

# An Autonomous Metric (Polytope-Convex Hull) For Relative Comparisons of MIQ

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**Abstract.** A means of measuring machine intelligence is presented. The technique is based on geometric procedures and works best on relative comparisons across different entities, rather than absolute comparisons of intelligence.

**Key Words:** Measuring Machine Intelligence

## I. INTRODUCTION

Defining, evaluating, and obtaining viable metrics for the measurement of autonomy, machine intelligence quotient (MIQ), or intelligence, in general, is a nontrivial task [1-9]. It is generally agreed that intelligence must be a high dimensional vector involving multiple attributes of a human or machine (Meystel [1]). Defining the relevant dimensions is also not a trivial task and much controversy exists. Even the discussion on how testing on intelligence is performed with humans creates controversy on which mental abilities constitute intelligence. The relevant issues include whether the IQ obtained, e.g. by the Stanford-Binet Intelligence Scale or the Wechsler Scales, are fair measures. Additional controversy also exists that certain less privileged racial, ethnic, or social groups do not have fair representations on the test questions pertinent to their living environments.

Albus [2] defines intelligence as having many dimensions. He also recognizes degrees or levels of intelligence. Some of the influencing parameters in describing features of intelligence for unmanned ground vehicles include, but are not limited to:

- (1) The computational power and memory capacity of the system's brain (or computer),
- (2) The sophistication of the processes the system employs for sensory processing, world modeling, behavior generation, value judgment, communication, and,

- (3) The quality and quantity of information and values the system has stored in its memory.

The measure of intelligence is also predicated on the success in solving problems, anticipating the future, and acting so as to maximize the likelihood of achieving goals. Obviously intelligence is goal oriented and related to success. The presumption is that different levels of intelligence produce dissimilar probabilities of success in the accomplishment of specific missions.

In studying autonomous systems [3], there are numerous (analogous) systems that can be examined for attributes both within and across processes that relate to autonomy. Some of these systems include living things (birds, fish, insects), intelligent highway vehicle systems, mobile robots, control of satellites in orbit, underwater vehicles systems, helicopters, tanks, human-machine interfaces, unmanned air vehicles, swarms of robots, and a host of other processes. In studying unmanned air vehicle systems (UAVs) [8,10] autonomy is desired since the goal is to maximize the ratio of UAVs/operators for a number of important reasons. The advantages include the significant reduction in cost, the elimination of the need to include a life support system (significantly reducing fuel and weight requirements), decreased vulnerability if the UAV is shot down or captured, enhancing reliability and robustness with multiple opportunities to achieve a mission, as well as other important traits. Again, in the design of UAVs, it is desired to

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have a metric to compare within and across different systems on the level of autonomy or intelligence designed in the aircraft.

It was pointed out in [4] that, at best, a measure of machine intelligence (MIQ) is a relative metric and it is difficult to have an absolute measure. This paper will discuss a relative means of determining how to contrast across different machines for comparative intelligence or autonomy. The goal is to have an objective measure to demonstrate that one machine has higher or lower degrees of intelligence or autonomy in comparison to another machine. Thus the designer can rate different machines in terms of their relative MIQ and investigate trade-offs between gain in MIQ versus cost and the benefits derived. It is cautioned that MIQ is very mission specific, and unless the mission can be accomplished with the appropriate level of success, then the machine may still not be appropriate. In other words, the appropriate tool has to be able to perform the given task. Success in a mission is the final measure that demonstrates that a machine has the appropriate MIQ for a given application. To understand the metric introduced here, some basics need to be reviewed and discussed to better grasp how the measure of MIQ was constructed herein.

## II. Some Basic Definitions

To understand the ensuing definition of MIQ, some basic concepts need to be reviewed. We present the fundamental nomenclature via key definitions.

