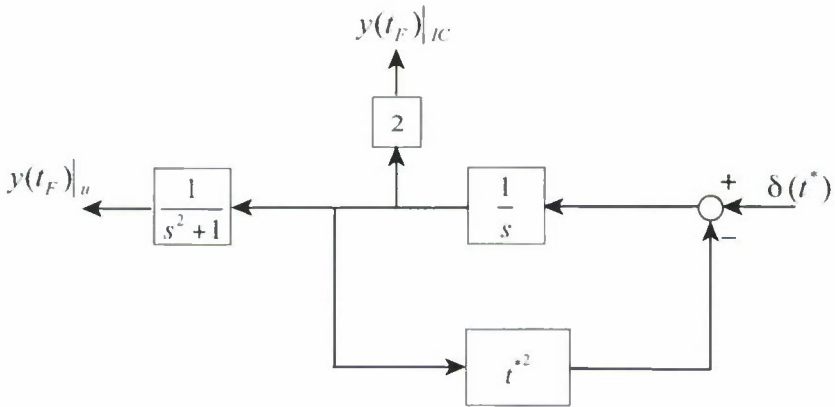


Figure 6. Block diagram representation of the associated adjoint system for deterministic input

It is clear from Figure 6 that the adjoint solution to the problem gives the sensitivity of $y(t_f)$ to each of the given inputs in one simulation run.

If, instead, the input to the system $u(t)$ is stochastic in nature and represented as a zero mean, white noise process characterised by the power spectral density ϕ_u , then the adjoint simulation diagram has the form presented in Figure 7, where the mean and variance of the output at time t_F are denoted by $\bar{y}(t_F)|_{IC}$ and $\sigma_y^2(t_F)|_u$, respectively.

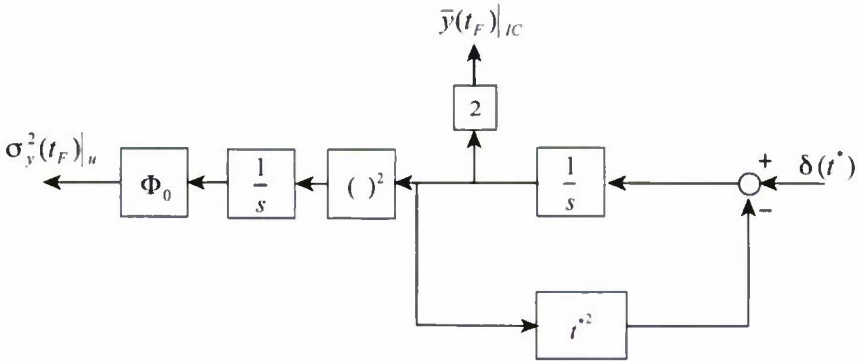


Figure 7. Block diagram representation of the associated adjoint system for stochastic input

4. The Adjoint Software Tool

For very complex systems with many feedback paths, application of the adjoint rules can be extremely tedious, time consuming and error prone. If the original simulation block diagram is implemented in Simulink, then the process required to construct the adjoint system can be conveniently automated to minimise these shortcomings. A tool has been developed to achieve this. This tool, which forms part of a suite of tools known as COVAD [50], is briefly described below.

4.1 COVAD Toolbox Overview

The COVAD toolbox has been designed as a general analysis package consisting of a Graphical User Interface (GUI) built using the Matlab 2007b GUI API. Via the GUI one is able to interact with Simulink models, such as a missile homing loop in order to perform a number of analysis types including the adjoint method or the Monte Carlo method.

The GUI also provides a means to store and retrieve the results of simulations conducted in MATLAB/Simulink by storing them within a doc block (a built-in

Simulink block) that is added into a Simulink model.

Viewing the results of the simulation is facilitated by the GUI via a link to the Matlab 2007b graph viewing facility. The GUI toolbox allows the user to filter from the simulated results a suitable subset for which graphs may need to be generated for analysis and then pipes this information across to the Matlab 2007b graph viewing facility.

