

**US AFOSR Research Report**

**Cold atoms, statistical physics and quantum simulations.**

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## Report Documentation Page

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## Summary

We proposed to develop a formalism allowing for the effective simulation of the dynamics of many-particle systems in regimes where quantum effects were significant. Using our expertise in the field of numerical simulation based upon techniques from quantum optics and, in collaboration with Anatoli Polkovnikov of Boston University, by developing analytic and novel numerical methods, we proposed to address specific, experimentally relevant problems, having both fundamental and practical significance, with an emphasis on cold atom systems. In particular, utilizing this seed funding, we planned to explore the dynamics of certain integrable and near-integrable many-body quantum systems that have recently been realized in the laboratory. The primary pay-off for this research was to be the development of robust theoretical techniques for the simulations of ultra-cold Bose gases and other quantum phenomena, such theoretical tools being essential for the future development of future technologies based upon current cold atom physics.

We have been successful in this program, as will be evidenced in our reporting against the detailed work plan presented below. In particular, we have; developed and implemented an efficient scheme for introducing next order quantum corrections into truncated Wigner approximation (TWA) simulation schemes; applied our TWA simulations to the study of atom-chip interferometry based motion sensors – going beyond our original proposal in significant, practical aspects; and studied the dynamics of coherence and decoherence in one-dimensional systems with particular emphasis on recent and next generation atom-chip interferometry experiments.

The work has resulted in one piece of published work (*Physical Review A* **79**, 063624), one manuscript submitted for publication (arXiv:0908.2930; currently under review with *Europhysics Letters*), a poster presentation at the *New Zealand Institute of Physics Conference* in Christchurch, New Zealand in July, and a further technical manuscript in preparation regarding the efficient numerical implementation of the quantum correction scheme. Aspects of the research will also be presented in poster form at the *Annual Dodd-Walls Symposium* to be held in Wellington, New Zealand in February 2010, the *International Conference on Atomic Physics* to be held in Queensland, Australia in July and through an invited talk at *Laser Physics* in Brazil, also in July. AFOSR/AOARD support was and will be acknowledged in all cases.

We therefore believe, given the modest budget of this research program, that this has been a highly successful project, laying valuable ground work towards future applications which should be of significant AFOSR interest. We thank the AFOSR/AOARD for their support.

## Report Against Detailed Work Plan

**December 2008**      **Begin work on simulation of one dimensional atom-chip based interferometers. Develop finite-temperature Truncated Wigner Approximation simulation technique. Work conducted in collaboration with visiting postdoctoral researcher.**

This work was begun in collaboration with Robin Scott, a postdoctoral research fellow visiting me from Nottingham University in the UK. Dr Scott worked with me from December until late April developing finite-temperature TWA simulation techniques. In particular we included Bogoliubov collective modes to previous zero-temperature codes. By thermally populating these modes we were able to simulate finite-temperature systems where previously only quantum, not thermal, noise had been included. This work resulted in the publication of *Quantifying finite-temperature effects in atom-chip interferometry of Bose-Einstein condensates*, R. G. Scott, *et al.*, Physical Review A, 063624 (2009). A copy of the paper is attached to this report. During the review process, a visit (April 2009) was made by the PI to Melbourne, Australia to where Dr Scott had moved in order to facilitate some quite major revisions to the manuscript. This visit was supported in part under this contract and partially by a New Economy Research Fund contract from the NZ Foundation for Research, Science and Technology.

**January 2009**      **Visit Polkovnikov (Boston University). Establish model for investigating efficient implementation of next order quantum corrections to Truncated Wigner Approximation.**

This visit was made and was funded by the New Zealand Government through International Science and Technology Linkages Fund contract. Extensive discussions ranged over schemes for implementing and assessing the introduction of leading order quantum corrections to the TWA simulations. A model system (single site model) was settled upon. In addition, during this time, the PI spent a significant period of time assisting a doctoral student of Polkovnikov's with the implementation of the Bogoliubov finite-temperature modes in their simulations. The student, Rafael Hipolito, is expected to graduate in the coming semester and has expressed an interest in pursuing post-doctoral research with our group at Otago, further strengthening these growing Otago-Boston University links.

**Feb-May 2009**      **Develop trial model. Investigate efficient implementation of next order corrections. Identify temporal position in classical evolution where “quantum jumps” – the leading order corrections – are most important. Develop a weighted sampling system to implement corrections in “real” simulations based upon this information.**

An MSc student, Kevin Hodder, was assigned this project, beginning on 1<sup>st</sup> March 2009. Mr Hodder was supported through a University of Otago Scholarship. He initially developed a simple single-site Hubbard model, which can be solved analytically, and implemented a numerical TWA simulation. He then included the quantum correction through the implementation of a series of quantum jumps. Including all possible jumps very effectively reproduced the exact analytical results, but was very time consuming. A finite, random sampling scheme was introduced which improved numerical efficiency at the cost of accuracy. Mr Hodder then made an analysis of the magnitude of the effect of a quantum jump on the final result depending upon its temporal location in the evolution of the simulation. This allowed a weighted-sampling scheme to be implemented giving a higher probability for a jump in a more significant temporal region of the simulation. This allowed greater efficiency with reduced reduction in accuracy of the simulations. A run-time assessment process for assessing the significance of individual jumps was then developed allowing an adaptive (during a given run) sampling scheme to be implemented. This proved highly efficient and was the subject of the poster presented by Mr Hodder at the NZIP Conference.