### **Definition 1 – Convexity:**

A subset  $A$  of  $\mathbb{R}^n$  is convex if, for any vectors  $\mathbf{x}$  and  $\mathbf{y}$  in  $A$  and scalars  $r$  and  $s$  with  $r \geq 0$  and  $s \geq 0$ ,  $r + s = 1$ , then every point  $r\mathbf{x} + s\mathbf{y}$  remains in  $A$ . In other words, if we have a convex set (2 dimensions)  $A$  with two points  $\mathbf{x}$  and  $\mathbf{y}$ , then if we draw a line from the point  $\mathbf{x}$  to  $\mathbf{y}$ , every point on the line remains inside the

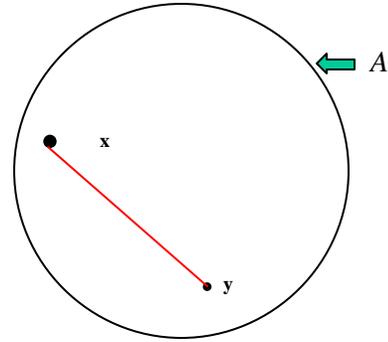


Figure 1 – The Convex Set  $A$  of a Circle

surface  $A$ . Figure 1 illustrates a circle in which the points  $\mathbf{x}$  and  $\mathbf{y}$  lie inside the circle. Drawing a line from the point  $\mathbf{x}$  to the point  $\mathbf{y}$  still remains inside the circle  $A$ . Also, every point along the line joining  $\mathbf{x}$  to  $\mathbf{y}$  also lies within the set  $A$  and no point on the line is outside the set  $A$ . Other examples of convex spaces in 3 dimensions include a cube, a sphere, etc. A cube is defined as follows:

$$\text{Cube} = A = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : |x_1| \leq 1, |x_2| \leq 1, |x_3| \leq 1 \right\} \quad (1)$$

It is also worthwhile to look at a surface which is not convex. Example 1 describes a set of points, which is not convex.

### Example 1—A set of points in a nonconvex set

The set  $A$  of points in  $\mathbb{R}^2$  defined by:

$$A = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : x_1 \geq 0, x_2 \geq 0 \quad \exists (x_1^{\frac{2}{3}} + x_2^{\frac{2}{3}})^{\frac{3}{2}} \leq 1 \right\} \quad (2)$$

Figure 2 is a plot of the nonconvex surface  $A$ . It is easily seen that a line cannot be drawn between any two points  $\mathbf{x}$  and  $\mathbf{y}$  in  $A$  and have every point on the line joining the points still lie in  $A$ . Thus the surface  $A$  in figure 2 is a nonconvex surface. Sometimes it is necessary to prove a surface is convex by the definition of its constituent elements. The

following alternative definition is useful for this purpose.

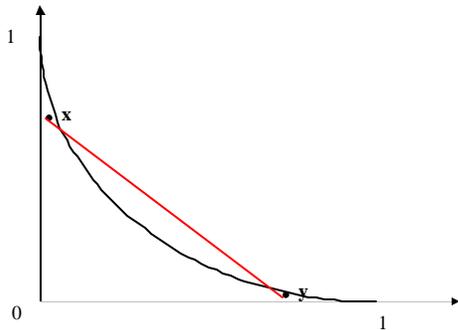


Figure 2 – A Nonconvex Set A

**Alternative Definition of Convexity:**

A function  $f(x)$  is convex if for all  $\mathbf{x}, \mathbf{y}$  and  $\lambda$  such that:  $0 \leq \lambda \leq 1$ ,

$$f(\lambda \mathbf{x} + (1 - \lambda) \mathbf{y}) \leq \lambda f(\mathbf{x}) + (1 - \lambda) f(\mathbf{y}) \quad (3)$$

The next three definitions will prepare for the appropriate definition of MIQ. Definition 2 refers to the outer surface (Convex Hull) that encloses the convex set.

**Definition 2 – Convex Hull:**

The convex cover (Convex Hull) of a convex set is what bounds the outside of the convex set. For figure 1, it is the circumference of the circle. For the cube of equation (1), the Convex Hull is the six surfaces of the cube. To define the Convex Hull more formally:

Let  $B$  be any subset of  $R^n$  and  $CH(B)$  is the convex hull of  $B$  if it contains all the convex combinations of the elements of  $B$ , i.e.

$$CH(B) = \{ \mathbf{x} : \text{there are elements } \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n \text{ in } B \text{ such that } \mathbf{x} \text{ is a convex combination of all of the } \mathbf{x}_i \text{ elements considered} \}$$

Hence the Convex Hull is the outside bounding surface of the convex set. The next definition generalizes this concept to multiple dimensions. Polytopes have many definitions,

e.g. with respect to classes of polynomials [11], with respect to matrices [12], and also with reference to general convex-compact sets [13]. Here the choice is made to use the term polytope with respect to geometric figures. For a set of points in  $R^n$  where  $n \geq 2$ , the concept of convexity is now extended to multiple dimensions.

