Diffusion Map Kernel Analysis for Target Classification

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Abstract— Given a high dimensional dataset, one would like to be able to represent this data using fewer parameters while preserving relevant information, previously this was done with principal component analysis, factor analysis, or feature selection. However, if we assume the original data actually exists on a lower dimensional manifold embedded in a high dimensional feature space, then recently popularized approaches based in graph-theory and differential geometry allow us to learn the underlying manifold that generates the data. One such manifold-learning technique, called Diffusion Maps, is said to preserve the local proximity between data points by first constructing a representation for the underlying manifold. This work examines binary target classification problems using Diffusion Maps to embed the data with various kernel representations for the diffusion parameter. Results demonstrate that specific kernels are well suited for Diffusion Map applications on some sonar feature sets and in general certain kernels outperform the standard Gaussian and Polynomial kernels, on several of the higher dimensional data sets including the sonar data contrasting with their performance on the lower-dimensional publically available data sets.

I. INTRODUCTION

THE central problem in high-dimensional data analysis is the trade-off between computational complexity and the resolution gained with either more features or pixels. Therefore, a typical first step in analyzing high-dimensional data is to find a lower-dimensional representation and the concise description of its underlying geometry and density. This is usually done however, with global dimension reducing techniques such as principal component analysis, and Multidimensional Scaling. These techniques in general work well with well behaved maximally variant data. What if the data is only locally correlated? Then these techniques do not provide informative embedded data. Alternatively, graph based manifold learning techniques offer to embed the data based on local relationship preservation, i.e., they generally preserve the neighborhood structure. Such techniques are Diffusion Maps [1] and [2], Local linear Embedding [3], Laplacian Eigenmaps [4], Hessian Eigenmaps [5], and Local Tangent Space Alignment[6].

In this paper we consider the manifold learning technique Diffusion Maps of Coifman et al. [1], [2] and analyze the neighborhood preserving effects of kernel selection on the resulting manifold for publicly available data sets. These effects are studied by looking at the classification results for each binary target data set in various embeddings.

A. Overview

Diffusion Maps are defined as the embedding of complex data onto a low dimensional Euclidian space, via the eigenvectors of suitably normalized random walks over the given dataset. It has been shown, both theoretically in [1] and by examples in [2] how this embedding can be used for dimensionality reduction, manifold learning, geometric analysis of complex data sets and fast simulations of stochastic dynamical systems.

Diffusion Maps are said to preserve the local proximity between data points by first constructing a graph representation for the underlying manifold. The vertices, or nodes of this graph, represent the data points, and the edges connecting the vertices, represent the similarities between adjacent nodes. If properly normalized, these edge weights can be interpreted as transition probabilities for a random walk on the graph. After representing the graph with a matrix, the spectral properties of this matrix are used to embed the data points into a lower dimensional space, and gain insight into the geometry of the dataset. It has been shown in [1] and [2] that the eigenfunctions of Markov matrices can be used to construct coordinates called Diffusion Maps that generate these efficient representations of the complex geometric structures and the associated family of diffusion distances, obtained by iterating the Markov matrix, defines the multiscale geometries that prove to be useful in the context of data parameterization and dimensionality reduction. The process of constructing these Diffusion Maps as described in [1] and [2] is discussed in sections II.B through II.E.

B. Construction of a Random Walk on the Data

Given a data set Ω with a distribution μ of the points on Ω and a kernel $k : \Omega \times \Omega \rightarrow \mathbb{R}$ that satisfies the following properties:

- k is symmetric: k(x, y) = k(y, x),
- *k* is positivity preserving: $k(x, y) \ge 0$.

This kernel represents some notion of affinity or similarity between points of Ω as it describes the relationship between pairs of points in this set and in this sense, one can think of the data points as being the nodes of a symmetric graph whose weight function is specified by *k*. The kernel constitutes an a priori presumption of the local geometry of Ω , and since a given kernel will capture a specific feature of the data set, its choice should be guided by the application

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that one has in mind; this will be discussed later.