Mr Hodder has since generalized the model to a multi-site Hubbard model and is currently in the process of writing his MSc thesis. A technical publication based upon his thesis will be published in due course, with AFOSR/AOARD support acknowledged.

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**Milestones:**

- By end of May:
- a] Have published (at least) one paper applying Truncated Wigner Approximation to atom interferometers based on one dimensional cold gases. **[Result: Achieved]**
  - b] Have working model to investigate efficient numerical implementation of quantum corrections. **[Result: Achieved]**
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**June – September    Develop full Truncated Wigner Approximation simulations of one dimensional systems including next order quantum corrections.**

As discussed above, a one-dimensional Hubbard model based upon the TWA with next order quantum corrections has been implemented. This code is currently being further utilized to investigate effects of disorder in one-dimensional systems – activity that goes beyond the original scope of this research project.

**July                            Potential visit to Otago by Polkovnikov (partially funded by NZ government). Discussion of connection between phase space methods (Truncated Wigner Approximation) and traditional condensed matter based non-equilibrium techniques.**

This visit took place in August 2009. Prof. Polkovnikov visited Otago for two weeks. This was funded by the New Zealand Government through International Science and Technology and New Economy Research Fund contracts held by the PI. Extensive discussions were had regarding connections between phase-space methods, of which the TWA is an example, and other representations. In particular the relationship between path-integral methods for quantum Monte Carlo simulations, the classical limit and phase-space methods were discussed. Possibilities for using the ideas developed from the next order quantum corrections to TWA for evaluating non-equal-time correlation functions in path-integral quantum Monte Carlo simulations were discussed in detail, resulting in a research proposal by one of the PI's post-doctoral researchers. This was successful, resulting in a University of Otago Research Grant contract under which the post-doc, Dr Daniel Schumayer, will visit Massachusetts in the coming year. This development will allow Monte Carlo methods to be used for certain non-equilibrium simulations, in particular quenches, which are of significant current interest. This would be a significant advance.

**Sept. – Dec.                    Application of simulations to current and future experiments on one dimensional integrable and near-integrable systems. Characterisation of such systems hopefully leading to active involvement in development of next generation experiments.**

Again in collaboration with Dr Scott of Nottingham University, we applied our new techniques to analyze recent experiments regarding one-dimensional integrable and near-integrable systems. We were able to, for example, reproduce the sub-exponential scaling of coherence between two separated one-dimensional systems seen in experiments at Heidelberg and in Vienna. We extended this to investigate the growth of coherence between coupled one-dimensional systems and also to investigate effects of increasing the strength of a two-dimensional optical lattice to split an homogeneous system into an array of one-dimensional “wires”. These, especially the latter, have important implications for future wire-chip interferometry experiments and we are actively engaged in communication with the leading experimentalists in this area.

This work has now been written up for publication and submitted to *Europhysics Letters*. A copy of the manuscript is attached to this report.

**November                    Visit to Polkovnikov (Boston University). Further discussion of connections between semi-classical phase space methods and traditional condensed matter techniques. Collaboration on writing of manuscripts for dissemination of results.**

This visit was not made for personal (PI) reasons which are detailed in the covering letter. A visit to Polkovnikov by the PI is now planned for February of 2010 when final details regarding remaining joint technical publications will be discussed.

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**Milestones:**

- By end of December: a) Have full working simulation code for one dimensional systems including at least first order quantum corrections to the Truncated Wigner Approximation. **[Result: Achieved]**
- b) Have investigated current and future experiments on integrable and near-integrable one dimensional cold gas experiments. **[Result: Achieved]**
- c) Have disseminated results through at least two further archived publications.  
**[Result: Not Achieved – At least one further publication will be forthcoming after the completion of Hodder’s thesis and the February visit to Polkovnikov. This milestone will then be Achieved.]**
- d) Have a clearer understanding of the connections between semi-classical phase space methods and traditional condensed matter techniques (e.g. Keldysh). **[Result: Achieved]**
- e) Develop clear plan to assist with development of next generation atom-interferometry experiments with emphasis on sensor technology. **[Result: Achieved]**
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In conclusion, apart from the caveat regarding December Milestone (c), all milestones for this project have been achieved. Ultimately the publication output and general dissemination of this work through conference presentations will exceed our expectations entering this program. We therefore believe this has been a highly successful project and once more thank the AFOSR and the AOARD in particular for their support of our work.

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