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Abstract: Fragility curves are becoming increasingly common components of flood risk assessments. This report introduces the concept of the fragility curve and shows how fragility curves are related to more familiar reliability concepts, such as the deterministic factor of safety and the relative reliability index. Examples of fragility curves are identified in the literature on structures and risk assessment to identify what methods have been used to develop fragility curves in practice. Four basic approaches are identified: judgmental, empirical, hybrid, and analytical. Analytical approaches are, by far, the most common method encountered in the literature. This group of methods is further decomposed based on whether the limit state equation is an explicit function or an implicit function and on whether the probability of failure is obtained using analytical solution methods or numerical solution methods. Advantages and disadvantages of the various approaches are considered.

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Preface

This study of methods used in estimating fragility curves was sponsored by the U.S. Army Corps of Engineers' (USACE) Water Resources Infrastructure (WRI) research area.

This investigation was conducted during the period March to December 2009. Principal investigator for the study was Dr. Martin T. Schultz, Environmental Laboratory (EL), U.S. Army Engineer Research and Development Center (ERDC). The report was prepared by Dr. Schultz, along with Ben P. Gouldby and Jonathan D. Simm, HR Wallingford, Ltd., and Dr. Johannes L. Wibowo, ERDC Geotechnical and Structures Laboratory (GSL).

The USACE International Research Office provided support for the participation of Gouldby and Simm in the *Workshop on Reliability and Fragility Curves*, which was conducted during the course of this research.

Direction for this research was provided by Dr. Michael K. Sharp, ERDC Technical Director for Water Resources Infrastructure; Dr. Maureen K. Corcoran, Assistant Director for WRI; Dr. Monte L. Pearson, Chief, Geotechnical Engineering and Geosciences Branch; Bartley P. Durst, Chief, Geosciences and Structures Division; and Dr. David W. Pittman, Laboratory Director (all of GSL); along with Warren P. Lorentz, Chief, Environmental Risk Assessment Branch; Dr. Richard E. Price, Chief, Environmental Processes and Engineering Division; and Dr. Elizabeth Fleming, Laboratory Director (all of EL).

COL Gary E. Johnston was Commander and Executive Director of ERDC. Dr. Jeffery P. Holland was Director.

Summary

Fragility curves are increasingly common components of risk assessments. They appear to have received the most attention in the seismic risk assessment literature, but are becoming more widely used in flood risk assessment. The introduction of fragility curve concepts into U.S. Army Corps of Engineers (USACE) guidance can be traced back to a 1991 Policy Guidance Memorandum, which suggested that a function similar to a fragility curve could be used in estimating the economic benefits of flood protection (USACE 1991). Since then, probabilistic reliability assessment methods and fragility curves have been addressed in several USACE guidance documents (e.g., USACE 1997; USACE 1999) and have been used in flood risk assessments and in engineering studies (e.g., IPET 2009; Ebeling 2008). Since first introduced, methods used in developing fragility curves for flood protection infrastructure have evolved and show evidence of converging with the methods used in other risk assessment fields. However, for the most part, the quality and sophistication of the methods used to develop fragility curves for geostructures lags that used in other fields. There is a great opportunity to apply what has been learned in these other fields of study.

This report reviews the literature on structures and risk assessment to identify practical examples of fragility curves. The literature on seismic risk assessment offers the largest number of relevant examples. Approaches to developing fragility curves can be classified as judgmental, empirical, analytical, or hybrid. Judgmental fragility curves are based on expert opinion or engineering judgment. Empirical fragility curves are based on observational data obtained through natural or scientific experiments. Analytical fragility curves are based on models. Hybrid fragility curves employ two or more of these three approaches. Both advantages and disadvantages are associated with each approach, and no one approach will satisfy all purposes. In selecting an approach, one considers what raw materials are available in terms of data and models, how well the failure modes of interest are understood, and the time and funding that are available.

Analytical approaches are the most common in the peer-reviewed literature. Examples can be classified based on the expression of the limit state

equation and based on what solution methods are used to estimate the probability of failure. The limit state equation may be either an explicit function or an implicit function. The failure probability may be obtained using either analytical or numerical solution methods. Analytical solution methods include (1) first-order second-moment analysis, (2) first-order reliability method, and (3) second-order reliability method. Numerical solution methods include (1) Monte Carlo simulation and (2) the response surface method. There are many variations on these basic analytical approaches. In terms of an overall trend in what analytical methods are being used to develop fragility curves, numerical solution methods are gaining prominence over analytical solution methods. This is attributed to a need to overcome simplifying assumptions of the analytical approaches and the decreasing costs of computational work.

1 Introduction

Probabilistic risk assessment methods have been evolving in the U.S. Army Corps of Engineers (USACE) for more than three decades. These methods continue to evolve as the understanding of risk analysis concepts increases and the computing power required to carry out the required calculations becomes ever more widely available. In the course of this evolution, fragility curves have become integral components of probabilistic risk assessments. Fragility curves describe how the reliability of a structure changes over the range of loading conditions to which that structure might be exposed. The primary objectives of this report are to introduce the concept of the fragility curve and show how the fragility curve is related to more familiar reliability concepts, such as the deterministic factor of safety and the relative reliability index. This report reviews examples of fragility curves and describes the advantages and disadvantages of the various methods that have been used to characterize fragility.

Probabilistic reliability concepts have appeared in the water resources literature for many years. These concepts have been used to address issues in water supply and reservoir management (Hashimoto et al. 1982; Burn et al. 1991), water quality (Tung 1990; Maier et al. 2001), flood risk management (Patev and Leggett 1995), and the design and evaluation of flood defenses (Buijs et al. 2004; Steenbergen et al. 2004; Moellmann et al. 2008; Merkel and Westrich 2008). In general, these applications have focused on point estimates of reliability rather than a full characterization of reliability over the range of loads to which a system might be exposed. Reliability assessment methods have also been discussed in the geotechnical engineering literature. For example, Christian et al. (1994) and Duncan (2000) demonstrate how a probability of structural failure can be estimated by characterizing uncertainty in the factor of safety using data that are commonly available in geotechnical engineering practice and efficient analytical approximation methods. Geotechnical practice has tended to favor the deterministic factor of safety and the relative reliability index over probabilistic approaches. Duncan (2000) attributes this to a lack of familiarity with the terms and concepts that are used in reliability analysis. However, he points out that reliability analysis can enhance a conventional analysis by providing information about uncertainty in the deterministic factors of safety.

Reliability methods were first introduced into structural engineering in the 1930s. These methods have traditionally focused on assessing system reliability in relative terms at a single design load to ensure compliance with structural design codes and standards (Ellingwood et al. 2004; Ellingwood 2008). The procedures for assessing structural reliability involve calculating a reliability index, which is interpreted as a nominal or relative probability of failure at a particular design point. This relative measure is suitable for ranking structural alternatives, but is not to be interpreted probabilistically unless some important restrictive assumptions can be satisfied (USACE 1997; Melchers 1999). Since the introduction of reliability assessment methods, much has changed. In particular, performance-based engineering design methods and risk-informed decision-making approaches require estimates of failure probabilities that can be interpreted in absolute terms (Ellingwood 2008). Fragility curves have been developed as a way of providing that information.

Fragility curves are functions that describe the conditional probability of system failure over the full range of loads to which that system might be exposed. In contrast to nominal failure probabilities estimated from reliability indices, fragility curves provide a richer, much more comprehensive perspective on system reliability because they are functions rather than points and because they are interpreted in terms of absolute probabilities rather than nominal probabilities, implying knowledge of the underlying probability distributions. Simm et al. (2009) note that, in moving toward probabilistic thinking, it is important not to confuse traditional representative load and strength information with mean values in probabilistic analysis. Traditional representative load and strength information is already adjusted to account for uncertainty and variability in mean values whereas, in probabilistic analysis, the objective is to represent the actual mean value of load or strength.

This report is primarily concerned with understanding what methods might be employed to construct fragility curves for the purpose of evaluating the design of earthen levee structures and informing flood risk assessments. This task is approached by reviewing examples of fragility curves that have been developed in the literature. Methods of constructing fragility curves appear to be most highly evolved in the literature on seismic risk assessment. This is perhaps because fragility curves were first introduced for conducting seismic risk assessments at nuclear power plants (Kennedy et al. 1980; Kaplan et al. 1983). Therefore, while this report is concerned with how fragility curves might be developed for earthen levees that provide flood protection, many examples considered in this review come from the seismic risk assessment literature and address other types of structures, primarily buildings and bridges. In examining what methods are used in those fields, the objective is to learn what methods might be appropriate for earthen levees.

The USACE has made progress on developing methods to characterize fragility curves for water resources infrastructure and use these in risk assessment. For example, the Interagency Performance Evaluation Task Force (IPET) developed fragility curves for levees and floodwalls using first-order approximation methods and empirical data on failure rates during Hurricane Katrina (IPET 2009). Ebeling et al. (2008) used numerical methods to develop fragility curves for concrete gravity dams founded on a sloping rock base considering sliding and overturning failure modes. While the underlying models of failure mechanisms used in this example would be inappropriate for earthen levees, the overall numerical approach to the simulation of fragility curves could be used for earthen levees given an appropriate set of models. Ebeling et al. (2008) note that, within USACE, the term “system response curve” has been adopted to describe fragility curves, but this terminology has not been adopted in the technical literature.

This report is organized as follows. Chapter 2 introduces fragility curves, discusses their interpretation, and describes how they are used in risk assessment. Chapter 3 describes what approaches are used to estimate fragility curves in practice, based on a review of examples taken from the literature on structures and risk assessment. While there are many variations on any one approach, the examples can be grouped into four basic approaches: judgmental, empirical, hybrid, and analytical. Analytical approaches are, by far, the most common method encountered in the literature. This group of methods is further decomposed based on whether the limit state equation is an explicit or an implicit function and what approaches are used to estimate the probability of failure. The advantages and disadvantages of the various approaches are considered.

2 Overview of Key Concepts

This chapter of the report provides a brief introduction to uncertainty and risk and then describes the relationship between factors of safety, the reliability index, and fragility curves. The chapter concludes with a discussion of how fragility curves are used in risk assessment.

Uncertainty and risk

Uncertainty is the lack of knowledge about a quantity. Uncertainty can be described as either aleatory or epistemic. Aleatory uncertainty is attributed to natural variability over space and time or to inherent randomness. Examples of uncertain quantities that are aleatory in nature include river flow rates, rainfall amounts, and time between extreme events. Aleatory uncertainty cannot be reduced by obtaining more information; therefore, aleatory uncertainty is sometimes also known as irreducible uncertainty. Epistemic uncertainty is uncertainty attributed to a lack of knowledge. Epistemic uncertainties can, in principle, be reduced by obtaining more information, although in practice it may be very difficult, expensive, or physically impossible to do so. An example of an uncertain quantity that is epistemic in nature is the crest elevation at a particular point in a levee reach. An estimate of the crest elevation might be obtained from measurements at nearby locations, but only precise measurements at the exact location of interest will resolve that uncertainty. Uncertainty in a quantity is often a mixture of aleatory and epistemic uncertainty.