**Definition 3 – Polytope:**

Given the subset  $A$  of  $R^n$  which is a polytope if, for any vectors  $\mathbf{x}$  and  $\mathbf{y}$  in  $A$  and scalars  $r$  and  $s$  with  $r \geq 0$  and  $s \geq 0$ ,  $r + s = 1$ , then every point  $r \mathbf{x} + s \mathbf{y}$  still remains in  $A$ . This generalizes for  $n \geq 2$  and all points can be connected in  $A$ . Figure 3 illustrates a triangle as a 2 dimensional convex set and figure 4 generalizes this result to 3 dimensions. The goal is to increase  $n$  to any number greater than 2 and triangles or geometric figures with vertices will be used in each dimension.

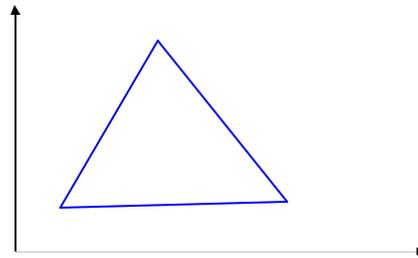


Figure 3 – A Triangle as a 2-dimensional Polytope

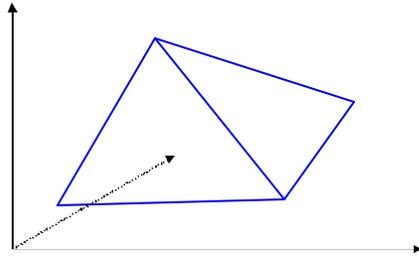


Figure 4 – A Triangle Extended as a 3-dimensional Polytope

**Definition 4 – MIQ as a Polytope:**

The prior definitions have provided some valuable tools to help in the definition of a measure of MIQ in multi-axes, as is necessary since intelligence is such a multidimensional process. There is a 3 step process in developing this methodology.

**Step 1:** Consider a minimum of 3 attributes for a 2 dimensional definition of MIQ.

**Step 2:** Generalize this result to 4 or more attributes in this 2-dimensional (planar space). In the two dimensional space, the map now extrapolates with any number of features necessary to complete the mission.

**Step 3:** The last step takes the generalization to a third or higher dimension. In all cases all the figures constructed are Convex Hulls or polytopes. Thus comparisons can always be made within any dimension involving two or more machines to be considered. To explain this better, figure 5 illustrates the Step 1 process with the 3 attributes of intelligence [2] being defined as: goals achieved (task performance), uncertainty in the environment, and sensors available. Figure 6 now

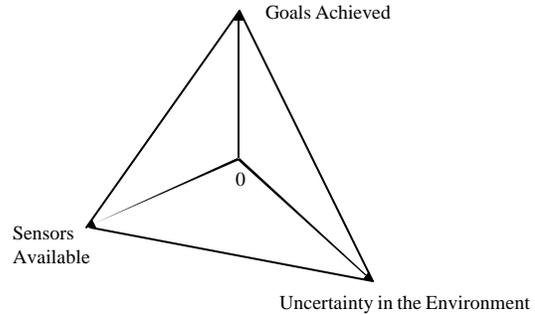


Figure 5 – First Definition of MIQ with 3 Axes (Intelligence Attributes)

extrapolates the previous figure to include a total of 5 attributes in the planar dimension with the addition of two more attributes of intelligence selected including: actuators controlled and *a priori* knowledge. Finally figure 7 generalizes to 3 dimensions with the addition of three additional intelligence attributes in the third dimension, including: accuracy level, time efficiency, and energy

