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US Army Corps  
of Engineers  
Water Resources Support Center

# ***RISK-BASED EVALUATION OF FLOOD WARNING AND PREPAREDNESS SYSTEMS***

## ***Volume 1 - Overview***



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# RISK-BASED EVALUATION OF FLOOD WARNING AND PREPAREDNESS SYSTEMS

*Volume 1 - Overview*

by

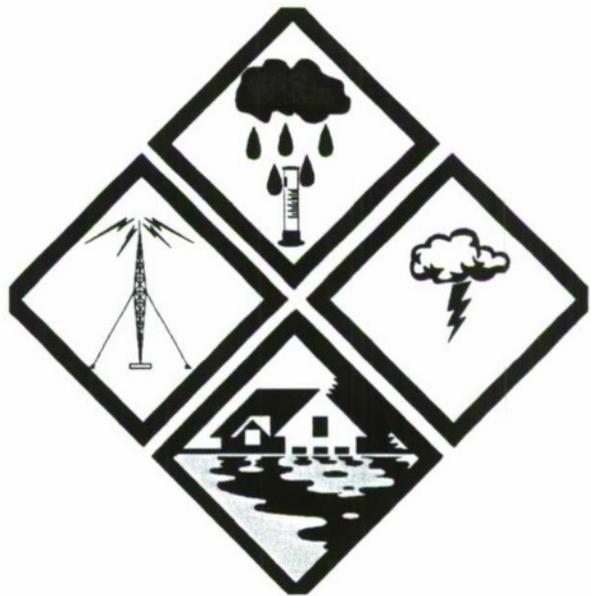
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## Preface

This report is a product of the U.S. Army Corps of Engineers' Risk Analysis for Water Resources Investments Research Program. The program is managed by the Institute for Water Resources, which is a unit of the Water Resources Support Center. The report was prepared to fulfill part of several work units in the research program. These work units focused on developing and applying the concepts of risk preference and risk communication to water resources issues. The report conforms to the basic planning model and to the risk and uncertainty analysis recommendations presented in *Economic and Environmental Principles and Guidelines for Water Related Land Resources Implementation Studies* (P&G).

The risk analysis framework encompasses the four basic steps in dealing with any risk: characterization, qualification, evaluation, and management. The purpose of conducting these analyses is to provide additional information to both Federal and non-Federal partners on the engineering and economic performance of alternative investments that address water resources problems. The goal is to produce better informed decisions and to foster the development of the idea of rational joint consent by all parties to an investment decision.

This report, entitled *Risk-Based Evaluation of Flood Warning and Preparedness Systems*, represents a synthesis and elaboration of three earlier technical reports<sup>1</sup> to the Institute for Water Resources prepared by Environmental Systems Modeling, Inc. The results presented here have as a unifying theme that design and evaluation of structural and nonstructural measures for flood mitigation, including flood warning and preparedness systems, is an integrative, holistic process that requires an understanding of the contribution each type of measure makes to the performance of the overall system. The models rely on concepts of multiobjective decisionmaking, tradeoff analysis, and the risk of extreme events. This report is divided into OVERVIEW and TECHNICAL sections. Each of the four OVERVIEW sections summarizes in a nontechnical style a methodology developed for the integration of flood warning and preparedness systems into the design and evaluation process. The four methodologies are (1) integration of structural measures and flood warning/preparedness systems, (2) multiobjective decision-tree analysis, (3) performance characteristics of a flood warning system, and (4) selection of optimal flood warning threshold. Each OVERVIEW section describes the main features of the model, case study, or example. The four TECHNICAL sections correspond to the sections of the OVERVIEW and contain the mathematical details that would be needed in an application of the methodologies. In addition to being a consultant for this report, Prof. Roman Krzysztofowicz is the sole author of the TECHNICAL section of Part 3—Performance Characteristics of a Flood Warning System; the OVERVIEW section of Part 3 is excerpted and edited from the same TECHNICAL section. The contribution and description of case-study data in Section 4—Selection of Optimal Flood Warning Threshold—also is adopted from work of Krzysztofowicz.

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<sup>1</sup>Performance Characteristics of a Flood Warning System and Selection of Optimal Warning Threshold (September 1990); Case Studies in Selecting Optimal Flood Warning Threshold (March 1992); and Integration of Structural Measures and Flood Warning Systems for Flood Damage Reduction (March 1992)

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Mr. Stuart A. Davis, of the Institute for Water Resources' Technical Analysis and research Division, provided technical review for this report. Dr. Eugene Z. Stakhiv, Chief of the Policy and Special Studies Division, reviewed the earlier technical reports on which this report is based. Dr. David A. Moser, of the Technical Analysis and Research Division, is the principal investigator for the Risk Analysis Research Program. The Chief of the Technical Analysis and Research Division is Mr. Michael R. Krouse, and the Director of the Institute for Water Resources is Mr. Kyle E. Schilling. Mr. Robert M. Daniel, Chief of the Economics and Social Analysis Branch, Planning Division; Mr. Earl E. Eiker, Chief of the Hydrology and Hydraulics Branch, the Operations, Construction, and Readiness Division at the Headquarters of the U.S. Army Corps of Engineers serve as technical monitors for the research program. Numerous field reviewers provided valuable insights and suggestions to improve early drafts.

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## Introduction



Each of the four methodologies developed in this report contributes an important dimension to risk-based evaluation of systems for flood damage reduction -- which is incomplete without accounting for both structural and nonstructural measures. The unifying theme of these results is that the design and evaluation of structural and nonstructural measures for flood mitigation, including flood warning and preparedness systems, is an integrative, holistic process that eventually must build on an understanding of the contribution of each type of measure to the performance of the overall system. Furthermore, the design of flood mitigation is tied to multiple objectives of minimizing cost and risk and maximizing performance. Consideration of the risk of extreme events is an essential element in the evaluation of design tradeoffs.

The four methodologies developed here for the modeling and evaluation of flood warning and preparedness systems are:

- (1) Integration of structural measures and flood warning/preparedness systems,
- (2) Multiobjective decision-tree analysis,
- (3) Performance characteristics of a flood warning system, and
- (4) Selection of optimal flood warning threshold.

The assumptions, main functions, and limitations of the four methodologies are summarized in Table 1.

## Multiple Objectives

The single-objective models that had been advanced in the fifties, sixties, and seventies are today considered by many to be unrealistic, too restrictive, and often inadequate for most real-world complex problems. The proliferation of books, articles, and conferences and courses during the last decade or two on what has come to be known as multiple-criteria decisionmaking (MCDM) is a vivid indication of this somber realization and of the maturation of the field of decisionmaking [see Chankong and Haimes 1983]. In particular, an optimum derived from a single-objective mathematical model, including that which is derived from a decision tree, often may be far from representing reality -- thereby misleading the analyst(s) as well as decisionmaker(s). Fundamentally, most complex problems involve, among other things, the minimization of costs, the maximization of benefits (not necessarily in monetary values), and the minimization of risks

**Table 1. Assumptions, Main Functions, and Limitations of the Four Methodologies**

	<b>Assumptions</b>	<b>Main Functions</b>	<b>Limitations</b>
<i>Integration of structural measures and flood warning/preparedness systems</i>	Knowledge of flood frequency, discharge, stage, and damage relationships for various combinations of structural and flood warning/preparedness systems.	Determine the optimal design options from among alternative combinations of structural and flood warning/preparedness measures in a multiobjective framework, including cost, the expected flood loss, and risk of extreme floods .	No operational issues associated with warning/preparedness systems and structural measures are considered.
<i>Multiobjective decision-tree analysis</i>	Knowledge of the probabilities for the underlying distributions of water level. Knowledge of severity of loss with alternative decisions at various time stages.	Determine the optimal sequential decisions in an individual flood event based on the observation of water stage.	No flood forecast is taken into consideration.
<i>Performance characteristics of a flood warning system</i>	Knowledge of the joint probability description of flood forecast and actual flood crest.	Provide an evaluation model of the performance of a flood forecast system. In particular, the ROC curve characterizes the tradeoff between the probabilities of detection and false warning.	Interactions between successive flood events through the dynamics of the community response fraction are not taken into account.
<i>Selection of optimal flood warning threshold</i>	Knowledge of the joint probability description of flood forecast and actual flood crest. Knowledge of the loss to the community associated with flood stage. Knowledge of the dynamics of the community response fraction.	Find the optimal threshold level at which to issue a flood warning in order to balance the desire for high present-flood-loss reduction with the possibility of high future flood loss being inevitable.	The derived optimal threshold may not be stationary; i.e., the optimal threshold may vary in different flood events even if the community response fraction is the same.

of various kinds. For example, decision trees, which are a powerful mechanism for the analysis of complex problems, can better serve both the analysts and the decisionmakers when they are extended to deal with the above multiple objectives.

### **Impact Analysis**

On a long-term basis, managers and other decisionmakers are often rewarded not because they have made many optimal decisions in their tenure; rather, they are honored and thanked for avoiding adverse and catastrophic consequences. If one accepts this premise, then the role of impact analysis -- studying and investigating the consequences of present decisions on future policy options -- might be as important, if not actually more so, than generating an optimum for a single-objective model or identifying a Pareto-optimum set (a *Pareto-optimum*, or non-inferior, alternative cannot be improved in any one objective without seeing a corresponding loss with respect to one or more other objectives) for a multiobjective model. Certainly, when the ability to generate both is present, having an appropriate Pareto-optimum set and knowing the impact of each Pareto-optimum on future policy options should enhance the overall decisionmaking process.

### **The Risk of Extreme and Catastrophic Events**

Risk, which is a measure of the probability and severity of adverse effects, has until recently been commonly quantified via the expected-value formula. This formula essentially precommensurates events of low frequency and high damage with events of high frequency and low damage. Although learned students of risk analysis recognize the disparity between the above fallacious representation of extreme and catastrophic events and the perception of these events by individuals or the public at large, many continue to use this approach. The trend, however, is moving toward the conditional-expected-value approach, where extreme and catastrophic events are partitioned, isolated, quantified in terms of conditional expectation (e.g., using concepts from the statistics of extremes), and then evaluated along with the common expected value of risk or damage [Asbeck and Haimes 1984; Haimes 1988; Karlsson and Haimes 1988].

The partitioned multiobjective risk method (PMRM) developed by Asbeck and Haimes [1984] separates extreme events from other noncatastrophic events, and thus provides the decisionmaker(s) with additional valuable and useful information. In addition to using the traditional expected value, the PMRM generates a number of conditional expected-value functions, termed here risk functions, which represent the risk, given that the damage falls within specific probability ranges (or damage ranges).

Combining either a conditional expected risk function or the unconditional expected risk function with the cost objective function creates a set of multiobjective optimization problems in which the tradeoffs between cost and the risk arising from the various ranges of damage are analyzed. This formulation offers more information about the probabilistic behavior of the problem than the single multiobjective formulation that minimizes only the cost and the expected damage. The tradeoffs between the cost function and any risk function allow decisionmakers to consider the marginal cost of a small reduction in the risk objective, given a particular level of risk assurance for each of the partitioned risk regions, and given the unconditional risk function.

## **Flood Forecasting and Warning/Preparedness Systems**

Flood control can be provided by either structural or nonstructural measures or a combination of both. Structural flood control measures, such as an increase in dam height, affect the flood-frequency relationship. Nonstructural measures, such as a flood warning/preparedness system, do not have an impact on the flood-frequency relationship; however, they modify the flood-damage relationship.

The benefits of flood forecasts have been studied and systems approaches to flood forecasting have been pursued by many research scholars for more than twenty years [NACOA 1972; Bhavnagri and Bugliarello 1965; Bock and Hendrick 1966; Day and Lee 1976; Lee et al. 1975; Sniedovich et al. 1974; Sniedovich and Davis 1977]. Curtis and Schaake [1988] evaluated flood warning benefits both on a national (or regional) scale and on a specific site problem. Prediction models for loss of life from floods were studied by Lee et al. [1986] and Shabman [1987]. Barrett et al. [1988] developed categories for flood warning systems based on types of flood forecasting systems and flood response systems.

Predicting the future behavior of a time-dependent random variable is a major research task in the theory and applications of stochastic processes. Critical events occur when the level of the random variable crosses a given high level (e.g., flooding level). An alarm is set off when the random variable exceeds a specified threshold level. An alarm system is considered optimal if it detects catastrophes with an acceptable level of probability and at the same time yields a minimum expected number of false alarms [Lindgren 1979, 1980, 1985; de Maré 1980]. The paper by de Maré [1980] indicates that when judging the performance of an alarm system, it is not very interesting to know, in the mean, how close the prediction is to the actual process; however, it is important for a system to be able to detect catastrophes without causing too many false alarms.

In a series of papers, Krzysztofowicz and his colleagues [Alexandridis and Krzysztofowicz 1985; Ferrell and Krzysztofowicz 1983; Krzysztofowicz 1983a, b; 1985; Krzysztofowicz and Davis 1983a, b, c, d; 1984] conceptualized the flood forecast-response process in the form of a total system. This system is defined as a cascade coupling of two components: (1) the forecasting system, which includes data collection, flood forecasting, and forecast dissemination; and (2) the response system, which encompasses decisionmaking and action implementation. Based on the above mathematical description of the physical flood forecast-response process, Krzysztofowicz and his colleagues establish performance measures of flood warning systems.

Paté-Cornell [1986] presents a method for assessing the performance of the forecasting system and human response, given the memory that people have kept on the quality of previous alerts. The tradeoff between the rate of false alerts and the length of the lead time is studied to account for the long-term effects of "crying wolf." An explicit formulation of benefits from warning systems is derived under the above considerations.

## **Toward Implementation of the Methodologies**

An immediate and most worthwhile challenge is the refinement of the four methodologies of this report for the operational setting. For instance, a decision support system for the risk-based evaluation of

flood warning systems might be developed to integrate these methodologies in a framework consistent with Corps of Engineers planning procedures [HEC 1988].

## **Organization of the Report**

The body of this report has two volumes: OVERVIEW (Volume 1) and TECHNICAL (Volume 2). Each of the four sections in the OVERVIEW Volume summarizes in a nontechnical style a methodology developed for the integration of flood warning systems into the design and evaluation process. The OVERVIEW Volume describes the main features of the model, case study, or example. The four TECHNICAL Volume sections correspond to the sections of the OVERVIEW and contain the mathematical details that would be needed in an application of the methodologies. The OVERVIEW's present an excerpted group of the figures and tables used in the TECHNICAL sections.

## **Part 1**

# **Integration of Structural Measures And Flood Warning Systems: Overview**



## **Introduction**

In most cases, the maximum flood loss reduction can be only achieved through an optimal combination of both structural and nonstructural flood control measures, since the adoption of integrated measures will certainly enlarge the feasible region of flood control measures in comparison with situations where only structural or nonstructural measures alone are considered. Structural measures include the construction of reservoirs, levees, and flood walls. Nonstructural measures include floodplain land use planning, flood insurance, flood warning systems, floodproofing, and permanent relocation. Various flood control measures prevent inundation of the floodplain in different ways and have different impacts on the flood damage-frequency relationship. A structural measure, such as an increase in reservoir height, affects the frequency-discharge relationship; levees and flood walls confine the discharge within certain channels, thus changing the relationship between discharge and elevation; most nonstructural measures, such as a flood warning system, modify the stage-damage relationship.

The idea of combining both structural and nonstructural measures in flood control is not new. Various research results have been reported in which structural measures are combined with nonstructural measures, such as zoning, floodproofing, and flood insurance. Readers can refer to Thampapillai and Musgrave [1985], which provides a comprehensive survey in reviewing integrated structural and nonstructural measures in flood damage mitigation. To date, however, no other research work on combining structural measures with flood warning systems has appeared in the literature.

Issues of both design and operation are involved in structural measures as well as nonstructural ones. Building a reservoir is a structural measure in flood control. Determination of the height of the reservoir is a design issue, while determination of the amount of the release on a monthly or daily basis is an operational issue. Installing a flood warning system is a nonstructural measure in flood control. Determination of an acceptable reliability of a warning system is a design issue with a consideration of the system cost, while determination of the flood warning threshold for various flood events is an operational issue. It is important to note that operational issues can only be addressed in a framework of dynamic optimization. For example, different levels of flood warning thresholds will cause different probabilities of missed forecast and false alarm, thus affecting the fraction of the community's future response. In this part, we analyze design options for the combination of structural measures and flood warning systems, thus building on and extending the existing methodology of computing flood loss for a given structural measure developed by the Army Corps of Engineers to incorporate flood warning systems.

For the computation of flood loss for a given flood-control structural measure, a widely-used procedure developed by the Army Corps of Engineers investigates the relationships between discharge vs. frequency, discharge vs. elevation, and damage vs. elevation, such that the damage-frequency curve can be generated for an average annual flood loss. An integrative approach is developed in this part to combine the calculation of flood loss reduction through flood warning systems with the calculation of flood loss for a given flood-control structure, thus facilitating the evaluation of combined structural measures and flood warning systems in reducing flood loss. Recognizing the inadequacy of the expected value as the sole

measure of risk, the partitioned multiobjective risk method (PMRM), which was employed in an earlier Army Corps of Engineers study on dam safety and which provides an added risk measure of extreme events, is adopted in this study. The conventional expected value of loss, the conditional expected value of extreme loss generated by the PMRM, and the cost associated with the structural measures and flood warning systems both separately and together are traded-off and analyzed in a multiobjective framework. An example problem that uses data from case studies performed for or by the U.S. Army Corps of Engineers demonstrates the efficacy and contribution of the developed integrated methodological framework. In particular, integrated structural measures for flood control and flood warning systems demonstrate clear and undisputed advantages over each system separately. The integrated system also provides a wider range of alternatives that makes flood protection more cost effective. The developed integrated methodological framework is simple to understand and use, since it builds on existing Army Corps of Engineers practices and uses accepted and already adopted procedures. The new methodology, however, brings an additional measure of risk to the Corps analyses -- the risk of extreme events in a multiobjective framework -- and thus makes the entire analysis more comprehensive and meaningful for decisionmaking purposes.

## **Features of the Model**

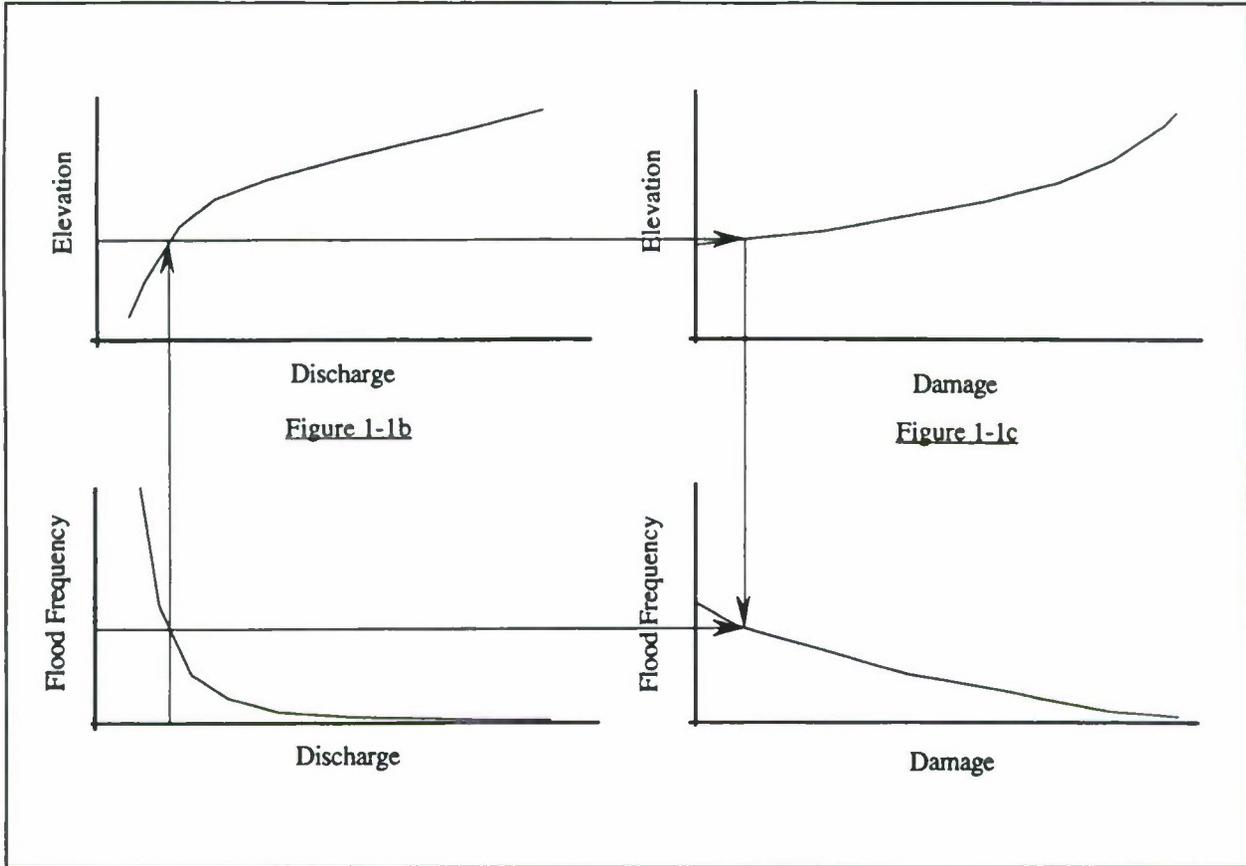
In the procedure for computing flood damage that has been developed by the Army Corps of Engineers, four functional relationships (or curves -- Fig. 1-1) are needed to completely quantify each alternative of structural measures:

- 1) curve of frequency vs. discharge,
- 2) curve of elevation vs. discharge,
- 3) curve of elevation vs. damage, and
- 4) curve of frequency vs. damage.

Note here that the fourth curve can be derived if the other three are known. In general, the relationships of frequency vs. discharge, elevation vs. discharge, and damage vs. elevation are constructed from real data such that the curve of frequency vs. damage can be derived in order to compare the expected flood damage for structural measures. The approach of discrete enumeration of all possible combinations of both structural and nonstructural measures is adopted in our development.

In this report we subscribe to a premise that the introduction of a flood warning system will not affect the relationships between the frequency and discharge and between the elevation and discharge. It will, however, alter the curve of elevation vs. damage, thus changing the relationship between frequency and damage.

To evaluate the flood loss reduction by installing a flood warning system, the concept of category-unit loss function detailed by Krzysztofowicz and Davis [1983] is adopted.



**Figure 1-1.** Curves for Relating Flood Frequency, Discharge, Elevation, and Damage

One of the critical parameters used in this study to evaluate flood loss is the fraction of people in a community who respond to flood warnings. The cost function of evacuation is assumed to be a linear function of the response fraction.

The flood loss function without a warning system is essentially given by the curve of elevation vs. damage (Figure 1-1c) for each given structural measure. Alternatively, the flood loss function without a warning system can be defined by reference to a unit damage function specifying the fraction of the maximum possible damage that occurs for a given depth of flooding.

The flood loss with a warning system is taken to be a function of all of the parameters considered in the case of no warning system and, additionally, the evacuation costs of the response fraction of the community less the losses avoided by an evacuation.

The value of the maximum flood loss for a community and the functional form of the unit damage function can be obtained from the curve of elevation vs. damage when structural measures are evaluated. The only additional information required to calculate the relationship of elevation vs. flood loss reduction through a flood warning system are the value of maximum evacuation cost, the value of response fraction in the community and the losses avoided by an evacuation. The resulting curve of elevation vs. flood loss

reduction can be viewed as a function parametrized by the response fraction. We should note, however, that the flood loss reduction is a linear function of the response fraction  $\theta$ .

Reducing the value of damage in the curve of damage vs. elevation for each structural measure by a function of the elevation and the response fraction yields a new relationship between elevation and damage when a flood warning system is introduced. Setting the response fraction equal to one yields a maximum achievement of flood loss reduction. Combining this new curve of elevation vs. damage with other two curves of frequency vs. discharge and elevation vs. discharge provides us with a new relationship between frequency and damage for a combined structural measure and a flood warning system.

Although the relationship of damage vs. frequency provides the most complete evaluation for each flood control alternative, it is necessary to compress information to generate a risk measure when various flood control alternatives are compared. The most commonly used risk measure is the expected value of the flood loss. Although the expected-value approach indicates the central tendency of flood damage for each flood control alternative, it fails to separate extreme, catastrophic flood events from the rest. The partitioned multiobjective risk method (PMRM) [Asbeck and Haines 1984] adopts the concept of conditional expectation, which enables us to isolate, quantify, and evaluate the impact of each flood control alternative on extreme, catastrophic flood events.

Multiobjective analysis is performed to evaluate the various flood control alternatives. In the PMRM [Asbeck and Haines 1984], for each flood control alternative we calculate three objective functions: the cost, the expected value of damage, and the conditional expected value of flood damage from extreme floods whose return periods are greater than a preset value. Both the expected damage and the conditional expected damage can be derived from the curve of damage vs. frequency for each flood control alternative.

The conditional expected damage gives information about the risk of extreme events that is overlooked in analyses where only the expected value of damage is considered. A flood control decision may have different impacts on the expected flood loss and the conditional expected flood loss from floods whose return period exceed a certain threshold level. The evaluation of risk in the multiobjective framework of the PMRM provides more decision aids to determine the optimal flood control strategy. The added tradeoff information between the cost and the risk of extreme flood loss addresses explicitly the public concern about catastrophic flood loss. The importance of including information on the risk of extreme events will be demonstrated through the multiobjective tradeoff analysis presented in the case study below.

## **Case Study–Moorefield, WV**

This section develops an example to illustrate the integrated approach described in the previous section. A study undertaken at the South Fork and South Branch Potomac Rivers at Moorefield, West Virginia, in 1990 was selected as the basis for the development of the example.

### Local Flood Protection at South Fork and South Branch Potomac Rivers at Moorefield, West Virginia

The documents provided for this study were a reconnaissance report dated September 1987 and an Integrated Feasibility Report and Environmental Impact Statement dated March 1990. The latter consists of a main report and 13 appendices [U.S. Army Corps of Engineers 1990].

The following extracts from the main report [U.S. Army Corps of Engineers 1990] provide some background for this example:

The town of Moorefield in Hardy County, West Virginia, is subject to flooding from the South Fork and South Branch Potomac River. Serious floods have occurred in March 1936, June 1949, and November 1985 .....

In response to the flooding problem, the Corps of Engineers and the Interstate Commission on the Potomac River Basin initiated a cost-shared feasibility study in February 1988 to identify and evaluate possible solutions .....

A range of possible structural and nonstructural measures was examined. These measures included levees, floodwalls, channel improvements, bridge modification, and nonstructural alternatives. The most effective measures were combined into plans for comparison to the without project condition .....

In the example developed here, two given structures of flood control will be investigated. Plan 1 is the zero-cost plan, that is, the without-project-condition alternative. Plan 4 is a structural plan and includes levees and floodwalls to protect residential areas, industrial plants, businesses, schools, and commercial areas in both North and South Moorefield. A detailed description of this plan is provided on page 67 of the main report [U.S. Army Corps of Engineers 1990]. More details about the two plans can be found in Part 1 of the technical volume.