The probability of structural failure is a function of both uncertainty in the capacity and uncertainty in the demand. The capacity of a structure to withstand a load is a function of its geometry and material properties. These are fixed and can potentially be known, but it may be very difficult to evaluate them. Therefore, when evaluating the reliability of an existing structure, uncertainty in structural capacity is epistemic. If the strength of materials is also a function of environmental variables such as temperature, humidity, or moisture content, these are inherently variable and the uncertainty in structural capacity is both aleatory and epistemic. Similarly, uncertainty about what loads will be exerted on a structure can be either aleatory or epistemic. For example, hydraulic loads on earthen levees are a function of head differentials and the level of water against the structure. Because water levels are inherently variable, uncertainty in hydraulic loads is aleatory. However, if the load resulting from a head differential can only be approximated through modeling, for example, then the uncertainty is both epistemic and aleatory.

A risk is a potential outcome with an adverse consequence of uncertain severity. This definition includes the term “potential outcome” to indicate that a risk is an outcome that may or may not be realized in the future. The term “adverse consequence” is used to indicate that the potential outcome involves a loss of some sort. The term “uncertain severity” indicates there is a lack of information on how big a loss might be realized. Risks are characterized by a distribution of probabilities over the range of all possible outcomes or consequence levels. An example of a risk associated with living near water is property damage caused by flooding. Flood risk might be described either by a distribution of probabilities over potential water elevations (e.g., a stage-frequency curve) or by a distribution of probabilities over potential economic damages to structures, infrastructure, and other property caused by inundation (e.g., damage-frequency curve).

While risks are fully defined by probability distributions over consequence levels, they are often summarized in expected value terms. For example, flood risks can be summarized in terms of an expected annual damage (EAD) estimate. Expected annual damages are calculated by integrating a probability distribution describing variability (aleatory uncertainty) in potential water levels with a stage-damage relationship that describes the economic damages should that water level occur. Residual uncertainty in an estimate of EAD typically reflects epistemic sources of uncertainty that have not been resolved.

Risk assessment is the process of obtaining a distribution of probabilities over potential outcomes. This is typically accomplished through some form of systems-level modeling. For example, procedures for flood risk modeling are well developed in the USACE and have been outlined in guidance documents (e.g., USACE 1996). Where flood-prone areas are protected from flooding by structures such as levees, floodwalls, or dams, fragility curves are increasingly being used in risk assessments to describe how the probability of failure changes as the hydraulic load on the structure increases. Fragility curves can also be developed to represent the probability of failure given multiple failure modes and multiple loads. As discussed above, fragility curves are related to other reliability concepts.

Therefore, this introduction to fragility curves continues with reference to two of these concepts.

Design factor of safety

The adequacy of geotechnical structures has traditionally been evaluated using a factor of safety. A structure is adequate if it can perform its intended function satisfactorily. The design factor of safety, *FS*, is the ratio of resistance, *R* (i.e., capacity), the maximum load under which a system can perform its intended function, and the resultant stress, *S* (i.e., load or demand), placed on a system under design conditions:

$$FS = \frac{R}{S} \tag{1}$$

If $FS > 1$, a margin of safety exists. Structures are typically designed to a factor of safety greater than one to provide a margin of safety. The margin of safety, *Z*, is the difference between resistance and load:

$$Z = R - S \tag{2}$$

This function is known as a limit state equation or a performance function. If capacity exceeds demand, $Z > 0$, there is residual capacity and the system is in a survival state. If demand exceeds capacity, $Z < 0$, the system is in a failure state. The condition $Z = 0$ is the limiting state. For systems that are brittle and well understood, capacity and demand may be well known. More often than not, there is uncertainty about the capacity of a system to withstand that load. There may also be uncertainty in what load is placed on the system under design conditions.

When there is uncertainty in capacity or demand, *R* and *S* take the form of random variables, and uncertainty in these variables is described by probability distributions: $F_R(r)$ and $F_S(s)$. In the presence of uncertainty, the state of the system (failure or survival) can only be evaluated with some probability. Reliability, *r*, is the probability that the structure is in a survival state:

$$r = 1 - p_f \tag{3}$$

The term p_f is the probability of failure calculated from a joint probability density function for resistance and load:

$$p_f = p(Z \le 0) = p(FS \le 1) = \iint_{R \le S} f_{RS}(r,s)drds \tag{4}$$

If R and S are independent, as is often assumed, then $f_{RS}(r,s) = f_R(r) f_S(s)$. The safety margin, Z, is evaluated using the limit state equation. The density function is the derivative of the probability distribution function with respect to the random variable: $f_R(r) = \frac{\partial F_R(R)}{\partial r}$ and $f_S(s) = \frac{\partial F_S(s)}{\partial s}$.

Reliability index

A frequently used measure of reliability is the reliability index. Like the factor of safety, the reliability index has also been addressed in numerous USACE guidance documents (USACE 1997; 1999) and in USACE reports (Wolff et al. 2004). If capacity and demand are normally distributed, the reliability index can be calculated as the ratio of the mean and standard deviation of the safety margin:

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2 - 2\rho_{RS}\sigma_R\sigma_S}} \tag{5}$$

where μ_z and σ_z are the mean and standard deviation of the safety margin, respectively. Assuming a normal distribution for R and S, these two moments of the safety margin can be derived from the first and second moments of R and S. If R and S are uncorrelated, the denominator simplifies to $\sqrt{\sigma_R^2 + \sigma_S^2}$. The probability of failure is then calculated from the reliability index using the standard normal distribution function (Φ):

$$p_f = 1 - \Phi(\beta) = \Phi(-\beta) \tag{6}$$

This method of reliability assessment is known as the first-order second-moment (FOSM) method because the safety margin is a linear (first-order) function of capacity and demand variables, and only the first- and second-moments of the random variables are used in estimating the reliability index. If the assumptions of normality are satisfied, the probability of failure can be interpreted in absolute terms.

Figure 1a illustrates uncertainty in capacity, demand, and the resulting distribution in the safety margin. In this figure, probability distributions characterizing uncertainty in capacity and demand are used to obtain a probability distribution characterizing uncertainty in the safety margin (Figure 1b). The distribution for the safety margin has mean μ_z and standard deviation σ_z. The reliability index, β, is the ratio of μ_z and σ_z. The probability of failure, p_f, is the area under the curve to the left of zero (0) on the *x*-axis.

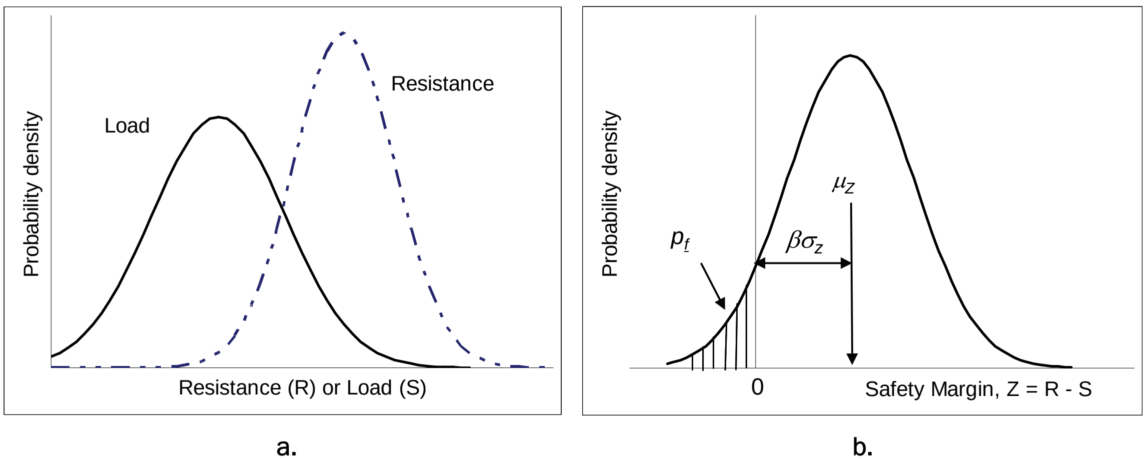

Figure 1. Reliability index. Information about uncertainty in capacity and demand (a) is used to derive a probability distribution for the safety margin (b).

If the capacity and demand random variables follow a lognormal distribution, then FOSM can again be used to calculate the reliability index and an absolute probability of failure:

$$\beta = \frac{\ln(m_R/m_S)}{\sqrt{\ln(1+V_R^2)+\ln(1+V_S^2)-2\rho_{RS}\sqrt{\ln(1+V_R^2)\ln(1+V_S^2)}}} \qquad (7)$$

The variables m_R and m_S are the medians of capacity and demand, respectively, and *V* is the coefficient of variation: $V = \sigma/\mu$. Again, if it is assumed that R and S are uncorrelated, the denominator simplifies to $\sqrt{\ln(1+V_R^2)+\ln(1+V_S^2)}$. Because a lognormal random variable becomes normally distributed when subjected to a natural-log transformation, the standard normal function can be used to compute a probability of failure using the standard normal density function.

If the distribution of capacity and demand is neither normal nor lognormal or if their distributions are unknown, the reliability index (β) can be approximated using the following equation:

$$\beta \approx \frac{\ln(\mu_R/\mu_S)}{\sqrt{V_R^2 + V_S^2}} \tag{8}$$

The approximation can be used regardless of the underlying probability distributions. However, the estimated probability of failure can only be interpreted in nominal or relative terms. The accuracy of this approach depends upon how closely the underlying distributions for capacity and demand actually follow a normal distribution, but this approach is often used in the absence of sufficient information to evaluate this condition. If the condition is not met, β serves as a nominal or relative index of reliability because it varies monotonically with the p_f. However, the probability estimate has no useful meaning in absolute terms (Melchers 1999). The reliability index is the number of standard deviations, σ_z, between the estimated mean margin of safety and the failure point

With respect to estimating the probability of structural failure, FOSM is a restrictive method because it requires assumptions about the distribution of uncertainty in system variables. Unless these assumptions can be met in practice, estimates of the probability of failure based on FOSM should be interpreted only in relative terms. In the face of complicating factors, there are a number of analytical and numerical solution methods that might be used to solve for the probability of failure. These methods, which are discussed in a variety of sources (e.g., Melchers 1999), often require a great deal more effort than the methods described above. Therefore, they should only be pursued when relative or nominal estimates are not sufficient to support decision-making. Because fragility curves are used to convey conditional probabilities that are interpreted in absolute terms, we briefly describe some of these analytical and numerical methods of solving the failure integral later in this report.

Fragility curves

Fragility curves are functions that describe the probability of failure, conditioned on the load, over the full range of loads to which a system might be exposed. Although they are closely related to the relative reliability index, they differ in several respects. In particular: (1) they are functions rather than point estimates, (2) the loads are treated deterministically, so fragility curves express a probability of failure that is conditional on the load rather than an overall probability of failure, and (3) the probabilities are generally interpreted in absolute terms. Fragility curves provide a richer and more comprehensive perspective on system reliability than nominal failure probabilities based on traditional reliability index because they convey more information about the reliability of the system.