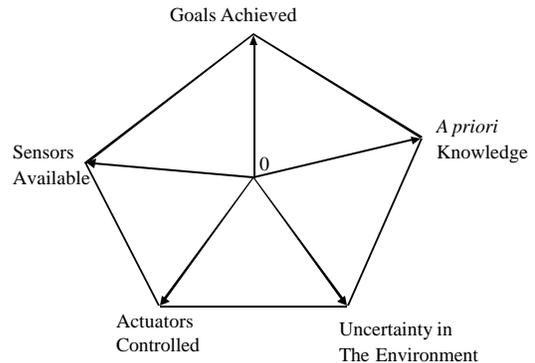


Figure 6 – Generalization of Figure 5 to now Include 5 Attributes

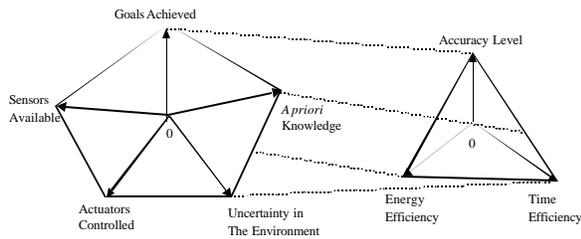


Figure 7 – Generalization of Figure 6 to 3 Dimensions With 8 Attributes.

efficiency [9]. To this point, the process has been an abstraction; in the next section a comparison is made of relative examples to illustrate how to use this methodology.

### Methodology to Compare Across Machines

To illustrate how to use the methodology, four examples are considered with (presumed) increasing levels of intelligence (machine or nonmachine). They include:

- (1) A toaster.
- (2) A washing machine with fuzzy logic to detect quality of cleaning.
- (3) An insect (ant).
- (4) A human operator.

Due to the complexity of representation, figure 8 portrays a comparison of the washing machine with fuzzy logic to the toaster using the simplified planar representation

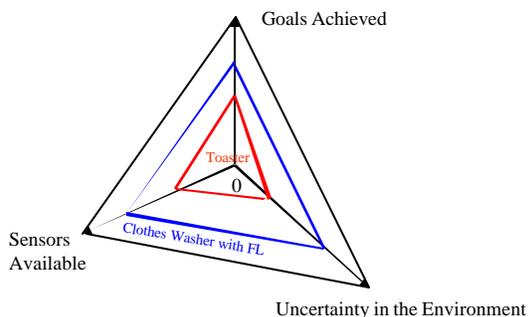


Figure 8 – Comparison of a Toaster and Clothes Washer With Fuzzy Logic

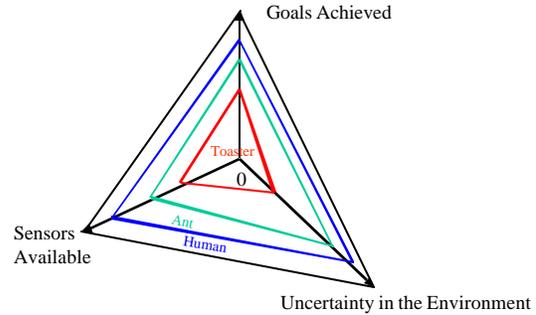


Figure 9 – Comparison of a Human, Ant and Toaster

introduced in figure 5. Obviously the more intelligent machine is further displaced from the origin and due to the convexity of the polytope, it is seen that, in general, the fuzzy logic system appears to have greater machine intelligence (area measure). In figure 9, the evaluation of MIQ is now made between the mixture of living things and machines. The comparison involves a human, an ant, and a toaster. Here the relative hierarchy is specified by the amount of area or volume contained in each polytope. Thus the intelligence measure is very relative (not absolute) to compare across living things and machines. To summarize the results so far, the following paradigm is suggested on how to synthesize this MIQ metric:

### Steps in Synthesizing the MIQ Paradigm:

- (i) For the specific mission, define the axes of the polytope to be relevant to the performance of the mission under consideration (e.g. a toaster cannot clean a rug, nor can a washing machine toast a piece of bread).
- (ii) Define the scales of each axes of the polytope relevant to the mission of interest.
- (iii) Plot alternative machines on the same axes.
- (iv) The hypervolume resulting will provide a relative (not absolute) comparison of the efficacy of a particular machine to perform certain missions.

Recall there is no absolute standard (however, an existing machine could be a

baseline for comparison purposes) and, at best, the relevance of each machine to perform a specific mission can be better understood via this procedure.

### III. Summary and Conclusions

Using properties such as convexity and relative measures of machine intelligence, the effectiveness to perform specific missions under various conditions can be determined. It is difficult to obtain an absolute measure of MIQ but by comparison to baseline or existing machines in use, there is some value in the relative comparison. The results can be extended to any level of complexity by considering convex polytopes in a multiple dimensional space.

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