It should be noted that the GUI is built with the intention of supporting both the novice and the advanced analyst. For the novice analyst, a wizard is used to guide the user through the steps required to configure a model and to perform the desired analysis type. As the model is configured, a display panel on the GUI is updated so that a quick summary of the state of the model configuration is always easily viewable by the analyst. For the advanced user, a scripting space has been introduced within the GUI, where the analyst can write, within some guidelines, M-code that is able to configure the model in a more flexible way. Even with this scripting facility, it has been broken into steps so that after a gap between analysis efforts, the time to familiarise oneself with progress so far is not tedious. There is even the ability to mix and match between script and wizard, thanks to the modular design under the hood, employed when configuring the model via steps.

While the toolbox is envisaged to work across different versions of Matlab, presently it is able to function in 2007b and upwards only, since much of it was developed within Matlab 2007b from scratch.

4.2 Use-Case for Adjoint Analysis

1. User invokes the COVAD GUI via the MATLAB command line.
2. User then creates a project. A project is a space where multiple models on which different sets of related analyses are packaged together in a bundle.
3. Once the project is created, the user starts to populate the project with Simulink models.
4. For each model, the user then selects the analysis type to be applied to that model via a list of possible analysis options. With respect to this report, the user would select the Fwd + Adjoint analysis option.
5. This then configures the wizard to cater for the particular analysis type. The steps of the wizard will alter depending on the analysis combination chosen.

6. Now the user walks through the wizard steps and then setup of the inputs, outputs, and model parameters of the Simulink model subject to analysis via the GUI. The setup can involve setting up the ranges for the variables that are parameters for the various model blocks and selective enabling of the noise and target manoeuvre inputs to be included in the model. Via a Create Adjoint button found on one of these types, the user will invoke a script that will then automatically build the adjoint for the selected model in accordance with the rules described earlier in the paper.
7. Once this is complete, the user is able to go ahead and run the simulation on the model via the Simulate button on the GUI. This will generate run data that will be displayed on the GUI as well as being saved into Doc Block, added to the model, which will serve as a repository of simulation run history for the model with associated analysis type.
8. Finally the user can take the run results and then, depending on the data collected, be able to select columns for the x and y axis to produce relevant plots, e.g. miss distance versus flight time, for both forward and adjoint time simulation runs and subsequent analysis.

4.3 Verification and Validation of the Toolbox

It is envisaged that the COVAD toolbox will be verified and validated against results given in the open literature such as those given by Zarchan [2]. The verification and validation steps are:

- a. Compare the simulink homing loop block diagrams for various configurations to those given by Zarchan [2] in both forward and adjoint time to verify that the models have been built correctly.
- b. Compare the forward run and adjoint run simulation results with corresponding results given by Zarchan [2] for validation purposes.

5. Missile Applications

Consider the planar missile-target engagement geometry depicted in Figure 8. Here we have a radio frequency (RF) guided missile intercepting an incoming target.

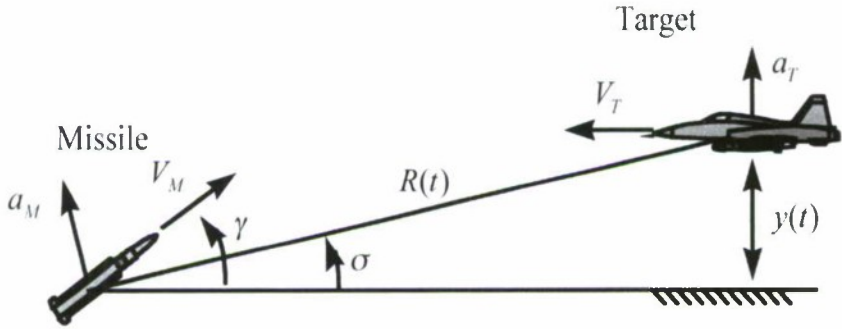


Figure 8. Missile target engagement geometry

Both missile and target speed are assumed constant. Initially, both missile and target are on a collision course. The governing equations describing small perturbations about the collision course are given as follows [2] (assuming a near head-on case).

By inspection of Figure 8, the relative acceleration is approximately

$$\ddot{y} = a_T - a_M \quad (13)$$

Similarly, the expression for the line of sight angle σ can be approximated by

$$\sigma = y / R(t) \quad (14)$$

For a head-on case, the closing velocity V_c reduces to

$$V_c = V_M + V_T \quad (15)$$

The linearised range equation $R(t)$ is approximated by the expression

$$R(t) = V_c(t_F - t) \tag{16}$$

where t_F denotes the total flight time of the engagement.