It is known that to any reversible Markov process, one can associate a symmetric graph. In addition, the converse is also true, i.e., from the graph defined by (Ω, k) , one can construct a reversible Markov chain on Ω . This technique is known as the normalized graph Laplacian construction. The steps are as follows: define

$$d(x) = \int_{\Omega} k(x, y) d\mu(y)$$
(1)

to be a local measure of the degree of node x in this graph and define P^t to be an $n \times n$ matrix whose entries are given by

$$p_t(x,y) = \frac{k(x,y)}{d(x)}$$
(2)

which is the probability of transition from x to y in one time step. For t = 1 this can be interpreted as the first-order neighborhood structure of the graph.

C. Powers of P and Multiscale Geometric Analysis of Ω

The matrix P contains geometric information about the data set Ω . The transitions that it defines directly reflect the local geometry defined by the immediate neighbors of each node in the graph of the data. In other words, $p_1(x, y)$ represents the probability of transition in one time step from node x to node y and it is proportional to the edge-weight k(x, y). For $t \ge 0$, the probability of transition from x to y in t time steps is given by $p_t(x, y)$, the kernel of the t^{th} power P^t of P. Larger powers of P, allows the integration of the local geometry and therefore will reveal relevant geometric structures of Ω at different scales, i.e., larger neighborhoods.

D. Spectral Analysis of the Markov Chain

Powers of P constitute an object of interest for the study of the geometric structures of Ω at various scales. A classical way to describe the powers of an operator is to employ the language of spectral theory, namely eigenvectors and eigenvalues. Although for general transition matrices of Markov chains, the existence of a spectral theory is not guaranteed, the random walk constructed here exhibits very particular mathematical properties, i.e., if the graph is connected, which we now assume, then the stationary distribution is unique and we have

$$\lim_{t \to \infty} p_t(x, y) = \phi_0(y) \tag{3}$$

where the Markov chain has a stationary distribution given by

$$\phi_0(y) = \frac{d(y)}{\sum_{z \in \Omega} d(z)}.$$
(4)

The chain is reversible, i.e., it follows the detailed balance condition:

$$\phi_0(x)p_1(x,y) = \phi_0(y)p_1(y,x).$$
(5)

The vector ϕ_0 is the top left eigenvector of P. The spectral analysis of the Markov chain is governed by the following eigen-decomposition

$$p_t(x, y) = \sum_{l \ge 0} \lambda_l^t \psi_l(x) \phi_l(y), \tag{6}$$

where $\{\lambda_l\}$ is the sequence of *eigenvalues* of *P* (with $|\lambda_0| \ge |\lambda_l| \ge |\lambda_2| \ge \cdots$) and $\{\psi_l\}$ and $\{\phi_l\}$ are the corresponding biorthogonal right and left eigenvectors.

E. Diffusion Distances and Diffusion Maps

The spectral properties of the Markov chain can now be linked to the geometry of the data set Ω . As previously mentioned, the idea of defining a random walk on the data set relies on the following principle: the kernel *k* specifies the local geometry of the data and captures some geometric feature of interest. The Markov chain defines fast and slow directions of propagation, based on the values taken by the kernel, and as one runs the walk forward, the local geometry information is being propagated and accumulated the same way local transitions of a system can be integrated in order to obtain a global characterization of this system.

Running the chain forward is equivalent to computing the powers of the operator P. For this computation, we could, in theory, use the eigenvectors and eigenvalues of P. Therefore, we are going to directly employ these objects in order to characterize the geometry of the data set Ω . The family of diffusion distances $\{D_t\}_{t \in N}$ is given by

$$D_t^2(x,z) = \sum_{y \in \Omega} \frac{\left(p_t(x,y) - p_t(z,y)\right)^2}{\phi_0(y)}.$$
(7)

In other words, $D_t(x, z)$ is a functional weighted l_2 distance between the two posterior distributions $p_t(x, \cdot)$ and $p_t(z, \cdot)$. For a fixed value of t, D_t defines a distance on the set Ω . By definition, the notion of proximity that it defines reflects the connectivity in the graph of the data. Indeed, $D_t(x, z)$ will be small if there is a large number of short paths connecting xand z, that is, if there is a large probability of transition from x to z and vice versa. The main interesting features of diffusion distance are: 1) the points are closer if they are highly connected, 2) $D_t(x, z)$ involves summing over all paths and is therefore robust to noise perturbations, 3) the distance takes into account all evidence relating x and z. $D_t(x, z)$ does not have to be computed explicitly. It can be computed using the eigenvectors and eigenvalues of P:

$$D_{t}^{2}(x,z) = \sum_{l \ge 1} \lambda_{l}^{2t} (\Psi_{l}(x) - \Psi_{l}(z))^{2}.$$
(8)

