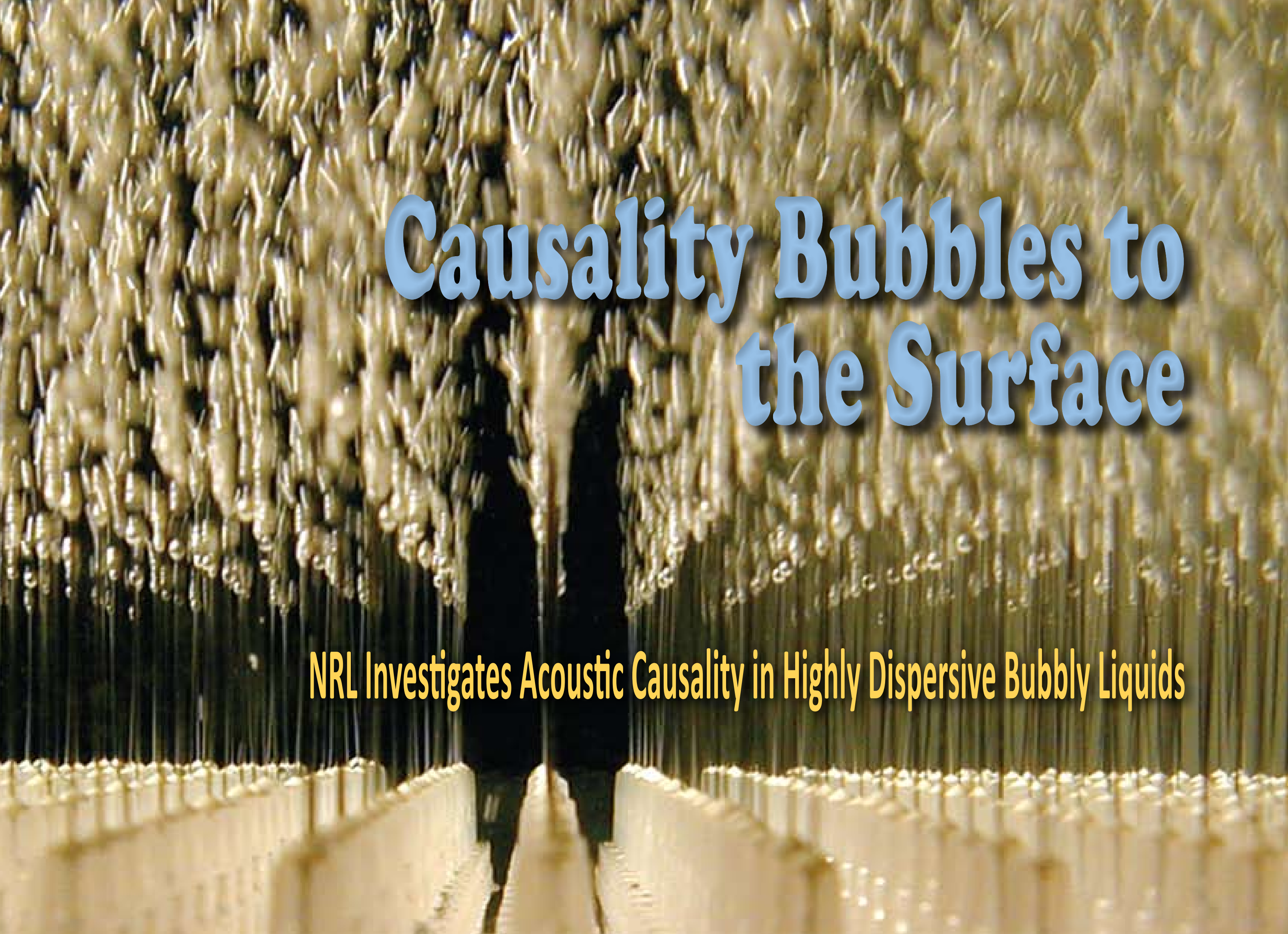
Causality Bubbles to the Surface

NRL Investigates Acoustic Causality in Highly Dispersive Bubbly Liquids



The most fundamental property exhibited by any physical system is causality: in essence, cause must precede effect. The modern interpretation of causality sprung forth from Albert Einstein's classic, yet at the time of its publication controversial, work on Special Relativity, wherein he postulated that the travel time of any signal cannot exceed the speed of light in a vacuum. The same underlying physical and mathematical principles pioneered by Arnold Sommerfeld and Léon Brillouin nearly a century ago that were used to quell Einstein's critics and prove his assertions continue to provide essential and intriguing insights into scientific phenomena of vital importance to the U.S. Navy: acoustic propagation through highly dispersive subsurface bubbles and bubble clouds in the ocean.

Since the commissioning of NRL's unique Salt Water Tank Facility in the late 1990s, NRL scientists have conducted many experiments designed to investigate different aspects of acoustic propagation in bubbly liquids. One of the major accomplishments of this facility in 2008 provided experimental verification of an important correction to contemporary theories of acoustic propagation in bubbly media. This correction has resulted in a causal self-consistent theory verified by both higher frequency data taken in the Salt Water Tank Facility and historical data. Further observations have elucidated some additional features that have significant implications upon acoustic signal propagation and suggest that despite many decades of scientific investigation, we have only begun to sketch out a comprehensive description of the physical phenomena surrounding acoustic propagation in the strongly scattering highly dispersive environment of bubbly media.

Investigating Acoustic Causality in Highly Dispersive Bubbly Liquids

G.J. Orris, M. Nicholas, and D.K. Dacol
Acoustics Division