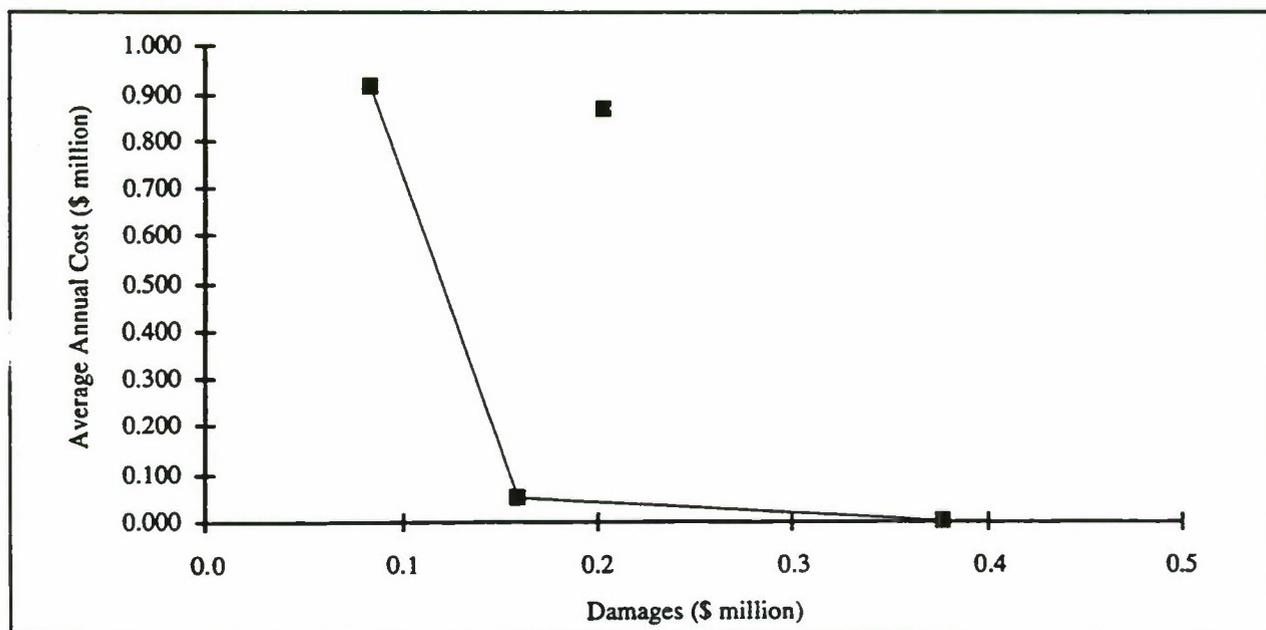
### Tradeoff Analysis

Once the conditional and unconditional expected values for the different plans are computed as described in the technical section, we can perform a tradeoff analysis in terms of costs and damages. The compiled results are shown in Table 1-1.

**Table 1-1. Summary of Results: Tradeoffs Among Cost, Expected Damage ( $f_5$ ), and Risk of Extreme Events ( $f_4$ ) for the Four Alternatives**

	Average Annual Cost (\$ million)	$f_5(L)$	$f_4(L \alpha=0.9)$	$f_4(L \alpha=0.99)$
Plan 1	0.000	0.377	3.428	6.657
Plan 1+W	0.050	0.159	1.586	4.086
Plan 4	0.865	0.204	2.515	5.705
Plan 4+W	0.915	0.084	1.412	3.348

Since plan 1 is the option of doing nothing, it does not have an associated cost. The average annual cost for plan 4 is given as \$0.865 million for a 50-year level of protection [Table I-18, page I-29, U.S. Army Corps of Engineers 1990]. The average annual cost of the flood warning system is assumed to be \$50,000. Figure 1-2 shows the resulting tradeoffs when using the expected value alone. The solid line shows the *Pareto optimal* frontier. (An option is Pareto optimal, or *noninferior*, if there is no other option that improves on any particular objective function without loss of ground in the other objective functions.) Figures 1-3 and 1-4 show the tradeoffs between annual cost and flood losses for  $\alpha = 0.9$  and  $0.99$  respectively, the  $\alpha$ 's corresponding to two levels of the risk of extreme events. Figure 1-5 shows these same tradeoffs together. In these figures,  $f_3$  denotes the expected value of loss and  $f_5$  denotes the conditional expected loss, a measure of the risk of extreme events. As explained in the technical volume, the conditional expected loss  $f_4$  with  $\alpha = 0.9$  is the average loss over the worst ten percent of floods; similarly,  $f_4$  with  $\alpha = 0.99$  is the average loss over the worst one percent of floods. Note from Figure 1-5 that the option of plan 4 without the warning system is noninferior, or a viable option, in considering only structural measures; the same option becomes inferior when considering the warning system options. Plan 1, plan 1+W, and plan 4+W constitute the noninferior set of options in the combined structural/nonstructural analysis. The combined analysis of structural and nonstructural measures, incorporating the risk of extreme events, clearly demonstrates the relative inefficiency of plan 4 without the warning system, an important result that would not have come from a traditional approach.



**Figure 1-2. Cost vs. Damage Tradeoff for the Expected Value ( $f_3$ )**  
(Note that there are three Pareto optimal (efficient) alternatives)

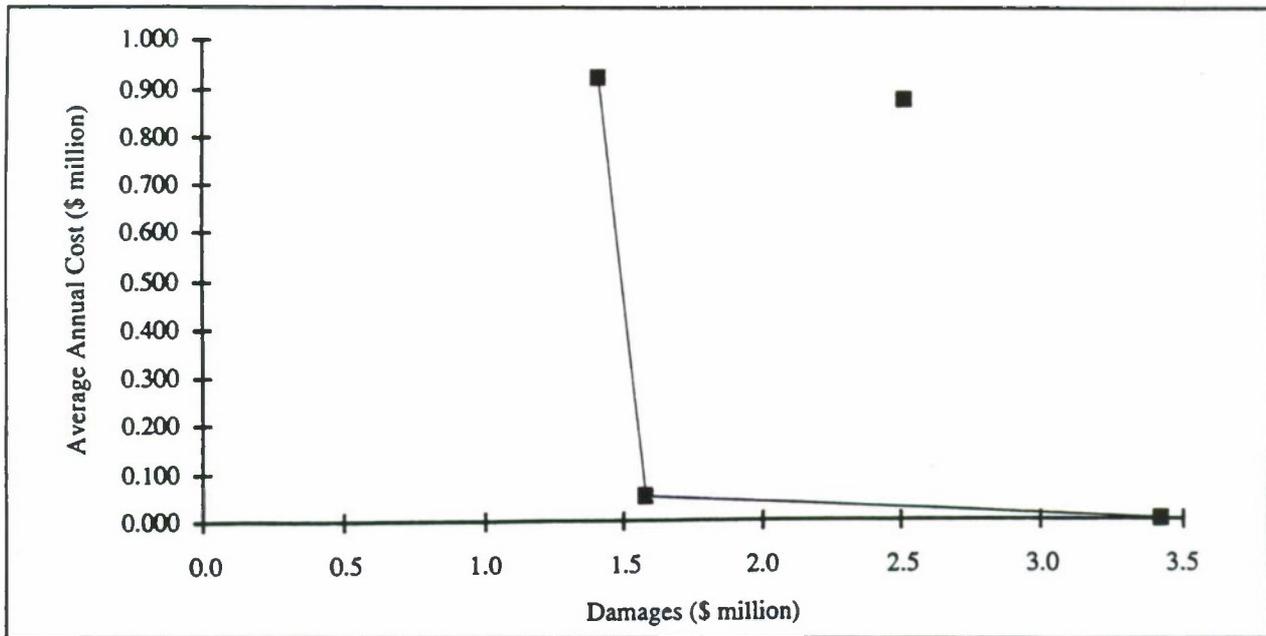


Figure 1-3. Cost vs. Conditional Expected Damage ( $f_d$ ) Tradeoff for  $\alpha = 0.9$   
(Note the three Pareto optimal (efficient) alternatives)

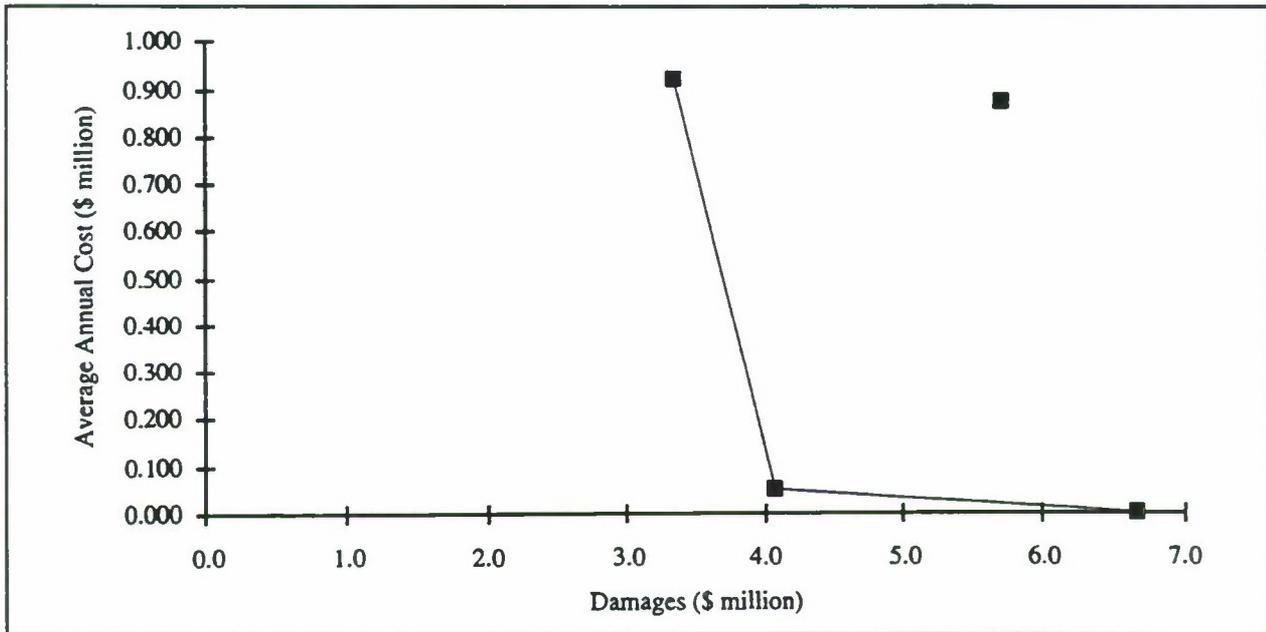
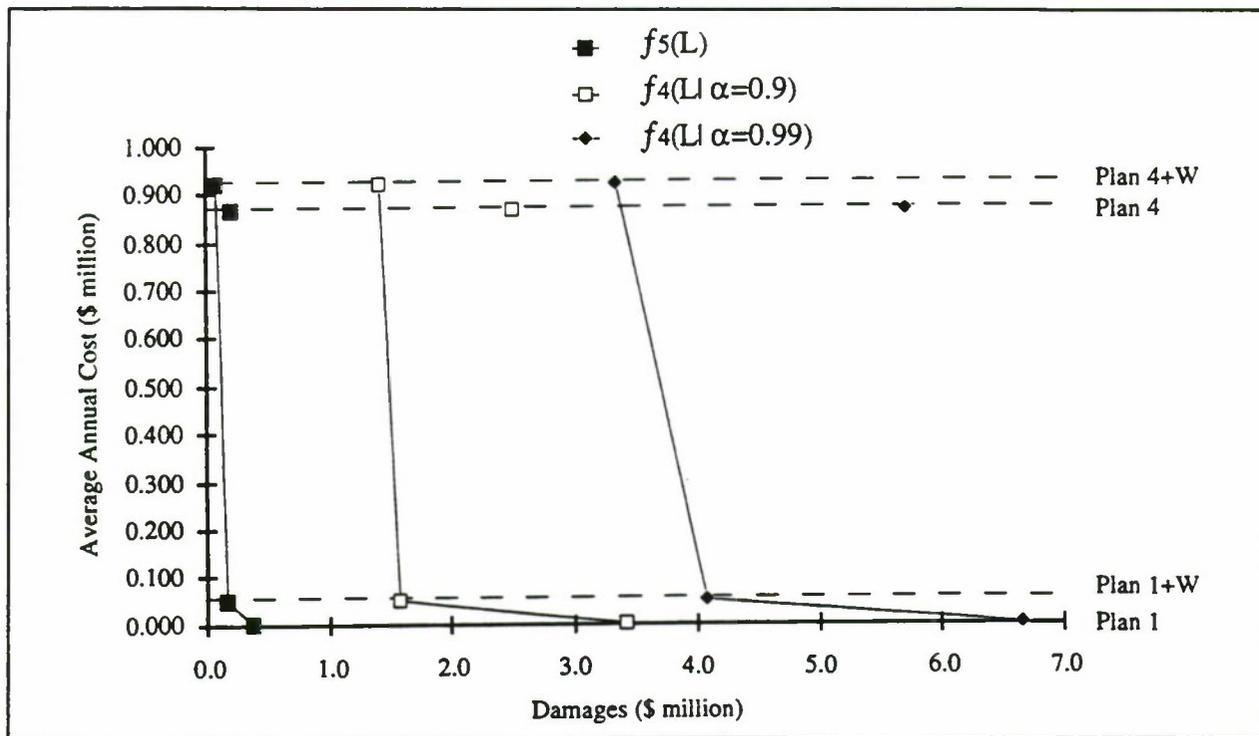


Figure 1-4. Cost vs. Conditional Expected Damage ( $f_d$ ) Tradeoff for  $\alpha = 0.99$   
(Note the three Pareto optimal (efficient) alternatives)



**Figure 1-5.** Tradeoffs among Cost, Expected Damage ( $f_5$ ), and Conditional Expected Damage ( $f_4$ ) for the Four Alternatives: Risk of Extreme Events ( $f_4$ ) Evaluated at Two Partitioning Levels ( $\alpha = 0.9, \alpha = 0.99$ )



## **Part 2**

# **Multiobjective Decision-tree Analysis: Overview**

## **Introduction**

### **Overview**

Decision-tree analysis, which is especially useful when related actions are taken in a sequence, has emerged over the years as an effective and useful tool in decisionmaking. More than two decades ago, Howard Raiffa [1968] published the first comprehensive and authoritative book on decision-tree analysis. Ever since, its applications to a variety of problems from numerous disciplines have grown by leaps and bounds [see Sage 1977 and Hamburg 1970]. Advances in science and in scientific approaches to problem solving are often made on the basis of earlier works of others. In this case, the foundation for Raiffa's contributions to decision tree analysis can be traced to the works of Bernoulli on utility theory [see von Neumann and Morgenstern 1944; Edwards 1967; Savage 1954; Adams 1960; Arrow 1963; Shubik 1964; Luce and Suppes 1965; and others]. This chapter, in an attempt to build on the above seminal works, extends and broadens the concept of decision-tree analysis to incorporate: (a) multiple, noncommensurate and conflicting objectives, (b) impact analysis, and (c) the risk of extreme and catastrophic events [Haimes et al. 1990]. In contrast, the current practice often involves one-sided use of decision trees -- optimizing a single objective function and commensurating infrequent catastrophic events with more frequent noncatastrophic events using the common unconditional mathematical expectation.

We consider below (see Section 2.3) the application of multiobjective decision trees to a flood-warning and evacuation problem. Three possible actions, (a) evacuation, (b) issuing a flood watch, and (c) doing nothing, are under consideration at each of two decision periods. There are costs associated with the first two options. Furthermore, the cost associated with each option is a function of the period in which the action is taken. The evaluative performance measures used include the expected and conditional expected costs (losses) and the expected and conditional expected loss of lives resulting from a series of flood watch/evacuation decisions. The conditional expected value (of cost and of lives lost) is a measure of the risk of extreme events that is generated in the partitioned multiobjective risk method (PMRM) described below.

### **Multiple Objectives**

The single-objective models that had been advanced in the fifties, sixties, and seventies are today considered by many to be unrealistic, too restrictive, and often inadequate for most real-world complex problems. The proliferation of books, articles, and conferences and courses during the last decade or two on what has come to be known as multiple-criteria decisionmaking (MCDM) is a vivid indication of this somber realization and of the maturation of the field of decisionmaking [see Chankong and Haimes 1983]. In particular, an optimum derived from a single-objective mathematical model, including that which is derived from a decision tree, often may be far from representing reality -- thereby misleading the analyst(s) as well as the decisionmaker(s). Fundamentally, most complex problems involve, among other things, the minimization of costs, the maximization of benefits (not necessarily in monetary values), and the

minimization of risks of various kinds. Decision trees, which are a powerful mechanism for the analysis of complex problems, can better serve both the analysts and the decisionmakers when they are extended to deal with the above multiple objectives.

### **Impact Analysis**

On a long-term basis, managers and other decisionmakers are often rewarded not because they have made many optimal decisions in their tenure; rather, they are honored and thanked for avoiding adverse and catastrophic consequences. If one accepts this premise, then the role of impact analysis -- studying and investigating the consequences of present decisions on future policy options -- might be as important, if not actually more so, than generating an optimum for a single-objective model or identifying a Pareto optimum set for a multiobjective model. Certainly, when the ability to generate both is present, having an appropriate Pareto optimum set and knowing the impact of each Pareto optimum on future policy options should enhance the overall decisionmaking process within the decision-tree framework.

### **The Risk of Extreme and Catastrophic Events**

Risk, which is a measure of the probability and severity of adverse effects, has until recently been commonly quantified via the expected-value formula. This formula essentially precommensurates events of low frequency and high damage with events of high frequency and low damage. Although learned students of risk analysis recognize the disparity between the above fallacious representation of extreme and catastrophic events and the perception of these events by individuals or the public at large, many continue to use this approach. The trend, however, is moving toward the conditional-expected-value approach, where extreme and catastrophic events are partitioned, isolated, quantified in terms of the conditional expectation (e.g., using concepts from the statistics of extremes), and then evaluated along with the common expected value of risk or damage [Asbeck and Haines 1984; Haines 1985; Karlsson and Haines 1988a, 1988b; Haines et al. 1988].

The partitioned multiobjective risk method (PMRM) developed by Asbeck and Haines [1984] separates extreme events from other noncatastrophic events, and thus provides the decisionmaker(s) with additional valuable and useful information. In addition to using the traditional expected value, the PMRM generates the conditional expected-value, which represents the risk, given that the damage falls within an extreme range of exceedance probability (or range of damage). The conditional expected value represents the risk with low probability of exceedance and high damage. Combining each of the conditional expected value and the unconditional expected value with the competing objective of cost results in a set of multiobjective optimization problems from which an optimal balance of cost and risk can be found. The tradeoffs between the cost objective and either risk objective (conditional and unconditional expected value) enable decisionmakers to consider the marginal cost of a small reduction in a risk objective, given a particular level of risk assurance for extreme events and given the unconditional risk function. The PMRM is integrated with the multiobjective decision tree in the *continuous-case* example of Section 2.3 below.

## Methodological Approach

### Extension of the Decision Tree to Multiple Objectives

Similar to the decision-tree in conventional single-objective analysis, a multiobjective decision tree (Fig. 2-1) is composed of decision nodes, chance nodes and a time sequence of related actions and consequences [Haimes et al. 1990]. Following Raiffa [1968], a decision node is designated by a small square and a chance node by a small circle. But now each path through the tree, instead of having a single measure of evaluation as has been usual, is characterized by a vector-valued (multiobjective) performance measure.

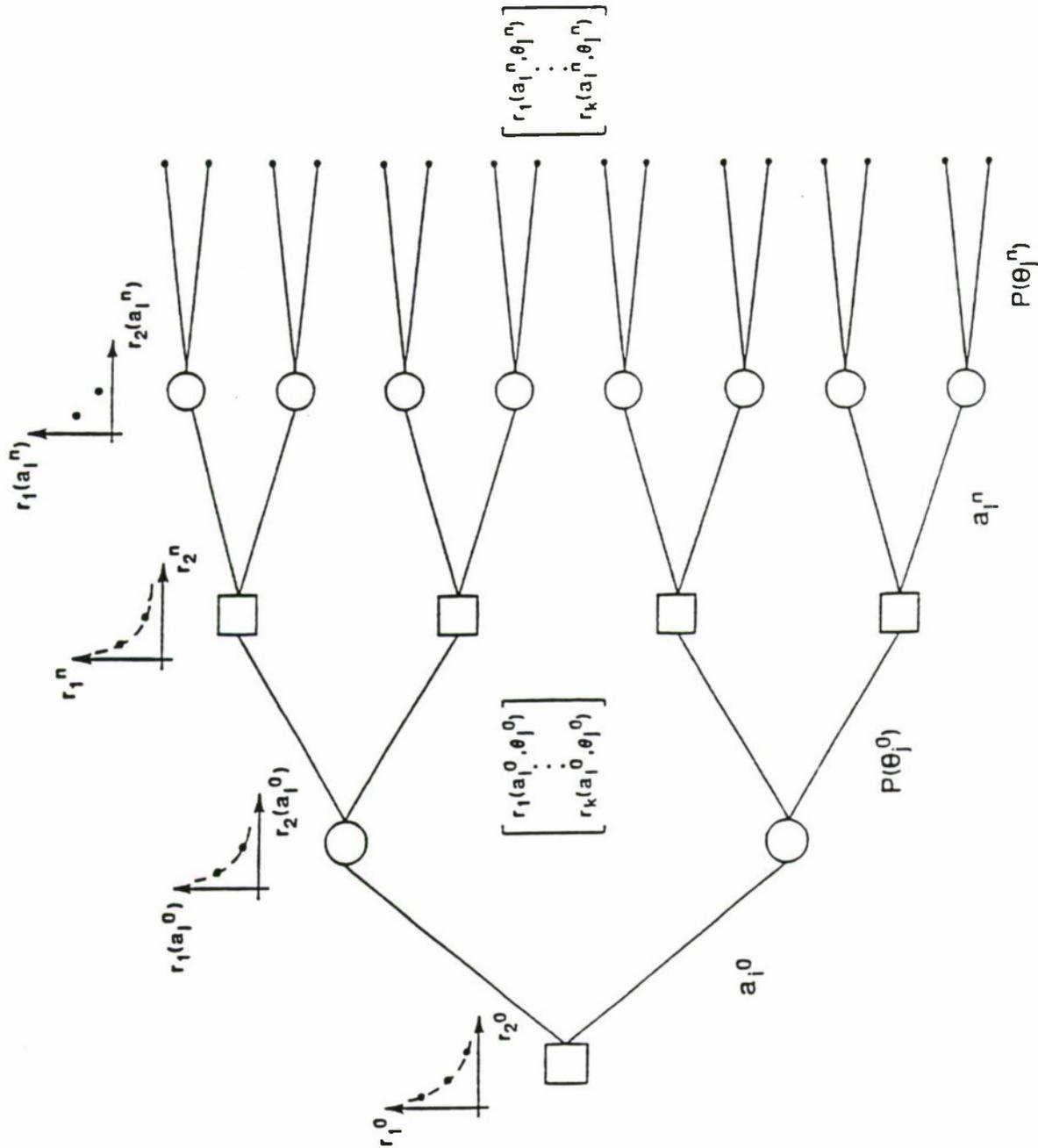
At a decision node, the decisionmaker selects one course of action from the feasible set of alternatives. We assume that there are only a finite number of alternatives at each decision node. These alternatives are shown as branches emerging to the right side of the decision node. The performance vector associated with each alternative is written along the corresponding branch. Each alternative branch may lead to another decision node, a chance node, or a terminal point.

A chance node, indicates that a chance event is expected at this point; that is, one of the states of nature may occur. We consider two cases in this study: a) a discrete case, where the number of states of nature is assumed finite; and b) a continuous case, where the possible states of nature are assumed continuous. The states of nature are shown on the tree as branches to the right of the chance nodes, and their known probabilities are written above the branches. The states of nature may be followed by another chance node, a decision node, or a terminal point.

Allowing for the evaluation of the multiple objectives at each decision node constitutes a significant extension of the average-out-and-folding-back strategy used in conventional single-objective decision-tree methods. The procedure for multiobjective decision trees is similar to that of a single-objective tree. At each decision node and at each branch emerging to the right side of the decision node, we find the corresponding set of vector-valued performance measures (multiple objectives) for each alternative and identify the set of noninferior solutions. At a noninferior solution (or Pareto optimum solution), one cannot achieve a decrease in any single objective without observing an increase in at least one other objective.

In multiobjective decision-tree analysis, instead of having a single optimal value associated with a single-objective decision tree, we have a set of vector-valued objective values of noninferior decision alternatives at each decision node. In single-objective decision-tree analysis, there is no choice process at the chance nodes, since only an averaging-out process takes place there. In multiobjective decision-tree analysis, a set of Pareto optimum (noninferior) alternatives is associated with each branch emerging from a chance node. A vector minimization is performed to discard from further consideration the resulting inferior combinations. Finally, a straightforward solution technique is applied repeatedly until the set of noninferior solutions at the starting point of the tree is obtained.

Together with the following comments (*Impact of Experimentation* and *Extension to Multiple Risk Measures*), the examples on flood watch and evacuation decisions described below provide an overview of the new methodology that incorporates multiple objectives and the PMRM into decision trees.



**Figure 2-1.** Structure of Multiobjective Decision Trees

### **Impact of Experimentation**

The impact of an added piece of information (obtained, e.g., through experimentation) on different objectives is now addressed, and the value of the information is quantified by a vector-valued measure. In conventional decision-tree analysis, whether or not an experiment should be performed depends on an assessment of the expected value of experimentation, which is the difference between the expected loss without experimentation and the expected loss with experimentation. If the expected value of experimentation is negative, experimentation is deemed unwarranted; otherwise, the experiment that yields the lowest loss is selected. In multiobjective decision-tree analysis, the monetary index does not constitute the sole consideration; rather, the value of experimentation is judged in a multiobjective way where, in many cases, the noninferior frontiers generated with and without experimentation do not dominate each other. The added experimentation in these cases reshapes the feasible region (and thus the noninferior frontier) and generates new and better options for the decisionmakers (Fig. 2-2).

### **Extension of the Decision Tree to Multiple Risk Measures**

Multiobjective decision-tree analysis calls for the adoption of multiple-risk measures. Often, the expected value, by itself, provides insufficient information for risk management. The expected value of adverse effects, which has been most commonly used in conventional decision-tree analysis, is in many cases inadequate, since this scalar representation of risk commensurates events that correspond to all levels of losses and to their associated probabilities. The common expected-value approach is particularly deficient for addressing extreme events, since these events are concealed during the amalgamation of events of low probability and high consequence with events of high probability and low consequence. The synthesis of several approaches -- single-objective decision-tree analysis, multiobjective optimization, the partitioned multiobjective risk method (PMRM), and the statistics of extremes -- has led to the development of the multiobjective decision-tree method. This new form of decision-tree analysis can handle different risk functions including the common expected value, the conditional expected value for extreme events, and the event with maximum probability, thus providing the decisionmaker(s) with more comprehensive knowledge and a robust decision policy.

Determining the folding-back strategy associated with conditional expected values is substantially different from such an operation using the conventional expected value. Unlike the latter, which is a linear operator, the conditional expected-value operator is nonlinear. This nonlinearity represents an obstacle in decomposing the overall value of the conditional expected value and in calculating it at different decision nodes. Thus, in calculating conditional risk functions, all performance measures at the different branches are mapped to the terminal points where the partitioning is performed.