As previously mentioned, the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_N$ tend to 0 and have a modulus strictly less than 1. As a consequence, the above sum can be computed to a preset accuracy $\delta > 0$ with a finite number of terms: if we define as the number of elements retained to meet this accuracy. Then, up to relative precision δ , we have

$$D_{t}(x,z) = \left(\sum_{l\geq 1}^{s(\delta,t)} \lambda_{l}^{2t} (\psi_{l}(x) - \psi_{l}(z))^{2}\right)^{\frac{1}{2}}.$$
(9)

We can therefore introduce a family of diffusion maps $\{\Psi_t\}_{t\in\mathbb{N}}$ given by

$$\Psi_{t}: x \to \begin{pmatrix} \lambda_{1}^{t} \Psi_{1}(x) \\ \lambda_{2}^{t} \Psi_{2}(x) \\ \vdots \\ \lambda_{s(\delta,t)}^{t} \Psi_{s(\delta,t)}(x) \end{pmatrix}$$
(10)

Each component of $\Psi_t(x)$ is termed diffusion coordinate. The map $\Psi_t: \Omega \to \mathbb{R}^{s(\delta,t)}$ embeds the data set into a Euclidean space of $s(\delta, t)$ dimensions. This method constitutes a universal and data driven way to represent a graph, or any generic data set, as a cloud of points in a Euclidean space. Moreover, $s(\delta, t)$ depends on the properties of the random walk and not on the number of features of the original representation.

III. KERNEL FUNCTIONS

The kernel constitutes our prior definition of the local geometry of Ω , and since a given kernel will capture a specific feature of the data set, its choice should be guided by the application that one has in mind. Below is the list of kernels used here:

• Laplacian Kernel: $k(x, y) = \exp(-\left|\left|\left|x - y\right|\right| - \mu \left| / b \right) / 2b$,

• Gaussian Kernel:
$$k(x, y) = \exp(-||x - y||^2 / 2\sigma^2)$$

• Rayleigh Kernel:
$$k(x, y) = \frac{\|x - y\| \exp(-\|x - y\|^2 / 2\sigma^2)}{\sigma^2}$$
,

• Polynomial Kernel: $k(x, y) = (1 + \langle x, y \rangle)^d$

where the Gaussian and Polynomial kernels are most familiar from support vector machines. The Laplacian and Rayleigh were introduced previously in [7].

IV. EXPERIMENTS

A. Experimental Setup

The problem here is to analyze the effects on resultant diffusion maps of certain kernel functions for the classification of select databases. Each database is divided into ten groups that are as equal as possible, 10-fold cross validation. Nine groups are set aside for the training set and one group for the dedicated testing set. This procedure is continued until all groups have represented as a testing set. The average performance overall 10-folds is presented as the probability of classification (P_c) , or sensitivity, and the probability of false alarm (P_{FA}), or specificity. This is done to demonstrate the trade-off between correctly classifying true cases versus incorrectly classifying false cases. Each kernel uses the same groups for each data set so that the possibility of poor individual performance due to the distribution of the draw is eliminated. In addition, each experiment is done ten times and the results are averaged over these runs.

A Linear Discriminant Analysis (LDA) classifier is used to evaluate the enhancement provided by the individual kernels to the diffusion map process. A LDA classifier assumes the classes have equal covariance matrices. In this case, the decision boundaries between classes is linear, and can in general be a hyperplane. The general form for LDA is

$$\delta_{k}(x) = x^{T} \Sigma^{-1} \mu_{k} - \frac{1}{2} \mu_{k}^{T} \Sigma_{k}^{-1} \mu_{k} + \log \pi_{k}$$
(11)

and the decision rule is $G(x) = \operatorname{argmax}_k \delta_k(x)$. Where the parameters are estimated from the training data as follows:

• $\pi_k = N_k / N$, where N_k is the number of class-k; observations;

•
$$\mu_k = \sum_{g_i=k} x_i / N_k$$
;
• $\hat{\Sigma} = \sum_{k=1}^{K} \sum_{g_i=k} (x_i - \mu_k) (x_i - \mu_k)^T / (N - K)$

For example, the LDA rule classifies to class 1 if

$$x^{T} \hat{\Sigma}^{-1}(\hat{\mu}_{1} - \hat{\mu}_{0}) > \frac{1}{2} \mu_{1}^{T} \hat{\Sigma}^{-1} \mu_{1} - \frac{1}{2} \mu_{0}^{T} \hat{\Sigma}^{-1} \mu_{0} + \dots$$

$$+ \log(N_{0} / N) - \log(N_{1} / N)$$
(12)

).

and class 0 otherwise.