Causality in nearly all physical systems has been a recurrent subject, often causing apparent paradoxes since before the 20th century. Linear acoustic propagation through subsurface bubble clouds in the ocean offers an especially challenging physical system within which to investigate issues of causality, and in the past has had several competing fundamental theories. Signal travel times and absorption in such a system exhibit enormous variations depending on the acoustic signal frequency, bubble size distribution, void fraction, and other ambient physical parameters. We have found a correction to some contemporary theories of acoustic propagation in bubbly media that has brought these theories into compliance with the physical law of causality. In doing so we have created a self-consistent theory that also matches higher-frequency data taken in the NRL Salt Water Tank Facility, as well as historical data. We have experimentally investigated this new theory and have observed some additional features that have significant implications upon acoustic signal propagation and suggest that we have only begun to scratch the surface of providing a comprehensive description of the physical phenomena surrounding acoustic propagation in the highly dispersive environment of bubbly media.

INTRODUCTION

Einstein's postulate that no physical process can travel faster than the speed of light was disputed by several leading physicists shortly after its publication. The main objection to the theory was based on studies of existing dispersive metal compounds with measured values for their phase velocities, where the derived group velocities were greater than the velocity of light in a vacuum.[1] The erroneous assumption in the argument against Einstein's postulate was based on the original work of Stokes that stated the group velocity, $\partial\omega/\partial k$, determined the velocity of propagation of wavelike signals in any medium. The argument used to object to Einstein's postulate neglected two important and omnipresent suppositions. The first supposition was that any signal that can send information is by necessity of finite length and, therefore, of infinite frequency extent. The second supposition was concerned with the experimental system, based on the interaction of electromagnetic waves with metals, which generated the data that were used to counter Einstein's postulate. This system had a significant degree of dispersion and a highly frequency-dependent dispersion function. This manifests itself within the model of electrons in metals, which at that time were considered to behave classically as damped harmonic oscillators.

