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DIVERSITY TECHNIQUES FOR SPECTRUM SENSING IN FADING ENVIRONMENTS (POSTPRINT)

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14. ABSTRACT
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15. SUBJECT TERMS
spectrum sensing, dynamic spectrum access, cognitive radio, diversity techniques

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Abstract—A key feature of a cognitive radio (CR) is a reliable spectrum sensing technique which enables the CR to detect the so-called white spaces in the frequency band. This allows opportunistic access of the unlicensed (secondary) users to these white spaces without causing undue interference to licensed (primary) users. In many scenarios the CR may operate in a multipath fading environment where spectrum sensing must cope with the fading effects of the unknown primary signal. In this paper we first study the effects of multipath fading of the performance of the autocorrelation-based spectrum sensing algorithm in [1]. The results show that Rayleigh fading causes significant degradation in the detection and false alarm. This motivates us to investigate three diversity combining techniques, namely equal gain combining, selective combining, and equal gain correlation combining. For Rayleigh fading channels we evaluate the performance of these three techniques through simulation. The results show that for detection probabilities of interest (e.g., > .9), a system with a four-branch diversity achieves an SNR gain of more than 5 dB over the no-diversity system that uses the same number of received signal samples.

Index Terms—Spectrum Sensing, Dynamic Spectrum Access, Cognitive Radio, Diversity Techniques

I. INTRODUCTION

The ever increasing demand for radio spectrum in both civilian and military applications has led to a new paradigm in spectrum management often referred to as opportunistic spectrum access (OSA). In OSA an unlicensed (a.k.a secondary) user can identify, and "opportunistically" operate in, the licensed spectrum bands that are currently unutilized by the licensed (a.k.a. primary) user. Cognitive radio is proposed as the enabling technology for OSA. A key feature of a cognitive radio is its spectrum sensing capability which enables the CR to

sense its environment and detect the presence or absence of the primary signals. Spectrum sensing algorithms have been heavily investigated in recent years.

In many scenarios the cognitive radio is deployed in a multipath fading environment and hence must cope with the fading effects on the unknown primary signal. Recently several authors have considered the problem of spectrum sensing in a fading environment. In [2], the authors have investigated several diversity techniques in order to improve the performance of the energy detectors. In [3], it is shown that equal gain combining is optimal for energy detectors in the infinitesimally small signal-to-noise ratio regime. In [4], collaboration of secondary users is employed in order to improve the performance of the energy detectors and simulation results are obtained for one-out-of-n fusion rule. In [5], the authors consider a collaborative scheme whereby each secondary user makes a local decision and then transmits its decision to a fusion center. At the fusion center, the decisions are linearly combined and used to obtain a global decision. Wideband spectrum sensing under frequency selective fading has been investigated in [5], [6] using a multiband OFDM-like system.

A new approach for spectrum sensing based on the autocorrelation of the received signal was recently proposed in [1] and analyzed in the case of additive white Gaussian noise channel. The autocorrelation-based method is effective in an oversampled system (i.e., when the sampling rate exceeds the primary signal bandwidth), making it a suitable technique for detecting the primary signals of unknown bandwidth. Also, it was shown that, contrary to energy detectors, the performance of the autocorrelation-based detector does not suffer from the absence of an accurate estimate of the noise power. In this paper we first show that, as expected, the performance of the autocorrelation-based detector deteriorates significantly for Rayleigh fading channels. We then consider three receive-diversity combining techniques and

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show that most of the performance loss due to fading can be recovered using a four-branch diversity receiver. The first two are commonly employed diversity combining methods, namely equal gain combining and selective combining. These methods combine the decision statistics computed from each branch. The last method, which we refer to as equal gain correlation combining, combines the autocorrelation estimates from each branch. In other words, the autocorrelation-based decision statistic is computed with improved autocorrelation estimates

The rest of the paper is organized as follows. Section II formulates the spectrum sensing problem. Section III presents the autocorrelation-based method from [1]. In this section, the detection performance of the autocorrelation-based method for Rayleigh fading channel is evaluated via analysis and simulation. Section IV introduces the three diversity combining methods with their performance analysis under AWGN channel and in Section V, the performance of the diversity combining methods are evaluated for Rayleigh flat fading channel via simulation. Finally, conclusive remarks are given in Section VI.