The shape of a fragility curve describes uncertainty in the capacity of the system to withstand a load or, alternatively, uncertainty in what load will cause the system to fail. If there is little uncertainty in capacity or demand, the fragility curve will take the form of a step function, illustrated in Figure 2a. A step function has a $p_f = 0$ below the critical load and a $p_f = 1$ above the critical load. The step function communicates absolute certainty that the system will fail at a critical load and is appropriate for brittle and well-understood systems. For elastic, poorly understood, or complex systems, there is uncertainty in the capacity of the system to withstand a load. In these cases, the fragility curve takes the form of an S-shaped function, as shown in Figure 2b. The S-shaped function implies that, over a certain range of demand, the state of the system can only be evaluated with some probability. The S-shaped fragility curve is appropriate when there is uncertainty in the capacity of the system to withstand a load.

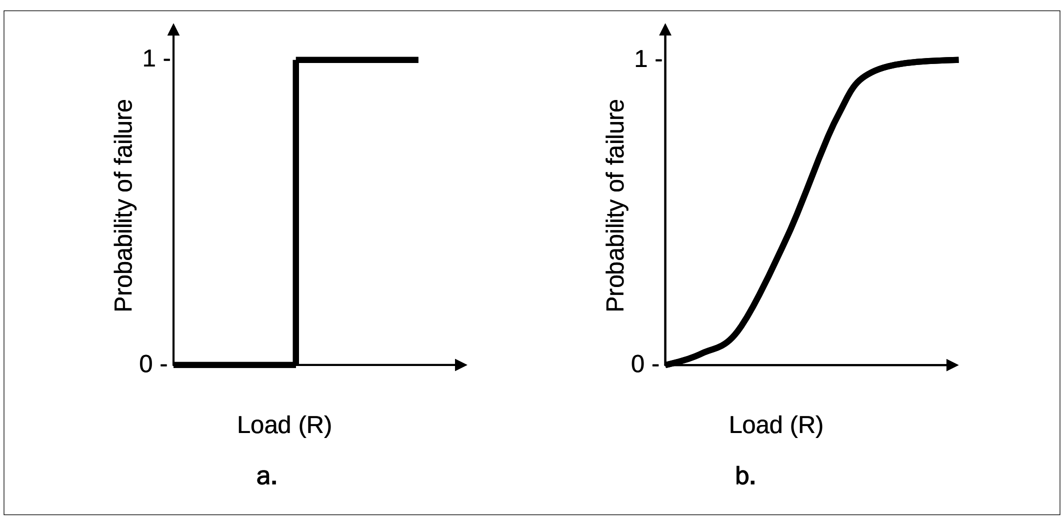

Figure 2. A conceptual fragility curve. The fragility curve is a step function (a) for very well understood or brittle systems. A fragility curve is an S-shaped function (b) for poorly understood or elastic systems.

Fragility curves can be derived from the reliability index by assuming that all of the uncertainty is in the capacity term and varying the demand parametrically. This approach is illustrated in Figure 3, which plots several fragility curves under the assumption that uncertainty in the capacity term follows a lognormal distribution. Therefore, the fragility curve also follows a lognormal distribution. This choice of a lognormal form for the fragility curve has been supported by many recent studies (Ellingwood et al. 2007). The conditional probability of failure is estimated from the following relationship:

$$p[Z \le 0 \mid S = s] = F_R(s) = \Phi(-\beta) = \Phi(\ln(s/m_R)/\sigma_{\ln R}) \quad (9)$$

The $F_R(s)$ is a cumulative distribution function that gives the probability of failure conditional on the demand that is placed on the system, $p[Z \le 0 \mid S = s]$. The variable m_R is the median of a probability distribution characterizing uncertainty in capacity and $\sigma_{\ln R} = \sqrt{1+V_R^2}$.

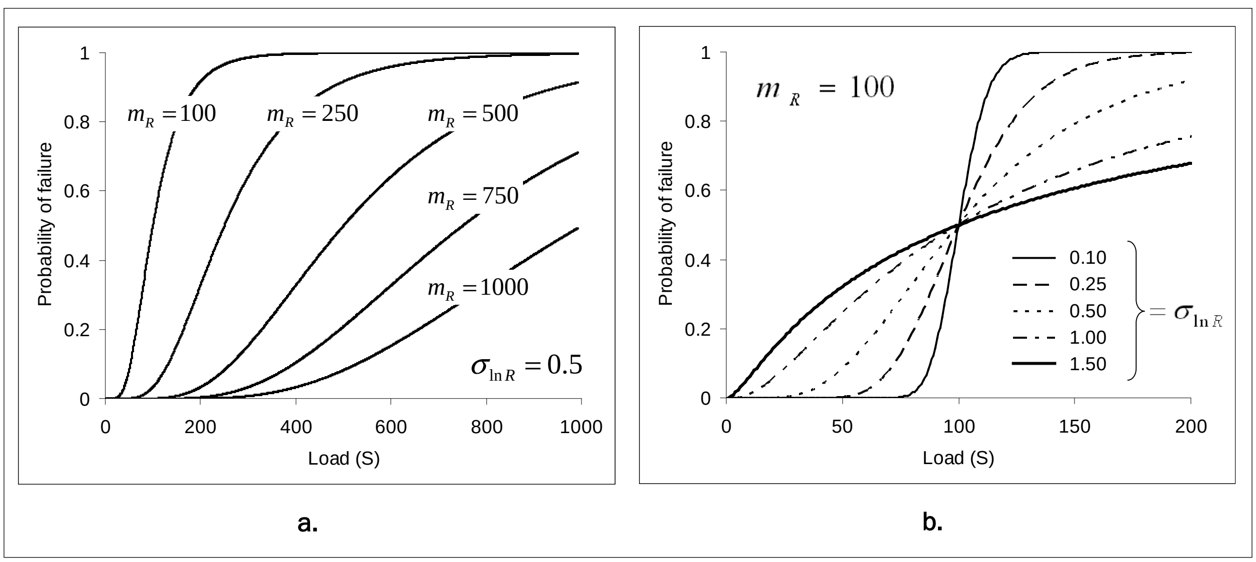

Figure 3. Examples of fragility curves derived from the reliability index. This example assumes a lognormal distribution for the capacity term. In plot a, m_R is varied from 100 to 1000 while $\sigma_{ln\,R} = 0.5$ is held constant. In plot b, m_R = 100 is held constant while $\sigma_{ln\,R}$ is varied from 0.1 to 1.5.

In Figure 3a, m_R assumes different values from 100 to 1000, while $\sigma_{\ln R}$ is held constant at 0.5. This plot shows that the probability of failure depends upon the relationship between capacity and demand. As demand increases relative to capacity, the probability of failure approaches one. When $m_R = s$, β = 0 and $p_f = 0.5$, indicating that the system has a 50%

chance of being in a failure state. Increasing m_R, the median load at which failure occurs, tends to reduce the conditional probability of system failure. Varying $\sigma_{\ln R}$ from 0.1 to 1.5 represents increasing levels of uncertainty in the capacity of the system. As $\sigma_{\ln R}$ increases, $p[Z \leq 0 \mid S = s]$ increases below m_R and decreases at loads greater than m_R. However, in each case, $F_R(s) = 0.5$. This concept is illustrated in Figure 3b, which shows realizations of fragility for different values of $\sigma_{\ln R}$ while m_R is held constant. Increasing the variance of the capacity term (i.e., increasing the amount of uncertainty in system capacity) reduces the conditional probability of failure at higher loads.

The term “failure” is a relative term that means that the capacity of a structure to provide a designated level of service has been exceeded.[1] It does not necessarily imply catastrophic failure of the structure (i.e., the structure has fallen apart). Rather, it implies that the value of the performance function is less than a predefined critical limit state. Among studies that develop fragility curves for seismic risk assessment, there appears to be a trend of considering fragility curves for multiple performance levels simultaneously. The structure is said to “fail” when it does not meet a limit state condition, which is a serviceability criterion (e.g., Ellingwood 2008). Following exposure to a seismic load, structures may take one of a number of mutually exclusive damage states: (1) fully serviceable, (2) serviceable, but impaired, (3) not serviceable, and (4) collapsed. Seismic risk assessors are typically interested in the probability that a structure will be in one of several possible damage states following exposure to a seismic load.

This discussion has shown that there is a direct linkage between the three concepts: factor of safety, reliability index, and fragility curves. The fragility curve is a more valuable characterization of system reliability than either the factor of safety or the reliability index. The factor of safety is often used deterministically to evaluate the adequacy of system under a design load, but assumes that capacity is known. The reliability index introduces the concepts of uncertainty in capacity and demand, but only provides information about reliability relative to a single design point. The fragility curve provides a characterization of system reliability over the full range of loads to which a system might be exposed. Thus, it provides more

[1] In USACE guidance documents, the term probability of unsatisfactory performance has sometimes been used instead of the probability of failure. This is to emphasize that a failure is not necessarily catastrophic (USACE 1997).

information than the reliability index. Fragility curves can be derived from the reliability index under the restrictive assumption that capacity and demand are uncorrelated and normally distributed random variables. When the distributions for capacity and demand do not satisfy these assumptions or are unknown, other methods of developing fragility curves are available.

Use of fragility curves in risk assessment

Fragility curves were specifically identified as important components of risk assessments in a National Research Council review of USACE flood risk methodology (NRC 2000). Fragility curves are important components of accurate risk assessments under the following conditions: (1) the loads (e.g., the demands) placed on a system are either variable or uncertain; (2) the capacity is uncertain because there is spatial or temporal variability in material strengths, the system is inherently elastic, or the system is poorly understood; and (3) the system is brittle, but poorly understood.

The earliest description of how fragility curves could be used in flood risk assessment is provided in a 1991 USACE Policy Guidance Letter (USACE 1991; Simm et al. 2009; Vorogushyn et al. 2009). Since then, fragility curves have become increasingly common components of risk assessments in the literature. Examples include Hall et al. (2003) and Gouldby et al. (2008), who used fragility curves in flood risk assessment models in the United Kingdom. Apel et al. (2004) used fragility curves to model flood risks on the Rhine River below Cologne, Germany. The USACE developed fragility curves for earthen levees and floodwalls to model flood risks in New Orleans (IPET 2009). Fragility curves are included as components of HAZUS-MH, the U.S. Federal Emergency Management Agency's software for estimating potential losses from floods, hurricane winds, and earthquakes. Fragility curves are also an integral component of the Hydrologic Engineering Center's Flood Damage Assessment software (HEC-FDA).

In risk assessment, potential losses, L, are uncertain. The objective is to calculate the probability that losses will exceed some potential level, l. The total probability that losses will exceed a level l, $p[L \geq l]$, could be calculated as follows:

$$p[L \geq l] = \sum_{S} p[L \geq l \mid S = s] p[Z \leq 0 \mid S = s] p[S = s] \qquad (10)$$

The term $p[S = s]$ is the probability of realizing a hazardous event, *S*, of severity, *s*. The term $p[Z \leq 0 \mid S = s]$ is the conditional probability of exceeding the capacity of the system given the occurrence of a hazard event of severity *s*. This probability is based on the fragility curve. The overall probability of system failure is $p_f = p[Z \leq 0 \mid S = s]p[S = s]$. The term $p[L \geq l \mid S = s]$ is the probability that losses, which are often a function of event severity, exceed some amount, *l*, given severity of the event. Because fragility is a probability between 0 and 1, the effect of ignoring the fragility curve in a risk assessment will be to overestimate the probability of exceeding the loss of interest, $p[L \geq l]$.