These two points and their physical implications were pointed out in a now classic set of companion papers published in 1914 by Sommerfeld and Brillouin that laid the theoretical groundwork for countless developments in physics, information theory, and engineering throughout the 20th century.[2,3] Their resolution to the neglected suppositions was based on a detailed analysis of the definition of a wave pulse of finite length and the signal's frequency content for a wave travelling in a dispersive medium. Using the theory of complex analytic functions, they observed that the dispersion formula, and, hence, the Fourier integral defining the wave pulse at a future time and position separated from the origin of the pulse, could be shown to be analytic in the upper half complex frequency plane. Thus it could be demonstrated that all electromagnetic signals are undetectable until at least such a time $t = x/c_0$ has elapsed, where x is the distance from source to receiver, and c_0 is the velocity of light in a vacuum in the infinite frequency limit. Due to the nature of the medium, instrumentation of infinite precision might be needed to detect the incoming signal moving at that velocity for a given dispersive medium and pulse carrier frequency. This requirement of analyticity led Kramers and Krönig to separately derive their now famous relation between the real and imaginary parts of the dispersion formula.

From the standpoint of acoustic wave propagation in any fluid, bubbles represent one of the strongest, most ubiquitous, and complex extinction mechanisms. These mechanisms can be modeled in much the same way as the semi-classical electrodynamics systems used to model the propagation of electromagnetic waves in metals. Thus we are naturally led to ask if there is an equivalent theory of causality, or in effect a "speed limit," of acoustic waves in a continuum mechanical system, e.g., dispersive bubbly media, to that of the electromagnetic case first investigated over 100 years ago. Understanding the complex dynamics of bubbles and the acoustic interaction with a bubble field is a necessary first step in trying to model and measure the propagation of an acoustic signal in or near a bubbly medium or when creating new acoustic devices using bubbles.

The ubiquity of bubbles in the ocean can be imagined by noting that a single plunging breaker at the ocean surface can generate hundreds of millions of bubbles. While many of these bubbles are relatively large and rise quickly to the surface where the bubble wall collapses and the bubble ceases to exist, a significant fraction of the bubbles are relatively small, with average radii typically between 20 and 50 μm. These small bubbles have a small buoyancy and can be mixed into the upper surface of the ocean to form larger clouds that can persist for many seconds and be forced down to depths of 20 m or more by circulation cells. The clouds typically have complex shapes and population densities with horizontal scales from 1 to 100 m.

We are thus left with the following complex picture of acoustic wave propagation in the upper ocean: the acoustic medium is itself a semi-regular collection of clouds with complex time-dependent spatial extent. Not only does each bubble interact with an incoming acoustic wave approximately as a damped harmonic oscillator, but the injection of a gas into the liquid fundamentally changes the compressibility of the medium in and near the bubble/bubble clouds. This changes the acoustic phase velocity in and near the clouds, causing them to become effective scatterers/resonators and in so doing, provide for a broader range of scattering scale sizes. Complicating the picture even further is the fact that the phase velocity and associated medium attenuation are highly dependent on the incident wave frequency in a nontrivial and nonlinear way that causes significant dispersion to occur. Thus it is imperative that the dispersion and the fundamental limits that causality places upon any theory of wave propagation in a dispersive bubbly acoustic medium be understood in full.

SINGLE BUBBLE OSCILLATIONS

Physically, a bubble exists because of the molecular/atomic effects of surface tension at an interface between two fluids. While large bubbles in water often exhibit large fluctuations in the shape of this interface as they are deformed due to nonuniform surface drag forces in their eventual rise towards the surface, the majority of oceanic bubbles (particularly those that persist) are small, and small radial pulsations about an equilibrium value are by far the most significant mode of oscillation of the bubble interface. Furthermore, it has been experimentally demonstrated that even extremely deformed bubbles radiate acoustic energy very much as though they were spherical, and undergoing a small radial oscillation. Thus, the acoustic signature and natural frequency of oceanic bubbles can be well approximated as small radial pulsations alone governed by the Rayleigh-Plessett equation,[4,5]

$$\left(1-\frac{1}{c_0}\frac{dR}{dt}\right) R\,\frac{d^2R}{dt^2}+\frac{3}{2}\left(1-\frac{1}{3c_0}\frac{dR}{dt}\right)\left(\frac{dR}{dt}\right)^2 = \frac{1}{\rho}\left(1+\frac{1}{c_0}\frac{dR}{dt}+\frac{R}{c_0}\frac{d}{dt}\right)(p_B-P)$$