In this example, the quantity of interest is the miss distance. This is defined as the relative separation between missile and target at the end of the flight, that is,

$$MD = y(t_F) \tag{17}$$

The above equations can be represented in block diagram form as shown in Figure 9

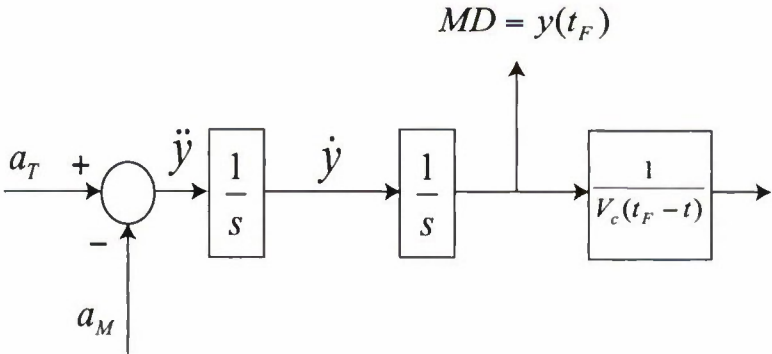


Figure 9. Block diagram representation of linearised geometry equations

For homing guidance, the missile relies on its RF seeker to track the target. Thus to complete the homing loop in Figure 9, a model of the seeker is required. The signals from the seeker are then used by the guidance computer to generate the guidance commands for the autopilot. The autopilot stabilises the missile and deflects the fins to cause the desired manoeuvre. The missile then reacts aerodynamically and alters the flight path.

In this example, the seeker, autopilot and aerodynamic response of the missile are modelled using transfer functions. The guidance law used is the proportional navigation guidance law and has the form $a_c = N^1 V_c \dot{\sigma}$, where N is the effective navigation ratio, V_c is the closing velocity, $\dot{\sigma}$ is the rate of change of sight line angle and a_c denotes the missile commanded acceleration.

The complete linearised homing loop model for this example is represented in block diagram form in Figure 10.

Radar noise, which contaminates the seeker's measurement of the line of sight to the target, is added in the model in the form of glint and receiver (fading) noise. These are modelled as zero mean white noise processors.

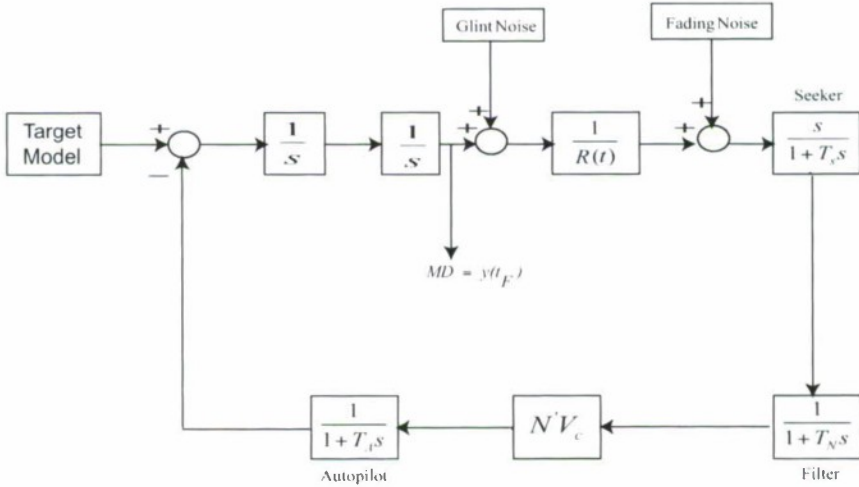


Figure 10. Linearised homing loop model

The target model for this example is developed as follows. Prior to intercept, the target is assumed to execute, at random, an evasive step manoeuvre of magnitude A . The random time T for execution of the target manoeuvre is uniformly distributed between zero and final homing time t_f with probability density function given by

$$p_T(t) = \begin{cases} \frac{1}{t_f} & , 0 \leq t \leq t_f \\ 0 & , otherwise \end{cases} \quad (18)$$

It has been shown in [24-26] that the above target model can be represented by a linear shaping filter driven by white noise. This model, which is shown in Figure 11, has the same statistical characteristics, up to second order, as that based on equation 18. The power spectral density of the zero mean, input white noise processor is given by $\Phi_n = A^2/t_f$.