II. SPECTRUM SENSING PROBLEM

The cognitive radio is assumed to be equipped with K antennas to enable diversity combining. The primary user signal is denoted (in complex notation) by $s(t) = \tilde{s}(t)e^{j2\pi f_c t}$, where $\tilde{s}(t)$ is the complex baseband signal occupying the frequency band $(-f_b, f_b)$ Hz. The signal $r_k(t)$ received by the kth antenna is observed over a bandwidth of $2f_{bw}$ Hz centered around f_c. The signals $r_k(t)$, $k = 1, 2, \cdots, K$, are assumed to have undergone independent flat fading which remains constant during the observation period $[0, T]$. Consequently the kth received signal is given by

$$r_k(t) = \eta h_k s(t) + V_k(t), \ 0 \le t \le T, \qquad (1)$$

where $\eta \in \{0, 1\}$ determines the presence or absence of the primary signal $s(t)$, and where h_k is a complex-valued Gaussian random variable with i.i.d. components denoting the fading parameter of the kth channel. The processes $\{V_k(t)\}$, $k = 1, 2, \cdots, K$, are assumed to be independent, complex-valued white Gaussian noise processes.

The signal $r_k(t)$ is first downconverted to the baseband signal $x_k(t)$ which is then sampled as depicted in Fig. 1 to obtain the discrete-time baseband signal $\{x_{k,n}\}$. The lowpass filter used in the downconverter is assumed to be an ideal "brick" filter so that the subsequent sampling process with period $T_s \triangleq (2f_{bw})^{-1}$
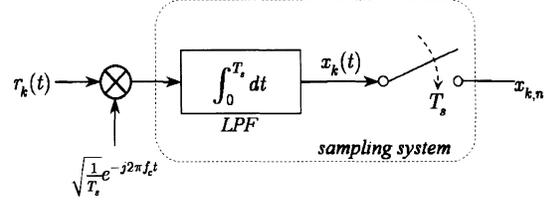

Fig. 1. Simplified block diagram of the k-th RF front-end.

is alias-free. The resulting complex baseband signals $\{x_{k,n}\}$, $k = 1, 2, \cdots, K$, are given by

$$x_{k,n} = x_k(nT_s) = \eta h_k s_n + v_{k,n} \qquad (2)$$

where $s_n = \tilde{s}(nT_s)$ and where for $k = 1, 2, \cdots, K$, $\{v_{k,n}\}$ are independent iid complex-valued Gaussian random processes with mean zero and autocorrelation function $r_{v_k v_k, l} = \sigma_v^2 \delta_l$.

The primary signal $\{s_n\}$ is unknown to the receiver and is modeled as a complex-valued zero-mean wide-sense stationary (WSS) process, characterized by its autocorrelation function $r_{ss,l} \triangleq E[s_n s_{n-l}^*]$. Furthermore, $\{s_n\}$ is assumed to be band-limited in the frequency range $(-\omega_b, \omega_b)$ rad/sample where $\omega_b = 2\pi f_b/f_s$ and $0 < \omega_b < \pi$. The band-limited nature of $\{s_n\}$ guarantees that it is non-white, i.e., $r_{ss,l} \neq \sigma_s^2 \delta_l$ where σ_s^2 is the signal power and δ_l is the Kronecker delta function. The average SNR of $x_{k,n}$ is denoted by

$$\bar{\gamma} \triangleq \frac{\sigma_s^2 E[|h_k|^2]}{\sigma_v^2}. \qquad (3)$$

Assuming that $\{s_n\}$ is uncorrelated from the noise processes $\{v_{k,n}\}$, we express the conditional autocorrelation function of $\{x_{k,n}\}$ as

$$r_{x_k x_k, l | H_\eta} = \eta |h_k|^2 r_{ss,l} + \sigma_v^2 \delta_l \qquad (4)$$

The detection of the primary signal is now described by the following binary hypotheses testing problem.

$$\begin{cases} H_0 : & \eta = 0, \text{ primary signal absent} \\ H_1 : & \eta = 1, \text{ primary signal present} \end{cases} \qquad (5)$$

Spectrum sensing operation is performed on N samples of $x_{k,n}$ from each of the K antennas. Denoting the set of samples collected by the k-th antenna by $\mathbf{x}_k = (x_{k,0}, x_{k,1}, \cdots, x_{k,N-1})$, the algorithm forms a decision statistic $T(\mathbf{x}_1, ..., \mathbf{x}_K)$ and compares it to a threshold λ, i.e.,

$$T(\mathbf{x}_1, ..., \mathbf{x}_K) \begin{matrix} < \lambda & \text{decide } H_0 \\ \ge \lambda & \text{decide } H_1. \end{matrix} \qquad (6)$$