The experimental variable values are listed below.

Experimental Variables	
$\delta = 1e-7, \ \alpha = 1, \ b = 2, \ \mu = 1, \ \sigma^2 = 3, \ d = 3.$	

Where δ is the diffusion threshold, α is the diffusion probability distribution scaling, *b* is the Laplacian kernel scaling parameter, μ is the mean for the Laplacian kernel, σ^2 is the variance for the Gaussian kernel and the square of the mode for the Rayleigh kernel, and *d* is the polynomial kernel degree.

B. Data Sets

The experiment discussed above tests the kernels and their embeddings for classification enhancement on the resulting Diffusion Maps over eight publically available data sets [8]:

• Pima Indian: **Pima Indian Diabetes** Connectionist Bench Sonar • Sonar1: Wisconsin Diagnostic Breast Cancer • WDBC: • WPBC: Wisconsin Prognostic Breast Cancer • Clev. Heart: Heart Disease Data Set. Cleveland • Wisc. BC: Wisconsin Breast Cancer Original Shallow Water Acoustic Toolset [9] • Sonar2: • Sonar3: Shallow Water Acoustic Toolset [9]

For each data set listed above, Table 1 below includes the number of samples, the class distribution, and the number of features, or attributes.

C. Results

The experimental results for the kernel effects on the resultant diffusion maps are shown below in Table 2 through Table 9. The tables are listed per database with each kernel given a column. The rows correspond to the original and reduced dimension pairs.

Table 1 shows that for the Pima Indian database the Polynomial and Gaussian kernels have a better P_C than the

Laplacian and Rayleigh kernels with a trade-off of a slightly worse P_{FA} . For the Sonar1 database, all of the kernels are fairly consistent with the Rayleigh kernel slightly outperforming, on average, the Laplacian kernel with an average P_C 72.6%. The Laplacian kernel outperforms the other three kernels on the WDBC database with an average P_C 98%, however as for an more acceptable P_{FA} the Rayleigh kernel offers a sound alternative with a decrease average P of 95.5%. This result differs from the WPBC database with the Rayleigh kernel resulting in an average P_C of 66% and all four kernels failing overall to capture the embedding appropriately.

Results for the Clev. Heart database show that the Rayleigh kernel captures the embedding with an average P_C of 77.3% and a slightly higher P_{FA} than the Gaussian kernel. For the Wisc. BC database the Gaussian kernel outperforms the other three with a P_C of 98.5% with a 0.4% increase in P_{FA} as compared to the next best Laplacian result. For the Sonar2 database the Rayleigh kernel outperforms the other three by a minimum of 13% for an average P_C of 95%. This demonstrates the superiority of this kernel to capture the embedding of this particular feature set. The performance on the Sonar3 database leaves much to be desired, however. With an average P_C of 79.4% the Rayleigh kernel demonstrates a marked improvement over the Gaussian with an average P_C of 51.7%, nevertheless the gain comes with an increased P_{FA} of 13.3%.

V. CONCLUSIONS AND FUTURE WORK

As the experiments demonstrate, the choice of kernel effects the resultant diffusion map. Overall, the Laplacian and Rayleigh kernels outperformed the standard Polynomial and Gaussian kernels on all of these databases, with a few exceptions such as the Pima Indian and Wisc. BC datasets. It appears that the Laplacian and Rayleigh kernels perform best on the higher dimensional non-Gaussian datasets and the standard kernels work well with lower-dimensional data. Therefore, for enhanced target recognition capability and an acceptable P_{FA} the Rayleigh kernel appears the appropriate choice to best capture the embedding distribution to enhance the diffusion map process.