Figure 11. Shaping filter equivalent of random step manoeuvre

Table 1 summarises the parameter values used in this example.

Table 1. Nominal values of system parameters

Parameter	Nominal Value	Parameter	Nominal Value
Time of Flight, t_f	5 s	Tgt Man Mag, A	3 g
Seeker Time Const., T_s	0.05 s	Glnt Noise PSD, Φ_{G_i}	2 m ² /Hz
Filter Time Const., T_N	0.1 s	Fading Noise PSD, Φ_f	10 ⁻⁶ rad ² /Hz
Autopilot Time Const., T_A	0.1 s	PN Ratio, N	4

After inclusion of the target model into the homing loop block diagram of Figure 10 and entering the parameter values, given in Table 1, into COVAD, we obtain the GUI information displayed in Figure 12.

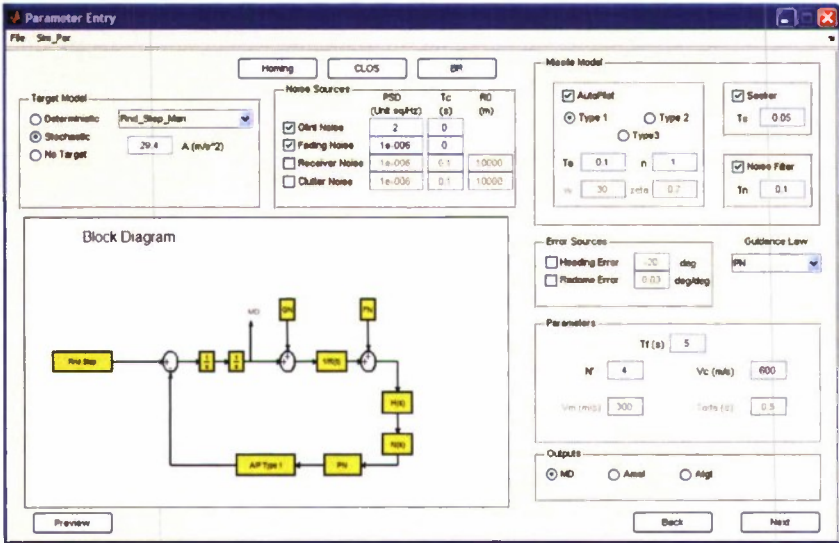


Figure 12. Parameter values entered into COVAD

The preview pane in the figure displays a functional representation of the entered data and is there to provide the user with an indication of the structure of the simulation model that will be built in block diagram form in Simulink. Once the Next button is pressed, the Simulink model representation of this data is automatically generated. This is shown in Figure 13.

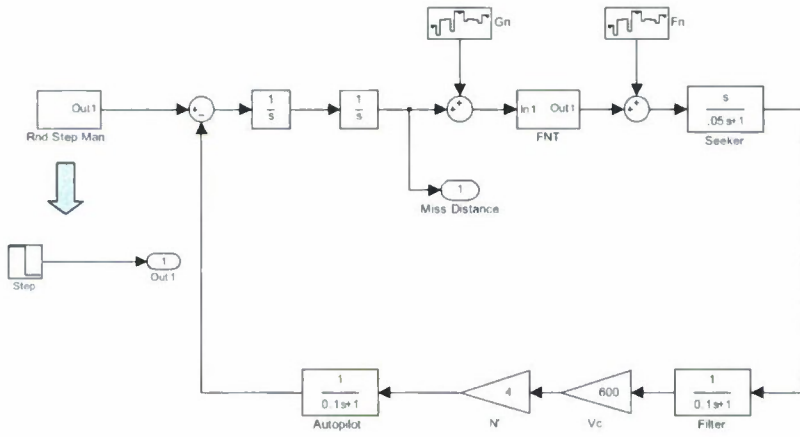


Figure 13. Simulink implementation of the missile homing loop

The adjoint toolbox in COVAD may now be invoked to automatically convert the Simulink model of the original system into an associated Simulink adjoint system for further analysis. The Simulink adjoint system is shown in Figure 14.