where c_0 is the phase velocity of sound in a quiescent medium, $R(t)$ is the radius of the bubble as a function of time, p_B is the static equilibrium pressure inside the bubble given by the static quiescent pressure external to the bubble and the Laplace pressure $p_L = (2\sigma)/(R_0)$, and $P(t)$ is the driving external pressure (acoustic) field. Since the Laplace pressure is inversely proportional to the radius of the bubble, it can have a substantial impact on the dynamics of small bubbles. This added pressure also causes the smaller bubbles to dissolve into the solution at an ever-increasing rate, as the leakage of the entrapped gas into the liquid is a strong function of the internal bubble pressure. This causes the bubbles to become smaller yet, increasing the internal pressure even more, and hastening their dissolution. The second effect is that the bubble's natural frequency is modified as the bubble becomes effectively stiffer, effectively increasing the apparent resonance frequency. The Rayleigh-Plesset equation results in additional first-order terms in the radial deflection that are complex and thus result in energy dissipation.

The second dissipative effect on the acoustic field when it encounters a bubble is the liquid's viscosity. The forced oscillations of the bubble wall necessarily deform the liquid around the bubble wall. A volume element of liquid near the bubble wall will deform as the bubble expands and contracts, becoming thicker with less solid angle upon contraction and thinner with larger solid angle upon expansion. Energy in the process is not conserved, as all deformations of a volume element of a viscous liquid demand work. For general small radial oscillations, the associated dissipation can be shown to result in a damping term of the form $4\pi\mu$, where μ is the dynamic coefficient of viscosity of the liquid.[6]

The third dissipation mechanism of bubble dynamics is the thermodynamics of the heat generated as a byproduct of expansion and contraction of the gas trapped in the bubble volume. This heat is conducted from the internal vapor into the greater liquid thermal bath (ocean). Early bubble acoustics either assumed that the bubble oscillated in either an isothermal or adiabatic process depending on the frequency of ensonification. However, the actual heat transfer is significantly more complicated and has far-reaching consequences for acoustic propagation in a bubbly liquid. Since the heat conduction takes some time to travel to and from the bubble wall, there is a phase delay between the total pressure field and the bubble radius (or, equivalently, volume). If it is assumed that the internal bubble gas can be treated as a polytropic gas, then for small linear radial oscillations, the bubble radius can be considered to have a form of $R(t) = R_0\,(1 + X(t))$, where $X(t)$ is a dimensionless small harmonic perturbation. The pressure can then be given similarly as $P(t) = P_0\,(1 + \Phi X(t))$, where the factor Φ is a complex function of the frequency, the equilibrium bubble radius, the ratio of specific heats of the bubble gas, and the diffusivity of the bubble gas through[6,7]

$$\Phi(\omega, R_0) = \frac{3\gamma_g}{1 + 3(\gamma_g - 1) i\chi(\omega, R_0)\left[\sqrt{1/i\chi(\omega,R_0)}\coth\left(\sqrt{1/i\chi(\omega,R_0)}\right) - 1\right]},$$

where $\chi(\omega, R_0) = \dfrac{D_g}{\omega R_0^2}$ and $D_g = \dfrac{(\gamma_g - 1)\kappa_g T_g}{\gamma_g\, p_\infty}$,

where the subscripts "g" refer to the gas phase and p_∞ is the quiescent pressure minus surface tension effects.

FROM THE ACOUSTICS WITH A SINGLE BUBBLE TO THE ACOUSTICS OF A CLOUD

The transition from a single bubble to a collection of bubbles follows along that of the two-phase fluid. This results in a generalized wave equation for the pressure in the liquid phase with an added term involving the expansion and contraction of the bubbles, represented as a total change in macroscopic void fraction,[8]

$$\frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} - \nabla^2 P = \rho \frac{\partial^2 \beta}{\partial t^2},$$