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TABLE 1. EXPERIMENTAL DATA SETS					
Data Set	# Samples	# Class 0	# Class 1	# Attributes	
Pima Indian	768	268	500	8	
Sonar1	208	97	111	60	
WDBC	569	212	357	30	
WPBC	198	151	47	33	
Clev. Heart	303	164	139	13	
Wisc. BC	699	458	241	9	
Sonar2	22263	21154	1109	60	
Sonar3	3562	3512	50	60	

TABLE 2. EXPERIMENTAL RESULTS FOR PIMA INDIAN				
		K	ernel	
Dimension (Original,Final)	Gaussian	Laplacian	Rayleigh	Polynomial
(8,2)	PC: 0.558	PC: 0.694	PC: 0.672	PC: 0.748
	FA: 0.5187	FA: 0.3881	FA: 0.347	FA: 0.4366
(8,3)	PC: 0.704	PC: 0.704	PC: 0.672	PC: 0.714
	FA: 0.6567	FA: 0.3918	FA: 0.347	FA: 0.4067
(8,4)	PC: 0.702	PC: 0.678	PC: 0.666	PC: 0.716
	FA: 0.4813	FA: 0.3806	FA: 0.3507	FA: 0.4067
(8,5)	PC: 0.724	PC: 0.684	PC: 0.676	PC: 0.716
	FA: 0.4739	FA: 0.3246	FA: 0.3545	FA: 0.403

(8,6)	PC: 0.74	PC: 0.692	PC: 0.674	PC: 0.706
	FA: 0.4515	FA: 0.291	FA: 0.3619	FA: 0.4216
(8,7)	PC: 0.744	PC: 0.692	PC: 0.7	PC: 0.704
	FA: 0.4627	FA: 0.2873	FA: 0.306	FA: 0.3582
(8,8)	PC: 0.738	PC: 0.684	PC: 0.692	PC: 0.692
	FA: 0.4813	FA: 0.2836	FA: 0.2873	FA: 0.3358

TABLE 3. EXPERIMENTAL RESULTS FOR SONAR1					
		Ke	ernel		
Dimension (Original,Final)	Gaussian	Laplacian	Rayleigh	Polynomial	
(60,2)	PC: 0.5676	PC: 0.5856	PC: 0.6396	PC: 0.5225	
	FA: 0.2887	FA: 0.3711	FA: 0.4227	FA: 0.3505	
(60,3)	PC: 0.6126	PC: 0.7658	PC: 0.7387	PC: 0.6577	
	FA: 0.2577	FA: 0.2268	FA: 0.2577	FA: 0.2165	
(60,4)	PC: 0.7117	PC: 0.7568	PC: 0.7387	PC: 0.7568	
	FA: 0.268	FA: 0.2474	FA: 0.2887	FA: 0.2577	
(60,5)	PC: 0.7748	PC: 0.7477	PC: 0.7027	PC: 0.7297	
	FA: 0.2784	FA: 0.268	FA: 0.299	FA: 0.2577	
(60,6)	PC: 0.7477	PC: 0.7297	PC: 0.7748	PC: 0.7297	
	FA: 0.299	FA: 0.2268	FA: 0.2784	FA: 0.2371	
(60,7)	PC: 0.7477	PC: 0.7477	PC: 0.7568	PC: 0.7297	
	FA: 0.3196	FA: 0.2165	FA: 0.268	FA: 0.2268	
(60,8)	PC: 0.7477	PC: 0.7297	PC: 0.7297	PC: 0.7477	
	FA: 0.3299	FA: 0.2371	FA: 0.268	FA: 0.2371	

TABLE 4. EXPERIMENTAL RESULTS FOR WDBC					
		Kei	mel		
Dimension (Original,Final)	Gaussian	Laplacian	Rayleigh	Polynomial	
(30,2)	PC: 0.9468 FA:	PC: 0.9748	PC: 0.9496	PC: 0.9356	
	0.1462	FA: 0.1651	FA: 0.09906	FA: 0.184	
(30,3)	PC: 0.958 FA:	PC: 0.9692	PC: 0.958	PC: 0.9888	
	0.1604	FA: 0.1132	FA: 0.08491	FA: 0.2028	
(30,4)	PC: 0.9468 FA:	PC: 0.9804	PC: 0.9608	PC: 0.972	
	0.1698	FA: 0.1038	FA: 0.08962	FA: 0.1321	
(30,5)	PC: 0.958 FA:	PC: 0.9832	PC: 0.958	PC: 0.9776	
	0.1368	FA: 0.1179	FA: 0.08962	FA: 0.1179	
(30,6)	PC: 0.9524 FA: 0.09906	PC: 0.9832 FA: 0.1226	PC: 0.9496 FA: 0.09434	PC: 0.9748 FA: 0.1274	
(30,7)	PC: 0.9496 FA:	PC: 0.9888	PC: 0.944	PC: 0.9748	
	0.1085	FA: 0.09434	FA: 0.08019	FA: 0.1226	
(30,8)	PC: 0.9552 FA: 0.09434	PC: 0.9804 FA: 0.09434	PC: 0.9636 FA: 0.08019	PC: 0.9776 FA: 0.1368	