Note that the impulse response of the adjoint system has been computed by imposing an initial condition of unity on integrator2 in the diagram. Standard Simulink tools can now be utilised for further analysis of the system.

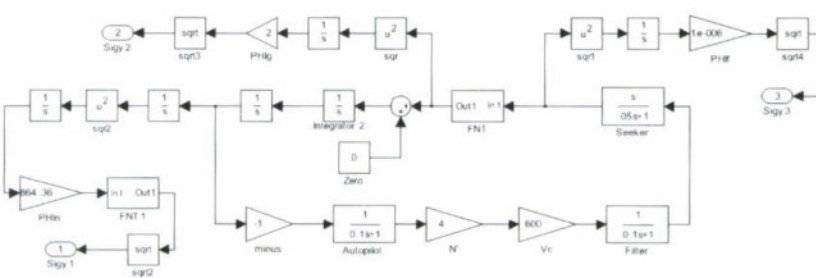


Figure 14. Corresponding adjoint system in Simulink

Figure 15 compares the results from the adjoint simulation with those generated using a Monte Carlo method. The results give the RMS miss distance of the engagement as a function of flight time. The adjoint results were obtained in one simulation run of the Simulink system. The Monte Carlo results are based on the physical model of the target manoeuvre (as opposed to the shaping filter approach required for the adjoint model) and required 200 runs for each value of flight time t_f considered.

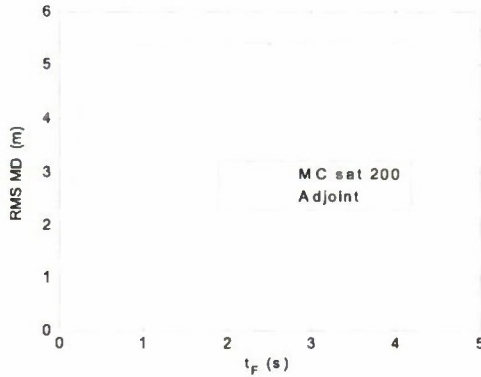


Figure 15. RMS miss distance comparison – Adjoint vs Monte Carlo results

In addition to the total RMS miss distance results, the adjoint solution also provides, within the same simulation run, an error budget consisting of the contributions that each input has on total system performance. The error budget information is displayed in Figure 16.



Figure 16. Adjoint solution generates error budget

It is clear from the error budget plot that, in this example, glint noise is the biggest contributor to miss distance.

As another example, let us now use the software to investigate the performance of the missile against other possible target manoeuvres. If instead of the random step manoeuvre, the target executes a sinusoidal manoeuvre at some time prior to intercept, where, as previously, the time T at which the manoeuvre is initiated is assumed to be uniformly distributed over the interval $[0, t_f]$ with probability density function given by equation (18), then such a manoeuvre can be modelled as shown in Figure 17.

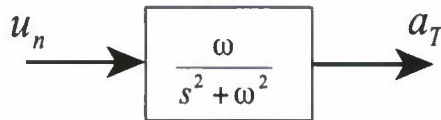


Figure 17. Random sinusoidal target manoeuvre model.

The parameter ω appearing in the figure denotes the frequency, in rad/s, of the sinusoidal target manoeuvre. The RMS miss distance adjoint simulation results for this form of the target manoeuvre are summarised in Figure 18 for selected values of the parameter ω . Note that all other nominal values for the system parameters as given by table 1 remain the same here.