In summary, we adhere to the following rules when calculating the conditional expected value in the folding-back procedure of decision trees:

- 1) Partition and calculate the conditional expected value of extreme damage at terminal points according to the conditional probability density function.
- 2) Fold back and perform at each chance node the operation of the expected value.

The justification and elaboration of these steps is given in the technical part.

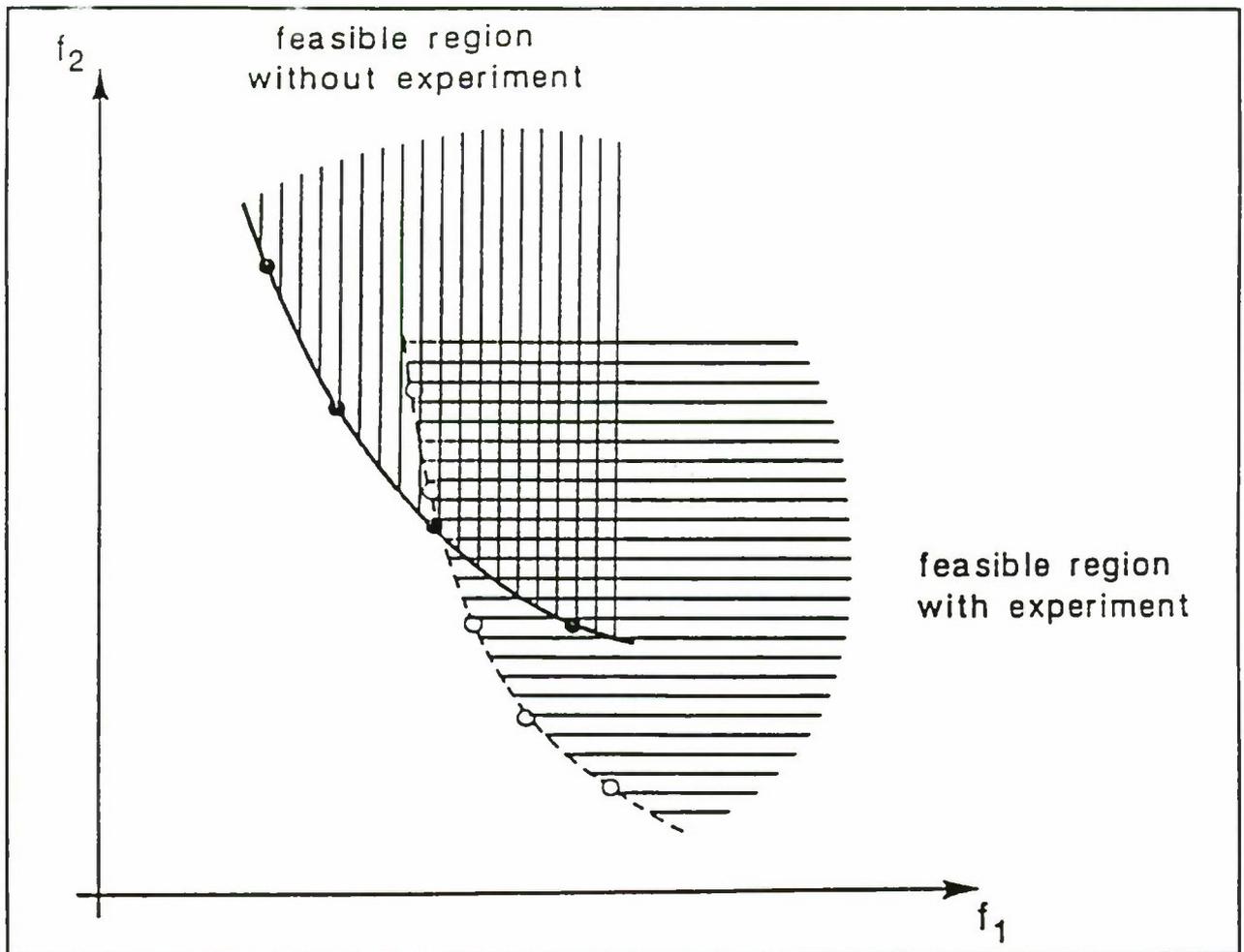


Figure 2-2. Re-shape of the Feasible Region by an Experimentation

Note that although reducing the variance (the uncertainty) of the risk may not contribute much to reducing the expected value, it often markedly reduces the conditional expected value associated with extreme events (see Fig. 2-3). Two benefits that result from additional experimentation include reducing the expected loss and reducing the uncertainty associated with decisionmaking under risk. However, in most cases, these two dual aspects of experimentation conflict with each other. The general framework of multiobjective decision-tree analysis proposed here provides a medium with which these dual aspects can be captured by investigating the multiple impacts of experimentation.

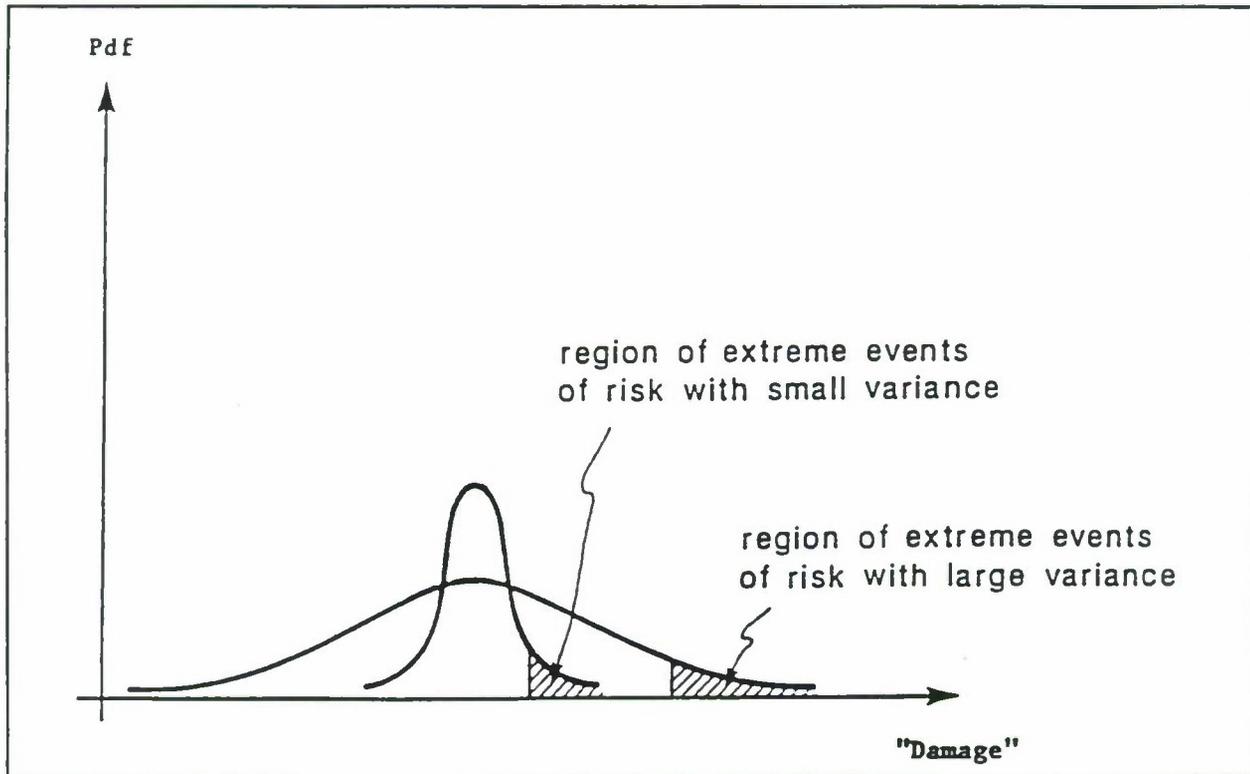


Figure 2-3. Variance and Region of Extreme Events

## Conclusions

Multiobjective decision-tree analysis is an extension of the single-objective-based decision-tree analysis formally introduced more than two decades ago by Howard Raiffa [1968]. This extension is made possible by making a synthesis of the traditional method and more recently developed approaches used for multiobjective analysis and for the risk of extreme and catastrophic events. Successful applications of single-objective decision-tree analysis to numerous business, engineering, and governmental decisionmaking problems over the years have made the methodology into an important and valuable tool in systems analysis. Its extension – incorporating multiple noncommensurate objectives, impact analysis, and the conditional expected value for extreme and catastrophic events – might be viewed as an indicator of growth in the broader field of systems analysis and in decisionmaking under risk and uncertainty. Undoubtedly, there remain several theoretical challenges that must be addressed to fully realize the strengths and usefulness of the extended methodology. In this sense, the multiobjective decision-tree analysis proposed here constitutes the first, albeit important, step in the direction of developing improved and more representative models and decisionmaking tools.

## Example Problem for the Discrete Case

### Problem Definition

The example problem discussed here concerns a simplified flood warning and evacuation system. Three possible actions, (a) evacuation, (b) issuing a flood watch, and (c) doing nothing, are under consideration. There are cost factors associated with the first two options. The decision tree covers two time periods, and the cost associated with each option is a function of the period in which the action is taken. The complete decision tree for the problem is shown in Fig. 2-4. The following assumptions are made:

- a) There are three possible actions with associated costs for the first period:
  - 1) issuing an evacuation order at a cost of \$5 million [EV1],
  - 2) issuing a flood watch at a cost of \$1 million [WA1], and
  - 3) doing nothing at no cost [DN1].
- b) For the second period the actions and the corresponding costs are:
  - 1) issuing an evacuation order at a cost of \$3 million [EV2],
  - 2) issuing a flood watch at a cost of \$0.5 million [WA2], and
  - 3) doing nothing at no cost [DN2].
- c) The flood stage is reached at water flow ( $W$ ) = 50,000 cfs.
- d) There are three possible underlying probability density functions (pdfs) for the water flow:
  - 1)  $W \sim \text{lognormal}(10.4, 1)$  represented as  $LN_1$ ; where the parentheses designate statistical parameters of the lognormal distribution of flow as (*mean flow*  $\times 10^3$  cfs, *standard deviation*  $\times 10^3$  cfs);
  - 2)  $W \sim \text{lognormal}(9.1, 1)$ , represented as  $LN_2$ , and
  - 3)  $W \sim \text{lognormal}(7.8, 1)$ , represented as  $LN_3$ .

The prior probabilities that any of these pdfs is the actual pdf are equal.

- e) There are four possible events at the end of the first period:
  - 1) A flood ( $W > 50,000$  cfs) occurs.

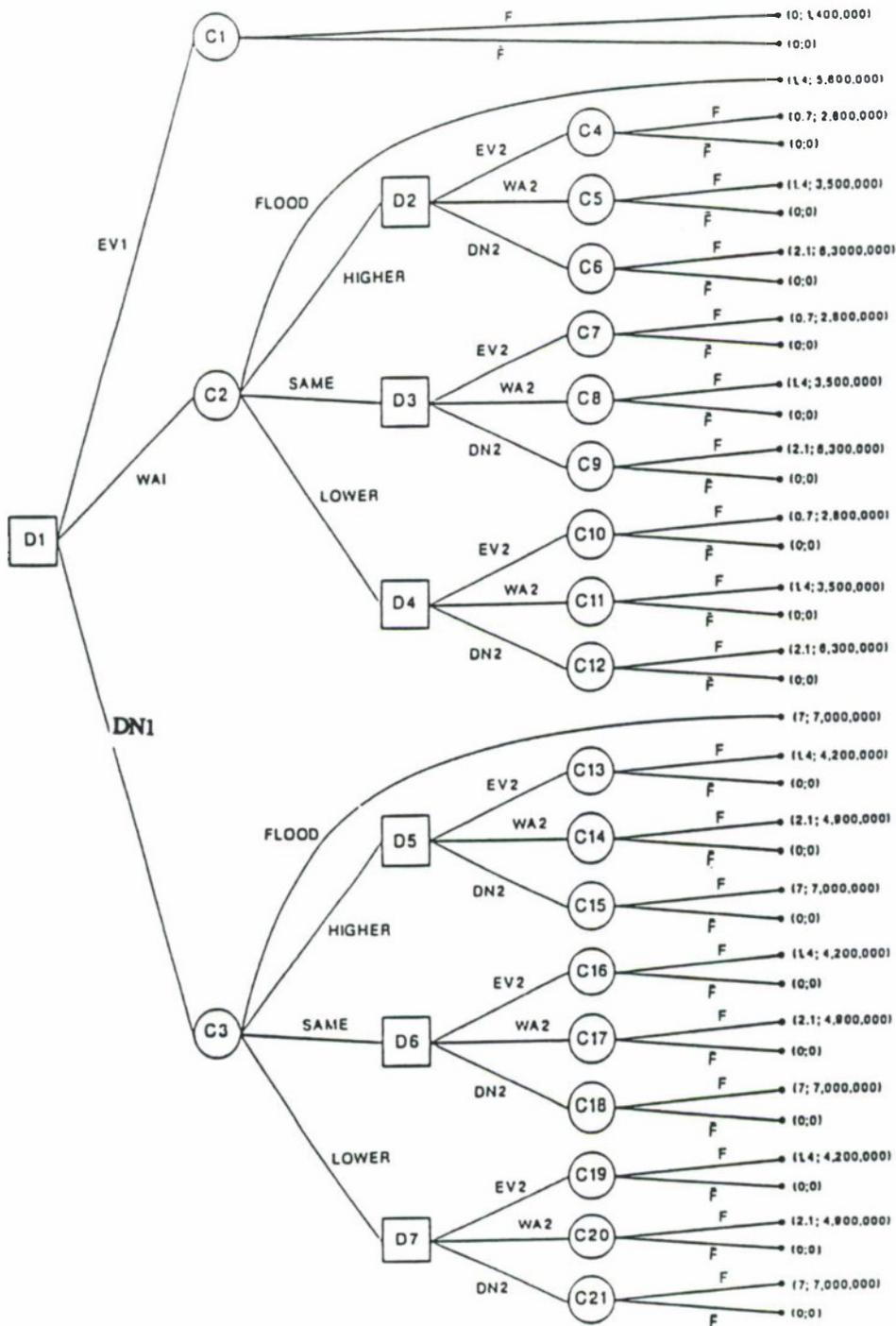


Figure 2-4. Decision Tree for the Discrete Case

- 2) The water flow is greater than that of the previous period ( $15,000 \text{ cfs} \leq W \leq 50,000 \text{ cfs}$ ), represented as W1.
  - 3) The water flow is in the same range as that of the previous period ( $5000 \text{ cfs} \leq W \leq 15,000 \text{ cfs}$ ), represented as W2.
  - 4) The water flow is lower than that of the previous period ( $W \leq 5000 \text{ cfs}$ ), represented as W3.
- f)  $L = 7$  and  $C = \$7,000,000$  are respectively the maximum possible loss of lives and properties, given no flood warning. All other costs are shown in Fig. 2-4.

### Summary of the Results

The decision tree is simplified and solved for this example as described in the technical section. Table 2-1 presents the values of the loss vectors for the second-period decision arcs. In *folding back* at each decision node, the vector-valued performance measures are compared to eliminate all dominated (inferior) policies. Consider, for example, decision node D2. The vector corresponding to the decision DN2 is inferior to the vector corresponding to the decision WA2

$$\begin{bmatrix} 0.3058 \\ 1,264,579 \end{bmatrix}_{WA2} < \begin{bmatrix} 0.4588 \\ 1,376,241 \end{bmatrix}_{DN2}$$

where 0.3058 is the expected loss of life and \$1,264,579 is the expected flood loss for the decision to issue a warning WA2. Note that the two evaluative measures for WA2 are each of lesser value than the corresponding measures for the *do nothing* option DN2, which is the condition for identifying an inferior policy.

Table 2-2 presents the noninferior decisions for the second-period decision arcs. Averaging-out at the chance nodes for the first period, each noninferior decision corresponding to each arc is multiplied by the probability for that arc, yielding a single decision rule for the first-period decision node. For example, we have 18 different combinations at WA1, one of which is (EV2 | higher, EV2 | same, EV2 | lower). The value of the loss vector for this combination is:

$$\begin{aligned} & \left[ \begin{array}{c} 0.1779 + 0.1529 * 0.2466 + 0.0704 + 0.2686 + 0.0150 * 0.3577 \\ (1,000 + 711.76 + 3,611.663 * 0.2466 + 3,281.526 * 0.2686 + 3,060.043 * 0.3577)1,000 \end{array} \right] \\ & = \begin{bmatrix} 0.2399 \\ 4,578,391 \end{bmatrix} \end{aligned}$$

**Table 2-1. Expected Value of Loss Vectors for the Second-period Decision Arcs (Discrete Case)**

Node	Arc	L	C
D2	EV2	0.1529	3,611,663
	WA2	0.3058	1,264,579
	DN2	0.4588	1,376,241
D3	EV2	0.0704	3,281,526
	WA2	0.1408	851,908
	DN2	0.2112	633,434
D4	EV2	0.0150	3,060,043
	WA2	0.0300	575,054
	DN2	0.0450	135,097
D5	EV2	0.3058	3,917,494
	WA2	0.4588	1,570,410
	DN2	1.5292	1,529,157
D6	EV2	0.1408	3,422,289
	WA2	0.2112	992,671
	DN2	0.7038	703,815
D7	EV2	0.0300	3,090,065
	WA2	0.0450	605,076
	DN2	0.1501	150,108
C2	F	0.1779	711,760
C3	F	0.8897	889,700

L -- loss of lives  
C -- cost (\$)

**Table 2-2. Noninferior Decisions for the Second-period Decision Nodes (Discrete Case)**

Node	Noninferior decisions
D2	EV2, WA2
D3	EV2, WA2, DN2
D4	EV2, WA2, DN2
D5	EV2, WA2, DN2
D6	EV2, WA2, DN2
D7	EV2, WA2, DN2

where the first and second elements represent an expected loss of lives of 0.2399 and expected cost of \$4,578,391, respectively. Table 2-3 presents the values of the vector of objectives for the first-period decision node. Note from Table 2-3 that a total of nine noninferior decisions are generated for action WA1. Similarly, there are five noninferior solutions for action DN1 (by comparison of all vectors for that action), and fourteen noninferior solutions after comparing all decisions for all actions (see Fig. 2-5). Fig. 2-6 depicts the graph of all noninferior solutions. The decisionmaker, in interaction with the analyst, can usually identify a most-preferred policy from among the noninferior set.

## Example Problem for the Continuous Case

### Problem Definition

The problem developed above for the discrete case is modified here to handle continuous loss functions and extreme random events. The set of objective functions is extended to include the *conditional* expected loss of lives and *conditional* expected cost in addition to the expected values. The main difference between the discrete and the continuous cases lies in calculating the damage vector for the terminal nodes, which can be determined using the expected value and/or the conditional expected value of extreme events. The subsequent computations are similar to those carried out for the discrete case. In addition, the assumption (f) of the *discrete* case above is modified as:

- f') The parameters L and C are now defined as, respectively, the possible loss of lives and cost, given that no flood warning is issued; they are linear functions of the water flow W as defined in the technical section.

The complete decision tree for this case is shown in Fig. 2-7. The loss functions L and C are used in calculating the unconditional expected-value denoted by  $f_3(\bullet)$ , and/or the conditional expected-value denoted by  $f_4(\bullet)$ , which represents the risk of extreme events. Note that each of the risk measures the conditional expected value  $f_4(\bullet)$  and the unconditional expected value  $f_3(\bullet)$  is composed of two components (or expectation values) -- cost and loss of lives.

### Summary of the Results

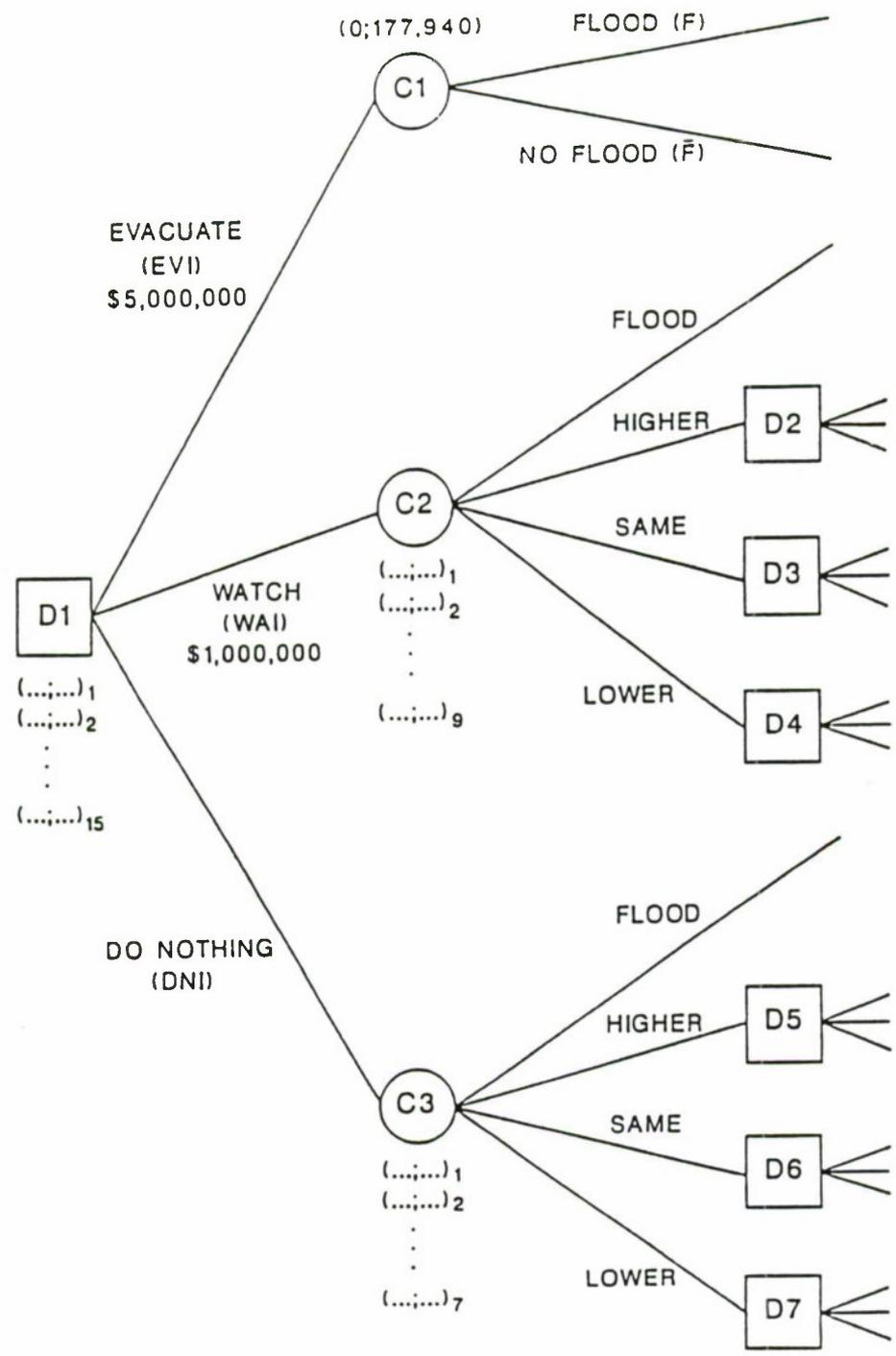
The decision tree is simplified and solved for this *continuous* example as described in the technical section. Note that regardless of whether a watch (WA1) was issued or a do-nothing (DN2) action was followed at the first period, the same three possible actions are evaluated at the second period: evacuate, issue another flood watch, or do nothing. Depending on the actions taken at the first and second periods and the water flow level at the second period, different values of losses are generated for each terminal chance node. Recall that there are three equally probable underlying pdfs for the water flow for the first period.