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For ease of notation, in the following we will drop the dependence of the decision statistic on the sample data $\{\mathbf{x}_k\}_{k=1}^{K}$. The performance of spectrum sensing algorithm is determined by the false alarm and detection probabilities given by

$$P_{fa}(\lambda) \triangleq Pr\{T > \lambda | H_0\} \tag{7}$$

and

$$P_d(\lambda) \triangleq Pr\{T > \lambda | H_1\}. \tag{8}$$

III. AUTOCORRELATION-BASED SPECTRUM SENSING ALGORITHM

The spectrum sensing algorithms in this paper are based on the estimates of the autocorrelation function in (4). Using the samples from the k-th antenna, $\mathbf{x}_k$, the autocorrelation function at some lag l can be estimated from

$$\hat{r}_{k,l} \triangleq \begin{cases} \frac{1}{N-l}\sum_{n=0}^{N-l-1} x_{k,n+l}x_{k,n}^*, & l \geqslant 0 \\ \hat{r}_{k,-l}^*, & l < 0 \end{cases} \tag{9}$$

Under the null hypothesis, $\hat{r}_{k,l} = 0$ for all $l \neq 0$. On the other hand, assuming that $\{s_n\}$ is non-white, under the alternate hypothesis, $\hat{r}_{k,l} \neq 0$ for some $l \neq 0$. By further assuming that the primary baseband signal $\{s_n\}$ is lowpass and complex-valued with independent real and imaginary components, we form the following autocorrelation-based decision statistic for the k-th antenna:

$$T_k \triangleq \sum_{l=1}^{L} w_l \frac{\text{Re}\{\hat{r}_{k,l}\}}{\hat{r}_{k,0}}. \tag{10}$$

Scaling by $\hat{r}_{k,0}$ ensures that the algorithm is a constant false-alarm rate (CFAR) detector. The parameter L should be chosen so that $r_{ss,l} > 0$ for all $l \leq L$. While the weighting coefficients w_l could be chosen to achieve the optimal performance if the transmitted signal statistics are known to the receiver, we use $w_l = 1$ for all $l = 1 : L$ to further reduce the numerical complexity.

Next, we present the performance of the algorithm for the case of $K = 1$ (no diversity) where T_1 is used as the decision statistic.

A. False-Alarm Rate

Clearly the false-alarm rate is independent of the fading effects of the channel. Moreover, for the autocorrelation-based detector, the false-alarm rate is not influenced by the receiver noise since the detector is a CFAR detector. Consequently the probability of false alarm is found in [1] to be

$$P_{fa} = Q\left(\lambda\left[\frac{\lambda^2 + \Sigma}{N}\right]^{-\frac{1}{2}}\right) \tag{11}$$

where $Q(x)$ is the Gaussian Q-function and

$$\Sigma = \frac{1}{2}\sum_{i=1}^{L} w_i^2. \tag{12}$$

B. Detection Rate Over AWGN Channel

For the AWGN channel, in (2), we have $h_k = 1$ and, consequently, SNR is a constant ($\gamma = \sigma_s^2/\sigma_v^2$). The probability of detection can then be expressed by [1]

$$P_d = Q\left(\frac{\lambda - \mu_1}{\sqrt{\sigma_1^2}}\right) \tag{13}$$

where

$$\mu_1 = \frac{p\gamma}{\gamma + 1} \tag{14}$$

and

$$\sigma_1^2 = \frac{(1 + \gamma)\lambda^2 - 4p\gamma\lambda + (\Sigma + q\gamma)}{N(\gamma + 1)^2}. \tag{15}$$

Here,

$$p = \sum_{i=1}^{L} w_i \rho_i(\omega) \tag{16}$$

and

$$q = \sum_{i=1}^{L}\sum_{j=1}^{L} w_i[\rho_{i-j}(\omega) + \rho_{i+j}(\omega)]w_j \tag{17}$$

where $\rho_l = r_{ss,n}/\sigma_s^2$, which is the correlation coefficient of the transmitted signal.[1]

C. Detection Rate Over Rayleigh Fading Channel

We can view the fading effect as the fluctuation in the power of the received signal. Denote the resulting instantaneous SNR as $\gamma = |h_k|^2\sigma_s^2/\sigma_v^2$. Then the average detection probability can be determined by evaluating

$$\bar{P}_d = \int_0^\infty P_d(\lambda, x) \, p_\gamma(x) \, dx \tag{18}$$

where $P_d(\lambda, x)$ is the detection probability for the AWGN channel as defined in (13) with explicit indication of its dependence on λ and and the SNR value x, and where in the case of Rayleigh fading $p_\gamma(x)$ is given by

$$p_\gamma(x) = \frac{1}{\bar{\gamma}}\exp\left(-x/\bar{\gamma}\right), \quad x > 0. \tag{19}$$

The complexity of (13) makes it extremely difficult to find a closed-form solution for (18). Therefore to evaluate (18), we approximate the argument of the Q-function in (13) by the first two terms of its Taylor series

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[1]In [1], there is an error in (57), the expression for σ_1^2. Equation (15) in this paper is the corrected expression.