Systems-level risk models can take many forms, so there are a number of different ways that fragility curves might be incorporated into these models. Many risk assessment models conform to a source-pathway-receptor framework in which the source is the physical cause of the hazard; the receptor is the object that is potentially harmed if exposed to the hazard; and the pathway is the route by which the receptor is exposed to the hazard. When used in a source-pathway-receptor type framework, the fragility curve modifies the hazard pathway in the model.

The expense and effort needed to develop fragility curves can be justified by increased precision in the results of a risk analysis. Absent the use of fragility curves, risk models can assume that a structure never fails ($p_f = 0$) or that a structure always fails ($p_f = 1$). If a structure never fails, expected losses are underestimated. If a structure always fails, expected losses are overestimated.

3 Examples of Fragility Curves from the Literature

This chapter describes approaches to developing fragility curves based on examples in the literature. Different approaches to developing fragility curves are identified and described. Examples of fragility curves found in the literature are classified based on what approaches and methods were used in developing them. The advantages and disadvantages of the various approaches are described. Examples that involve developing fragility curves for earthen levees are discussed.

Four approaches to developing fragility curves

Approaches to developing fragility curves can be classified into four broad categories: judgmental, empirical, analytical, and hybrid (Jeong and Elnashai 2007). Judgmental approaches are based on expert opinion or engineering judgment. Empirical approaches are based on observations. Analytical approaches are based on models. Hybrid approaches combine two or more of the other approaches. Each approach differs in terms of the level of effort required to implement it and the precision that is attached to the results. However, no one approach is always best. The choice of what approach to use involves making a trade-off between cost and precision that is appropriate for the application.

Of the four approaches, analytical methods are the most common type encountered in the peer-reviewed literature. Analytical methods are further decomposed into several distinct groups based on whether the limit state equation is an explicit function or an implicit function and on whether the probability of failure is obtained using analytical solution methods or numerical solution methods. Advantages and disadvantages of the various approaches and methods are considered.

Judgmental approaches

Fragility curves that are based on some form of expert opinion are classified as judgmental. There are no limits to the number of methods that may be used to elicit judgments from experts, and these procedures can vary widely in terms of the level of rigor with which they are implemented. When relying on expert opinion, it is important to devise replicable and verifiable procedures to elicit the opinions from experts.

Judgmental approaches are often used as a last resort because of limitations in the availability of observational data and models. While the primary advantage of the judgmental approach is that it is not limited by the quantity and quality of available data, the absence of data with which to validate results is its primary disadvantage because it is difficult to qualify elicitation results.

Many potential sources of bias exist in methods based on expert judgment. For example, an expert's opinion is influenced by his individual experience, which in turn may be influenced by factors specific to the location in which the expert works (Jeong and Elnashai 2007). If such factors are known and understood in advance, it may be possible to control for these factors in an elicitation protocol. However, it may be very difficult to control for such factors because there may be an insufficient number of experts and these biases may tend to go undetected.

An early example of a judgmental fragility curve is discussed in a 1991 USACE Policy Guidance Letter (USACE 1991) and a 1993 USACE Engineer Technical Letter (USACE 1993). These letters introduced the concept of a fragility curve to characterize the reliability of existing levees for estimating the economic benefits of flood protection. However, these documents did not refer to these functions as fragility curves. The instructions for developing reliability functions were to identify two locations (elevations) on each levee slope: the probable nonfailure point (PNP) and the probable failure point (PFP). The PNP is the water elevation below which it is "highly likely" that the levee *would not* fail, defined as $p_f \cong 0.15$. The PFP is the water elevation at which it is "highly likely" that the levee *would* fail, defined as $p_f \cong 0.85$. The fragility curve is sketched by drawing a straight line between the points. Selection of PNP and PFP was to be based on knowledge of past performance, but not necessarily hard data. USACE (1993) goes further by providing a template for assessing the PNP and PFP locations on levee embankments in the absence of knowledge of past performance or the presence of material changes in field conditions since performance was observed.

Empirical approaches

Empirical fragility curves are based on observational data documenting the performance of structures under a variety of loads. Observations may be obtained systematically through controlled experiments or may be collected in an opportunistic fashion, which is uncontrolled. An advantage of systematic experiments is that structural characteristics, load-structure interactions, and structure-environment interactions can be controlled for in the observations. However, this assumes that a sufficient number of observations can be made and that the important characteristics and interactions are known in advance. Databases that contain observations that have been collected opportunistically may be more difficult to analyze statistically. Observational data tend to be highly specific to their source situations and may be sparse in the domain representing the more extreme events, which may also tend to be the events of most interest (Jeong and Elnashai 2007).

Empirical methods are the most common approach used in evaluating fragility curves for mechanical, electrical, and electronic parts because it is relatively easy to replicate specimens and test them to failure. However, the approach is generally limited to situations in which a sufficient quantity of data can be collected, and lack of data is often cited as a barrier to using the approach. Sometimes, natural events can yield a sufficient number of observational data points to meaningfully estimate fragility curves. Working independently, Shinozuka et al. (2000) and Tanaka et al. (2000) each developed fragility curves from bridge inspection data collected at bridges following the Hyogoken Nambu earthquake. Casciati et al. (2008) notes that fragility curves developed through empirical methods are also more-or-less specific to the observed structures, making it difficult to use these fragility curves to model the reliability of other structures. Fragility curves developed using an empirical approach generally cannot be confirmed through laboratory testing because it would require bringing multiple physical models to failure, an activity that he regards as too expensive and too time consuming.

Analytical approaches

Analytical fragility curves are based on structural models that characterize the performance limit state of the structure. The performance of the structure is a function of some vector of "basic" variables, $\mathbf{X}$. These variables determine both the capacity of a structure to withstand a load and the demand placed on the structure. Basic variables include material properties, geometry, or dimensions; they could also include environmental variables (such as temperature or humidity) that might in some way affect capacity. The limit state equation, also known as the performance function, can be expressed as the difference between capacity, $G_R(\boldsymbol{X})$, and demand, $G_S(\boldsymbol{X})$:

$$Z = G(R,S) = G(\mathbf{X}) = G_R(\mathbf{X}) - G_S(\mathbf{X}). \tag{11}$$

The solution space consists of three regions: $G(R,S) < 0$ is a failure state; $G(R,S) = 0$ is the limiting state; and $G(R,S) > 0$ is the survival state. Basic variables can be either random variables or deterministic variables. The probability of failure is given by integration of a multivariate density function for the n-dimensional vector of basic random variables over the failure domain, $G(\mathbf{X}) \leq 0$:

$$p_f = p[G(\mathbf{X}) \leq 0] = \int \cdots \int_{G(\mathbf{X}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{12}$$

If some of the basic random variables are deterministic, p_f is conditioned on the value of these variables. The models used in analyzing reliability can range from simple to complex, and simplified representations of more complex models are often used as substitutes for numerical models. A fragility curve is constructed by calculating a probability of failure under loads ranging from those at which failure is highly unlikely to those at which failure is almost certain.

Analytical approaches can be decomposed into four distinct groups based on whether the limit state function is an explicit function or an implicit function and whether the probability of failure is obtained using analytical solution methods or numerical solution methods. Limit state functions may be either explicit or implicit. An explicit limit state function is one that could be written explicitly in terms of basic random variables. An implicit limit state function is one that cannot be written in closed form as a function of basic variables, but is implied through a numerical model. Analytical approaches can be further decomposed into subgroups based on whether analytical or numerical solutions are used to calculate the probability of failure. Three analytical solution methods and two numerical solution methods are described herein:

- Analytical solution methods
  - First-order second-moment (FOSM) analysis
  - First-order reliability method (FORM) (a.k.a., advanced first-order second-moment (AFOSM) method)
  - Second-order reliability method (SORM)
- Numerical solution methods
  - Monte Carlo simulation (MCS)
  - Response surface method (RSM)

First-order second-moment: In the FOSM approach, basic random variables are described only by their first and second moments (mean and standard deviation). The random variables are usually assumed to be normally distributed, but their distributions are often unknown. The variables are usually transformed to the standard normal distribution, giving the limit state a standardized multivariate normal distribution. If the random variables are uncorrelated, the standardization is trivial. However, if the basic variables are correlated, an intermediate transformation is required. The Rosenblatt and Nataf transformations are often used in this regard (Rosenblatt 1952; Nataf 1962; Melchers 1999). FOSM is most convenient to implement if the limit state equation is linear (hence the name first order) and a range of approaches can be employed for this linearization. A common method is to approximate the moments of the limit state by employing a Taylor series about a convenient point (e.g., a vector of means of the random variables). Having undertaken the transformations and the linearization, the probability of failure can be obtained from standard analysis of the multivariate normal density. Patev and Leggett (1995) used this method to analyze the reliability of a reinforced concrete drainage structure.

First-order reliability method: The FORM method extends the FOSM approach to include additional information relating to probability distributions of the capacity random variables. In the FOSM method, only the first two moments of random variables are used to calculate the reliability index. If the distributions are non-normal, methods are available to transform those distributions to normal distributions. Perhaps the most commonly employed transformation is that of lognormal to normal distributions. Melchers (1999) indicates that transformations are available for other distributions as well, but their transformations may not be as straightforward as that of the lognormal distribution. Once the transformation is complete, the FORM approach proceeds by calculating the reliability index and computing the probability of failure using the standard multivariate normal distribution. FORM was used to develop the fragility curves in Figure 3.

Second-order reliability method: Both FOSM and FORM are applied in the case of a linear limit state function, which may not be a realistic portrayal of the limit state equation. The degree of approximation when using a linearized limit state equation depends upon the curvature in the underlying limit state relationship that is being modeled. If the approximation error introduced by linearization is regarded as unacceptable, second-order methods can be used to obtain an analytical solution to the probability of failure. The most common second-order approach is to fit a parabolic, quadratic, or higher order function to the limit state surface centered on the design point. However, SORM introduces additional complexity in evaluating the probability of failure. An asymptotic procedure for evaluating this probability density is described by Breitung (1984). In this procedure, the curvatures of the limit state surface are estimated at the design point. However, this approach is only suitable if the limit state function does not have a high degree of curvature (Melchers 1999). Franchin et al. (2003) used this method to develop fragility curves to evaluate the performance of reinforced concrete structures under seismic loads.

The simplifications and assumptions that are required to obtain analytical solutions to the probability of failure can limit the value of the analytical methods described above. These simplifications and assumptions are as follows: (1) the limit state function is assumed to be linear, or nearly so; (2) an explicit correlation structure must be assumed for basic variables; and (3) there are severe limits with regard to the probability distributions that can be used to characterize uncertainty in the basic random variables. Numerical methods such as Monte-Carlo simulation do not suffer from these limitations.

Monte Carlo simulation: In general, these approaches work by generating samples, or realizations, of the random variables (and dependencies) from their specified distributions and evaluating the limit state functions to determine whether failure occurs. This process is repeated many thousands of times, and the probability of failure is approximated by the fraction of failures conditional on one or more load variables. A fragility curve is constructed using this approach by varying the load parametrically if it is deterministic, or otherwise deriving the load conditions for each realization of the model(s). Monte Carlo simulation is the most general reliability method available. Efficient sampling methods such as Latin hypercube and importance sampling are sometimes used instead of Monte Carlo sampling to reduce computational effort; however, the basic approach to estimating reliability is the same. Apel et al. (2004) used a Monte Carlo simulation approach to develop fragility curves for levees. The fragility curve estimated the probability of a levee breach conditional on two independent load variables: overtopping height and overtopping duration. The authors obtained 10^4 realizations of the limit state condition for selected combinations of independent load variables and then constructed a three-dimensional failure surface.