Table 2-4 summarizes the values of the unconditional and conditional expected loss vectors  $f_3(\bullet)$  and  $f_4(\bullet)$  for the decision arcs corresponding to the second period. Once these values are calculated, the noninferior decisions for each node are calculated by folding back the same way as in the discrete case. Table 2-5 yields the noninferior decisions for the second-period decision arcs. Averaging-out at the chance nodes for the first period follows the same procedure used in the discrete case. Consider, for example,

**Table 2-3. Decisions for the First-Period Decision Node (Discrete Case)**

First-period decision	Second-period decision			Loss vector	
	Higher	Same	Lower	L	C
* EV1	-	-	-	0.0000	5,177,940
* WA1	EV2	EV2	EV2	0.2399	4,578,391
* WA1	EV2	EV2	WA2	0.2452	3,689,511
* WA1	EV2	EV2	DN2	0.2506	3,532,138
WA1	EV2	WA2	EV2	0.2588	3,925,796
* WA1	EV2	WA2	WA2	0.2641	3,036,916
* WA1	EV2	WA2	DN2	0.2695	2,879,543
WA1	EV2	DN2	EV2	0.2777	3,867,113
WA1	EV2	DN2	WA2	0.2830	2,978,233
* WA1	EV2	DN2	DN2	0.2884	2,820,860
WA1	WA2	EV2	EV2	0.2776	3,999,600
WA1	WA2	EV2	WA2	0.2829	3,110,720
WA1	WA2	EV2	DN2	0.2883	2,953,347
WA1	WA2	WA2	EV2	0.2965	3,347,005
* WA1	WA2	WA2	WA2	0.3018	2,458,125
* WA1	WA2	WA2	DN2	0.3072	2,300,752
WA1	WA2	DN2	EV2	0.3154	3,288,323
WA1	WA2	DN2	WA2	0.3207	2,399,442
* WA1	WA2	DN2	DN2	0.3261	2,242,070
DN1	EV2	EV2	EV2	1.0136	3,880,297
DN1	EV2	EV2	WA2	1.0190	2,991,417
DN1	EV2	EV2	DN2	1.0566	2,828,675
DN1	EV2	WA2	EV2	1.0325	3,227,701
* DN1	EV2	WA2	WA2	1.0379	2,338,821
DN1	EV2	WA2	DN2	1.0756	2,176,079
DN1	EV2	DN2	EV2	1.1648	3,150,115
DN1	EV2	DN2	WA2	1.1702	2,261,235
DN1	EV2	DN2	DN2	1.2078	2,098,493
DN1	WA2	EV2	EV2	1.0513	3,301,506
DN1	WA2	EV2	WA2	1.0567	2,412,626
DN1	WA2	EV2	DN2	1.0943	2,249,884
DN1	WA2	WA2	EV2	1.0702	2,648,910
* DN1	WA2	WA2	WA2	1.0756	1,760,030
* DN1	WA2	WA2	DN2	1.1132	1,597,288
DN1	WA2	DN2	EV2	1.2025	2,571,324
DN1	WA2	DN2	WA2	1.2079	1,682,444
* DN1	WA2	DN2	DN2	1.2455	1,519,702
DN1	DN2	EV2	EV2	1.3153	3,291,333
DN1	DN2	EV2	WA2	1.3207	2,402,453
DN1	DN2	EV2	DN2	1.3583	2,239,711
DN1	DN2	WA2	EV2	1.3342	2,638,737
DN1	DN2	WA2	WA2	1.3396	1,749,857
DN1	DN2	WA2	DN2	1.3772	1,587,115
DN1	DN2	DN2	EV2	1.4665	2,561,151
DN1	DN2	DN2	WA2	1.4719	1,672,271
* DN1	DN2	DN2	DN2	1.5095	1,509,529

\* - noninferior decisions



**Figure 2-5. Decision Tree for the First Stage (Discrete Case)**

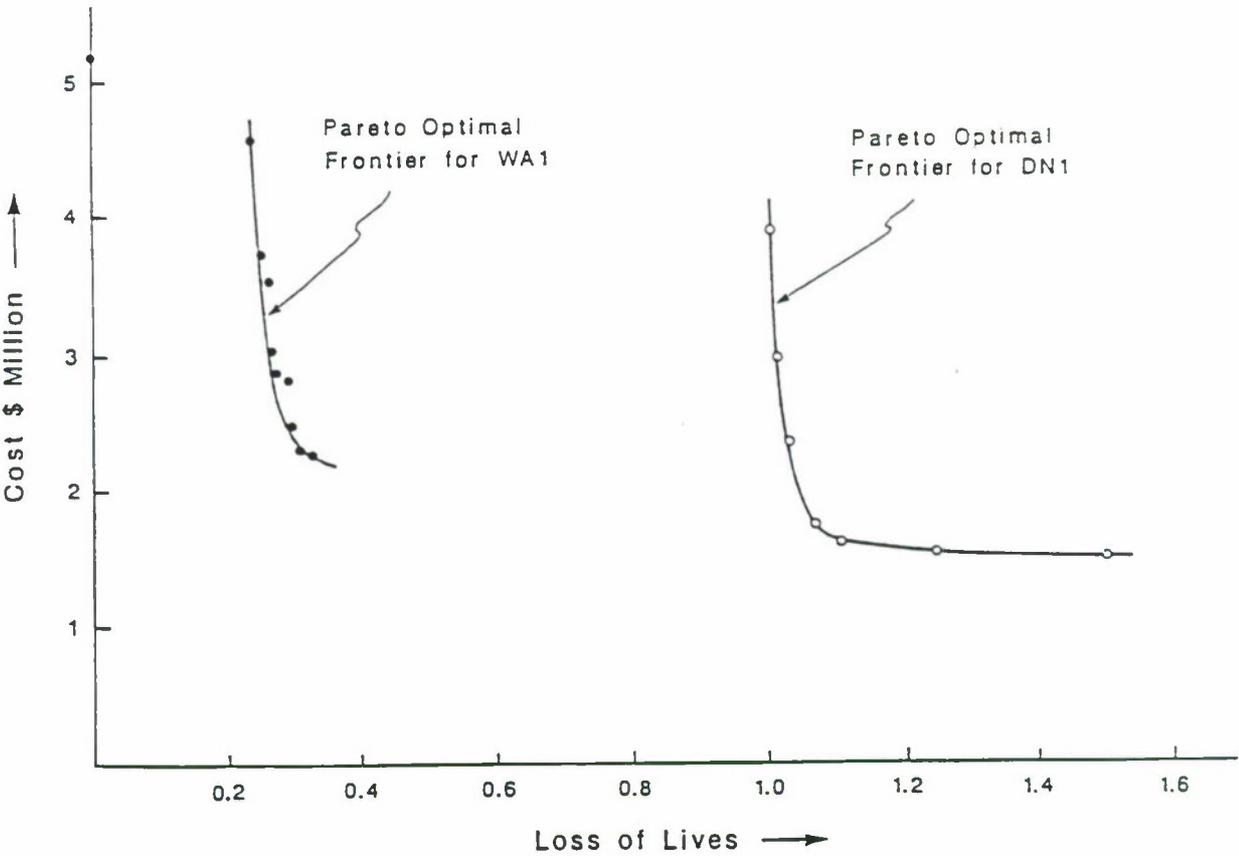
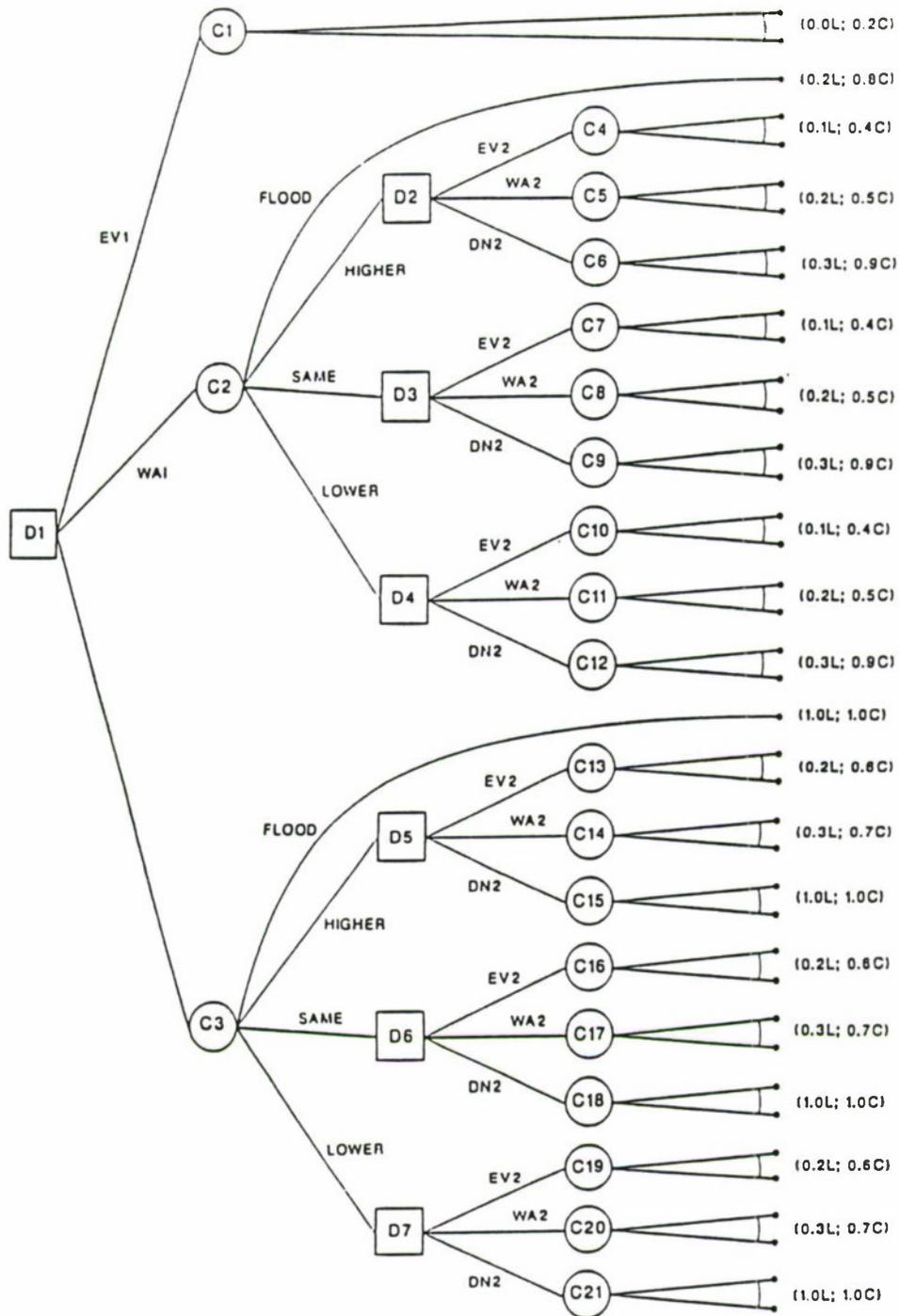


Figure 2-6. Pareto Optimal Frontier (Discrete Case)



**Figure 2-7. Decision Tree for the Continuous Case**

**Table 2-4. Loss Vectors for the Second-period Decision Arcs (Continuous Case)**

Node	Arc	$f_5(\cdot)$		$f_4(\cdot)$	
		L	C	L	C
D2	• * EV2	0.0798	3,319,034	3.5166	17,066,293
	• WA2	0.1595	898,792	7.0332	18,082,866
	• DN2	0.2393	717,826	10.5497	31,649,159
D3	• * EV2	0.0312	3,124,816	1.9595	10,838,047
	• * WA2	0.0624	656,020	3.9190	10,297,559
	• DN2	0.0936	280,835	5.8785	17,653,606
D4	• * EV2	0.0040	3,016,172	0.7598	6,039,120
	• * WA2	0.0081	520,215	1.5196	4,298,901
	• DN2	0.0121	36,387	2.2793	6,838,021
D5	• * EV2	0.1595	3,478,550	7.0332	24,099,439
	• WA2	0.2393	1,058,309	10.5497	25,116,012
	• DN2	0.7976	797,584	35.1657	35,165,732
D6	• * EV2	0.0624	3,187,223	3.9190	14,757,071
	• * WA2	0.0936	718,427	5.8785	14,216,583
	• DN2	0.3120	312,039	19.5951	19,595,118
D7	• * EV2	0.0081	3,024,258	1.5196	7,558,681
	• * WA2	0.0121	528,301	2.2793	5,818,461
	• DN2	0.0404	40,430	7.5978	7,597,801
C2	F	0.0886	354,298	4.4956	17,982,472
C3	F	0.4429	442,872	22.4781	22,478,090

• noninferior decisions using  $f_5(\cdot)$   
\* noninferior decisions using  $f_4(\cdot)$

**Table 2-5. Noninferior Decisions for the Second-period Decision Nodes (Continuous Case)**

Node	Noninferior decisions	
	$f_5(\cdot)$	$f_4(\cdot)$
D2	EV2, WA2, DN2	EV2
D3	EV2, WA2, DN2	EV2, WA2
D4	EV2, WA2, DN2	EV2, WA2
D5	EV2, WA2, DN2	EV2
D6	EV2, WA2, DN2	EV2, WA2
D7	EV2, WA2, DN2	EV2, WA2

action WA1. There are 27 different combinations when using the expected value  $f_5(\bullet)$ , and four different combinations when using  $f_4(\bullet)$ . Table 2-6 yields the values of the loss vectors for the first-period decision node using  $f_5(\bullet)$ , and Table 2-7 yields the values of the loss vectors using  $f_4(\bullet)$ . Note from Table 2-6 that for action WA1 there are a total of 10 noninferior decisions. Similarly for action DN1, there are 8 noninferior solutions resulting from the comparison of all vectors for action DN1, and 6 noninferior solutions after comparison of all decisions for all actions using  $f_5(\bullet)$  (see Fig. 2-8). Figure 2-9 depicts the graph of all noninferior solutions using the traditional *expected* losses  $f_5(\bullet)$ .

Note from Table 2-7 that there is only one noninferior action when considering the conditional expected losses  $f_4(\bullet)$ . Thus, the action EV1 is the most conservative from the point of view of extreme events. When the decisionmaker considers the risk of extreme events, the potential loss of property overshadows the cost of the warning system. Thus, the two objectives -- cost and loss of life -- do not conflict in this example when looking only at the risk of extreme events.

Table 2-6. Decisions for the First-period Decision Node Using  $f_3$  (Continuous Case)

First-period decision	Second-period decision			Loss vector	
	Higher	Same	Lower	L	C
* EV1	-	-	-	0.0000	5,088,574
* WA1	EV2	EV2	EV2	0.0408	3,781,716
* WA1	EV2	EV2	WA2	0.0423	2,888,912
* WA1	EV2	EV2	DN2	0.0437	2,715,847
WA1	EV2	WA2	EV2	0.0492	3,118,597
* WA1	EV2	WA2	WA2	0.0507	2,225,793
* WA1	EV2	WA2	DN2	0.0521	2,052,728
WA1	EV2	DN2	EV2	0.0575	3,017,822
WA1	EV2	DN2	WA2	0.0590	2,125,018
* WA1	EV2	DN2	DN2	0.0604	1,951,953
WA1	WA2	EV2	EV2	0.0604	3,184,884
WA1	WA2	EV2	WA2	0.0619	2,292,080
WA1	WA2	EV2	DN2	0.0633	2,119,015
WA1	WA2	WA2	EV2	0.0688	2,521,765
* WA1	WA2	WA2	WA2	0.0703	1,628,961
* WA1	WA2	WA2	DN2	0.0717	1,455,896
WA1	WA2	DN2	EV2	0.0771	2,420,990
WA1	WA2	DN2	WA2	0.0786	1,528,186
* WA1	WA2	DN2	DN2	0.0800	1,355,121
WA1	DN2	EV2	EV2	0.0801	3,140,258
WA1	DN2	EV2	WA2	0.0816	2,247,454
WA1	DN2	EV2	DN2	0.0830	2,074,389
WA1	DN2	WA2	EV2	0.0885	2,477,139
WA1	DN2	WA2	WA2	0.0900	1,584,335
WA1	DN2	WA2	DN2	0.0914	1,411,270
WA1	DN2	DN2	EV2	0.0968	2,376,364
WA1	DN2	DN2	WA2	0.0983	1,483,560
* WA1	DN2	DN2	DN2	0.0997	1,310,495
DN1	EV2	EV2	EV2	0.1153	2,851,964
DN1	EV2	EV2	WA2	0.1167	1,959,160
DN1	EV2	EV2	DN2	0.1270	1,784,649
DN1	EV2	WA2	EV2	0.1236	2,188,846
* DN1	EV2	WA2	WA2	0.1250	1,296,042
* DN1	EV2	WA2	DN2	0.1353	1,121,531
DN1	EV2	DN2	EV2	0.1823	2,079,690
DN1	EV2	DN2	WA2	0.1837	1,186,886
DN1	EV2	DN2	DN2	0.1940	1,012,375
DN1	WA2	EV2	EV2	0.1350	2,255,133
DN1	WA2	EV2	WA2	0.1364	1,362,329
DN1	WA2	EV2	DN2	0.1467	1,187,818

Table 2-6. (Continued)

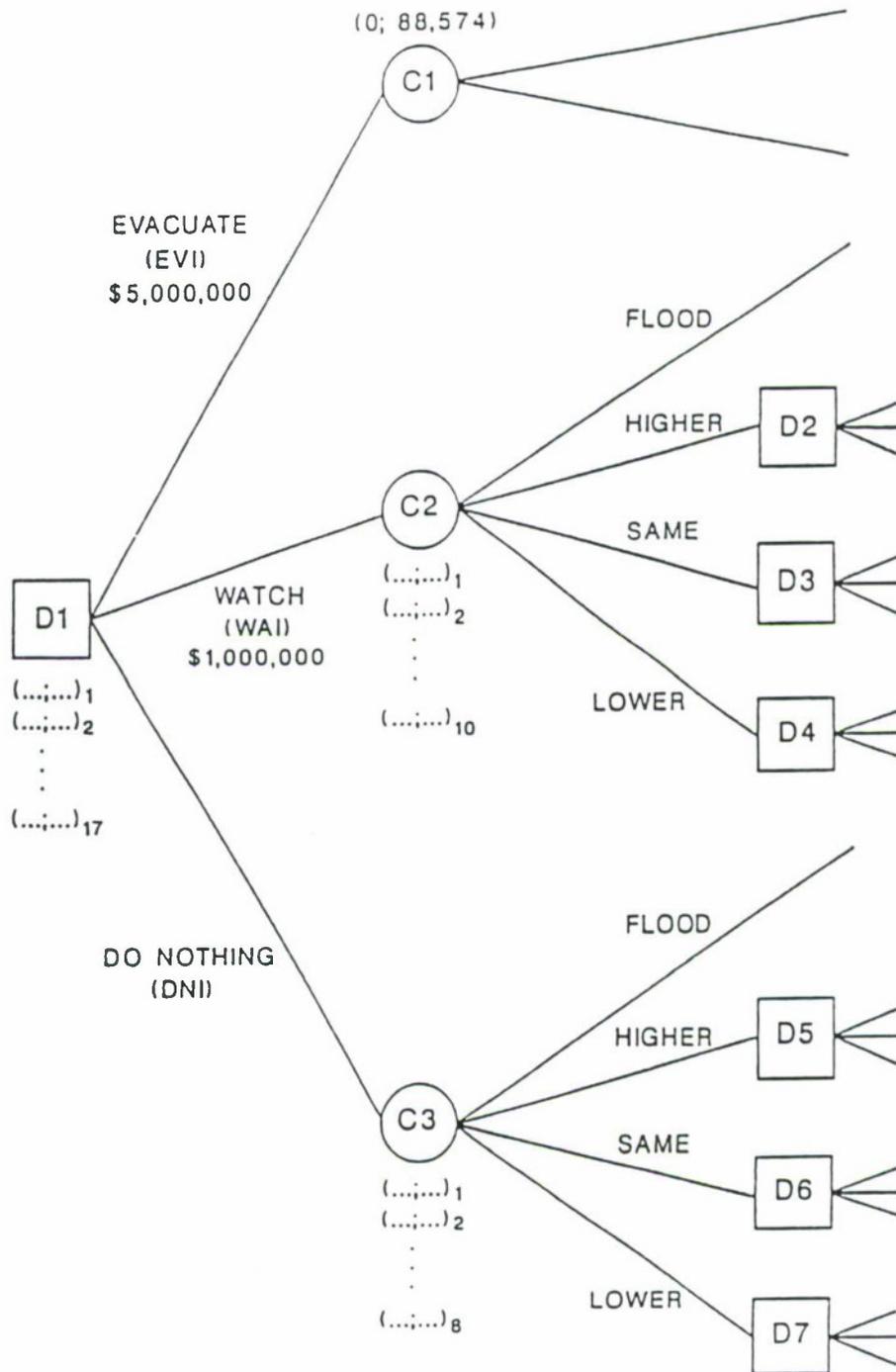
First Period Decision	Second period decision			Loss vector	
	Higher	Same	Lower	L	C
DN1	WA2	WA2	EV2	0.1433	1,592,015
* DN1	WA2	WA2	WA2	0.1447	699,211
* DN1	WA2	WA2	DN2	0.1550	524,700
DN1	WA2	DN2	EV2	0.2020	1,482,859
DN1	WA2	DN2	WA2	0.2034	590,055
* DN1	WA2	DN2	DN2	0.2137	415,544
DN1	DN2	EV2	EV2	0.2727	2,190,838
DN1	DN2	EV2	WA2	0.2741	1,298,034
DN1	DN2	EV2	DN2	0.2844	1,123,523
DN1	DN2	WA2	EV2	0.2810	1,527,720
DN1	DN2	WA2	WA2	0.2824	634,916
DN1	DN2	WA2	DN2	0.2927	460,405
DN1	DN2	DN2	EV2	0.3397	1,418,564
DN1	DN2	DN2	WA2	0.3411	525,760
* DN1	DN2	DN2	DN2	0.3514	351,249

\* noninferior decisions

Table 2-7. Decisions for the First-period Decision Node Using  $f_4$  (Continuous Case)

First-period decision	Second-period decision			Loss vector	
	Higher	Same	Lower	L	C
* EV1	-	-	-	0.0000	9,495,618
WA1	EV2	EV2	EV2	2.2367	12,565,412
WA1	EV2	EV2	WA2	2.5085	11,942,936
WA1	EV2	WA2	EV2	2.7630	12,420,237
WA1	EV2	WA2	WA2	3.0348	11,797,761
DN1	EV2	EV2	EV2	6.1876	15,467,376
DN1	EV2	EV2	WA2	6.4593	14,844,900
DN1	EV2	WA2	EV2	6.7140	15,322,201
DN1	EV2	WA2	WA2	6.9854	14,699,725

\* - Noninferior decisions



**Figure 2-8.** Decision Tree for the Second Stage Using  $f_5$  (Continuous Case)

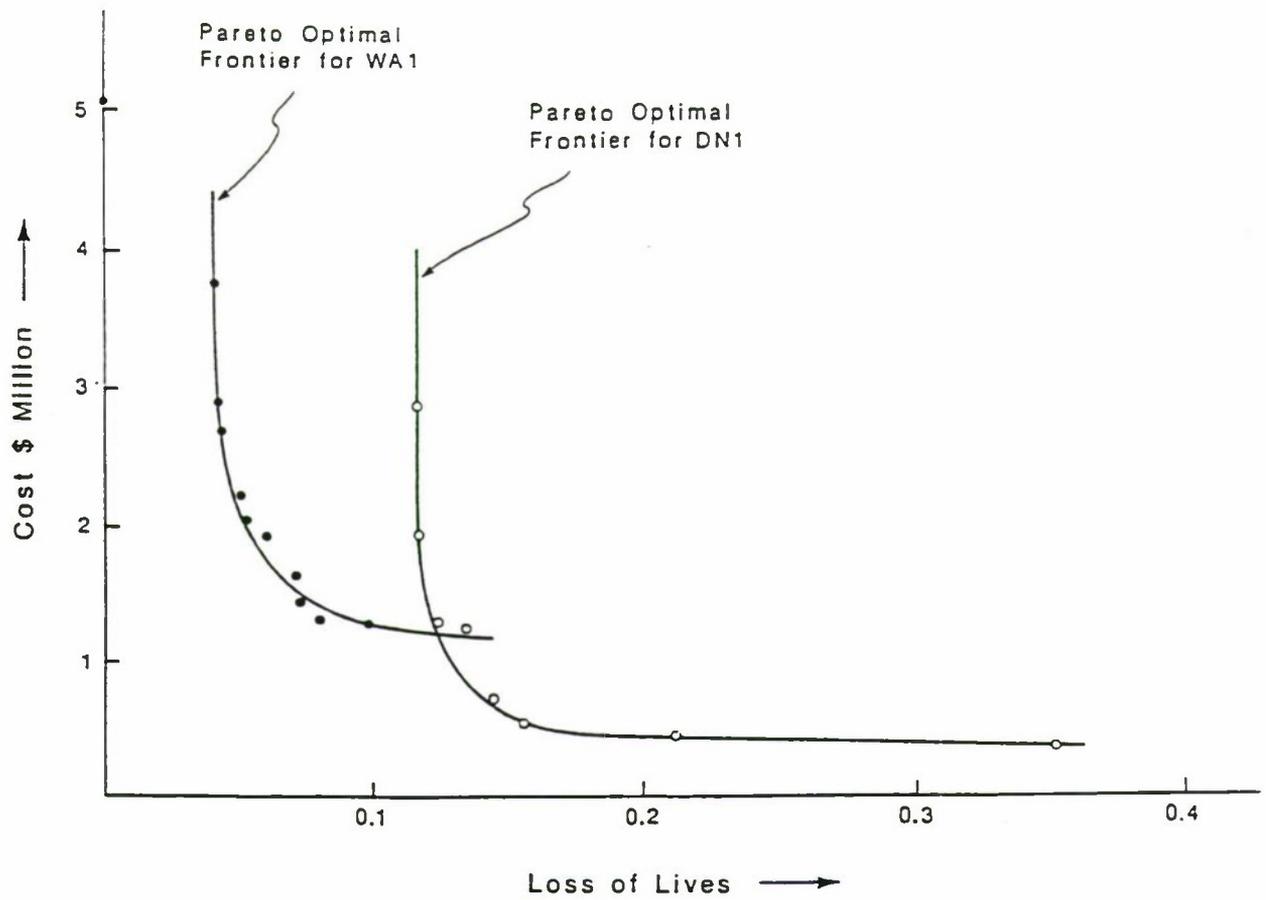


Figure 2-9. Pareto Optimal Frontier Using  $f_5$  (Continuous Case)

## Part 3

# Performance Characteristics of a Flood Warning System: Overview



### Introduction

From the utilitarian point of view, rooted in the Bayesian principles of rationality, the ultimate measure of performance of a flood warning system is the *ex ante* economic value. From the engineering point of view, there remains the need for auxiliary measures that characterize, perhaps only partially, the performance of various components of a flood warning system. The purpose of such measures is to aid the engineer in the process of planning and design.

One aspect of the performance of a flood warning system is its reliability. The following presents an overview of a model that outputs two measures of system reliability:

- the *relative operating characteristic* (ROC), which shows a relationship among (i) the probability of detection, (ii) the probability of a false warning, and (iii) the expected lead time of a warning, and
- the *performance tradeoff characteristic* (PTC), which shows a relationship among (i) the expected number of detections per year, (ii) the expected number of false warnings per year, and (iii) the expected lead time of a warning.

Each characteristic, ROC and PTC, can be displayed graphically in the form of a family of curves. The displays offer an aid to engineering planning and design of flood warning systems. The concept and interpretation of these displays are illustrated with a case study of the flood warning system for Milton, Pennsylvania.

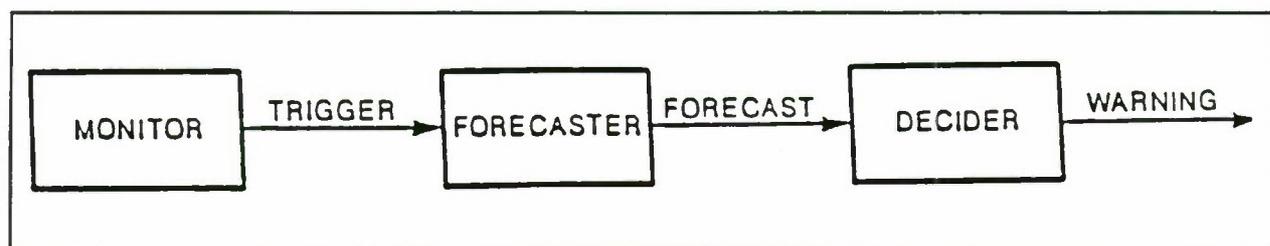
### Features of the Model

#### System Model

##### *Structure*

The model is tailored to a class of local warning systems which can be conceptualized as a cascade coupling of three components, shown in Figure 3-1: *monitor*, *forecaster*, and *decider*. The operation of such a system is idealized as follows.

Floods occur intermittently. For economic reasons, a flood data collection network, forecasting procedure, and emergency management do not operate continuously. Rather, their operation is triggered *only* when potential flood conditions are detected. To enable such detections, a system monitoring hydrometeorologic conditions operates continuously. When a set of predefined conditions is observed, the



**Figure 3-1.** Functional Structure of a Flood Warning System

monitor triggers operation of the forecast system. The flood data collection network is activated, and a forecast of the flood hydrograph is prepared. This forecast is supplied to the decision system -- a flood preparedness organization, or a floodplain manager -- who must then decide whether or not to issue a warning to the public.

### *Assumptions*

Principal definitions and assumptions underlying our model of a flood warning system are as follows:

1. A flood is the portion of a hydrograph above a flood stage, officially specified for a given river gauging station.
2. If a flood forecast is prepared, it is issued at a well-defined instant, consistently for every flood. The performance of a warning system is evaluated based on this one forecast.
3. The decision whether or not to issue a warning to the public is based on the forecasted flood crest.
4. The flood plain is divided into elevation zones. A flood warning is issued for a zone. Thus, depending on the forecast, it may be optimal to issue a warning for a lower zone, but not for an upper zone. Consequently, the performance characteristics are defined for a zone.

Mathematical models of the three system components are described below.

### *Monitor*

An all-important design decision is the choice of a forecast trigger -- an observable state that is likely to precede every flood and that, once observed, will trigger preparation of flood forecasts. Here are three examples of triggers:

(river stage) exceeds (threshold)

(rainfall intensity and duration) exceeds (threshold)

(meteorologic situation) is among a set of {potential flood situations}

A good monitor is particularly critical to local warning systems for flash floods in headwater areas with small watersheds and short concentration times. For instance, a flood developing rapidly during nighttime may occur undetected because of an equipment failure; a trigger may be false because the oncoming storm suddenly changes its track and bypasses the watershed. Consequently, the performance of the monitor may limit the performance of the total warning system, no matter how sophisticated its flood data collection network, forecasting procedure, and emergency management.