where β is the void fraction of the gas in the liquid and ρ is the density of the liquid. When the bubble field can be regarded in an isotropic and homogeneous system, then $\partial^2\beta/\partial t^2$ can be determined from the linearization of the Rayleigh-Plesset equation with the added effects of viscosity, surface tension, and finite conductivity of the bubble gas. This effective medium approach presents itself as a modified dispersion formula relating the wavevector to all of the physical parameters, and is necessarily a complex function reflecting the dissipation,[9]

$$k_{cm}^2 = \frac{\omega^2}{c_0^2}\left(1 + 4\pi c_0^2 \int_0^\infty dR_0 \frac{R_0 \rho_{BSD}(R_0)}{\dfrac{p_0 \Phi(\omega, R_0)}{\rho R_0^2} - \dfrac{2\sigma}{\rho R_0^3} - \dfrac{\omega^2}{1 - i\dfrac{\omega R_0}{c_0}} - i\dfrac{4\omega\mu_0}{\rho R_0^2}}\right)$$

where ρ_{BSD} is the bubble size distribution and ω is the radial frequency of the driving acoustic wave. Here it is evident that there are a significant number of physical parameters that affect acoustic wave propagation in bubbly water and that even this theory represents only a linear approximation to the physical reality. How faithful this treatment of the problem is, whether the behavior that this theory predicts represents a true system that is causal, and what theoretical predictions can be detected in the laboratory have been the focus of research at NRL's Salt Water Tank Facility, shown in Figs. 1 through 3.

At very low frequencies, the system becomes isothermal and the effect of the injection of a gas into the liquid can be grossly regarded as a change in the system's mean compressibility and mean density that result in Wood's equation for the phase speed of the combined liquid. The air (gas) phase in the two-phase fluid has a high degree of compressibility compared to the liquid. In the case of water, this ratio is approximately 10^4. However, the mass density ratio is approximately 10^{-3} and thus is not changed significantly by the inclusion of small amounts of gas. This combination frequently results in an acoustic phase speed that can be significantly lower than that of the gas alone.

If the bubble size distribution is relatively sharply peaked, then there is a frequency regime that can be identified with the resonance of a sizable portion of the bubbles. This is manifest as a dip in the phase speed curve to values even lower than that given by Wood's equation in the low frequency limit. Because the bubbles are now responding via the Rayleigh-Plesset equation as a damped resonating harmonic oscillator, the bubble pulsations and, hence, the total radial deflections maximize. As this occurs, the volume fluctuations maximize and, hence, thermal dissipation becomes a significant loss mechanism.

Just above the resonance frequency we enter the anomalous absorption regime where all of the loss mechanisms, viscous, thermal, and radiative (scattering), are significant. This area is characterized by three

FIGURE 1
Exterior view of the Salt Water Tank, with a small bubbler test tank on the left. The main tank measures 6 m x 6 m x 4 m deep, although typical water depth is 3 m. The interior of the tank has 50 precision-controlled air-flow ports to allow for various air injection mechanisms to be arranged for each experiment. Barely visible at the far left is the salt mixing tank used to mix salt into the main tank to any desired level of salinity. Visible through the Plexiglas windows of the large tank is an ITC2010 acoustic source, used for much of the experimental data below 10 kHz.