TABLE 5. EXPERIMENTAL RESULTS FOR WPBC				
		Ke	ernel	
Dimension (Original,Final)	Gaussian	Laplacian	Rayleigh	Polynomial
(33,2)	PC: 0.5532	PC: 0.5532	PC: 0.6596	PC: 0.5745
	FA: 0.3311	FA: 0.3311	FA: 0.3974	FA: 0.351
(33,3)	PC: 0.5745	PC: 0.5745	PC: 0.7021	PC: 0.5957
	FA: 0.3046	FA: 0.3377	FA: 0.3576	FA: 0.3775
(33,4)	PC: 0.5957	PC: 0.5319	PC: 0.6596	PC: 0.617
	FA: 0.3444	FA: 0.3709	FA: 0.3709	FA: 0.3377

(33.5)	PC: 0.5957	PC: 0.6383	PC: 0.6596	PC: 0.617
(55,5)	FA: 0.3245	FA: 0.351	FA: 0.351	FA: 0.3444
(33,6)	PC: 0.6383	PC: 0.617	PC: 0.6596	PC: 0.6383
	FA: 0.3311	FA: 0.3576	FA: 0.3377	FA: 0.3444
(33,7)	PC: 0.617	PC: 0.6383	PC: 0.6596	PC: 0.617
	FA: 0.3311	FA: 0.3576	FA: 0.3377	FA: 0.351
(33,8)	PC: 0.6383	PC: 0.617	PC: 0.617	PC: 0.617
	FA: 0.3113	FA: 0.3576	FA: 0.3311	FA: 0.3245

TABLE 6. EXPERIMENTAL RESULTS FOR CLEV. HEART				
		Ke	rnel	
Dimension (Original,Final)	Gaussian	Laplacian	Rayleigh	Polynomial
(13,2)	PC: 0.7266	PC: 0.7482	PC: 0.7626	PC: 0.741
	FA: 0.122	FA: 0.1951	FA: 0.2622	FA: 0.189
(13,3)	PC: 0.7266	PC: 0.7482	PC: 0.7626	PC: 0.7338
	FA: 0.128	FA: 0.189	FA: 0.2256	FA: 0.1768
(13,4)	PC: 0.7122	PC: 0.7266	PC: 0.7626	PC: 0.7194
	FA: 0.1402	FA: 0.1768	FA: 0.2134	FA: 0.1707
(13,5)	PC: 0.7194	PC: 0.7338	PC: 0.7626	PC: 0.7122
	FA: 0.1463	FA: 0.1646	FA: 0.1768	FA: 0.1646
(13,6)	PC: 0.7266	PC: 0.7554	PC: 0.7914	PC: 0.7626
	FA: 0.1341	FA: 0.1585	FA: 0.1768	FA: 0.1524
(13,7)	PC: 0.7554	PC: 0.7986	PC: 0.7842	PC: 0.7554
	FA: 0.1341	FA: 0.1646	FA: 0.1768	FA: 0.1463
(13,8)	PC: 0.741	PC: 0.7986	PC: 0.7842	PC: 0.7554
	FA: 0.1463	FA: 0.1707	FA: 0.128	FA: 0.1463