Perfect *diagnosticity* of the monitor means that every trigger is followed by a flood; in other words, the monitor provides a perfect diagnosis of a flood situation. Perfect *reliability* of the monitor means that every flood is preceded by a trigger; in other words, the monitor never fails to signal the oncoming flood. Both monitor attributes, *diagnosticity* and *reliability*, are defined as probabilities and can be anywhere in the range from zero, or worst, to one, or perfect.

#### *Forecaster*

The objective of modeling is to obtain a stochastic characterization of floods and forecasts in the form requisite for decisionmaking and performance evaluation. Toward this end, a *Bayesian processor of forecasts* is formulated following the principles laid down in earlier works of Krzysztofowicz [1983a, 1983b, 1985, 1987]. The inputs into the processor are a prior distribution describing natural flood events and a likelihood function describing the stochastic dependence between forecasted and actual flood events. The principal output from the processor is the posterior probability of flooding a given zone elevation, conditional on the forecast. This probability provides a basis for deciding the warning. In addition, the processor outputs several other probability distributions needed for system performance evaluation. The technical part outlines our approach to modeling the prior distribution and the likelihood functions.

#### *Decider*

When the trigger is observed and the forecast of the flood crest is prepared, the manager must decide whether or not to issue a flood warning for a zone of the floodplain. Thereafter the event takes place: either the zone is flooded or it is not. Each decision-event vector leads to outcomes whose undesirability (as they are mostly losses rather than gains) is evaluated in terms of a disutility function. The arguments of the disutility function are the actual flood crest and the lead time of the warning (to be defined later).

The objective of decision analysis is to find the *optimal warning rule*. According to the Bayesian postulates of rationality, the optimal rule should minimize the posterior expected disutility of outcomes. For a statistical, as contrasted with the economic, evaluation of system performance, it is not necessary to find the exact form of the optimal decision rule. It suffices to know its general structure. In practical cases, the optimal warning rule is that a warning should be issued for a given zone whenever the posterior probability of flooding that zone, conditional on the particular forecast, exceeds a predetermined threshold value.

### **Performance Measures**

#### *Performance Probabilities*

The vector of binary indicators of the status of the trigger, warning, flood, and zone flood can take on nine values which define four performance states of the warning system, as shown in Figure 3-2: *missed*

*flood* (M), *false warning* (F), *detection* (D), and *quiet*. (Q). These states are observable in the sense that one could count their occurrences over a period of time. In the limit, this count would give rise to conditional probabilities of incorrect system performance,  $P(M)$  and  $P(F)$ , and correct system performance,  $P(D)$  and  $P(Q)$ . Since  $P(M)$  has a simple relationship to  $P(D)$ , and, similarly,  $P(Q)$  to  $P(F)$ , it suffices to find the probability of detection,  $P(D)$ , and the probability of false warning,  $P(F)$ . The objective of modeling is to express these probabilities in terms of parameters and functions which characterize the warning system.

#### *Relative Operating Characteristic*

Different disutility functions may result in different threshold values for warning. With each threshold value, there is associated a probability of detection  $P(D)$  and a probability of false warning  $P(F)$ . A plot of  $P(D)$  versus  $P(F)$ , obtained by varying the threshold, is called the *relative operating characteristic* (ROC).

The ROC curve conveys the essential information about the tradeoffs that a given system offers between the probability of detection and the probability of false warning. However, the intuitive interpretation of these performance probabilities is not straightforward for they are conditional probabilities. Human intuition does not grasp easily such conditional events. Moreover, human cognition is generally not well trained in understanding and processing probabilities. Evidence of numerous and large biases in judgments involving probabilities is plentiful.

#### *Performance Tradeoff Characteristic*

In order to overcome the interpretive difficulties associated with the ROC, we propose to transform the probabilities of various states into the expected number of states per year. Given the expected number of floods per year the following quantities can readily be obtained:

*expected number of zone floods per year*

*expected number of detections per year for a zone*

*expected number of false warnings per year for a zone*

Once these expectation values are calculated, the ROC curve can be rescaled into a function relating the expected number of detections and the expected number of false warnings per year. This function will be called the *performance tradeoff characteristic* (PTC).

#### *Expected Lead Time*

The forecast time is the instant up to which hydrometeorologic observations for preparing the forecast are collected. The lead time of a warning for a given zone, conditional on the hypothesis that the zone will be flooded, is the time interval elapsed from the forecast time to the instant at which the flood waters reach the zone elevation. The lead time can be modeled as a random variable, and one can calculate its expected value.

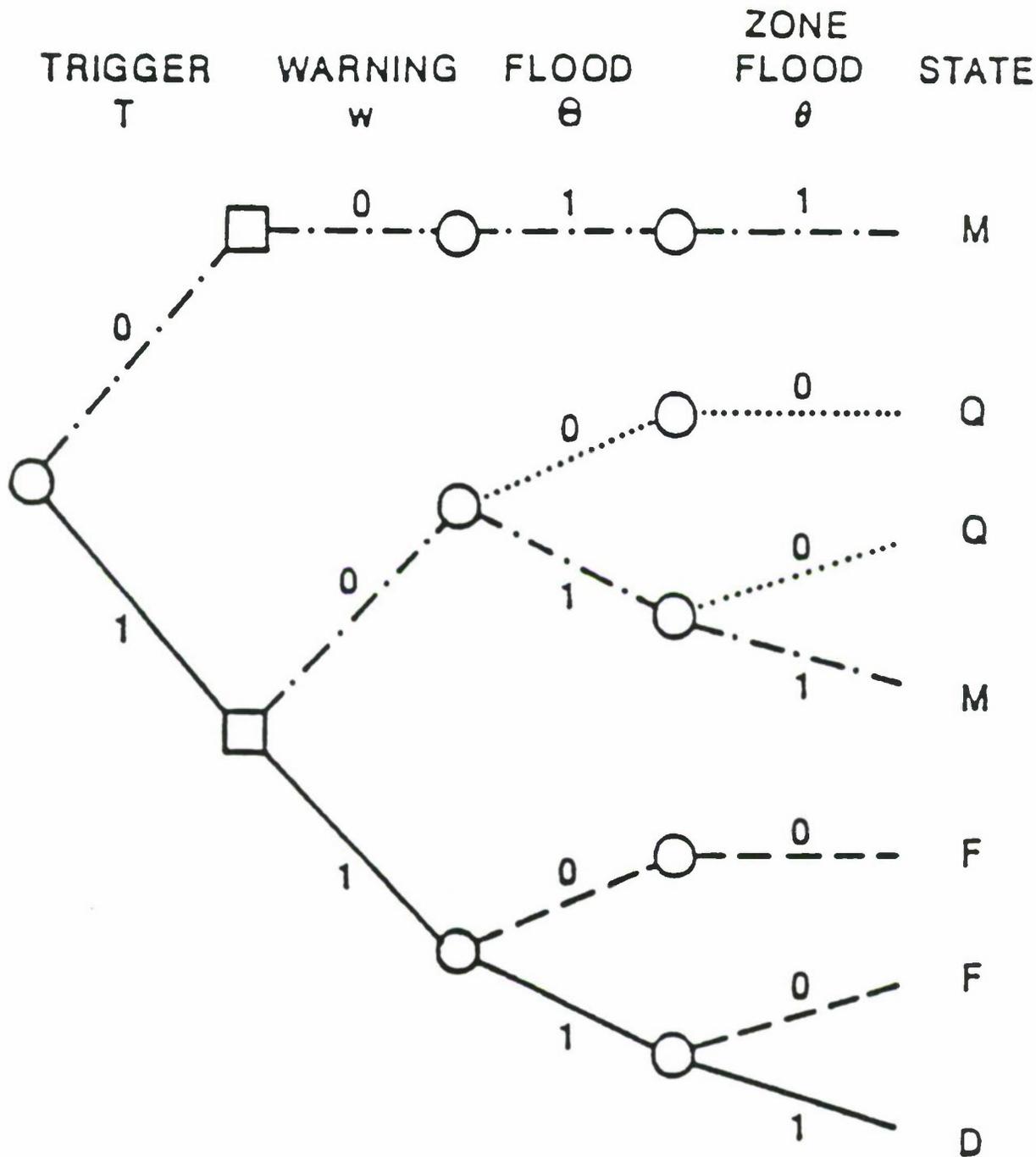


Figure 3-2. Tree of Events Leading to One of the Four Performance States of a Flood Warning System (Missed Flood (M), False Warning (F), Detection (D), and Quiet (Q))

The designer of a warning system can affect the lead time indirectly, through the definitions of (i) the forecast trigger and (ii) the forecast time. Each of these specifications may affect the probability densities of forecasted flood crest and the lead time. Consequently, any change in the design specifications may simultaneously affect the following performance measures: probability of detection  $P(D)$ , probability of false warning  $P(F)$ , expected number of detections per year, expected number of false warnings per year  $NF$ , and the expected lead time.

## Closure

The relative operating characteristic (ROC) and the performance tradeoff characteristic (PTC) are a part of a general theory of flood warning systems that is being developed. A number of questions are still awaiting answers. Among them is the connection between these statistical measures of performance and the *ex ante* economic value of a warning system. Such a connection is well known within the classical detection paradigm, but it remains to be investigated whether or not it extends to a much more complex paradigm of a flood warning system.

Applications of ROC and PTC concepts to other flood warning systems are also awaiting us. A number of applications would be desirable to systems with distinct hydrologic regimes, such as flash-flood streams and main-stem rivers, and distinct technologies, such as found in local warning systems and the forecast offices of the National Weather Service. Collectively, results of such case studies would offer useful guidance to engineers who plan and design flood warning systems.

## Case Study—Milton, Pennsylvania

### General Description

Properties of the ROC and PTC curves, and their potential role as aids to design analysis, will be illustrated through a case study of the flood warning system for Milton, Pennsylvania. The town has a population of about 8000 and is located on the West Branch of the Susquehanna River in northeastern Pennsylvania. The data used in the study were collected by Krzysztofowicz and Davis [1983]. The source of the flood and forecast data is the U.S. National Weather Service, River Forecast Center at Harrisburg, Pennsylvania. The forecast data are from the period 1959-1975. Thus the case studies reported herein are representative of the system performance during that period.

In all specifications of the parameters, the units of time are hours and the units of elevation are feet above the zero of the river gauge. The flood stage is at 19 ft, but almost all structures are located above the elevation of 22 ft. The densities in the model of the forecaster are assumed to follow the Gaussian law (normal probability distribution).

### Input Models and Parameters

*Record of floods.* The record of floods from the period 1885-1975 contains 20 flood events. From this record, we estimated the expected number of floods per year and the prior density of flood crest.

*Models of Monitor and Forecaster.* The parameters that must be estimated from the joint record of forecasted and actual floods are as follows:

*diagnosticity* of the monitor,

*reliability* of the monitor,

two likelihood functions of forecasted flood crest, and

–conditional on no flood occurring

–conditional on the height of the flood crest

expected lead time (for each zone elevation).

*Record of forecasts.* The forecast verification reports for the period 1959-1975 contain a record of 9 floods and 37 forecasts. The record does not contain information sufficient for the estimation of all parameters via statistical methods. Consequently, parameters of the monitor and parameters of the likelihood function that is conditional on no flood occurring had to be estimated subjectively based on a plausible interpretation and interpolation of the available information. On the other hand, the expected lead times and the parameters of the likelihood function that is conditional on flood crest were estimated from the data.

*System designs.* The monitor is assumed to trigger the forecaster when the river stage exceeds a specified threshold. Three alternative system designs are analyzed, in which the forecast trigger is defined as follows:

System S1: river stage exceeds 11 ft

System S2: river stage exceeds 15 ft

System S3: river stage exceeds 19 ft

The likelihood function conditional on no flood is assumed to be the same for each system. The remaining parameters vary with the design. Table 3-1 lists estimates of the estimated expected lead times and the calculated expected number of zone floods for four zones of the floodplain. The respective elevations of the zones are given in the same table.

*Interpretation.* Figure 3-3 shows the expected lead time plotted as a function of the elevation for each of the three systems. Clearly, when the threshold stage for triggering the forecaster is raised, the expected lead time is reduced uniformly for all elevations. Table 3-1 reveals further implications. When the lead time decreases, the diagnosticity of the monitor increases, since a higher threshold stage is always more diagnostic of the incoming flood. On the other hand, when the lead time decreases, the reliability also decreases. This is so because the observations of the river are made in 6-hour intervals, and it is possible for a rapidly rising river to exceed both the threshold stage and the flood stage within the 6-hour interval. In such an instance, flooding occurs prior to the preparation of a forecast. The likelihood of such an event increases as the threshold stage is raised closer to the flood stage; hence the reliability decreases.

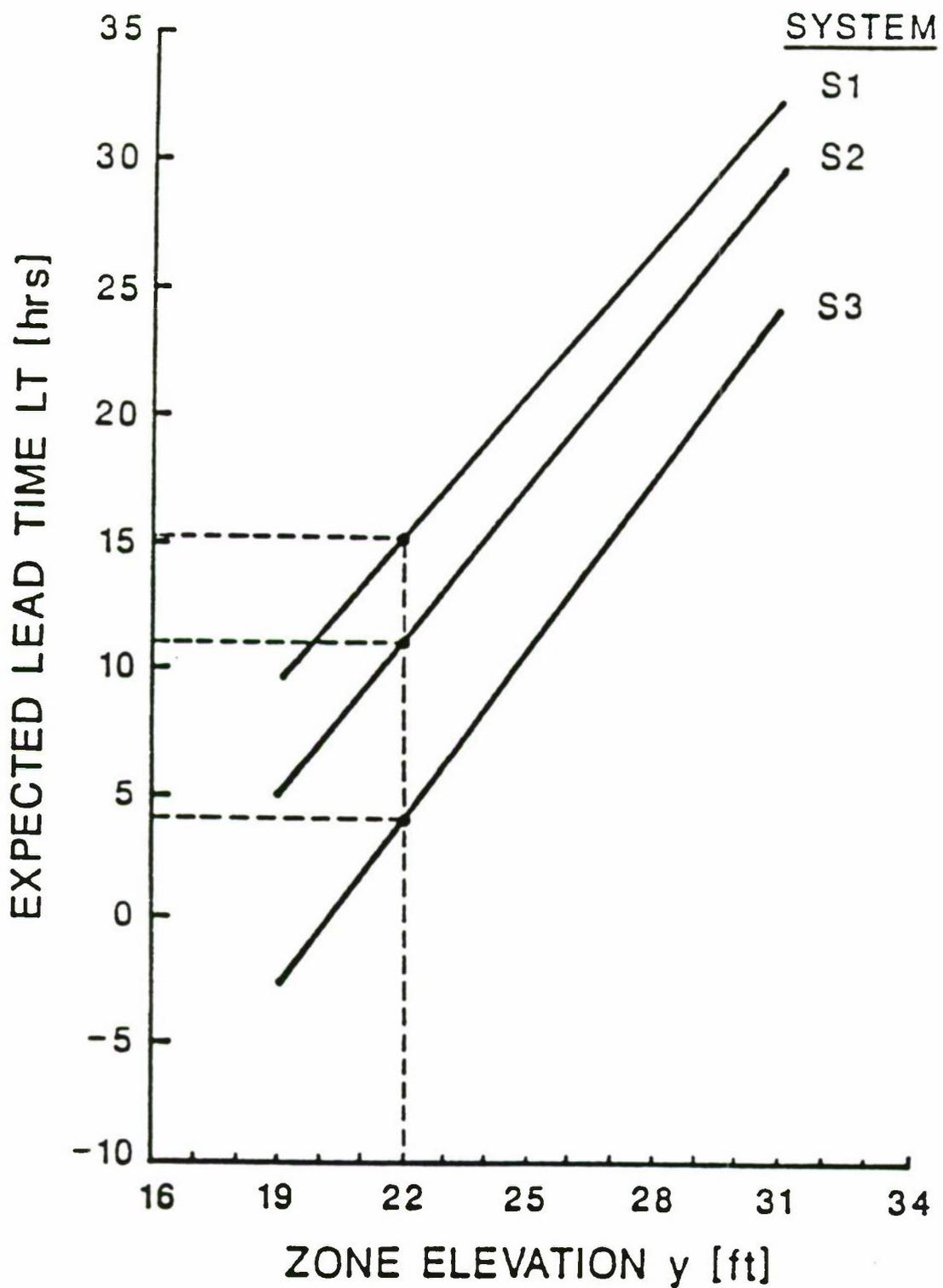


Figure 3-3. Expected Lead Time of a Flood Warning Versus the Elevation of the Floodplain for Three Warning Systems in Milton, Pennsylvania

**Table 3-1. Parameters of Three Alternative Designs of a Flood Warning System  
for Milton, Pennsylvania**

System Design	Monitor		Likelihood Function			Forecast Sufficiency Characteristic FSC
	Diagnosticity $\gamma$	Reliability $\rho$	Slope $a$	Intercept $b$	St. Dev. $\sigma$	
S1	0.80	1.00	0.44	10.65	3.06	6.95
S2	0.90	0.89	0.45	12.10	1.90	4.22
S3	1.00	0.83	0.64	8.48	2.21	3.45

**Table 3-2. Expected Number of Zone Floods and Expected Lead Times of Flood Warnings  
for Milton, Pennsylvania**

Zone Elevation $y$ [ft]	Expected Number* of Zone Floods $n$	Expected Lead Time LT [hrs]		
		System S1	System S2	System S3
19	47.1	9	5	-3
22	38.4	15	11	4
25	26.0	21	17	11
28	13.7	27	24	18

\*The expected numbers are for the period of 100 years.

When the expected lead time decreases, one also anticipates an increase in the quality of the flood crest forecasts. Table 3-1 reveals that the parameters of the likelihood function change their values with a change in lead time. But do these changes imply anything about the forecast quality? The answer to this question may be obtained via the *forecast sufficiency characteristic* (FSC). This measure is sufficient for comparing any two forecasters who produce forecasts and enables us to order forecasts in terms of their economic values. The FSCs calculated in the last column of Table 3-1, along with the expected lead times for the three systems in Table 3-2, confirm our hypothesis: when the lead time decreases, the quality of the flood crest forecasts increases.

### **Properties of the ROC and the PTC**

The ROC and PTC curves for design S1 are displayed in Figures 3-4 and 3-5. We shall highlight some general properties of these curves.

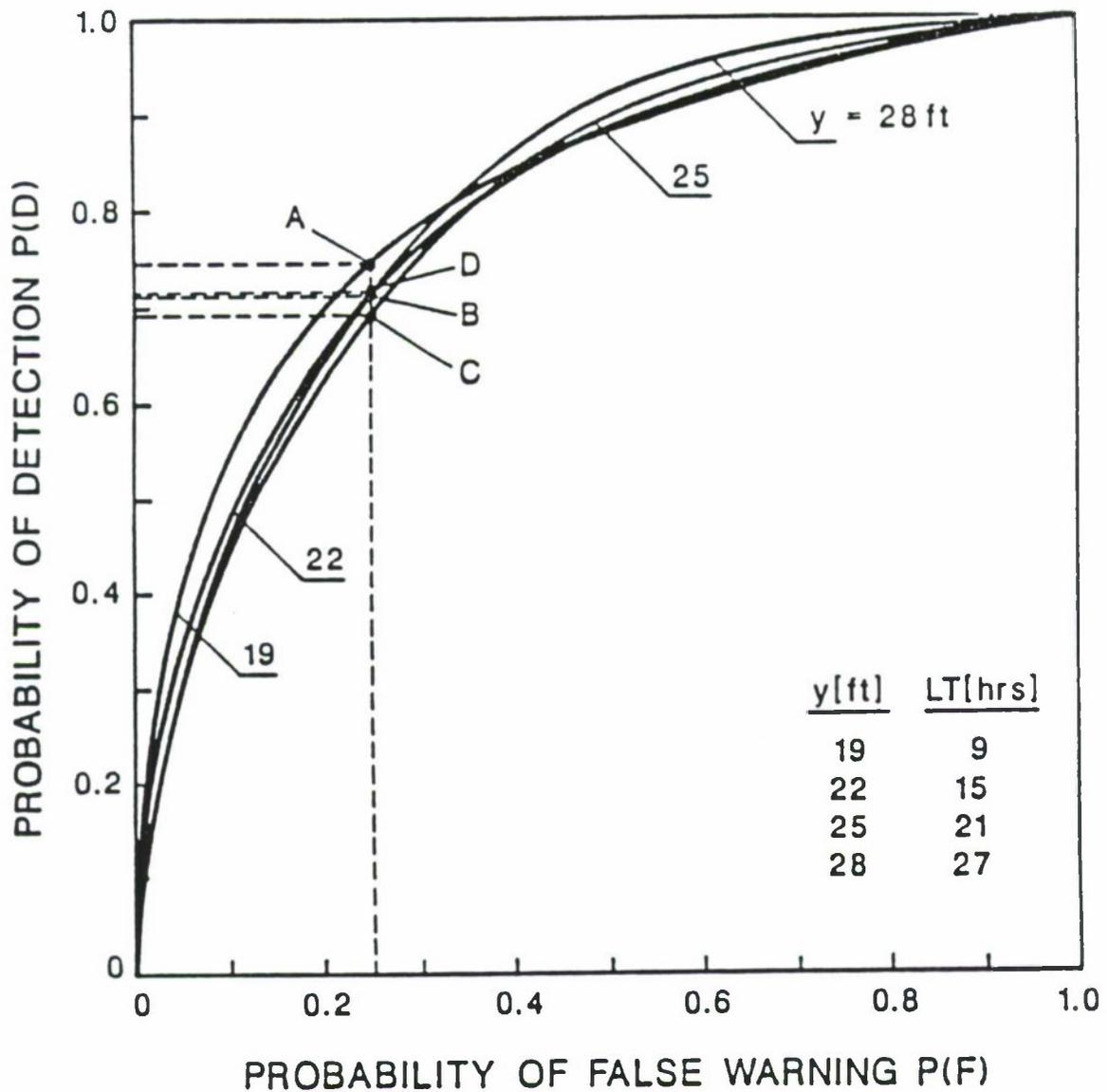
1. For a fixed zone elevation, the associated ROC is a concave function specifying a unique relationship between the probability of false warning,  $P(F)$ , and the probability of detection,  $P(D)$ . Probability  $P(F)$  may vary from 0 to 1, but probability  $P(D)$  is bounded from above by the reliability of the monitor  $\rho$ , which for design S1 happened to be 1.0. For a fixed zone elevation and a prior distribution of the flood crest, the shape of the ROC curve depends solely upon the design specifications for the monitor (via diagnosticity  $\gamma$  and reliability  $\rho$ ) and the design specifications for the forecaster (via the likelihood functions of flood crest forecast: one conditional on the flood crest, the other conditional on no flood occurring).

2. By mapping each point from the ROC in Figure 3-4 through relations (11)-(12) given in the technical part, we obtain the PTC shown in Figure 3-5. The PTC is also a concave function, increasing from the origin, which corresponds to  $P(F) = P(D) = 0$ , to a point which corresponds to  $P(F) = 1$  and  $P(D) = \rho$ . The expected number of detections per year,  $ND$ , never exceeds the expected number of zone floods  $n$ . On the other hand, the expected number of false warnings per year,  $NF$ , may exceed  $n$ . The PTC for zone elevations  $y = 25$  and  $y = 28$  do just that.

### **Performance Differences Between Zones**

1. The ROC curves for different elevation zones have generally similar shapes and cross each other. In other words, when the performance of a warning system is measured in terms of the probability of false warning  $P(F)$  and the probability of detection  $P(D)$ , all zones seem to be served equally well. However, the third performance measure -- the expected lead time of the warning -- illuminates the differences between the low-lying zones and the high-lying zones:  $LT = 9$  hours for  $y = 19$  and  $LT = 27$  hours for  $y = 28$ , a threefold difference.

2. The PTC curves are quite dissimilar, underscoring the fact that they convey different information than the ROC curves do. There are two main distinctions between the zones. First, there is the obvious distinction resulting from the elevations difference: the expected number of floods  $n$  in 100 years is 47.1 for  $y = 19$  and only 13.7 for  $y = 28$ . Second, there is a remarkable difference in terms of the expected number of false warnings  $NF$  associated with the maximum expected number of detections  $ND = n$ . This  $NF$  is equal to 19.2 for  $y = 19$ , and it increases to 52.5 for  $y = 28$ . In other words, to reach the upper limit of expected detections for the higher zone, one must accept a rate of false warnings  $NF = 52.5$ , which is 3.8 times higher than the rate of floodings  $n = 13.7$ .



**Figure 3-4.** Relative Operating Characteristics (ROC) of Warning System S1 for Four Zone Elevations,  $y$ , in Milton, Pennsylvania.  
(Symbol LT Denotes the Expected Lead Time of a Flood Warning)

3. To place these results in proper perspective, one has only to realize that floods reaching zone  $y = 28$  are more extreme and rare than floods reaching only zone  $y = 19$ . The PTC curves in Figure 3-5 inform us that a high detection rate for rare events comes at the price of a high rate of false warnings. This appears to be an inescapable tradeoff.

#### Operating Points

1. A point on the ROC, or PTC, is called an *operating point*. In Figure 3-4, we fixed an operating point for each zone such that for all zones the probability of false warning  $P(F) = 0.25$ . The probability of detection  $P(D)$  is different for each zone, but the differences are small. Table 3-3 lists the exact coordinates of these operating points on PTC. Figure 3-5 depicts these points, each of which has distinct NF and ND coordinates.

**Table 3-3.** Coordinates of Operating Points on the ROC and PTC Curves That Give the Same Probability of False Warning  $P(F)$  for Each Zone Elevation; System Design S1 for Milton, Pennsylvania

Operating Point	Zone Elevation $y$ [ft]	Probability of		Expected Number* of		Expected Lead Time LT [hrs]
		Detection $P(D)$	False W. $P(F)$	Detections ND	False W. NF	
A	19	0.75	0.25	35.1	4.8	9
B	22	0.71	0.25	27.3	6.9	15
C	25	0.69	0.25	18.1	10.0	21
D	28	0.72	0.25	9.9	13.3	27

\*The expected numbers are for the period of 100 years.

2. In general, the mapping between the operating points of ROCs and PTCs is one-to-one, with the following properties: (i) The operating points that have the same  $P(D)$  coordinate on the ROC have also the same ND coordinate on the PTC. (ii) The operating points that have the same  $P(F)$  coordinate on the ROC may have different NF coordinates on the PTC. We shall say that the mapping between the ROC and PTC is *nonorthogonal*.

3. The nonorthogonality of the mapping between the ROC and PTC should be taken as a caution: judgmental analysis of tradeoffs on the ROC, or PTC, is not a simple cognitive task. We recommend using the PTC as the primary aid to planning and design because the expected numbers ND and NF are easier to interpret and understand than the probabilities  $P(D)$  and  $P(F)$ , which are conditional probabilities.