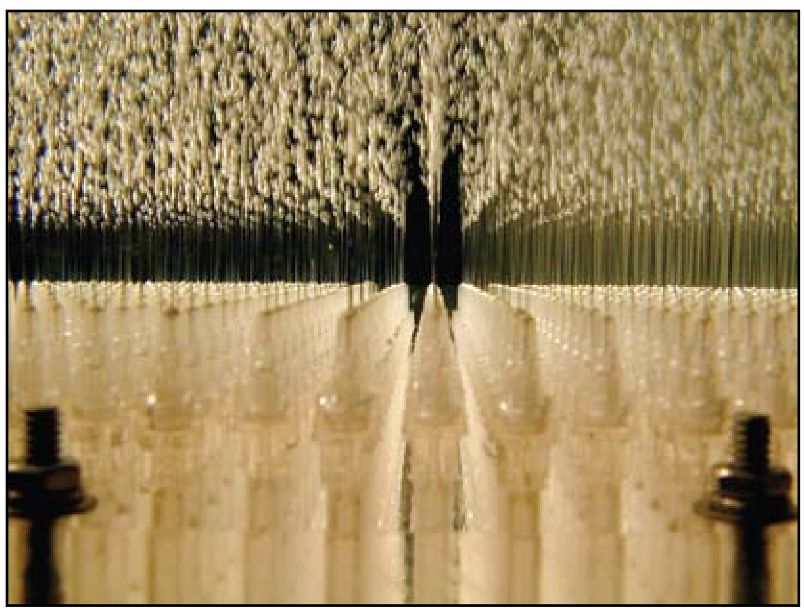


FIGURE 2
Precision bubbler. This bubbler consists of 3000 luer-lock square cut hypodermic needles glued into a 2.5-cm-thick Plexiglas sheet. This sheet is bolted over a plenum box weighted to be nearly neutrally buoyant. Since the pressure is nearly identical over all needles, the rate of bubble formation and size of bubbles is close to uniform.

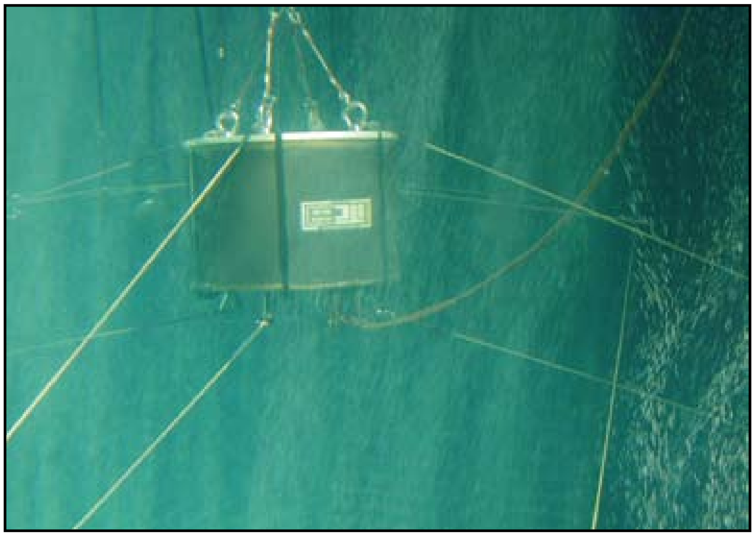


FIGURE 3
An ITC2010 acoustic source tethered in a bubble cloud produced by aeration tubes (which generate a log-normal bubble size distribution). Much of the data is collected under similar conditions to these. Here the void fraction is only 0.001 with a mean bubble radius of only 0.4 mm. Note that the ascent of so many bubbles drags enough fluid upwards to cause the source to move, and hence it is lashed to the side of the tank throughout our experiments: precise phase measurements can be made only when the source and receiver positions are well known. Typical data collection times can exceed a week for each set of physical parameters (temperature, salinity, void fraction, etc.).

dominant features. The first is the rise of the absorption to very high levels that can approach hundreds of dB/m for void fractions even as small as 10^{-4}. The second feature is a similar rise in the acoustic phase speed that can be tens of thousands of meters per second for similarly small void fractions (10^{-4}). This is due to the bubble's resonant frequency beginning to oscillate out of phase with the incident acoustic wave driving the small radial oscillations. As such, the medium stiffens considerably with respect to the phase speed. The third major feature is the significant departure of the acoustic signal velocity from the acoustic group velocity. The group velocity, as a quantity derived from behavior of the phase velocity, eventually passes through infinity to negative infinity and back: clearly the wave propagation velocity can no longer be approximated by the group velocity and we must rely on a complicated determination of the signal's velocity.

Far above resonance, the velocity of the signal asymptotes with both the group velocity and phase velocity to the quiescent medium's speed of sound. Furthermore, the attenuation becomes effectively constant with frequency, signifying the dominance of scattering as the loss mechanism at high frequencies. Here the wavelength of the incident acoustic field in the medium between the bubbles becomes significantly less than the size of the typical bubble.