Response Surface Method: In practical situations, the limit state equation may not be available in a closed form but is implied through a numerical model such as a finite element model of geotechnical failure, for example. The FOSM and FORM approaches are not directly applicable in this situation and, if the numerical model is computationally demanding, the Monte Carlo approach becomes impractical. The response surface method overcomes these challenges. In this method, the limit state function is evaluated at a relatively small number of points within the domain of capacity and demand variables and a function is fitted to these points using regression. Early response surfaces took the form of second-order polynomials (Bucher and Bourgund 1990). Recently, other methods of fitting functions such as neural networks have been used (Kingston et al. 2009). Once the limit state function has been approximated in this way, the FORM or Monte-Carlo simulation methods can then be employed to estimate the probability of failure. A fragility curve is constructed using the response surface approach by varying the load parametrically if it is deterministic or deriving the load conditions from each realization of the response surface. Iervolino et al. (2004) describe an alternative response surface approach. In this approach, a polynomial expansion of basic variables is fit directly to probabilities of failure estimated from simulation model outputs.

Generally, the literature suggests numerical solution methods are gaining prominence over analytical solutions that have typically assumed a lognormal distributional form for the fragility curve. The latter tend to suffer from simplifications assumed in the limit state function and the assumptions regarding independence among the random variables.

Sampling or Monte-Carlo methods do not suffer from these constraints, and increases in computational power over the past two decades have made these methods increasingly popular. To save on computation time, further developments based on simulation have been made. These approaches include stratified sampling methods such as Latin hypercube and importance sampling. If the limit state function is not available in a closed form but is implied through a numerical model of some kind, the response surface method (fitted to the values of the limit state function) has been developed. This surface can be approximated by fitting second-order polynomials or neural networks, for example. Although response surfaces reduce the computational effort, they introduce an added layer of approximation.

Hybrid approaches

A hybrid approach to developing fragility curves uses a combination of two or more of the three approaches discussed above in an attempt to overcome their various limitations (Jeong and Elnashai 2007). Empirical approaches tend to be limited by the availability of observational data; judgmental approaches tend to be limited by subjectivity of expert assessments; and analytical approaches tend to be limited by modeling deficiencies, restrictive assumptions, or computational burdens. There are many ways of implementing a hybrid approach. One approach is to construct a fragility curve using one approach over one segment of the load and a different approach over a remaining segment of the load. This approach is exemplified by fragility curves developed by IPET for modeling flood risk in New Orleans following Hurricane Katrina. The FOSM approach was used to construct fragility curves for nonovertopping water elevations, and an empirical approach was used to construct fragility curves for overtopping water elevations (IPET 2009). Another possibility is to combine fragility curves developed using judgmental or analytical approaches with observational data through Bayesian updating. For example, Singhal and Kiremidjian (1998) used observed building damage data to update analytical fragility curves for reinforced concrete frames. The Bayesian updating procedure was used to improve the robustness of the fragility curve and produced confidence bounds on estimates of the probability of failure. Jeong and Elnashai (2007) suggest calibrating analytical fragility curves to observational data as another hybrid approach.

Advantages and disadvantages of the approaches and methods

The previous discussion described and evaluated the methods for developing fragility curves. A review of the literature identified four main categories of methods: judgmental, empirical, analytical, and hybrid. Of these approaches, analytical methods comprise the largest family. Each of these methods has advantages and disadvantages. The following discussion of advantages and disadvantages is summarized in Table 1 and Table 2.

Judgment-based fragility assessments can be created with limited data so long as there is sufficient expertise or an appropriate rule-based method that can be replicated at different structures. A disadvantage of this method is that it is inherently subjective, making it difficult to verify or validate the results. In addition, it may be difficult to maintain consistency across applications of the method. Unless the procedures and logic for developing fragility curves can be documented, results are not auditable and it is difficult to resolve criticism or disagreement from other experts when defending fragility curves. Finally, there is no natural approach to improving the estimates over time. However, the method can be useful where data are limited or the consequences of the fragility curve being somewhat inaccurate are relatively unimportant. It can also be very useful as a way of helping practicing engineers to understand and check the appropriateness of fragility curves generated by other methods.

In principle, *empirical* fragility curves constructed from experimental and/or field observations are a desirable option and have the advantages that all practical details of the structure and load-structure and structure-environment interaction are taken into account in the empirical data. However, the review of the literature suggests that this approach is rarely used. The principal difficulty seems to be that it can be very difficult to get a sufficient number of samples. Data on the level of damage sustained at structures can sometimes be obtained following a natural disaster such as an earthquake, flood, or windstorm. For example, damage statistics can be collected from structures exposed to different seismic loads following an earthquake. However, many structures may be unique in terms of their design, construction, or setting within the environment (e.g., soil foundation type). Thus, a large number of data points may be needed to control for these differences. Data may also tend to be sparse in the domain representing the more extreme events, which are the ones of most interest. Finally, the applicability of empirical fragility curves tends to be limited to

Table 1. Advantages and disadvantages of approaches to developing fragility curves.

Approach	Advantages	Disadvantages
Judgmental	Not limited by data or models. Fast and cheap method if consequences of potential inaccuracy are small. Useful check on other fragility estimates.	Difficult to validate or verify. Subject to biases of experts. Not auditable. Cannot improve over time.
Empirical	Data may come from either controlled or natural experiments. Useful and flexible if data are available. Does not assume a correlation structure or a lognormal form for the fragility curve.	Data can be scarce and source-specific. Experiments can be expensive. Difficult to validate independently of the dataset. Difficult to extrapolate fragility curves to other structures.
Analytical	Based on physical models that can be validated and verified, enhancing transparency. Easier to extrapolate results to new situations. Facilitates a distinction between aleatory and epistemic uncertainty.	May be based on simplifications and assumptions. Requires the availability of data and models. More time consuming to implement. Requires a higher level of training.
Hybrid	Limitations of any particular approach can be overcome with a complementary approach. Modeling results and observations can be combined to improve the "robustness" of fragility estimates using Bayesian updating.	Limitations are the same as the individual approaches.

structures with design, material, and environmental characteristics similar to those considered in the data used to construct the original fragility curve.

Analytical approaches to developing fragility curves have many advantages over empirical and judgmental fragility curves because they are based on physical models, or at least explicit physical relationships between capacity and demand terms. Whether these models are simple or complex, they create transparency with regard to the assumptions and relationships among system components. The results are auditable and verifiable. Therefore, while not all experts may agree on the appropriateness or validity of a particular model, the sources of disagreement and

Table 2. Advantages and disadvantages of specific analytical and numerical solution methods.

Solution Method		Advantages	Disadvantages
Analytical	FOSM	Efficient and cheap method. Method is based on well-known approximations.	Requires a linear limit state equation and normal basic random variables. Assumes correlation structure and (usually) a lognormal form for the fragility curve. Result may be a rough approximation of the fragility curve due to oversimplification.
	FORM	Extends first-order approximation methods to handle non-normal basic random variables.	Transformations of variables. Assumes correlation structure and form for fragility curve.
	SORM	Extends first-order approximation methods to address nonlinear limit state equations. Can address non-normal basic random variables.	Requires approximation of limit state equation. Assumes correlation structure and form for fragility curve.
Numerical	MCS	Most general approach. Handles nonlinear limit state equation and non-normal basic variables. Correlation structure is explicit. Makes no assumptions about shape of fragility curve.	Computationally demanding, requiring many thousands of model runs. Application is limited to systems that can be adequately modeled.
	RSM	Overcomes challenges of an excessive computational burden using MCS.	Introduces an added layer of approximation. Response surface function obscures the underlying relationships among basic variables.

their importance can be discussed. Analytical approaches also provide mechanisms for addressing a wide range of uncertainties and parsing their effects in ways that distinguish between aleatory and epistemic uncertainties. The methods in the analytical category tend to be more time consuming to implement, are more data reliant (require models of failure modes, require data to populate the models), and require a higher level of training to implement.

There are advantages and disadvantages to the various solution methods used in the analytical approach. Table 2 outlines these advantages and disadvantages. Analytical solution methods yield exact solutions to the probability of failure, but require simplification of the limit state equations (e.g., linearization), strong assumptions about the how the basic random variables are distributed, and knowledge of the basic random variable correlation structure. While analytical solution methods are conceptually simple, the various transformations and other steps that may be required to apply these methods in practice are not necessarily straightforward. In addition, as discussed above, estimates of the probability of failure are valid only if the assumptions about distributions and correlations are satisfied in fact. Numerical solution methods yield approximate solutions, but do not require linearization of the limit state equations or knowledge of the correlation structure. These methods do require knowledge of the basic variable distributions, but the implementation of numerical solution methods does not require any transformation of these distributions. The main drawback of numerical methods is that realistic reliability problems tend to require a large number of model runs to implement, thus the approach tends to be limited by time and/or monetary constraints. In addition, the numerical approach assumes that well-informed models to simulate system failure exist in the first place.

The *hybrid* approach combines two or more of the above approaches (judgmental, empirical, and analytical) to compensate for the disadvantages associated with a single method. The advantages and disadvantages of these methods are the same as those of the various approaches that have been combined.

No method fits all purposes. The choice of method ultimately depends upon the characteristics of the problem and the purpose of the application. In selecting a method, it is important to consider

- How large are the potential risks?
- Do the risks justify the cost, time, and effort required to implement the method?
- Are the data, models, and other information required to carry out a fragility analysis available?
- What are the characteristics of these data and models?
- How accurate (or precise) does the fragility curve need to be to support decision making?

- What failure modes need to be addressed?
- Are there cultural preferences with regard to the methods?

While analytical fragility curves dominate the literature, there appears to be a general trend away from fragility curves developed using analytical solution methods toward fragility curves that are developed using numerical solution methods. This is attributed to the ever-decreasing cost of computing and reflects that numerical solutions often offer greater flexibility with fewer assumptions.

Classification of fragility curve examples

A review of the literature on structures and risk assessment was conducted to identify examples of fragility curves. These examples are classified in terms of the approaches and methods described herein. This review is limited to those papers that develop fragility curves. Papers that estimate reliability without defining a fragility curve were excluded from consideration. This review identified 45 examples of fragility curves in the peer-reviewed literature. While reliability methods date to the 1930s and the concept of fragility curves can be traced to 1980, most of the examples identified for this review were published within the past 3 years (since 2006). This trend is illustrated in Figure 4; however, it should be noted that this review is not at all exhaustive, and the trend may also reflect how the literature search was conducted.