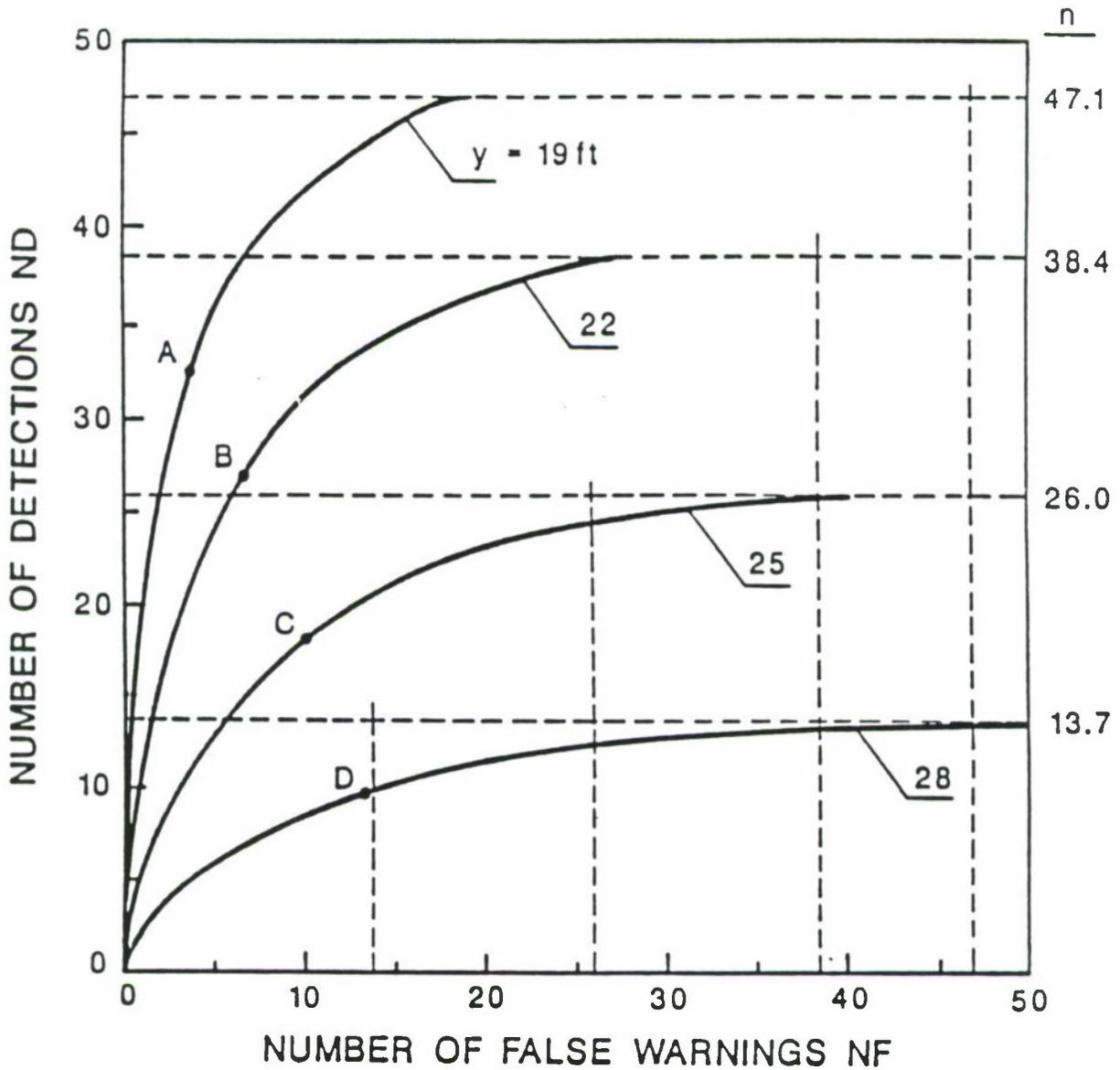


Figure 3-5. Performance Tradeoff Characteristics (PTC) of Warning System S1 for Four Zone Elevations,  $y$ , in Milton, Pennsylvania.  
(Symbol N Denotes the Expected Number of Zone Floods. All numbers, NF, ND, and  $n$  are for the Period of 100 Years.)

4. With each operating point on the PTC, or ROC, there is associated a unique threshold  $q^*$  in the warning rule. Thus, a specification of the operating point is equivalent to a specification of the rule for deciding warnings. To specify an operating point on the PTC for a given zone, one should consider a tradeoff between the expected number of detections ND and the expected number of false warnings NF. This tradeoff should encapsulate one's preferences for outcomes of all possible decision-event vectors for this particular zone. It follows that it would be irrational to fix the operating point based solely on a displayed PTC or ROC, without an in-depth analysis of all socioeconomic outcomes of every decision-event vector. That is why the PTC and ROC curves should be viewed only as aids to the planning and design process, rather than as a means of specifying the warning rule. The optimal warning rule should be found by minimizing the expected disutility of outcomes resulting from all possible decision-event vectors.

### **Performance Tradeoffs**

From a purely statistical point of view, which ignores the economic and social decision criteria, the engineer could consider the design process as an optimization problem with three criteria: maximize the expected number of detections ND, minimize the expected number of false warnings NF, and maximize the lead time LT. The ideal solution is an operating point at which:

- the expected number of detections ND is equal to the expected number of zone floods  $n$ ,
- the expected number of false warnings NF is equal to zero, and
- the expected lead time LT is equal to infinity.

In the absence of the ideal solution, tradeoffs must be made.

The kinds of tradeoffs that one may encounter are illustrated for the three alternative system designs, S1, S2, and S3. The ROC and PTC curves of these systems are compared in Figures 3-6 and 3-7 for zone elevation  $y = 22$  and in Figures 3-8 and 3-9 for zone elevation  $y = 28$ . An immediate observation is that designs S1 and S2 offer distinct performance characteristics. On the other hand, designs S2 and S3 have similar ROC and PTC curves over a range of operating points, while over the remaining range S2 outperforms S3. Together with the fact that S3 offers much shorter expected lead times LT than S2 does, it is unlikely that decisionmakers would prefer S3 over S2. This example illustrates then a screening analysis that may be performed on a large set of alternative designs before a few are selected for a detailed analysis of tradeoffs.

The ensuing discussion highlights the nature of performance tradeoffs that the PTC allows one to analyze between designs S1 and S2. The discussion concentrates on three operating points, labeled A, B, C, in Figure 3-7. Their coordinates (ND, NF, LT) are listed in Table 3-4.

1. A good way to start the analysis is to fix the expected number of false warnings NF at a level that appears acceptable, at least initially, say  $NF = 5.0$ , which means that one would be willing to accept 5.0 false warnings in 100 years, on the average. At this level of NF, design S1 ensures the expected detection of  $ND = 23.8$  floods out of the expected  $n = 38.4$  floods in 100 years. The expected lead time of a warning for each detected flood is  $LT = 15$  hours. The difference,  $n - ND = 38.4 - 23.8 = 14.6$ , is

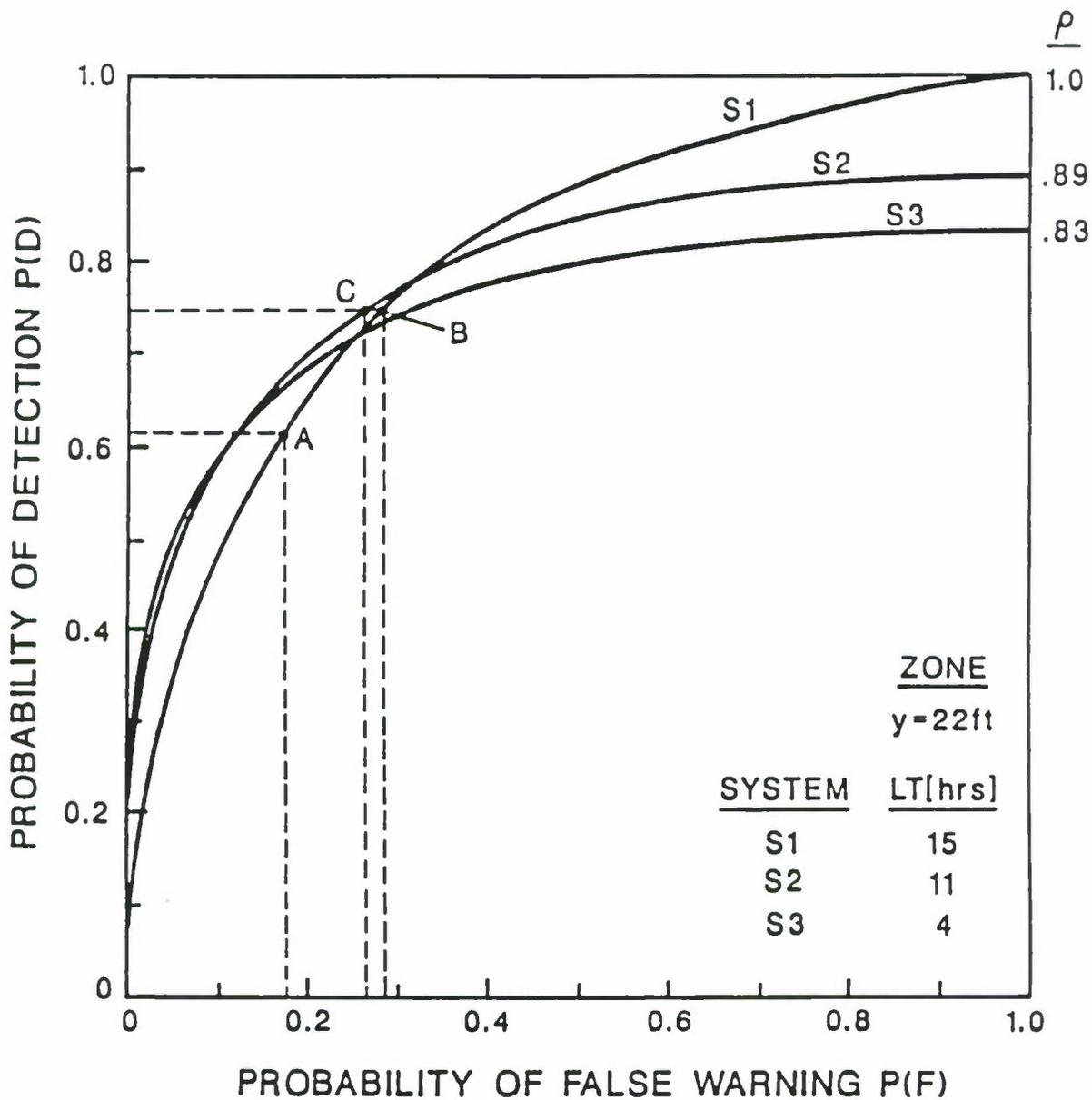


Figure 3-6. Relative Operating Characteristics (ROC) of Three Warning Systems, S1, S2, and S3 for Zone Elevation  $y = 22$  ft in Milton, Pennsylvania.  
(Symbol  $\rho$  denotes the reliability of the monitor)

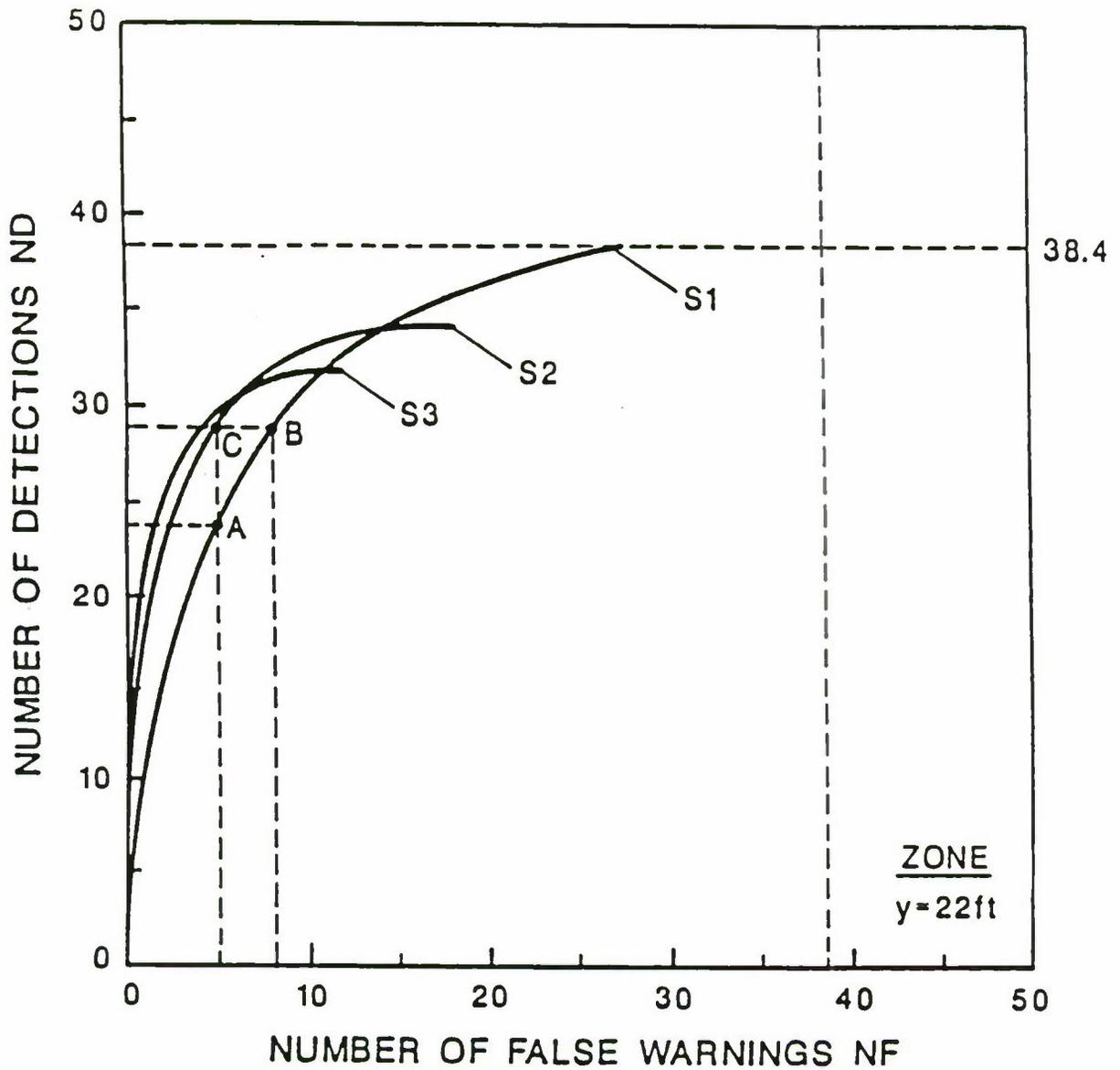


Figure 3-7. Performance Tradeoff Characteristics (PTC) of Three Warning Systems, S1, S2, and S3, for Zone Elevation  $y = 22$  ft in Milton, Pennsylvania.

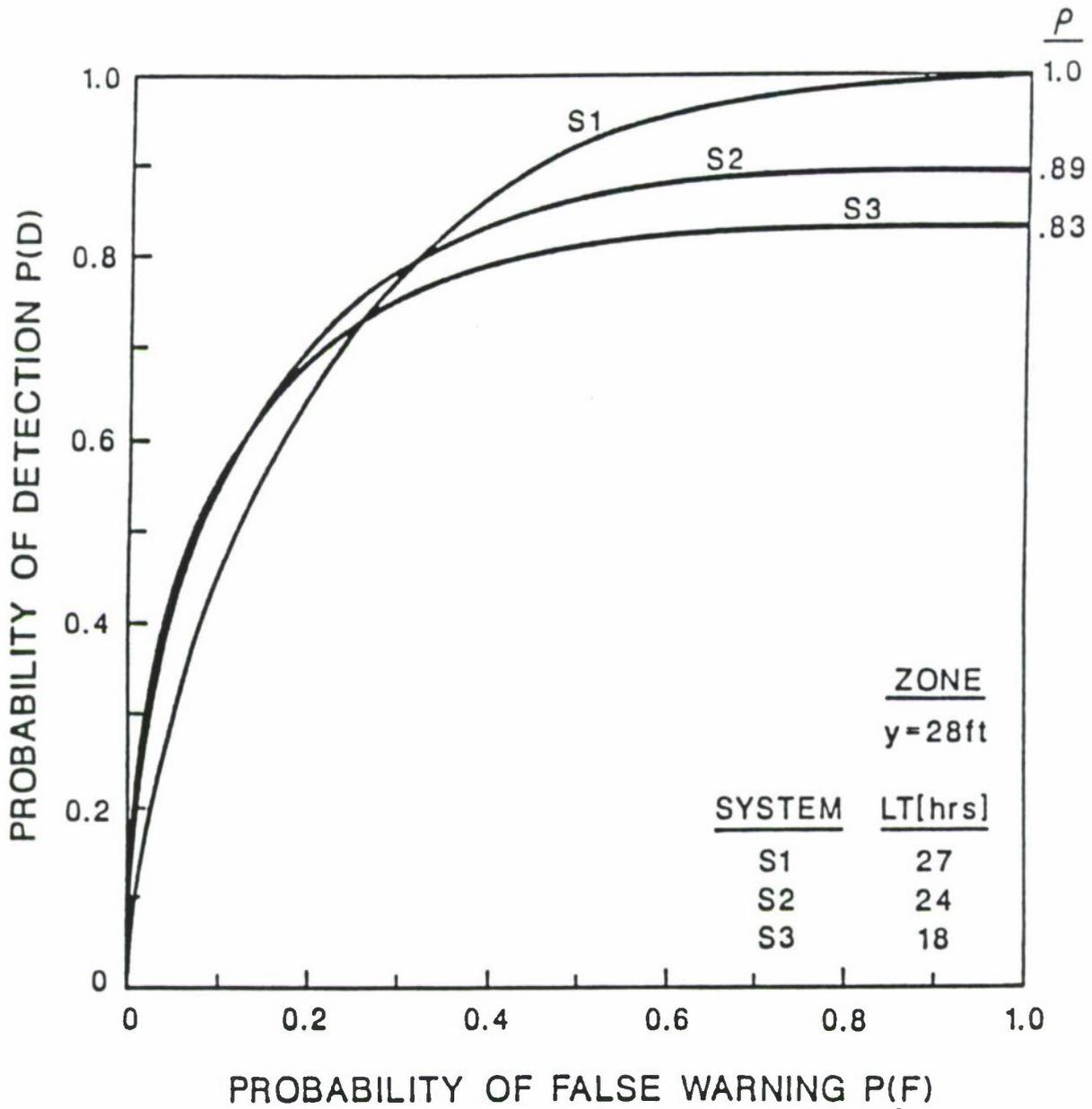


Figure 3-8. Relative Operating Characteristics (ROC) of Three Warning Systems, S1, S2, and S3, for Zone Elevation  $y = 28$  ft in Milton, Pennsylvania. (Symbol  $\rho$  denotes the reliability of the monitor)

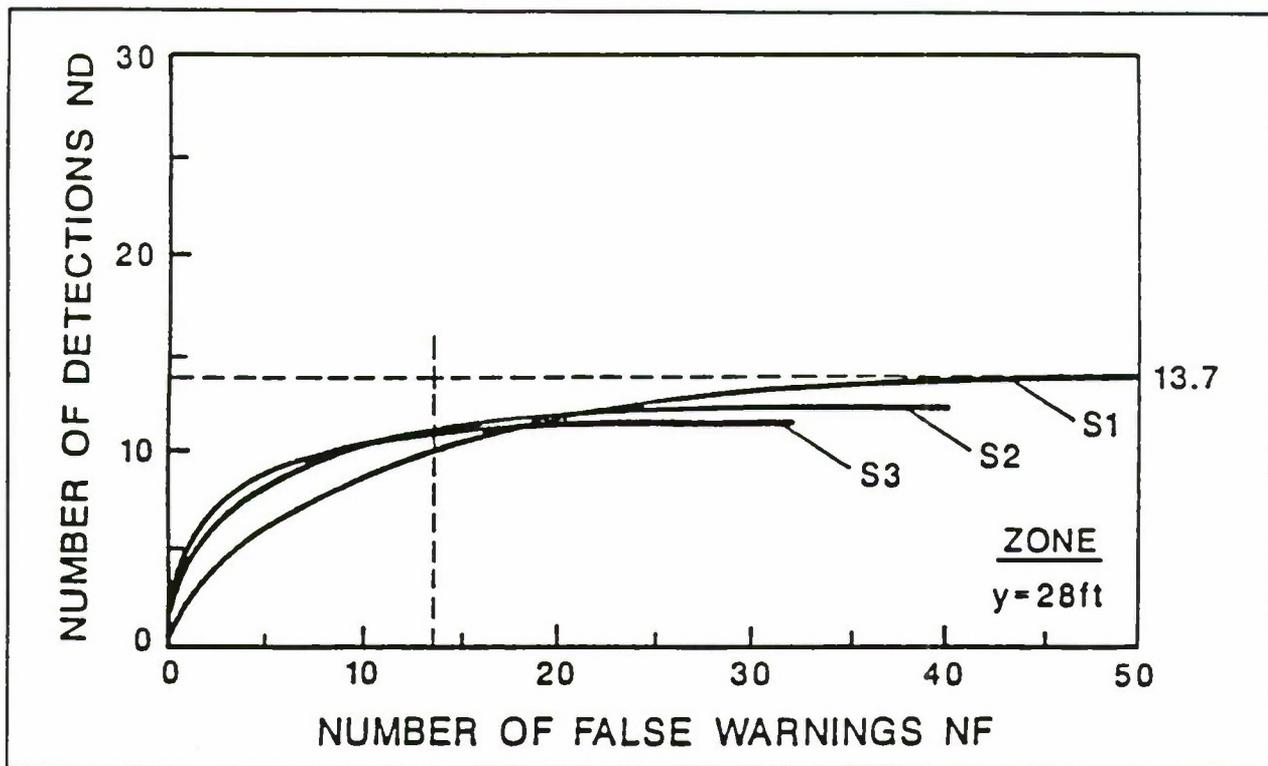


Figure 3-9. Performance Tradeoff Characteristics (PTC) of Three Warning Systems, S1, S2, and S3, for Zone Elevation  $y = 22$  ft in Milton, Pennsylvania

Table 3-4. Coordinates of Three Alternative Points on the ROC and PTC Curves for Zone Elevation  $y = 22$  ft

System Design	Operating Point	Probability of		Expected Number* of		Expected Lead Time LT [hrs]
		Detection P(D)	False W. P(F)	Detections ND	False W. NF	
S1	A	0.62	0.18	23.8	5.0	15
S1	B	0.75	0.29	28.9	8.1	15
S2	C	0.75	0.27	28.9	5.0	11

\*The expected numbers are for the period of 100 years.

the expected number of floods in 100 years that will arrive undetected, and thus will not be preceded by a warning to the public.

2. At the same level of  $NF = 5.0$ , design S2 ensures the expected detection of 28.9 floods in 100 years, with the expected lead time of a warning equal to 11 hours; the expected number of missed floods in 100 years is  $38.4 - 28.9 = 9.5$ . Thus, when comparing the operating points A and B, the following tradeoff should be considered: is it preferable or not to reduce LT from 15 to 11 hours in order to increase ND from 23.8 to 28.9 (or, equivalently to reduce the expected number of missed floods from 14.6 to 9.5)?

3. A similar analysis of tradeoffs may be carried out for a fixed expected number of detections ND, say 28.9 in 100 years. At this level of ND, the number of false warnings expected in 100 years is 5.0 for design S2 and 8.1 for design S1; the accompanying expected lead times of a warning are, respectively, 11 and 15 hours. Thus, when comparing the operating points B and C, the following tradeoff should be considered: is it preferable or not to reduce LT from 15 to 11 hours in order to decrease NF from 8.1 to 5.0?

4. The right endpoints of the PTC curves indicate that, given the present specifications for the monitor, design S1 can detect all  $n = 38.4$  floods expected in 100 years. However, design S2 has an upper limit of 34.2 expected detections in 100 years; thus, the minimum expected number of missed floods in 100 years is  $38.4 - 34.2 = 4.2$ . The upper limit of ND is achieved by each design at a different level of the expected number of false warnings NF, which is 27.8 for design S1, and 18.2 for design S2.

## **Part 4**

# **Selection of Optimal Flood Warning Threshold: Overview**



### **Introduction**

Flood control can be provided by either structural or nonstructural measures or a combination of both. Structural flood control measures, such as an increase in dam height, affect the flood-frequency relationship. Nonstructural measures, such as a flood warning system, do not have an impact on the flood-frequency relationship; however, they modify the flood-damage relationship.

In this chapter, flood warning systems are studied in a two-level hierarchical system framework. The interactions between the forecast subsystem and the response subsystem are investigated. Emphasis is placed on exploring the impact of the current selected flood warning threshold on the future response fraction of a flood warning. The probabilistic evaluation of a forecast system coupled with a stochastic dynamic model of the evolution of the response fraction in a community reveals that the desire for high present flood-loss reduction must be balanced with the possibility of high future flood loss. Multiobjective dynamic programming is used to select the optimal flood warning threshold. The proposed methodology is applied to case studies in Milton, Eldred, and Connellsville, Pennsylvania.

### **Features of the Model**

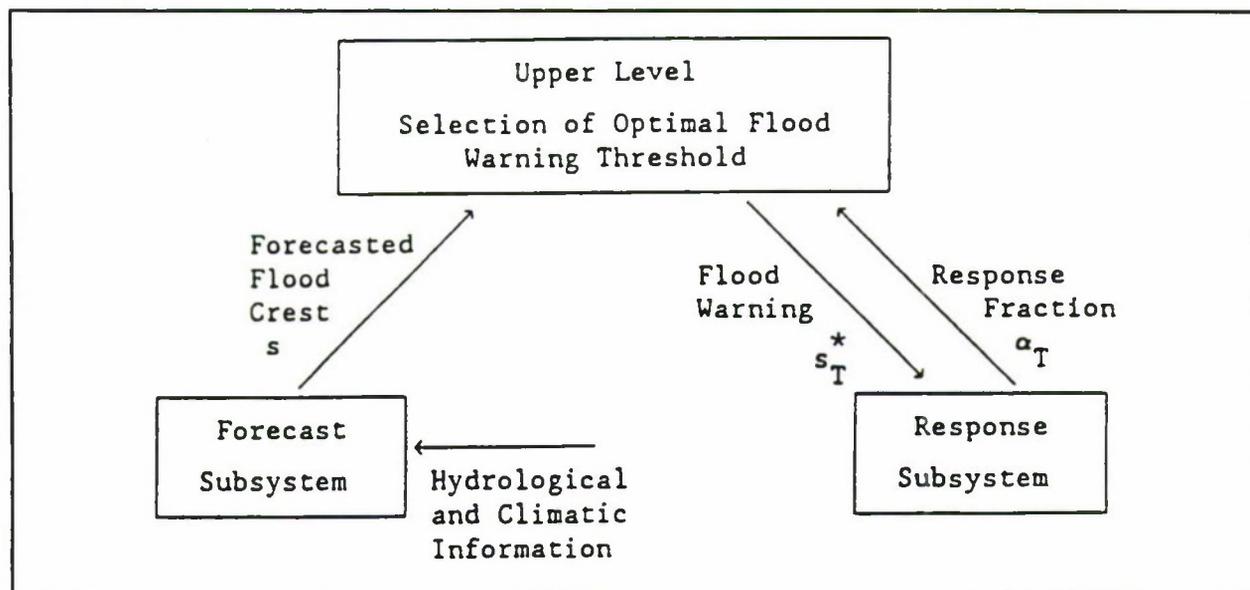
In general, the overall flood warning system can be viewed as and modeled in a two-level hierarchical system framework as is depicted in Figure 4-1. There are two subsystems at the lower level. One is the forecasting subsystem, which issues a flood forecast based on hydrological and climatic information. The other is the response subsystem, which includes decisionmaking and action implementation of a community in response to flood warning. At the upper level, it is assumed that a regional agency exists whose functions are to set a warning threshold, disseminate a flood warning to the community, provide transportation during the evacuation process, and collect statistical data of the warning system.