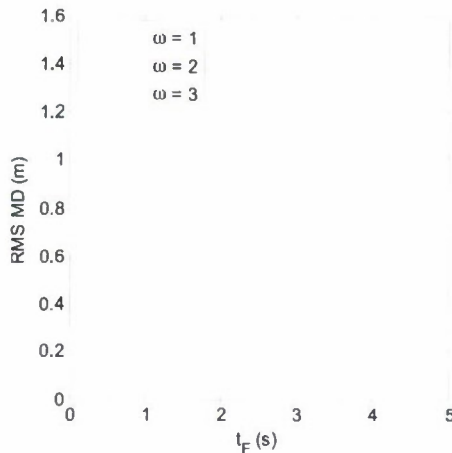


Figure 18. Random sinusoidal target manoeuvre effects on miss distance

As a final example, we investigate the performance of the interceptor when the target executes a random telegraph manoeuvre. The random telegraph manoeuvre represents a policy, starting at time zero, in which the target executes either a maximum positive or negative acceleration such that the number of sign changes per second follows a Poisson distribution, and the average number of sign changes is ν per second [26]. A typical realisation of a $3g$ Poisson target manoeuvre is shown in Figure 19.

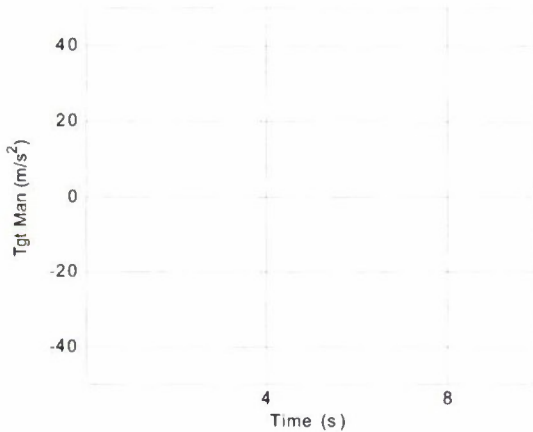


Figure 19. Random telegraph target manoeuvre

The shaping filter equivalent of a Poisson target manoeuvre can be represented by white noise through a low pass filter as shown in Figure 20 [61].

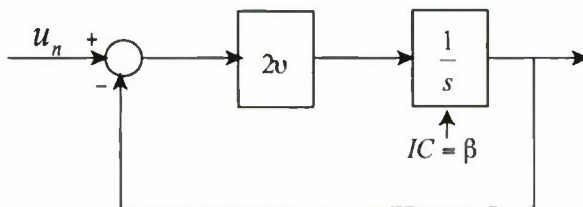


Figure 20. Shaping filter equivalent of random telegraph target manoeuvre

In the Figure, ν represents the average number of sign changes while β denotes the magnitude of the target manoeuvre, in this case, $3g$. Also in the figure, the white noise input has spectral density

$$\Phi_n = \frac{\beta^2}{\nu} \tag{19}$$

Using the same parameter values as previously, the adjoint block diagram can be readily constructed using COVAD. The results of the adjoint simulation for selected values of the parameter ν are given in Figure 21.

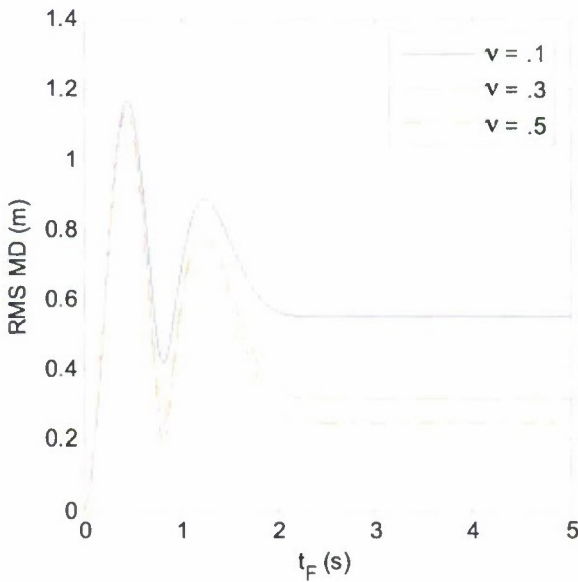


Figure 21. Random telegraph manoeuvre effects on miss distance

6. Conclusion

Mathematicians have long been paying attention to the adjoint operator and duality theorems in their studies of the existence of solutions to differential equations. In this report, the adjoint simulation method has been employed to investigate the response and sensitivity of linear time varying systems to a combination of deterministic and stochastic inputs. MATLAB/Simulink software has been developed to facilitate applications of the method to problems encountered in Aerospace/Defence science.

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