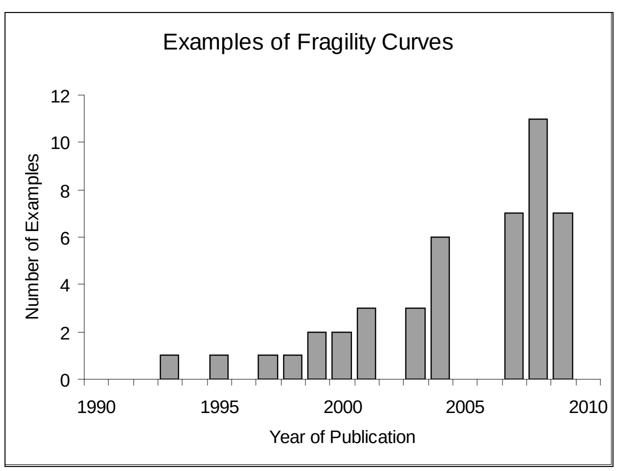

Figure 4. Number of fragility curve examples by year of publication

This study found that the analytical approaches to developing fragility curves are the most common approach and that numerical methods are exceedingly popular, perhaps because they are least restrictive in terms of assumptions and constraints on applications. Figure 5 shows the number of fragility curve examples by method, hazard domain, and structure type. The vast majority of examples (30 out of 45) come from the literature on

Method	Number
Judgmental	2
Empirical	5
Analytical	35
Hybrid	3
Total	**45**

a.

Hazard Domain	Number
Flood	12
Seismic	30
Fire	1
Wind	2
Total	**45**

b.

Structure Type	Number
Bridges	14
Buildings	15
Flood protection	12
Electrical	2
Other	2
Total	**45**

c.

Figure 5. Number of fragility curve examples by method, hazard domain, and structure type.

seismic risk assessment, which is the field in which fragility curves were first introduced (Kennedy et al. 1980). However, examples from the flood risk assessment literature are becoming much more common, particularly in Europe. Twelve studies developed fragility curves for flood protection infrastructure. This number includes dams subject to both seismic and hydraulic loads as well as natural embankments. Only three examples of fragility curves outside the flood and seismic hazard domains were identified, but this may also reflect how the literature search was conducted.

Table 3 lists the 45 examples of fragility curves considered in the course of this review. Examples are classified by approach, risk domain, and structure type. If an analytical approach was used, column three indicates whether an analytical solution method was used or a numerical solution method was used. If an analytical solution method was used, the limit state function was explicit and the method indicated is FORM or SORM. If a numerical solution method was used, the notes indicate whether the limit state function was explicit (E) or implicit (I) and whether Monte Carlo simulation (MC) or a response surface (RS) was used. If a response surface was used, Monte Carlo simulation was used on the response surface. Several of the papers employing analytical approaches used substantially different techniques and, for these papers, “Other” is entered in the column labeled “Method.” There is a great deal of variation among studies, and readers should consult the original papers for complete and accurate information on what methods were used and what hazards and structures were considered.

Table 3. Examples of studies that develop fragility curves.

No.	Paper	Approach	Method	Risk Domain	Structure Type	Description (Material, etc.)
1	Apel et al. (2004)	Analytical	E, MC	Flood	Flood protection	Earthen levees
2	Basoz and Kiremidjian (1997)[a]	Empirical	-	Seismic	Bridges	Unspecified
3	Casciati et al. (2008)	Analytical	E, MC	Seismic	Bridges	Cable-stayed bridge
4	Celik and Ellingwood (2008)	Analytical	FORM	Seismic	Buildings	Reinforced concrete frame structures
5	Choe et al. (2007)	Analytical	Other	Seismic	Bridges	Reinforced concrete columns
6	Choe et al. (2008)	Analytical	Other	Seismic	Bridges	Reinforced concrete columns with corrosion
7	Choi et al. (2004)	Analytical	FORM	Seismic	Bridges	Steel and concrete girder bridges
8	Der Kiureghian (2001)	Analytical	Other	Seismic	Electrical	Substation components: circuit breaker and transformer
9	Der Kiureghian (2001)	Analytical	Other	Seismic	Bridges	Reinforced concrete columns
10	Ebeling et al. (2008)	Analytical	E, MC	Flood	Flood protection	Concrete dams
11	Ellingwood and Tekie (2001)	Analytical	I, MC	Flood	Flood protection	Concrete dams
12	Ellingwood et al. (2004)	Analytical	FORM	Wind	Buildings	Light wood frame construction, residential
13	Ellingwood et al. (2004)	Analytical	E, MC	Seismic	Buildings	Light wood frame construction, residential
14	Ellingwood et al. (2007)	Analytical	FORM	Seismic	Buildings	Steel and reinforced concrete construction
15	Ellingwood (2008)	Analytical	FORM	Seismic	Buildings	Steel frame construction
16	Erberik and Elnashai (2004)	Analytical	I, MC	Seismic	Buildings	Concrete slab construction, residential
17	Franchin et al. (2003)	Analytical	SORM	Seismic	Buildings	Six-story, three-bay concrete frame structure
18	Gouldby et al. (2008)	Analytical	FORM	Flood	Flood protection	Various (levees, floodwalls, etc.)
19	Hall et al. (2003)	Judgment	-	Flood	Flood protection	Various (levees, floodwalls, etc.)
20	Iervolino et al. (2004)	Analytical	I, RS	Seismic	Industrial system components	Steel tank
21	IPET (2009)	Hybrid	-	Flood	Flood protection	Levees and floodwalls
22	Jeong and Elnashai (2007)	Analytical	FORM	Seismic	Buildings	Reinforced concrete frames
23	Kingston et al. (2009)	Analytical	I, RS	Flood	Flood protection	Levees

No.	Paper	Approach	Method	Risk Domain	Structure Type	Description (Material, etc.)
24	Li and Ellingwood (2007)	Analytical	FORM	Seismic	Buildings	Wood frame light construction, residential
25	Lin (2008)	Analytical	Other	Seismic	Buildings	Generic frame structures
26	Marano et al. (2008)	Analytical	Other	Seismic	Bridges	Reinforced concrete railway bridge
27	Murao and Yamazaki (1999)[b]	Empirical	-	Seismic	Buildings	-
28	Na and Shinozuka (2009)	Analytical	E, MC	Seismic	Seaport	Container terminal wharf
29	Nielson and DesRoches (2007)	Analytical	FORM	Seismic	Bridges	Multi-span concrete girder bridge
30	Padgett and DesRoches (2007)	Analytical	FORM	Seismic	Bridges	Multi-span concrete girder bridges
31	Padgett and DesRoches (2008)	Analytical	FORM	Seismic	Bridges	Multi-span concrete girder bridges
32	Pan et al. (2007)	Analytical	SORM	Seismic	Bridges	Multi-span steel girder bridge
33	Paolacci and Giannini (2009)	Analytical	Other	Seismic	Electrical	Substation component: High voltage switch
34	Patev and Leggett (1995)	Analytical	FORM	Flood	Flood protection	Reinforced concrete drainage structure
35	Rossetto and Elnashai (2003)	Empirical	-	Seismic	Buildings	Reinforced concrete
36	Shinozuka et al. (2000)	Empirical	-	Seismic	Bridges	-
37	Simm et al. (2009)	Hybrid	-	Flood	Flood protection	Various forms
38	Singhal and Kiremidjian (1998)	Hybrid	-	Seismic	Buildings	Reinforced concrete frames
39	Song and Kang (2009)	Analytical	Other	Seismic	Bridges	Multi-span steel girder bridges
40	Tanaka et al. (2000)[a]	Empirical	-	Seismic	Bridges	-
41	USACE (1993)	Judgment	-	Flood	Flood protection	Earthen levees
42	USACE (1999)	Analytical	FORM	Flood	Flood protection	Earthen levees
43	Vaidogas and Juocevičius (2008)	Analytical	E, MC	Fire	Buildings	Wood frame structure
44	van de Lindt and Dao (2009)	Analytical	I, MC	Wind	Buildings	Wood frame structure
45	Vorogushyn et al. (2009)	Analytical	E, MC	Flood	Flood protection	Earthen levees

[a] In Casciati et al. (2008).
[b] In Lin (2008).

Fragility curves for water resources infrastructure

Several of the examples described in Table 3 specifically consider earthen levees as well as other flood infrastructure. Since first introduced into flood risk assessment in 1991, there has been a noticeable evolution in the methods that are being employed to develop fragility curves. A chronological assessment of these examples reveals trends in both methods and interpretation. These trends include (1) judgmental approaches are being replaced by analytical approaches; (2) analytical approaches are being implemented with greater technical and statistical rigor; (3) numerical solution methods are gaining prominence over analytical solution methods; (4) the best approaches are explicit about the mechanisms by which failure occurs; (5) multiple failure modes are being simulated simultaneously; (6) the duration of exposure to a load is being explicitly accounted for; and (7) fragility curves are being developed to compare structural design alternatives and to assess operational reliability as well as to conduct risk assessment. The seven trends described here are revealed in the following examples.

The concept of using fragility curves for flood risk assessment was first introduced in a 1991 USACE Policy Guidance Memorandum (USACE 1991), and the methods were further developed in a 1993 USACE Engineer Technical Letter (USACE 1993). Their purpose was to support the estimation of economic benefits of flood protection by providing a method for assessing the reliability of existing levees that may not have been built to satisfy current requirements. The guidance recommended a judgmental approach that involved the identification of two elevations on the levee, one at which the levee would probably not fail ($p_f \cong 0.15$) and one at which the levee probably would fail ($p_f \cong 0.85$). The fragility curve was constructed by drawing a straight line between the points. Selection of these points was to be based on knowledge of past performance and material changes in field conditions since performance was last observed, but hard data were not necessarily required.

An Engineer Technical Letter (USACE 1997) outlined procedures for probabilistic reliability assessment in geotechnical engineering, but did not address the development of fragility curves. A later Engineer Technical Letter (USACE 1999, “Risk-based analysis in geotechnical engineering for support of planning studies”) updated the methods to be used in developing fragility curves for economic benefit assessment and superseded the earlier guidance. This guidance demonstrated how fragility curves could be developed using first-order reliability methods considering four failure modes: underseepage, through-seepage, slope stability, and surface erosion caused by excessive current velocity parallel to the slope. This document also discussed how information from multiple fragility curves could be combined into a single fragility curve.

Hall et al. (2003) developed fragility curves for national-scale flood risk assessment in the United Kingdom accounting for overtopping and breaching. In this study, the authors confronted the problem of needing to specify fragility curves for a very large number of structures. The authors approached this problem by developing generic fragility curves for 61 types of flood defenses, considering the type of structure and its condition. Fragility curves were developed for an overtopping failure mode accounting for uncertainty in what level of protection was actually provided by the defense as a result of variations in construction practices across the country and postconstruction settling rates. Fragility curves were developed for a breaching failure mode using a method similar to that proposed in USACE (1993). A combination of engineering judgment and analysis was used to develop fragility curves, but no explicit limit state equations or simulations were carried out as part of the study.

Apel et al. (2004) developed fragility curves for levees and incorporated these fragility curves into a flood risk assessment model applied to the Rhine River downstream of Cologne. The failure mechanism addressed in this study was breaching caused by overtopping and erosion. In this example, the conditional levee failure curves method described in USACE (1999) was extended by considering two independent load variables (overtopping height, d_h, and overflowing time, t_e). The limit state equation for breaching was defined as the difference between overflow (m^3/s) and a critical overflow (m^3/s). The failure mechanism was modeled at two locations using a deterministic model of “intermediate complexity.” Uncertain parameters included levee geometry and dimensions, and a nongeometric turf quality parameter. Uncertainty in these parameters was described using published values. Spatial variability in levee geometry and length effects was not considered. A Monte Carlo simulation procedure was used to generate 10^4 realizations of the model at selected combinations of the load variable values. The limit state equation was evaluated for each realization of the model. The fraction of realizations within the failure domain was determined for each combination of load variables. These conditional failure probabilities were used to construct a failure surface.