CAUSALITY AND THE ACOUSTIC SIGNAL VELOCITY

If the dispersion formula as stated above is used directly to calculate the phase speed, attenuation, and group velocity of an acoustic pulse in a bubbly liquid, then the problems seen a century ago by Sommerfeld and Brillouin are essentially repeated, albeit with a significantly more complex dispersion formula. The bubbly liquid dispersion formula can be shown to be analytic in the upper half complex frequency plane. Thus, Cauchy's integral theorem trivially proves causality, as the Fourier integral describing the propagation of an acoustic pulse is zero for times less than x/c_0. To determine the behavior of the signal as a function of time for times equal to or greater than this, the behavior of the dispersion formula in the complex frequency plane must be determined in detail, and in general the saddle point method can be used to determine the propagation characteristics as a function of time. This is because the lower half complex plane contains several branch cuts that must be included in any calculation.

An example of a typical integration path using this method is shown in Fig. 4. As in the electromagnetic case, there are three different phases of an incident signal's arrival: the Sommerfeld precursor, the Brillouin precursor, and the signal. The first two are dependent upon the behavior of the dispersion function in the complex plane near the point at infinity and the origin, respectively, and hence are independent of the source characteristics (i.e., frequency).[2,3,9] The actual signal arrival is determined by the crossings of the saddle point integration path (which is a function of time, distance, and the physics of the propagation medium) with the abscissa. If a portion of the signal's spectrum exists at this crossing, then there will be a simple pole in the Fourier integral, requiring an altering of the integration path around the pole to exclude it from the integration region. The consequences of this are a dominating contribution to the integral at that point and the arrival of energy with that frequency.[9]

EXPERIMENTAL EFFORT IN THE SALT WATER TANK FACILITY

Since the surface tension of water is affected by the presence of salt, the bubble size distribution found in saline environments is different from that found in

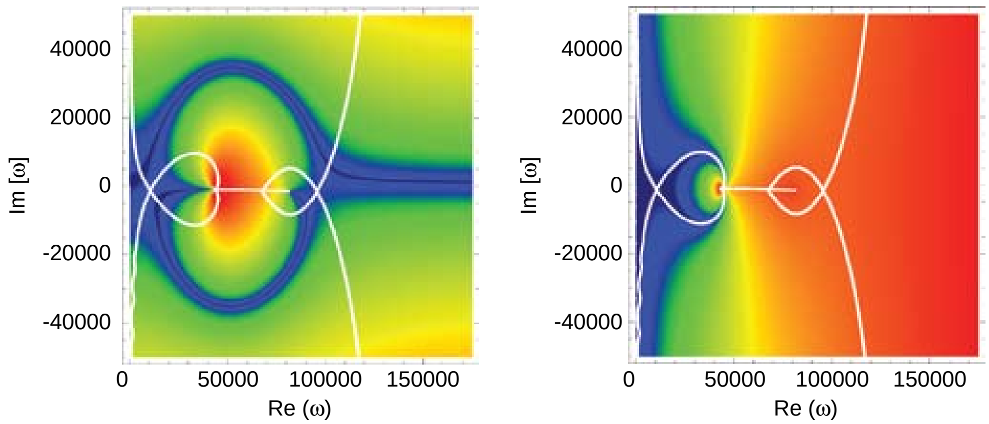


FIGURE 4
Real (left) and Imaginary (right) density plots of the complex phase function in the complex frequency domain. The Saddle Point Method of calculation of the Fourier integral defining the signal as it travels in time and space is determined by the shape and value of the functions shown here. The white loops around the branch cuts are the paths of integration for a specific time and distance from the source. Frequencies on the real axis that are intersected by the curves can have arrival times at distance x, thus defining the signal velocity of that frequency.

fresh water. However, the violent nature of the bubble entrainment mechanisms found in the open ocean tends to break up larger bubbles until the external turbulent forces no longer have sufficient force to overcome the surface tension. The Salt Water Tank Facility (SWTF) at NRL (see Figs. 1 through 3) was specifically constructed to test acoustic propagation in bubbly salt water to fully verify and validate theoretical predictions of acoustic wave propagation in bubbly water relevant to oceanic environmental conditions.