#### *Performance Measures of a Warning System*

Define a random variable which represents the actual flood crest and another random variable which represents the forecasted flood crest. If the prior probability density function of the flood crest is known and the conditional probability density function of forecasted crest, given the actual crest, is known, then the posterior probability density function of the actual crest, given the forecasted crest, can be straightforwardly obtained.

In this part both the prior distribution of the flood crest and the likelihood function are assumed to be of normal distributions, employing the so-called normal-linear model. With the assumption of the

normal-linear model, the probability distributions used throughout the analysis are of particularly simple forms.



**Figure 4-1.** Multilevel Structure of Flood Warning Systems

The flood warning threshold is defined by the fact that a warning is issued when the forecasted crest exceeds this threshold. A flood warning will be issued only when the forecasted flood crest exceeds this preassigned threshold level. For a given physical forecast system, the performance of the system can be evaluated by four probabilistic measures. There exist four possible outcomes that follow a flood warning decision: a correct warning, a false warning, a missed warning, and a correct quiet (the decision not to issue a warning). A correct warning is a warning followed by a flood. A false warning is a warning not followed by a flood. The probability of a false warning is also easily obtained. A missed forecast is a flood event which is not preceded by a warning. A correct quiet is the case of no warning and no flood. The probabilities of these four outcomes can be obtained from the assumptions concerning the forms of the probability distributions and likelihood functions. These four probabilistic measures are related to each other. Knowing one of them and the prior flood probability and the probability density function of the forecasted crest, the other three can be calculated.

There are two types of prediction errors of a forecast system -- Type I and Type II errors. Type I errors are those of missed predictions. Type II errors are those of false alerts. Clearly, the value of the selected warning threshold plays a key role in determining the probabilities of Type I and Type II errors. If the threshold is set lower, the forecast will have a lower probability value of a Type I error and a higher probability value of a Type II error. If the threshold is set higher, the forecast will have a higher probability value of a Type I error and a lower probability value of a Type II error.

Type I and Type II errors have different impacts on flood-loss reduction. A Type I error will result in an immediate flood loss. Thus, it has mainly a short-term impact. On the other hand, a Type II error will reduce the credibility of the forecast system. This cry-wolf consequence has mainly a long-term impact.

Such errors do not cause a flood loss at the present stage, but will discourage the response to flood warnings for future flood events. The present fraction of people who respond to a flood warning is certainly an indispensable factor in constructing the flood-loss function for a community. It thus affects the selection of the flood warning threshold. On the other hand, the response fraction fluctuates as time passes, based on the past performance of the warning system. The coupling between successive flood events is carried by dynamic evolution of the fraction of people who respond to a flood warning. The past performance of a flood warning system affects the present fraction of people who respond to the warning system. Therefore, it is necessary to investigate the operation of flood warning systems in a dynamic framework. Figure 4-2 shows the connection between the flood warning threshold and the response fraction of the community.

#### *A Model of the Response System*

In general, the response of a community to a flood warning system is affected by people's experience of flooding and their subjective evaluation of the past performance of the forecasting system.

The general interaction between a forecasting system and a response system can be described from the following considerations. The effectiveness of a forecasting system can be judged from its performance measures of Type I and Type II errors. The response of a community to a flood warning can be described by a state variable, that is, the fraction of people in the community who respond to a call for evacuation when warned. If a past flood event has been predicted, then confidence in the flood forecasting system will increase, and thus, future rates of response will also increase. On the other hand, a cry-wolf (Type II error) event will decrease confidence in the flood forecasting system, thereby decreasing future rates of response. People tend to have decreased confidence in a flood warning system when they have experienced a missed warning. However, the experience of flooding will increase people's alertness to the possibilities of future floods. For simplicity, it is reasonable to assume that the response fraction will remain unchanged after a missed warning has been experienced. It is also assumed that a correct quiet does not change the response fraction in the future. In view of the above discussion, the response fraction evolves dynamically, governed by some underlying law of transition.

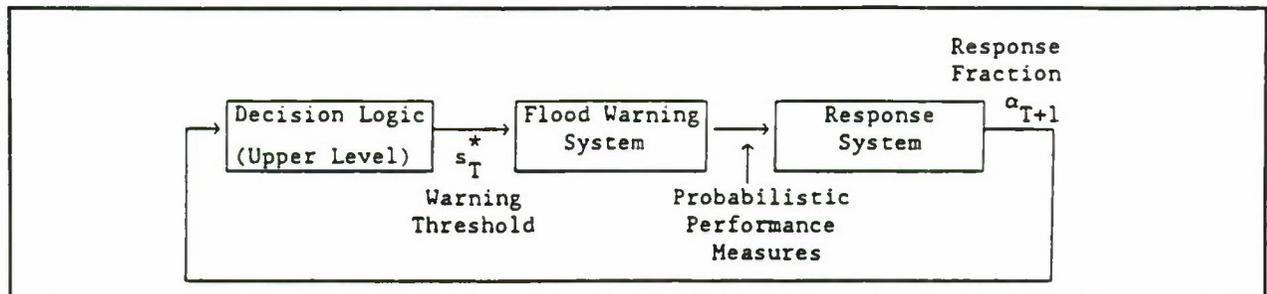
The response fraction in the community is described here as a controlled stochastic process in which the selected value of the warning threshold controls the transition probabilities. Knowing the present value of the response fraction, three possible transitions exist with given probabilities. The actual transition depends on the real outcome associated with the present warning decision. This stochastic system can be controlled in the sense of the expected value.

Note here that the feedback loop that encompasses the forecast and the response subsystems is closed only when the next flood event occurs. The present performance of a forecast system does not affect the response fraction at the present flood event, but it does affect the response fraction at the next flood event.

#### *Multiobjective Multistage Optimization Model*

A key aspect of flood warning systems is that the selection of the flood warning threshold cannot be viewed in isolation at each single flood event since the decisionmaker must balance the desire for high present flood-loss reduction with the possibility of high future flood loss. A multiobjective multistage optimization model is proposed in this part for finding the best value for the flood warning threshold of a

flood warning system. Evaluating the tradeoff between short- and long-term effects yields to an acceptable balance between the expected loss reduction at the current stage and the fraction of people who respond at the next flood event.



**Figure 4-2.** Interaction Between Forecast and Response Subsystems

At each flood event, the maximization of two noncommensurate and/or conflicting objective functions is considered. At a particular flood event, one objective is to maximize the expected property-loss reduction; the second objective is to increase the system's credibility by reducing the cry-wolf effect (i.e., increasing the fraction of people who respond in the future).

There are four possible, alternative losses associated with the warning decision: the expected loss when no warning is given and no flood occurs (assumed to be zero), the expected community property loss with no warning followed by a flood, the cost of an unneeded evacuation in the community (a warning and no flood), and the expected community property loss with a warning (warning followed by a flood). It is evident that the response fraction affects the cost of evacuation and the property loss with a warning. The technical section describes how to calculate the following measures of performance: (1) the expected property loss with a warning system operating at a selected threshold rule, (2) the expected property loss without a warning system, and (3) the expected property loss reduction realized from installing a warning system. All these loss functions, which are specific to an individual flood event, are described in detail in the technical section.

To construct a reasonable loss function, the concept of category-unit loss functions proposed by Krzysztofowicz and Davis [1983d] is adopted. The main modification is that the concept of the response degree of an individual, which was originally used in Krzysztofowicz and Davis [1983], is used in this study to represent the fraction of people who respond to the warning.

The N-stage multiobjective optimization problem of flood warning systems is then formulated to maximize the sum of the expected property-loss reductions of all flood events over the time horizon under consideration, and to maximize the forecast system's credibility, which is implicitly expressed by the expected fraction of people who respond to the warning beyond the time horizon under consideration. A third objective of maximizing the *expected life-loss reductions* of all flood events over the time horizon under consideration can also be included in this problem formulation when data are available.

Solving a multiobjective multistage optimization problem yields the set of noninferior solutions. A solution is noninferior if there is no other solution which improves on a single objective without seeing

losses in one or more other objectives. In this example, the selection of an optimal warning threshold from among the noninferior alternatives involves tradeoffs between the probabilities of Type I and Type II errors. The multiobjective optimization can be solved by the weighting method and dynamic programming.

## Case Studies

In the following case studies, historical data records obtained from the National Weather Service were used to estimate moments of the actual and forecasted crests; these estimates were next employed as parameter values in the normal-linear model of a forecast system. It must be stressed that this approach, while convenient analytically, offers at best an approximate representation of uncertainties in flood crests and their forecasts. Rather, the results presented in this report should be treated as hypothetical examples having some, but not all, realistic features.

### Application to Milton, Pennsylvania

System design S2 for Milton, Pennsylvania, described in this report in Part 3, Performance Characteristics of a Flood Warning System, is selected as the basis for the study in this subsection. In this case, the flood crest is of a normal distribution and the conditional probability density function of the forecasted flood crest, given the actual crest, is of a normal-linear form. It can be shown that (1) the marginal probability density function of the forecast is of a normal distribution and (2) the posterior distribution density function of the actual crest, given the forecasted crest, is of the normal-linear form. The probability of flooding given a particular forecasted crest is obtained by means of these distributions.

The four probabilistic measures of a forecasting system can be calculated for a given warning threshold and zone elevation. Table 4-1 shows those measures for two values of zone elevation and various values of warning threshold. A tradeoff between Type I and II errors can be clearly seen from Fig. 4-3. Different values of warning threshold are associated with different values of the probabilistic measures: probabilities of correct warning, correct quiet, false warning, and missed warning. They thus yield different impacts on the response fraction at the subsequent stage.

As in the other case studies, reasonable assumptions concerning the loss-function parameters are made that, along with the probabilities discussed above, enable the calculation of the expected flood-loss reduction.

Table 4-2 gives the calculated values of the expected flood-loss reduction for various response fractions and preselected warning thresholds where zone elevation is equal to 19 or 22 feet. The relationship between the expected flood-loss reduction and the warning threshold is also depicted in Fig. 4-4 for various response fractions and elevation levels. We consider a five-stage problem with the initial responding fraction equal to 0.7; that is, 70 percent of the population will respond to a warning initially. Two values of the elevation are considered, 19 and 22 feet.

**Table 4-1. Probabalistic Measures of the Warning System**

y = 19

$s^*$	$P_{11}(s^*, y)$	$P_{10}(s^*, y)$	$P_{01}(s^*, y)$	$P_{00}(s^*, y)$
15.0	0.8885870	0.1094329	0.0000862	0.0018939
15.5	0.8884591	0.1081553	0.0002140	0.0031715
16.0	0.8881721	0.1061974	0.0005011	0.0051294
16.5	0.8875671	0.1033221	0.0011061	0.0080047
17.0	0.8863676	0.0992858	0.0023056	0.0120411
17.5	0.8841263	0.0938829	0.0045469	0.0174439
18.0	0.8801783	0.0870001	0.0084949	0.0243263
18.5	0.8736137	0.0786737	0.0150595	0.0326531
19.0	0.8632919	0.0691255	0.0253813	0.0422013
19.5	0.8479231	0.0587649	0.0407501	0.0525619

y = 22

$s^*$	$P_{11}(s^*, y)$	$P_{10}(s^*, y)$	$P_{01}(s^*, y)$	$P_{00}(s^*, y)$
19.0	0.7171233	0.2152941	0.0077255	0.0598570
19.5	0.7108782	0.1958098	0.0139707	0.0793414
20.0	0.7009019	0.1734535	0.0239469	0.1016977
20.5	0.6858719	0.1490539	0.0389770	0.1260972
21.0	0.6644858	0.1237766	0.0603631	0.1513745
21.5	0.6357059	0.0989638	0.0891429	0.1761873
22.0	0.5990129	0.0759250	0.1258360	0.1992262
22.5	0.5546101	0.0557205	0.1702388	0.2194306
23.0	0.5035065	0.0390087	0.2213424	0.2361424
23.5	0.4474492	0.0259864	0.2773997	0.2491647

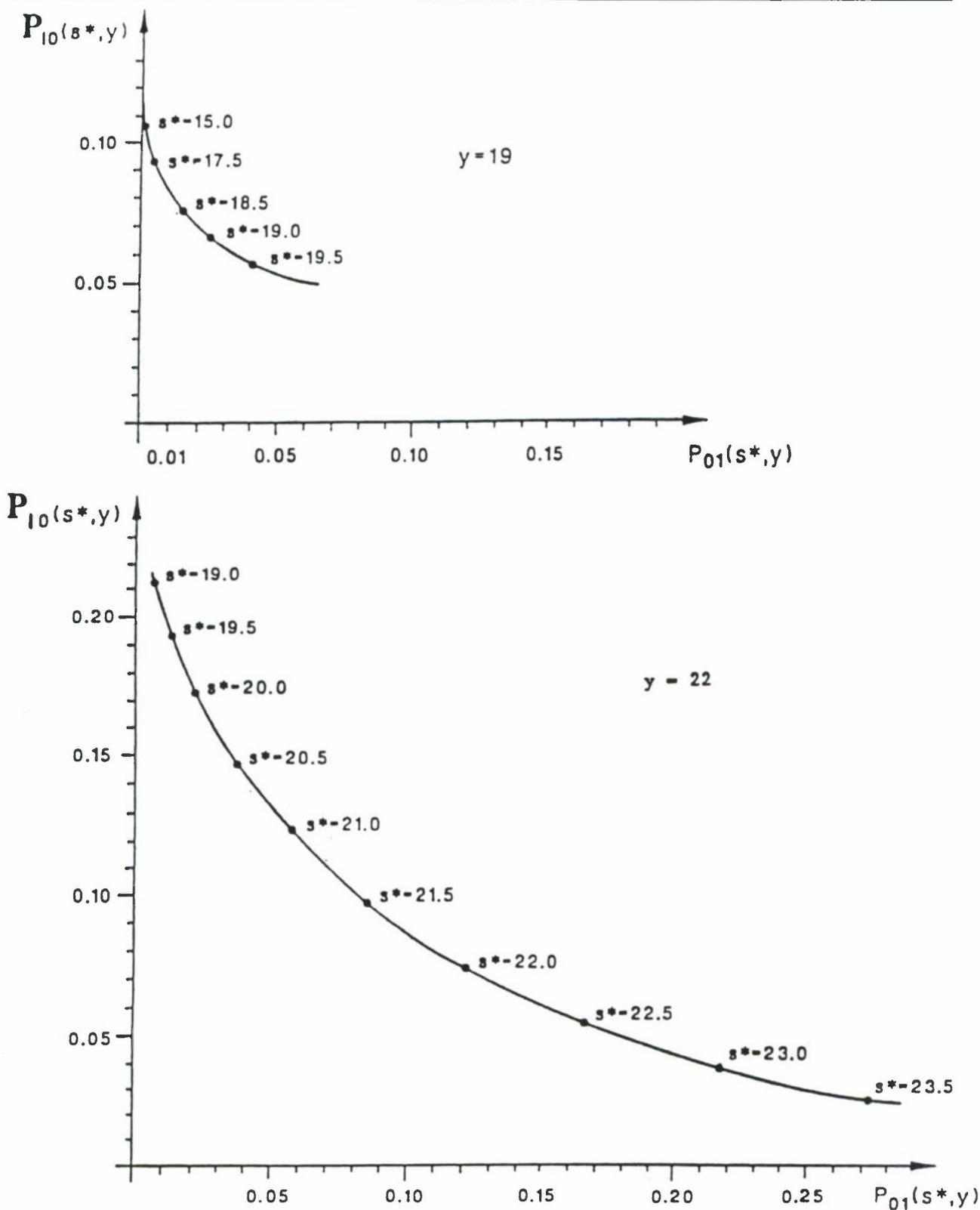


Figure 4-3. Trade-off Between Type I and Type II Errors

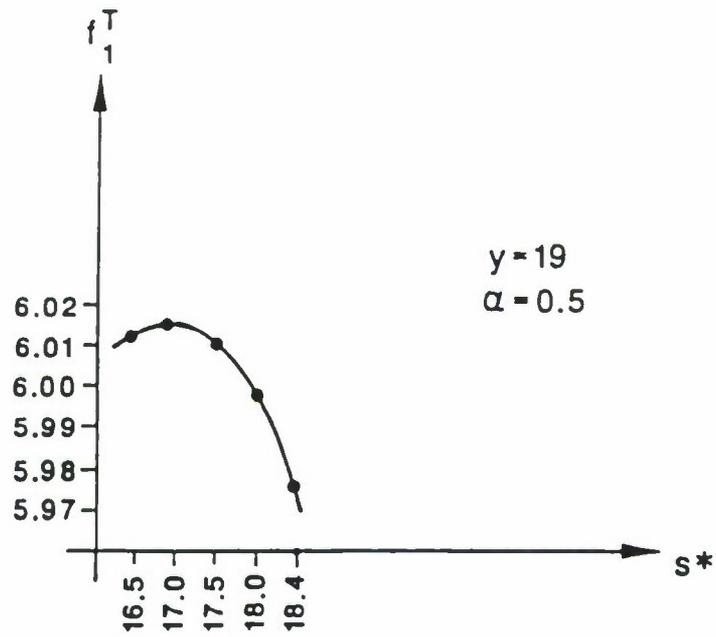
Table 4-2. Expected Flood-loss Reduction

y = 19

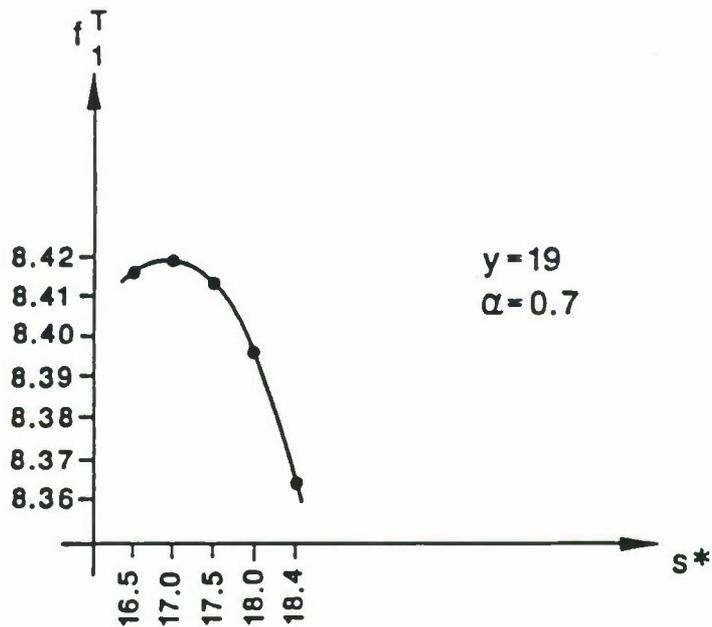
$s^*$ S	$\alpha$	0.50	0.60	0.70	0.80
16.5		6.013321	7.215985	8.418648	9.621314
17.0		6.014395	7.217274	8.420152	9.623033
17.5		6.011573	7.213888	8.416201	9.618517
18.0		5.998908	7.198690	8.398471	9.598253
18.4		5.975090	7.170107	8.365125	9.560144

y = 22

$s^*$ S	$\alpha$	0.50	0.60	0.70	0.80
19.0		3.592137	4.310565	5.028992	5.747422
19.5		3.595634	4.314761	5.033888	5.753016
20.0		3.574387	4.289264	5.004142	5.719021
20.5		3.511868	4.214242	4.916617	5.618992
20.9		3.420732	4.104879	4.789026	5.473174

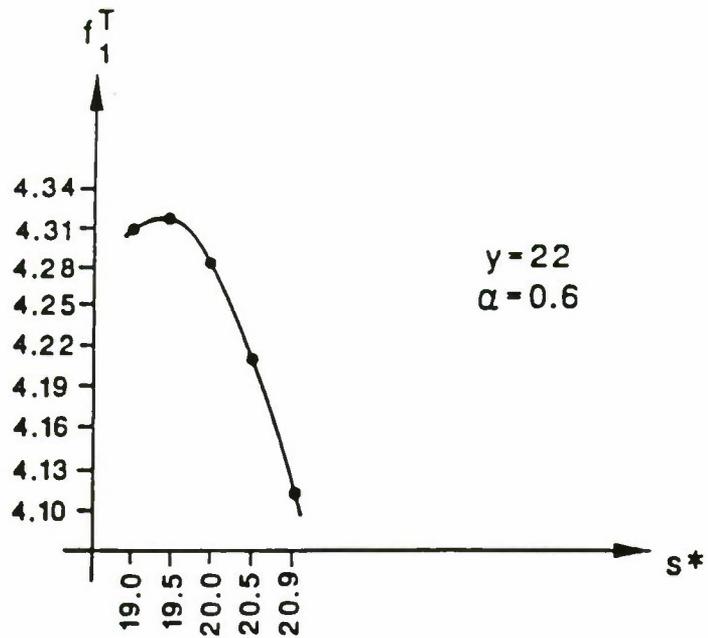


5(a)

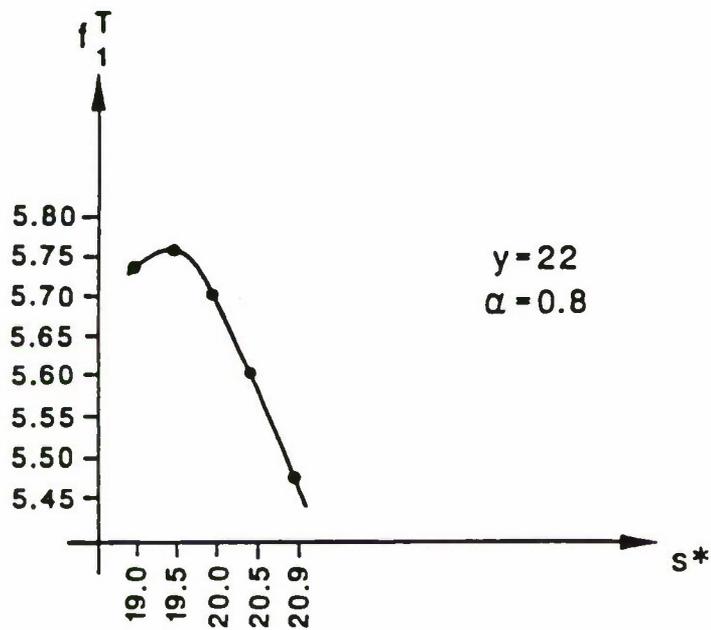


5(b)

Figure 4-4. Relationship Between the Expected Flood-loss Reduction and the Warning Threshold for Various Response Fraction and Elevation Levels



5(c)



5(d)

Figure 4-4. (continued)

The overall problem can now be posed as follows: choose the optimal warning threshold to maximize the sum of flood loss reductions in all five stages *and* the response fraction of the community after five stages. The optimization problem can be solved by the weighting method and dynamic programming.

(1) the lower the weighting coefficient associated with the first objective (loss reduction), the higher the value of the flood warning threshold will be set to avoid possible high Type II errors;

(2) in order to select a decision that maximizes the sum of flood-loss reductions, the flood warning threshold is set higher at the earlier stage than at the later stage (with respect to the same value of the response fraction) in order to reduce the probability of high loss at the later stages; and

(3) the higher the present response fraction, the more cautious the selection of the threshold is. That means that a higher value of threshold is set for a higher value of the present response fraction in order to avoid losing a larger number of the response population. We should note here that the third conclusion may be model-specific.

Figures 4-5, 4-6, and 4-7 show the optimal flood warning threshold as functions of the weighting coefficient, the stage, and the response fraction, respectively.

## **Application to Eldred, Pennsylvania**

### *Location*

Eldred is a small community situated on the upper Allegheny River in northern Pennsylvania, about three miles south of the state border with New York. The river gauge has a datum at 1417 feet and closes a drainage area of 50 square miles of mountainous terrain with its highest ranges towering at 2500 feet. The river flow at Eldred is essentially unimpaired natural runoff.

### *Data Records and Parameter Estimation*

Historical flood and forecast data were retrieved from the archives of the National Weather Service Forecast Office in Pittsburgh. The prior distribution of flood crests was estimated from a record of floods spanning 1942-1989. During these 48 years, 36 flood crests exceeded the gauge height of 11 feet (3 floods in every 4 years, on the average), and 14 had crests above the official stage of 17 feet (about one flood in every 3.5 years, on the average). The highest flood on record occurred in June 1972 and reached 29 feet.

The likelihood functions were estimated from a historical joint record of forecasted and actual flood crests. This record contained 12 floods that occurred in the period 1984-1988.