Gouldby et al. (2008) developed fragility curves for use in regional-scale flood risk management. As did Hall et al. (2003), these authors encountered the need to create fragility curves for a large number of structures. At the regional scale, there are potentially many types of flood protection infrastructure, and it is difficult to obtain the information necessary to develop fragility curves for each structure. The authors developed a database containing 600 generic fragility curves for the various types of structures using a FORM approach. The two failure modes considered in this study were piping leading to internal erosion and breaching, and breaching from overtopping and erosion on the protected side. Fragility curves were developed by characterizing uncertainty in basic random variables using expert judgment and then propagating uncertainty to the capacity term using standard methods. A reliability index was calculated treating the demand variable as deterministic, and a fragility curve was plotted varying the demand variable.

Fragility curves were developed for a flood risk assessment study in the New Orleans area following hurricane Katrina. This study was conducted by IPET operating under the auspices of the USACE (IPET 2009). Fragility curves were developed for New Orleans levees using a hybrid approach. An analytical approach was used below the crest of the levee considering a slope stability failure mode, and an empirical approach was used above the crest of the levee. The analytical portion of the fragility curve was developed for a slope stability failure mode using a FORM approach. A conditional failure probability was calculated from a reliability index using the standard normal distribution function at selected water elevations. The reliability index was based on a design factor of safety (adjusted by the study team) and coefficients of variation obtained from General Design Memoranda. The empirical portion of the fragility curve was based on observed failure rates during Hurricane Katrina. These fragility curves have been incorporated into a flood risk assessment tool for the City of New Orleans, described in Ayyub et al. (2009a; 2009b).

Simm et al. (2009) used hybrid fragility curves in a systems-level assessment of flood defenses to prioritize investments to improve flood protection in the Thames Estuary, United Kingdom. Fragility curves were developed for a set of 15 structures representing the range of levee types downstream of the existing Thames Barrier. For each exemplar structure, fragility curves were developed for five condition grades, allowing local differences by variations in crest levels, condition grade, and depth of postulated breaches. The approach used for determining which exemplar levee was most representative of the other levees was based on a comparison of basic information about the structural form of the levees (e.g., plans, sections and photographs, crest level). These authors employed The Reliability Tool (van Gelder et al. 2008), a prototype reliability calculator that is capable of generating fragility curves for foreshores, dunes and banks, embankments and revetments, walls, and point structures. The software considers 72 possible failure modes that may occur in response to hydraulic load, wave load, and lateral flow velocity. Most failure modes are represented by closed form limit state equations as described in Allsop et al. (2007). Monte Carlo simulation is used to analyze failure probabilities. The fault tree is used to calculate an overall conditional probability of failure for the structure considering the interaction among failure modes.

Kingston et al. (2009) describe a response surface method to estimate fragility curves when the failure mechanism can only be evaluated implicitly through a finite element model. In this case, the first- and second-order approaches are impossible and the more straightforward numerical methods are not practical. The method is to develop a response surface that approximates the limit state equation and reduces the computational burden associated with evaluating the fragility curve. In this study, an artificial neural network is used as an alternative to the more commonly encountered polynomial response surface function. Conditional failure probabilities are then evaluated through Monte Carlo simulation of the response surface function to obtain the fragility curve. The authors apply their method to the New Orleans 17th Street Canal floodwall.

Vorogushyn et al. (2009) describe development of fragility curves for earthen fluvial dikes considering two failure modes: piping in the dike foundation and slope stability failure caused by through-seepage. Physically based process models are used to simulate the failure mechanisms using a Monte Carlo simulation approach. Ultimate failure of the dike involves a sequence of dependent events that are taken into account using a fault tree approach. Typically, fragility curves do not account for the duration of load. These authors incorporate information about both water level and the duration of water impoundment to more realistically simulate dike failure probabilities under unsteady flood wave loadings.

The seven trends described above and highlighted in these examples are positive ones. Collectively, however, they suggest that there is increasing complexity in the theory, models, methods, and tools that are being used to confront the difficult task of characterizing system reliability. Correspondingly, the level of effort required to develop credible fragility curves is increasing. The upside of this trend is that those who use fragility curves to make decisions can use them to address a broader and more important set of decisions and can expect to have a much greater level of confidence in the outcomes of their decisions.

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Appendix A: Notes on Fragility Curve Examples Found in the Literature

This appendix provides a brief summary of the various papers that have been included in this review of the literature on estimating fragility curves. The emphasis in these notes is on describing what analytical approach was used in each study. However, no brief summary can fully describe these methods, and readers are advised to consult the original papers for authoritative summaries of these methods.

Apel et al. **(2004)**: See Chapter 3 of the main text.

Basoz and Kiremidjian (1997) developed empirical fragility curves using logistic regression and observations of bridge damage following the Northridge earthquake that struck San Francisco in 1994 (in Casciati et al. 2008).

Casciati et al. **(2008)** developed a fragility curve for the Bill Emerson Memorial Bridge, a cable-stayed bridge that crosses the Mississippi River at Cape Girardeau, Missouri. The fragility curve estimated the probability of failure from exposure to seismic loads and is developed using an analytical approach and Monte Carlo simulation considering multiple failure modes. The authors noted that performance thresholds are often uncertain and treated the performance threshold as a random variable assuming a lognormal distribution. A temporal dimension was introduced by considering multiple ground motion histories. The authors considered dependencies among different limit states, uncertainty in those limit states, and combined fragility curves for different structural members to produce a "global" fragility curve for the bridge.

Celik and Ellingwood (2008) developed fragility curves for reinforced concrete frame structures using an empirical model of the shear and bond-slip behavior of beam column joints exposed to seismic loads. Parameters of the model were calibrated to results of joint panel shear stress-strain experiments.

Choe et al. **(2007)** developed fragility curves for reinforced concrete columns using a closed form probabilistic model and demonstrated the use of Bayesian updating procedures to incorporate emerging information about system performance into the fragility curves. The authors found that the estimates were as accurate as more sophisticated mechanistic models developed from first principles. In principle, probabilistic models are applicable only within the ranges of data used to assess the models. Bayesian updating procedures make it possible to use the probabilistic models outside the range of conditions on which they have been trained. Choe et al. (2008) extended the work of Choe et al. (2007) by introducing a probabilistic time-dependent model of chloride-induced corrosion.

Choi et al. **(2004)** developed analytical fragility curves for four typical bridge types in the central and southeastern United States. Fragility curves were developed for five components of each bridge type (columns, steel bearings, expansion bearings, fixed dowels, and expansion dowels). Latin hypercube sampling was employed to obtain 100 earthquake-site-bridge samples for each bridge type considering uncertainty in seismic source, path attenuation, local soil conditions, and bridge components. A non-linear response history analysis was performed for each sample bridge, and a regression was fit to simulation results to estimate seismic demand on the structure. A damage state was assigned based on component response using predetermined damage indices. A reliability index was calculated assuming a lognormal distribution for structural capacity and seismic demand. The reliability index was calculated using the estimated seismic demand and a dispersion parameter based on the Federal Emergency Management Agency's HAZU.S. 97. The conditional failure probability is estimated using a standard normal distribution function. Fragility curves for each component were combined for each bridge type to estimate the conditional probability of failure for the bridge system as a whole.

Der Kiureghian (2001) described a Bayesian framework for structural fragility assessment that allows full use of available information, including results from mathematical model simulations, field and laboratory observations, and engineering judgment. The author demonstrated the method described in this paper through two examples. In the first example, a fragility curve is developed for an electrical substation exposed to seismic loads using performance data on substations affected by earthquakes and a purely empirical limit state equation. In this example, a purely empirical model is used because no mechanistic model is available to represent the limit state function. In the second example, a fragility curve is developed for reinforced concrete bridge columns exposed to seismic loads using a limit state equation that is based on simplified mechanics. In this example, the author's methods facilitate distinctions between aleatory and epistemic uncertainties in estimating the fragility curves.

Ebeling et al. **(2008)** developed fragility curves for concrete gravity dams considering uncertainty in strength, uplift parameters, silt-induced earth pressure, and post-tensioned anchor forces. These authors implemented a Latin Hypercube sampling program (DakotaLHS) to generate correlated random samples for the uncertain parameters. Uncertain parameters included effective cohesion, internal friction angle, earth pressure coefficient, allowable load, and uplift pressure. The samples were used to generate many realizations of an equilibrium system of equations, the outputs of which were used to develop fragility curves for sliding and overturning failure modes. The authors noted the use of the term "system response curve" to refer to fragility curves within the U.S. Army Corps of Engineers.

Ellingwood and Tekie (2001) developed fragility curves for a concrete gravity dam (Bluestone Dam, Hinton, WV) using a finite element model and Monte Carlo simulation techniques.

Ellingwood et al. **(2004)** developed fragility curves for lightweight wood frame construction (e.g., residential structures) considering both hurricane winds and seismic loads. The authors considered various roof configurations and construction practices, including roof type, slope, roof height, nailing pattern, connector type, and truss spacing. The authors developed performance goals and identified limit states by reviewing the performance of residential construction with respect to the subject loads. Fragility curves for hurricane winds were developed using first-order second-moment methods assuming the fragility curve followed a lognormal distribution. An explicit limit state equation was defined as a function of basic random variables, and parameters of the limit state equation were based on a variety of independent sources. Fragility curves for seismic loads were then obtained by fitting curves to Monte Carlo simulation results.

Ellingwood et al. **(2007)** considered the development of fragility curves for buildings in the central and eastern United States, where buildings are subject to levels of seismic loadings that range from low to moderate. The authors assumed that the fragility curve followed a lognormal distribution and used a FORM approach to estimate the fragility curve, accounting for both aleatory and epistemic sources of uncertainty in structural capacity. The aleatory uncertainty was taken to be a function of seismic demand uncertainty (uncertainty in the effect that ground motion would have on the structure) and uncertainty in structural capacity to withstand the load. Epistemic uncertainty was based on the accuracy of predicting the response of the structure to seismic loads.

Ellingwood (2008) derived fragility curves for a six-story steel frame construction using a FOSM approach. Numerical simulations were required to characterize uncertainty in the capacity term. The author derived the fragility curve using the equation for the reliability index assuming that inherent randomness in the seismic capacity of the structure follows a lognormal probability distribution. The author did not address epistemic uncertainty in the capacity or uncertainty in the seismic load. This paper also discussed the practical application of probability based engineering design.

Franchin et al. **(2003)** used a response surface approach to develop a fragility curve for a six-story three-bay reinforced concrete frame structure. In this example, the limit state function was available only in an algorithmic form, complicating the application of analytical reliability assessment techniques. A response surface approach was used to overcome the lack of an explicit limit state equation. The response surface, which often takes the form of a second-order polynomial expansion, is fit to a statistically rigorous sample of model inputs and outputs to obtain an expression that approximates the limit state equation. Reliability analysis is then performed using both FORM and Monte Carlo simulation of the response surface.

Gouldby et al. **(2008)**: See Chapter 3 of the main text.