TABLE 7. EXPERIMENTAL RESULTS FOR WISC. BC					
		Ke	rnel		
Dimension (Original,Final)	Gaussian	Laplacian	Rayleigh	Polynomial	
(9,2)	PC: 0.9793	PC: 0.971	PC: 0.9585	PC: 0.9378	
	FA: 0.04367	FA: 0.03493	FA: 0.02838	FA: 0.02838	
(9,3)	PC: 0.9793	PC: 0.9668	PC: 0.9585	PC: 0.9544	
	FA: 0.03712	FA: 0.03493	FA: 0.02838	FA: 0.03275	
(9,4)	PC: 0.9876	PC: 0.971	PC: 0.9585	PC: 0.9668	
	FA: 0.03712	FA: 0.03057	FA: 0.02838	FA: 0.03275	
(9,5)	PC: 0.9876	PC: 0.9668	PC: 0.9585	PC: 0.9627	
	FA: 0.03712	FA: 0.03493	FA: 0.0262	FA: 0.03275	
(9,6)	PC: 0.9876	PC: 0.971	PC: 0.9585	PC: 0.9668	
	FA: 0.03493	FA: 0.03493	FA: 0.02838	FA: 0.03275	
(9,7)	PC: 0.9876	PC: 0.9668	PC: 0.9627	PC: 0.9668	
	FA: 0.03493	FA: 0.03493	FA: 0.02838	FA: 0.03275	
(9,8)	PC: 0.9876	PC: 0.9668	PC: 0.9627	PC: 0.9585	
	FA: 0.03493	FA: 0.03493	FA: 0.03275	FA: 0.03275	

TABLE 8. EXPERIMENTAL RESULTS FOR SONAR2								
	Kernel							
Dimension (Original,Final)	Gaussian	Laplacian	Rayleigh	Polynomial				
(60,2)	PC: 0.4593	PC: 0.7703	PC: 0.9372	PC: 0.5458				
	FA: 0.03258	FA: 0.02744	FA: 0.09654	FA: 0.02247				
(60,3)	PC: 0.7013	PC: 0.7868	PC: 0.9434	PC: 0.7384				
	FA: 0.0269	FA: 0.02901	FA: 0.09768	FA: 0.03004				
(60,4)	PC: 0.7425	PC: 0.8033	PC: 0.9547	PC: 0.7714				
	FA: 0.02761	FA: 0.02685	FA: 0.08947	FA: 0.03198				

(60,5)	PC: 0.759	PC: 0.7951	PC: 0.9392	PC: 0.8208
	FA: 0.02825	FA: 0.02156	FA: 0.07472	FA: 0.02863
(60,6)	PC: 0.7765	PC: 0.7981	PC: 0.8877	PC: 0.8043
	FA: 0.03128	FA: 0.0208	FA: 0.05316	FA: 0.02744
(60,7)	PC: 0.7621	PC: 0.8012	PC: 0.898	PC: 0.7683
	FA: 0.02955	FA: 0.0208	FA: 0.05451	FA: 0.02048
(60,8)	PC: 0.7673	PC: 0.8023	PC: 0.898	PC: 0.7775
	FA: 0.02944	FA: 0.02075	FA: 0.05446	FA: 0.02064

TABLE 9. EXPERIMENTAL RESULTS FOR SONAR3							
	Kernel						
Dimension (Original,Final)	Gaussian	Laplacian	Rayleigh	Polynomial			
(60,2)	PC: 0.52	PC: 0.72	PC: 0.82	PC: 0.56			
	FA: 0.09937	FA: 0.1193	FA: 0.1532	FA: 0.0803			
(60,3)	PC: 0.58	PC: 0.68	PC: 0.82	PC: 0.54			
	FA: 0.08628	FA: 0.1233	FA: 0.1532	FA: 0.0660			
(60,4)	PC: 0.48	PC: 0.68	PC: 0.82	PC: 0.46			
	FA: 0.07489	FA: 0.1136	FA: 0.1509	FA: 0.0996			
(60,5)	PC: 0.48	PC: 0.68	PC: 0.84	PC: 0.58			
	FA: 0.07432	FA: 0.1079	FA: 0.1498	FA: 0.0896			
(60,6)	PC: 0.48	PC: 0.6	PC: 0.78	PC: 0.5			
	FA: 0.07346	FA: 0.1065	FA: 0.1102	FA: 0.07574			
(60,7)	PC: 0.54	PC: 0.54	PC: 0.74	PC: 0.46			
	FA: 0.06748	FA: 0.09539	FA: 0.1096	FA: 0.0674			
(60,8)	PC: 0.54	PC: 0.54	PC: 0.74	PC: 0.48			
	FA: 0.07318	FA: 0.09653	FA: 0.1091	FA: 0.0620			