The SWTF consists of a vinyl-coated steel walled tank measuring 6 m × 6 m × 4 m deep, with 12 clear Plexiglas windows in the tank walls, each window being 3 m × 1-1/2 m × 10 cm thick. Within this tank, we used 50 fabric-coated aeration tubes, each one 5 m long, placed 10 cm apart on the tank floor and fed from both ends by a filtered compressed air supply. These tubes continuously injected bubbles of a wide size distribution into the water volume, which would then rise to fill almost the entire volume of the tank. Depending on the air overpressure applied to the aeration tubes, the tank would contain from 7 to 70 million bubbles, which translates into void fractions of between 0.0002 and 0.002. At the center of the water volume we then used acoustic transducers to transmit narrowband acoustic signals, which we would then detect on hydrophones positioned at precisely measured distances from the sources. The time of flight and received intensity were measured directly from a comparison of the data recorded at a monitoring hydrophone placed next to the source and those recorded at the various receivers. The absolute phase of the signal's carrier frequency at the receiver can be inferred from the data and the phase velocity thus determined. Comprehensive acoustic data sets were collected for several void fractions between 0.0002 and 0.002, multiple salinities, and covering frequencies from 1 kHz (which is far below bubble resonance) up to 100 kHz (which is far above bubble resonance) in 100 Hz increments. Coincidentally, an underwater camera was used to capture bubble images alongside a calibration scale, and these were later used to generate numbers for the bubble size distributions, and to verify the void fraction measurements made from air flow meter and bubble rise time measurements.

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THE AUTHORS

GREGORY J. ORRIS received a double A.B. degree in applied mathematics and physics from the University of California in 1986. He then received a Ph.D. in physics from the University of California, San Diego in 1991. His dissertation concerned applications of renormalization group theory to classical field theory and the propagation of waves in random media. From September 1991 until July 1992, he was an ASEE Post doctoral Fellow at the Naval Research Laboratory researching applications of nonlinear dynamics theory to problems in acoustics. In August 1992, he joined NRL as a full-time employee in the Acoustics Division. He has worked on problems in acoustic wave propagation, bubble acoustics, acoustic signal processing, acoustic scattering theory, matched field processing, sonic crystals, acoustic metamaterials, sonomagnetism, and low-frequency sea-air transmission.

MICHAEL NICHOLAS received the B.Sc. degree (with Honors) from the University of Wales in 1978, and the M.S. and Ph.D. degrees from the University of Mississippi at Oxford in 1982 and 1988, respectively. All three degrees were in physics. His research was in experimental nuclear physics, with studies of Exotic atoms undertaken at the Los Alamos Meson Physics Facility (LAMPF) and at the Center for European Nuclear Research (CERN). After a brief period working in the oil and gas exploration industry at Hycalog in Gloucester, UK, he returned to the USA as a postdoctoral research associate at the National Center for Physical Acoustics. Here he studied basic physical mechanisms for bubble-related underwater ambient noise generation and acoustic surface scattering. In 1992 he became a research physicist at the Naval Research Laboratory, where he was initially involved in the Navy's Critical Sea Test, studying surface scattering and reverberation. He continues studies in bubble-related acoustics, and also works on underwater matched field processing, sonic crystals, underwater mine and intruder detection, laser-generated underwater acoustics, sonomagnetism, and low-frequency sea-air transmission.

DALCIO K. DACOL is a native of Brazil who has been in the USA since 1974 and is a naturalized U.S. citizen. He holds a Ph.D. in physics from the University of California at Berkeley (1980) and was an undergraduate student at the University of São Paulo in Brazil. His research interests are in the areas of wave propagation and scattering, wave scattering in random media, ocean acoustics, sonic crystals, sound scattering from surfaces and interfaces, and inversion of acoustic data for geoacoustic properties in the ocean.