### *Results of Case Study*

From the historical data, the flood crest is fitted by a normal distribution, and the conditional probability density function of the forecasted flood crest, given the actual crest, is also fitted by a normal distribution (the normal-linear model). It can be shown that

(a) the marginal probability density function of the forecasts is also of a normal distribution, and

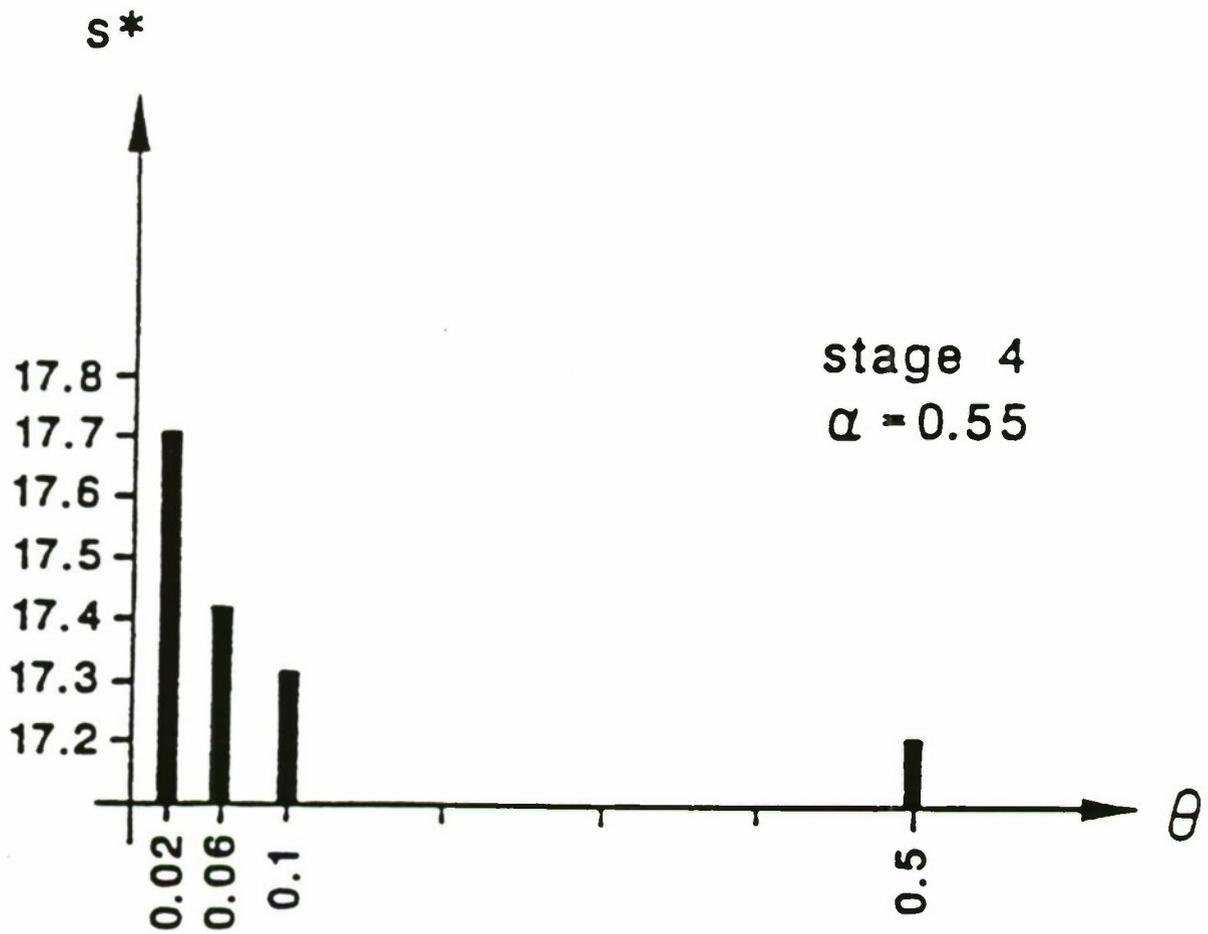


Figure 4-5. Relationship Between the Optimal Warning Threshold and the Weighting Coefficient for  $\alpha = 0.55$  at Stage 4

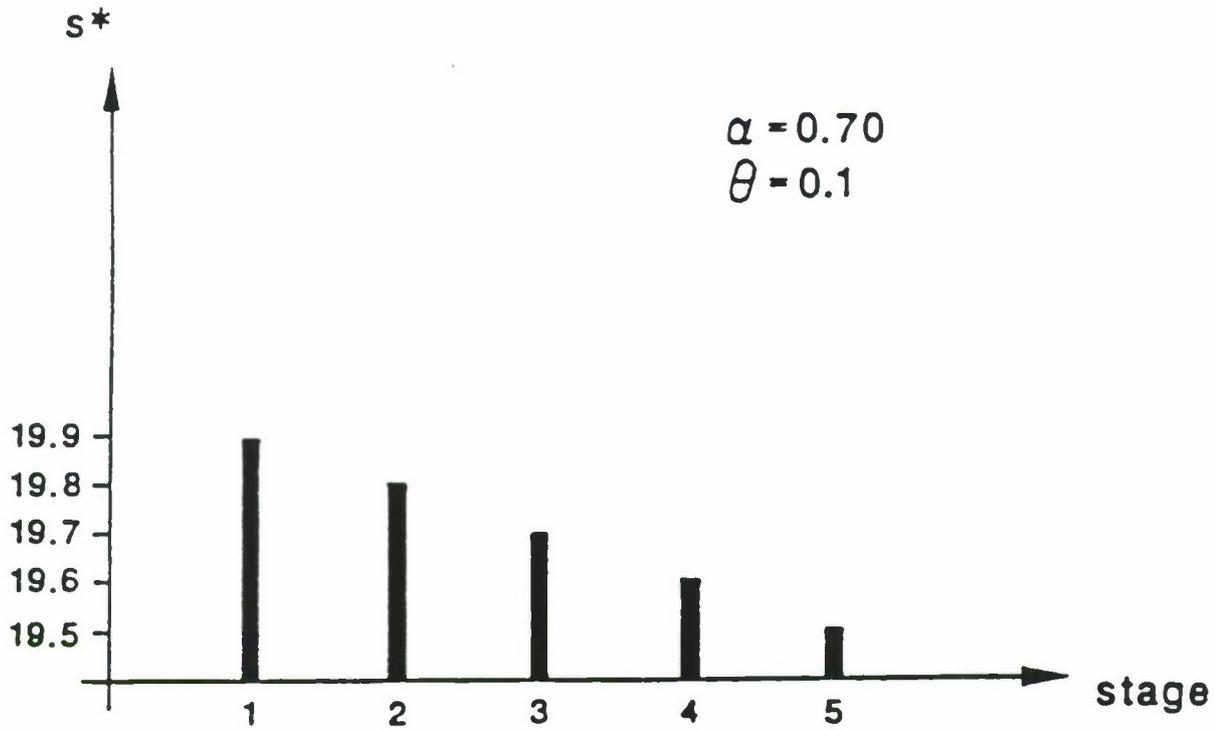


Figure 4-6. Relationship Between the Optimal Warning Threshold and the Stages for  $\alpha = 0.70$  and  $\theta = 0.1$

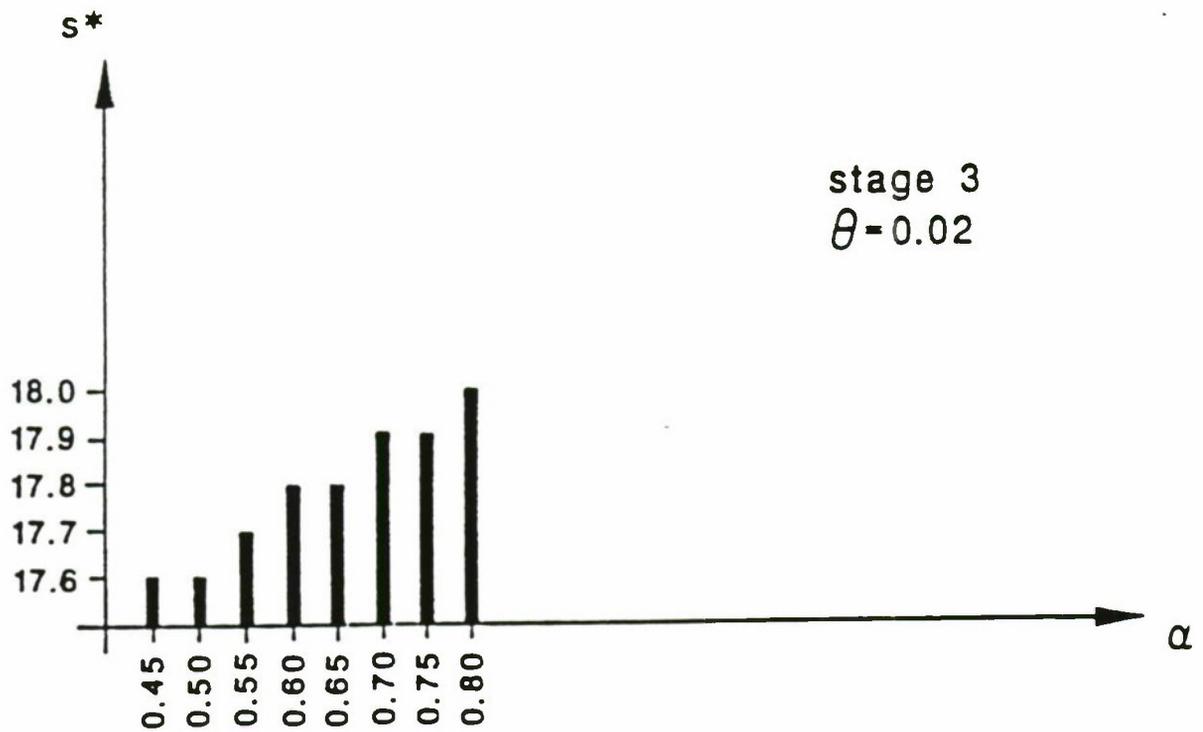


Figure 4-7. Relationship Between the Optimal Warning Threshold and the Response Fraction for  $\theta = 0.02$  at Stage 3

(b) the posterior distribution density function of actual flood crest given the forecasted crest is of the normal-linear form.

The four probabilistic measures of the forecasting system can be calculated for a given warning threshold and zone elevation. Different values of warning threshold are associated with different values of the four probabilistic measures: probabilities of correct warning, correct quiet, false warning, and missed warning. They thus yield different impacts on the response fraction at the subsequent flood stages. The flood-loss information is not available in the case study, and hence reasonable assumptions are made about the parameters of the loss functions.

### **Application to Connellsville, Pennsylvania**

#### *Location*

Connellsville, a town in southwestern Pennsylvania, embraces the banks of the Youghiogheny River—a tributary of the Monongahela River. The river gauge has a datum at 860 feet and closes a drainage area of 1326 square miles. The terrain varies from hilly to mountainous, especially in the eastern part of the basin where Mt. Davis reaches 3213 feet—the highest point in Pennsylvania.

#### *Reservoirs*

The river flow in Connellsville is partly regulated by storage reservoirs. The Deep Creek Reservoir, completed in January 1925, is used for hydroelectric power generation. It is owned and operated by the Pennsylvania Electric Company. The reservoir has a capacity of 93,000 acre-feet and closes a drainage area of 65 square miles, or about 5% of the total basin. Thus its influence on flood flows at Connellsville is insignificant.

The Youghiogheny Reservoir, downstream of the Deep Creek dam, was completed in October 1943. It serves multiple purposes and is operated by the U.S. Army Corps of Engineers. The reservoir has a capacity of 254,000 acre-feet, which equals 42% of the average annual runoff at the dam, and controls a drainage area of 434 square miles, which constitute 33% of the total basin. The length of the river between the dam and Connellsville is 29.4 miles. All these facts together suggest that the reservoir can only partially control floods at Connellsville.

#### *Two cases analyzed: present and hypothetical systems*

Two cases of the flood forecast system were analyzed and, accordingly, two sets of parameter estimates had to be constructed. The first case describes the present system which is composed of the Youghiogheny Dam and the National Weather Service (NWS) river forecasting technology. A flood forecast for Connellsville is prepared by routing the project regulated outflow from the dam and superimposing on it the predicted runoff from the drainage area between the dam and the forecast point. The second case describes a hypothetical system composed of the NWS river forecasting technology but without any influence of the Youghiogheny Dam on flood flows. Thus runoff from the entire basin must be predicted, as flow at Connellsville is unregulated.

*Data Records and Parameter Estimation*

Historical flood and forecast data were retrieved from the archives of the National Weather Service Forecast Office in Pittsburgh. For the present system, with the Youghiogheny Dam, the prior distribution of the flood crest was estimated from a record spanning 1943-1986. In these 44 years, 22 flood crests exceeded the official flood stage of 12 feet (thus, a flood occurred every two years, on the average). The highest flood on record occurred in October 1954 and reached 22 feet. The likelihood functions were estimated from a historical joint record of 6 forecasted and actual crests in the period 1984-1986.

For the hypothetical system, without the Youghiogheny Dam, the prior distribution of flood crests was estimated from a record spanning 1910-1942. During these 43 years, 22 flood crests were observed above the flood stage of 12 feet. The highest flood during that period occurred in March 1936 and exceeded 20 feet. Estimation of the likelihood functions presented a challenge since there is no historical joint record of forecasted and actual flood crests—a record that would correspond to the modern forecasting technology yet without any influence of the Youghiogheny Dam. The theory of sufficient comparisons of forecasts systems [Krzysztofowicz 1992] came to the rescue here. It seemed reasonable to assume that systems utilizing the same forecasting technology and operated for rivers with similar geomorphologic, hydrologic, and climatic characteristics should exhibit similar statistical characteristics of performance. In particular, their standardized sufficiency characteristics (SSC) should be similar. [For a definition and properties of the SSC see Krzysztofowicz 1992.] A flood forecast system for Milton, Pennsylvania, where river flows are unregulated, was taken as an analog. Its SSC was estimated from a historical joint record of forecasted and actual flood crests; this record contained 8 forecasts of floods that occurred in the period 1959-1975. Next, the variance estimate in the likelihood functions for Connellsville, the case with the dam, was adjusted so as to give the SSC for Connellsville the same magnitude as the SSC for Milton. In a sense, we have done a "statistical transfer" of a forecast system from Milton to Connellsville.

*Distribution of Damages*

The stage-damage function for Connellsville was estimated according to the methodology of Krzysztofowicz and Davis [1983a,b]. A crude inventory of establishments located at various elevations of the floodplain was extracted from the River Stage Data form prepared by the National Weather Service and the U.S. Geological Survey in May 1990. About 212 establishments were counted in the floodplain and the maximum possible damage for the community was estimated to be \$10,400,000 at the 1991 price level.

In order to construct the stage-damage function, the floodplain was discretized into five steps, whose elevations range from the flood stage at 12 feet to 20 feet, and the establishments were grouped into three structural categories: two-story house, commercial garage, and commercial store.

*Stage-Damage Function*

For each structural category, there is a unit damage function specifying the fraction of maximum possible damage to an establishment which occurs when the depth of flooding, measured from the first-floor level, is given. The general form of the unit damage function is polynomial, with the polynomial coefficients as given in the technical section for each of the three structural categories. Using all of the above information, the stage-damage function for the community may be constructed.

### *Results of Case Study*

In the case study of Connellsville, Pennsylvania, four different situations with structural and nonstructural flood prevention measures are investigated. The expected flood losses in the following four cases are calculated:

- (1) expected flood loss in the case with neither a dam nor a flood warning system,
- (2) expected flood loss in the case with a dam and without a flood warning system,
- (3) expected flood loss in the case without a dam and with a flood warning system, and
- (4) expected flood loss in the case with both a dam and a flood warning system.

When there is a dam, from the historical data the flood crest is fitted by a prior normal distribution, and the conditional probability density function of the forecasted flood crest, given the actual crest, is fitted by a normal-linear distribution. It can be shown that (a) the marginal probability density function of the forecast is of a normal distribution and (b) the posterior distribution density function of the actual crest, given the forecasted crest, is of the normal-linear form.

When there is no dam, from the historical data the flood crest is fitted by a prior normal distribution and the conditional probability density function of the forecasted flood crest, given the actual crest, is fitted by a normal distribution. It can be shown that (a) the marginal probability density function of the forecast is of a normal distribution and (b) the posterior distribution density function of the actual crest, given the forecasted crest, is of the normal linear form.

The four probabilistic measures of the forecasting system can be calculated for given warning threshold and zone elevation (Table 4-3). Different values of warning threshold are associated with different values of the probabilistic measures: probabilities of correct warning, correct quiet, false warning, and missed warning. They thus yield different impacts on the response fraction at the subsequent flood events.

The unit damage function is fitted from historical data, and the other loss-function parameters are set by means of reasonable assumptions described in the technical volume.

The technical volume gives the calculated values of the expected flood loss without a warning system for zone elevation equal to 12 and 14 feet for both cases with a dam and without a dam. It also gives the calculated values of the expected flood-loss reduction with full response for various preselected warning thresholds for elevations of 12 and 14 feet, in both cases with a dam and without a dam.

Table 4-3. Probabilistic Measures of the Warning System (Connellsville, Pennsylvania)

y = 12 (with a dam)

$s^*$	$P_{11}(s^*, y)$	$P_{10}(s^*, y)$	$P_{01}(s^*, y)$	$P_{00}(s^*, y)$
11.0	0.7763117	0.1021628	0.0014434	0.1200821
11.2	0.7748482	0.0879903	0.0029068	0.1342546
11.4	0.7722843	0.0735971	0.0054707	0.1486478
11.6	0.7681055	0.0594870	0.0096495	0.1627579
11.8	0.7617464	0.0462294	0.0160087	0.1760156
12.0	0.7526836	0.0343665	0.0250714	0.1878784
12.2	0.7405316	0.0243194	0.0372235	0.1979255
12.4	0.7251226	0.0163076	0.0526324	0.2059374
12.6	0.7065377	0.0103184	0.0712173	0.2119266
12.8	0.6850762	0.0061376	0.0926788	0.2161073

y = 14 (with a dam)

$s^*$	$P_{11}(s^*, y)$	$P_{10}(s^*, y)$	$P_{01}(s^*, y)$	$P_{00}(s^*, y)$
13.6	0.4459119	0.1342705	0.0045577	0.4152599
13.8	0.4421889	0.1087867	0.0082806	0.4407437
14.0	0.4363676	0.0851224	0.0141020	0.4644080
14.2	0.4278818	0.0640043	0.0225878	0.4855261
14.4	0.4163113	0.0460156	0.0341583	0.5035149
14.6	0.4014937	0.0314805	0.0489759	0.5180500
14.8	0.3835860	0.0204010	0.0668836	0.5291294
15.0	0.3630455	0.0124725	0.0874241	0.5370579
15.2	0.3405446	0.0071667	0.1099250	0.5423638
15.4	0.3168429	0.0038577	0.1336266	0.5456727

Table 4-3. (continued)

$y = 12$  (without a dam)

$s^*$	$P_{11}(s^*, y)$	$P_{10}(s^*, y)$	$P_{01}(s^*, y)$	$P_{00}(s^*, y)$
9.6	0.8598625	0.0954413	0.0079126	0.0367837
9.8	0.8577797	0.0913708	0.0099954	0.0408541
10.0	0.8552552	0.0870857	0.0125198	0.0451392
10.2	0.8522224	0.0826117	0.0155527	0.0496133
10.4	0.8486115	0.0779786	0.0191636	0.0542464
10.6	0.8443505	0.0732205	0.0234246	0.0590045
10.8	0.8393664	0.0683756	0.0284086	0.0638494
11.0	0.8335869	0.0634843	0.0341881	0.0687406
11.2	0.8269404	0.0585908	0.0408347	0.0736342
11.4	0.8193607	0.0537378	0.0484143	0.0784871

$y = 14$  (without a dam)

$s^*$	$P_{11}(s^*, y)$	$P_{10}(s^*, y)$	$P_{01}(s^*, y)$	$P_{00}(s^*, y)$
13.0	0.5588912	0.1818558	0.0335281	0.2257249
13.2	0.5522861	0.1680066	0.0401332	0.2395740
13.4	0.5447636	0.1543220	0.0476557	0.2532586
13.6	0.5362751	0.1409070	0.0561443	0.2666736
13.8	0.5267833	0.1278629	0.0656361	0.2797178
14.0	0.5162637	0.1152843	0.0761557	0.2922964
14.2	0.5047076	0.1032567	0.0877118	0.3043239
14.4	0.4921228	0.0918543	0.1002966	0.3157264
14.6	0.4785339	0.0811387	0.1138854	0.3264419
14.8	0.4639837	0.0711576	0.1284356	0.3364230

## GLOSSARY OF SYMBOLS

### Part 1. Integration of Flood Warning and Structural Measures

$C_E$	cost function of evacuation
$D$	flood discharge; used in the frequency-discharge-elevation curves
$E$	flood elevation; used in the discharge-elevation curve
$F$	flood frequency (exceedance probability)
$f(L)$	probability density function of damage $L$
$f_4$	conditional expected value of flood damage given exceedance of the flood with nonexceedance probability $a$ ; measure of the risk of extreme events in the PMRM
$f_5$	expected value of flood damage
$h$	flood stage
$L$	flood damage (millions \$)
$L_{RD}$	flood loss reduction defined as the difference between $L_{wo}$ and $L_w$
$L_w$	flood loss function with a warning system
$L_{wo}$	flood loss function without a warning system
$M$	number of feasible options involving only flood warning systems for flood mitigation
$MC$	maximum evacuation cost to community assuming full response
$MD$	maximum possible damage of the community due to flood of the highest magnitude
$MR(h - y)$	unit reduction function specifying the reduction of the maximum flood loss $MD$ when the depth of flooding is $(h - y)$ and full response of the community is made ( $q = 1$ )
$N$	number of feasible options involving only structural measures for flood mitigation
$p(L)$	probability of flood
$W$	denotes plans incorporating flood warning systems

$y$	elevation of the floodplain zone under consideration
$\alpha$	nonexceedance probability that partitions the range of extreme events; used in the definition of the conditional expected value $f_4$
$\delta(h - y)$	unit damage function specifying the fraction of MD that occurs when the depth of flooding is $(h - y)$
$\theta$	fraction of the community that responds to a flood warning; response fraction

## Part 2. Multiobjective Decision-Tree Analysis

$a_n$	action, or alternative, or option, at a decision node $n$
$C$	maximum possible loss of property (discrete case); possible loss of lives given no flood warning -- linear function of discharge $W$ (continuous case)
$C1, C2, C3$	chance nodes in the decision tree
$d_j^m$	number of elements in the the set $r_j^m$
$DN1, DN2$	do-nothing option in the first and second decision periods, respectively
$E[\bullet]$	expected value
$E^s[\ ]$	the $s$ th averaging-out strategy; for example $E^4$ denotes the conditional expected value of extreme events $f_4$
$EV1, EV2$	evacuation order in the first and second decision periods, respectively
$EVE$	expected value of experimentation; difference between expected loss without experimentation and expected loss with experimentation
$\Phi$	standard normal distribution function
$f_1$	cost objective function; balanced with the risk functions $f_2$ thru $f_5$ in the PMRM
$f_2, f_3$	conditional expected values
$f_4$	conditional expected value of the (damage) risk of extreme events
$f_4^*$	optimal value of $f_4$ , see Equation (2.17)
$f_5$	overall expected value of damage
$k$	dimension of the objective function vector

L	maximum possible loss of lives (discrete case); possible loss of lives given no flood warning -- linear function of discharge W (continuous case)
LN	lognormal distribution
$P_x$	cumulative distribution function of X
$p_x$	probability density function of X
r	the vector of objective functions in the decision tree $[r_1, \dots, r_k]$
$r_j^m$	set of Pareto optimum alternatives associated with each branch emerging from chance node m
W	actual flood level (cfs)
WA1, WA2	issuing a flood watch in the first and second decision periods, respectively
X	random variable of damage or loss
$\alpha$	partitioning nonexceedance for the conditional expected value $f_i$
$\alpha_i$	values of nonexceedance probability that partition the ranges of risk in the PMRM
$\beta_{ij}$	values of damage that partition the severity of risk in the PMRM for the $j$ th policy
$\lambda_{i1}$	tradeoffs between the cost objective function and the $i$ th risk function
$\mu$	mean of the discharge W
$\theta_n$	state of nature at node n of the decision tree (also used in unrelated context as parameters in the PMRM defined by Equation 2.4)
$\sigma$	standard deviation of the discharge W

**Part 3. Performance Characteristics of a Flood Warning System**

a, b	parameters of the normal-linear likelihood model f
D	detection (Equation 3.7)
F	false warning (Equation 3.7)
$f(s   h, \Theta = 1, T = 1)$	probability density of s conditional on the actual crest h, $\Theta = 1, T = 1$

FSC	forecast sufficiency characteristic, a measure sufficient for comparing any two forecasters who produce forecasts of the same variate
$g(h \mid \Theta = 1)$	prior probability density function of flood crest given flood occurs
$g(\lambda \mid \Theta = 1)$	probability density of $\lambda$ conditional on $\Theta = 1$
$h$	height of actual flood crest
LT	expected lead time
M	missed flood (Equation 3.7)
N	expected number of floods per year
N	normal probability distribution
$n$	expected number of zone floods per year
ND	expected number of detections per year for a zone
NF	expected number of false warnings per year for a zone
PTC	performance tradeoff characteristic, a plot of ND versus NF
$q$	quiet (Equation 3.7)
$q(s)$	$P(\theta = 1 \mid s, T = 1)$ , posterior probability of a flood in a given zone
$q^*$	optimal threshold associated with warning rule $W^*$
ROC	relative operating characteristic, a plot of $P(D)$ versus $P(F)$
$s$	forecasted flood crest
T	trigger indicator: trigger is not observed ( $T = 0$ ), trigger is observed ( $T = 1$ )
W	warning rule, $w = W(s)$ , where $w = 0$ and $w = 1$ denote "do not issue warning" and "do issue warning," respectively
$W^*(s)$	optimal warning rule (of the threshold type) minimizes expected disutility of outcomes
$y$	zone elevation
$\gamma$	$P(\Theta = 1 \mid T = 1)$ , <i>diagnosticity</i> conditional probability

$\kappa_0(s \mid \Theta=0, T=1)$	probability density of $s$ conditional on the forecast $\Theta = 0$ and $T = 1$
$\lambda$	lead time of a warning for a given zone, conditional on hypothesis that zone will be flooded
$\mu_h, \sigma_h$	mean and standard deviation of the prior density $g(h \mid \Theta = 1)$
$\mu_s, \sigma_s$	mean and standard deviation of the likelihood function $k_0$
$\theta$	zone flood indicator: zone flood does not occur ( $\theta = 0$ ), zone flood occurs ( $\theta = 1$ )
$\rho$	$P(T = 1 \mid \theta = 1)$ , <i>reliability</i> conditional probability
$\Theta$	flood indicator: flood does not occur ( $\Theta = 0$ ), flood occurs ( $\Theta = 1$ )

#### Part 4. Selection of Optimal Flood Warning Threshold

A, B, C	parameters used in the normal-linear likelihood model (Equations 4.7-4.9)
$c_1, c_2$	constants governing the evolution of the response fraction $a$
$D(h)$	stage-damage function for a community
$D_{00}$	expected loss when no warning is given and no flood occurs (zero)
$D_{01}$	expected community property loss without a warning
$D_{01}$	expected property loss without a warning conditioned on forecast $s$
$D_{10}$	cost of evacuation in the community
$D_{10}$	expected cost of community evacuation conditioned on forecast $s$
$d_{10}$	cost function of evacuation, linear function of response fraction $a$
$D_{11}$	expected community property loss with a warning
$D_{11}$	expected property loss with a warning conditioned on forecast $s$
$d_{11}$	loss function with a warning
erf	standard error function
$f(h \mid s)$	posterior distribution of $h$ given a forecast $s$

$f(s   h)$	conditional density of $s$ given $h$
$f_1$	sum of the expected property loss reductions over the planning horizon
$f_1^T$	expected property loss reduction (difference made by warning system)
$f_1^T$	expected property loss reduction at stage $T$
$f_2$	objective function representing credibility of forecast system: $E\{\alpha_{N+1}\}$
$g(h)$	prior probability density of flood crest $h$
$h$	flood crest
$k(s)$	marginal probability density of forecast $s$
MC	maximum evacuation cost with a full response
MD	maximum possible damage due to highest flooding with no response
$MR(h - y)$	unit reduction function--reduction of MD when the depth of flooding is $(h - y)$ and $a = 1$
$N$	number of successive flood events on planning horizon
$N(\mu, \sigma)$	normal distribution with mean $\mu$ and variance $\sigma^2$
$P_{00}$	probability of a correct quiet
$P_{01}$	probability of a missed forecast (Type I error)
$P_{10}$	probability of a false warning (Type II error)
$P_{11}$	probability of a correct warning
$q(s, y)$	probability that zone of elevation $y$ will be flooded conditioned on forecast $s$
$r = 1, 2, 3$	structural categories in the floodplain
$s$	forecasted flood crest
$s^*$	flood warning threshold; warning issued when $s \geq s^*$
$\phi$	noninferior decision sequence consisting of the set of warning thresholds for all decision periods in the planning horizon
$y$	elevation of a zone in the floodplain
$\alpha_T$	response fraction of the community in period $T$

$\delta(h - y)$	unit damage function specifies the fraction of MD when flood depth is $(h - y)$
$\delta_r(z)$	fraction of maximum possible damage to an establishment that occurs when the depth of flooding measured from the first floor level is $z$
$\Phi$	standard normal distribution function
$\mu_h, \sigma_h$	mean and standard deviation of the distribution $g(h)$

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