Hall et al. **(2003)**: See Chapter 3 of the main text.

Iervolino et al. **(2004)** developed a procedure for seismic vulnerability assessment of standardized industrial constructions in a probabilistic framework covering a range of components. Shell elephant foot buckling of steel tanks was considered in this example. Both seismic capacity and demand were considered probabilistically, with the latter assessed through a dynamic analysis. The authors described a response surface approach in which a conditional probability of failure was estimated from simulation model outputs, and a response surface (which is a polynomial function of basic variables) was fit to the conditional probability estimate.

IPET (2009): See Chapter 3 of the main text.

Jeong and Elnashai (2007) proposed developing a generic set of fragility curves as a means to avoid the need to conduct new rounds of simulation each time a structure or an element of that structure is modified or replaced in the course of design. A response surface method was used to develop the generic fragility curve. The response function was fit to basic variables affecting the shape of the fragility curve (stiffness, strength, and ductility). A previously compiled database consisting of pre-run "inelastic" response analyses over a wide range of values for basic variables was used to generate fragility curves. In this procedure, the first two moments of the system response quantities were estimated as a function of the demand variable, and the fragility curves were then derived using first-order methods. The authors validated their approximations by comparing fragility curves generated by the response surface method with numerically estimated fragility curves for example bridges and structures. The advantage of this approach is that it reduces the time and effort required to generate fragility curves for various limit states.

Kingston et al. **(2009, *under review*)**: See Chapter 3 of the main text.

Li and Ellingwood (2007) developed fragility curves for residential light-frame wood construction exposed to seismic loads. The authors estimated structural deformation (demand) as a function of spectral acceleration (i.e., earth movement) by fitting a regression equation to outputs of a finite element model simulating the response of structural and geotechnical systems subjected to seismic loads. The scatter around the mean prediction is interpreted as the aleatory uncertainty in structural demand due to random features of the ground motion ensemble and is assumed to have a lognormal distribution. The conditional probability that demand exceeds a critical capacity given a spectral acceleration was then derived using the first-order reliability methods.

Lin (2008) developed fragility curves for frame structures exposed to seismic loads using a novel statistical approach to modeling the capacity of structures. He represented the structures using multiple degree-of-freedom models and modeled the response of structures using a stationary

Gaussian random process. Using random vibration theory, the author then estimated the peak story drift of buildings and extended that solution to estimate both elastic and inelastic responses. In this paper, the author used this approach to construct fragility curves for a hypothetical four-story structure at four damage states ranging from slight structural damage to complete collapse.

Marano et al. **(2008)** developed analytical fragility curves for a railway bridge in Bari, Italy (the Viaduct of Corso Italia).

Na and Shinozuka (2009) developed analytical fragility curves for a seaport container wharf. The authors focused their analysis on the concrete walls of the wharf because permanent displacement of these walls would undermine the usefulness of the seaport. Other components of a seaport system are readily replaced or repaired. The authors used "pre-determined fragility curves" to estimate the probability of displacement at each berth and carried out a Monte Carlo simulation to estimate a fragility curve for the container terminal as a whole. The probability of failure for the container terminal was computed as the fraction of samples that exceeded the limit state. The authors used their results to estimate the economic cost of exposure to seismic loads and the potential economic benefits of retrofitting the walls of shipping berths.

Nielson and DesRoches (2007) developed fragility curves for bridge systems accounting for the fragility of individual bridge components. The approach used by these authors is similar to that used for many other seismic reliability analyses (e.g., Ellingwood et al. 2007). The capacity of the structure was estimated by sampling from uncertain material properties using Monte Carlo simulation. The probability of failure was estimated using the first-order reliability method assuming that capacity and demand follow a lognormal distribution. These authors found that the bridge as a system is more fragile than any one of its individual components. The authors tested the assumption that bridge columns can be used to represent an entire bridge system and found that this assumption resulted in errors as large as 50%.

Padgett and DesRoches (2007) addressed the need to have reliable fragility curves for a portfolio of multispan continuous concrete girder bridges. Bridge types were classified into groups based on features of design and construction, and fragility curves were developed for each class of bridge rather than for each bridge. Fragility curves for each class of bridge were used in systems-level risk assessments, such as those described by IPET (2009), Gouldby et al. (2008), and Hall et al. (2003). The classification of structures based on features of design and construction introduced a new source of uncertainty because structures within a given class may vary in terms of features that were not used in classification. For example, in this particular study, the bridge classification method did not account for differences in the geometry of bridges. These authors concluded that, with respect to seismic risk assessment, uncertainty in parameters describing material properties, which have traditionally been considered in fragility analyses, tended to be less important than uncertainty in the ground motion and geometry.

Padgett and DesRoches (2008) developed a methodology to compare retrofit alternatives for multispan continuous concrete girder bridges using fragility curves. Retrofits may have positive or negative effects on overall bridge reliability depending upon how they affect other bridge components.

Pan et al. **(2007)** developed a fragility curve for a multispan steel girder bridge characteristic of those found in the State of New York and evaluated the probability of bridge failure given exposure to seismic loads. These fragility curves accounted for uncertainty in estimating material strength, bridge mass, friction coefficient of expansion bearings, and expansion joint gap size. The authors compared the first-order and second-order reliability methods and concluded that second-order methods effectively reduced the confidence bands on estimated fragilities.

Paolacci and Giannini (2009) introduced a response surface approach to developing fragility curves for a high-voltage vertical disconnect switch, a component of electrical substations. The authors' method reduced the number of simulations that must be completed by fitting a response surface to the calculated reliability index so that the reliability index was a function of basic variables that have significant influence on the response of the structure to seismic loads. The authors tested and validated their response surface approach by comparing the fragility curves to Monte Carlo simulations.

Patev and Leggett (1995) estimated the reliability of an aged and visibly deteriorated reinforced concrete drainage structure constructed through an earthen levee. Although a full fragility curve was not developed, reliability was estimated for 10-, 50-, and 100-year flood events to reveal three points on the fragility curve. Modeling the exterior concrete wall as a concrete beam, the authors analyzed uncertainty in capacity in both moment and shear. Reliability estimates obtained using FOSM were compared with Monte Carlo simulations to demonstrate the adequacy of first-order approximations in this example.

Rossetto and Elnashai (2003) developed empirical fragility curves for European reinforced concrete buildings. The authors compiled 99 existing databases that were developed during post-earthquake damage assessments at approximately 340,000 reinforced concrete structures following 19 earthquakes. The authors developed a common damage scale to interpret the various databases, which differed in terms of how post-earthquake damage was described. The authors used these databases and the damage scale to develop fragility curves for seismic risk assessment of reinforced concrete buildings. The authors concluded that empirically determined fragility curves are not sufficient for accurate seismic risk assessments.

Shinozuka et al. **(2000)** developed empirical and analytical fragility curves for bridges. Empirical fragility curves were developed using bridge damage data obtained during bridge inspections following the 1995 Hyogoken-Nanbu earthquake. Analytical fragility curves are developed for bridges in the Memphis area using nonlinear dynamic analysis. In both cases, a two-parameter lognormal distribution is assumed for the fragility curve with the parameters obtained using the method of maximum likelihood.

Simm et al. **(2009)**: See Chapter 3 of the main text.

Singhal and Kiremidjian (1998) presented methods for updating analytical fragility curves based on information obtained through field observations and estimating confidence bounds on fragility curves. The authors developed analytical fragility curves for reinforced concrete frame structures using Monte Carlo simulation procedures and updated these fragility curves using observations of building response to seismic loads following the Northridge earthquake in Los Angeles, California. This method represented a hybrid approach because the analytical fragilities were modified using field observations.

Song and Kang (2009) proposed a matrix-based system reliability method to develop fragility curves for multispan steel girder bridges. They compared their results with fragility curves obtained using a Monte Carlo simulation approach with favorable results.

Tanaka et al. **(2000)** developed empirical fragility curves by fitting a two-parameter normal distribution to bridge damage data collected following the Hyogoken-Nanbu earthquake in Kobe, Japan. The database consisted of damage observations at 3683 bridges. Damage at each bridge was classified under one of five damage states.

USACE (1991) and USACE (1993): A USACE Policy Guidance Letter (PGL 26, USACE 1991) and a 1993 USACE Technical Letter (ETL 1110-2-328, USACE 1993) proposed using judgmental fragility curves to characterize the reliability of existing levees for the purpose of estimating the economic benefits of flood protection. The procedures were to identify a probable nonfailure point (PNP) at the elevation on the embankment below which it is "highly likely" that the levee *would not* fail, defined as $p_f \cong 0.15$, and the probable failure point (PFP) at the elevation on the embankment at which it is highly likely that the levee *would* fail, defined as $p_f \cong 0.85$. A straight line is drawn between the points. Selection of the PNP and the PFP was to be based on knowledge of past performance, but not necessarily on hard data. USACE (1993) goes further by providing a template for assessing the PNP and PFP on levee embankments where information about past performance is unknown or where material changes in field conditions have occurred since performance was last observed.

USACE (1999): A subsequent USACE Technical Letter (ETL 1110-2-556) updated previous guidance on how fragility curves should be developed for earthen levees. The guidance presented several "first-cut" examples illustrating how fragility curves could be developed for selected failure modes. A first-order second-moment approach was emphasized. Failure modes discussed in the ETL include underseepage, through-seepage, slope stability, and surface erosion caused by excessive current velocity parallel to the slope. The ETL also described how a composite fragility curve could be obtained using de Morgan's law: An overall probability of failure,

$p(f \mid S = s)$, conditioned on demand $S = s$, was calculated from the probabilities of failure for individual failure modes i = (1, 2, 3,....N), which were also conditioned on demand:
$p(Z \leq 0 \mid S = s) = 1 - \prod_{i=1}^{N}(1 - p(Z_i \leq 0 \mid S = s))$. Application of this function assumes independence of the failure modes. In other studies, fault trees have been used to address nonindependent failure modes.

Vaidogas and Juocevičius (2008) presented a method of estimating fragility curves for timber frame structures considering three failure criteria (loss of load bearing capacity, insulation, and integrity). A Monte Carlo simulation approach was used to develop the fragility curve.

van de Lindt and Dao (2009) presented a fragility curve for wood frame structures exposed to wind loads. The uplift capacity for panels having different nailing patterns and truss spacing was simulated using a detailed, nonlinear finite element model. The estimated capacity was fit to a statistical distribution to develop a closed-form limit state function. The probability of failure was estimated by computing the reliability index and using first-order reliability methods.

Vorogushyn et al. **(2009)**: See Chapter 3 of the main text.

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14. ABSTRACT

Fragility curves are becoming increasingly common components of flood risk assessments. This report introduces the concept of the fragility curve and shows how fragility curves are related to more familiar reliability concepts, such as the deterministic factor of safety and the relative reliability index. Examples of fragility curves are identified in the literature on structures and risk assessment to identify what methods have been used to develop fragility curves in practice. Four basic approaches are identified: judgmental, empirical, hybrid, and analytical. Analytical approaches are, by far, the most common method encountered in the literature. This group of methods is further decomposed based on whether the limit state equation is an explicit function or an implicit function and on whether the probability of failure is obtained using analytical solution methods or numerical solution methods. Advantages and disadvantages of the various approaches are considered.

15. SUBJECT TERMS

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