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expansion. The argument of the Q-function in (13) can be rewritten as:

$$f(\gamma) = \frac{a_1\gamma + a_2}{\sqrt{b_1\gamma + b_2}} \tag{20}$$

where

$$a_1 = \lambda - p \tag{21}$$

$$a_2 = \lambda \tag{22}$$

$$b_1 = \frac{\lambda^2 - 4p\lambda + q}{N} \tag{23}$$

$$b_2 = \frac{\lambda^2 + \Sigma}{N}. \tag{24}$$

The Taylor series expansion around a point γ_0 is given by

$$f(\gamma) = \sum_{n=0}^{\infty} \frac{f^{(n)}(\gamma_0)}{n!}(\gamma - \gamma_0)^n \tag{25}$$

The point γ_0 is chosen where $f(\gamma) = 0$, i.e., $\gamma_0 = -a_2/a_1$. This value of γ_0 centers the expansion about $f(\gamma) = 0$ which is in the middle of the $Q(f(\gamma))$ transition. The resulting approximation $\tilde{f}(\gamma)$ is found to be

$$\tilde{f}(\gamma) = a(\gamma - \gamma_0) \tag{26}$$

where

$$a \triangleq a_1\sqrt{\frac{a_1}{a_1b_2 - b_1a_2}} \tag{27}$$

Using (26), the average detection probability can be approximated by

$$\tilde{P}_d = \int_0^{\infty} Q(\tilde{f}(x))p_\gamma(x)dx \tag{28}$$

For Rayleigh fading, substituting (19) into (28) yields

$$\tilde{\tilde{P}}_{d,Rayl} = \int_0^{\infty} \frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right)Q[\tilde{f}(\gamma)] \tag{29}$$

Next this integral is evaluated to get

$$\tilde{\tilde{P}}_{d,Rayl} = \begin{cases} Q(-a\gamma_0) - \kappa_2 Q\left(\frac{1}{a\bar{\gamma}} - a\gamma_0\right), & a \leq 0 \\ Q(-a\gamma_0) + \kappa_2\left[1 - Q\left(\frac{1}{a\bar{\gamma}} - a\gamma_0\right)\right], & a < 0 \end{cases} \tag{30}$$

where

$$\kappa_2 \triangleq \exp\left(\frac{1}{2a^2\bar{\gamma}^2} - \frac{\gamma_0}{\bar{\gamma}}\right). \tag{31}$$
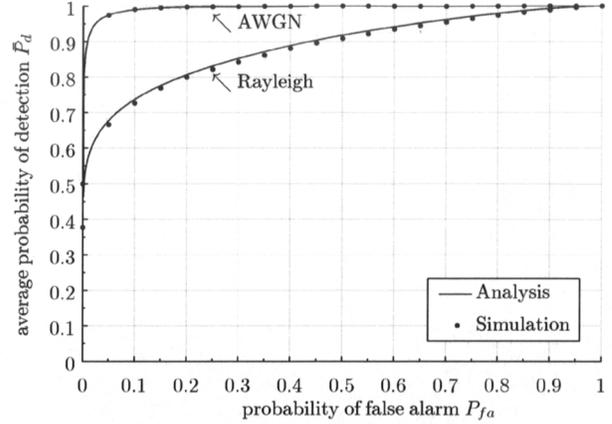

Fig. 2. ROC curve of non-diversity ($K = 1$) spectrum-sensing algorithm with $N = 1,000$, $L = 2$, and $\bar{\gamma} = -9$ dB.

D. Example: Performance Comparison for AWGN and Rayleigh Fading

To quantify the performance loss of the autocorrelation-based spectrum sensing due to fading, in this section we present an example for the AWGN and Rayleigh fading channels. The detector is set up with the following parameters: $N = 1,000$ and $L = 2$. The signal is sampled at $N_s = 3$ samples/symbol. Rectangular full-response pulse shaping is assumed which in the case of ASK, PSK, or QAM modulation schemes results in the following autocorrelation function for $\{s_n\}$:

$$r_{ss,l} = \begin{cases} \sigma_s^2\frac{N_s - |l|}{N_s}, & |l| < N_s \\ 0, & \text{otherwise} \end{cases} \tag{32}$$

In the simulations, the 16QAM modulation is used and the decision statistics are collected over $10,000$ trials.

The receiver operating characteristic (ROC) curves for the SNR value of -9 dB (average SNR for Rayleigh channel) is shown in Fig. 2. The performance loss due to the unknown channel gain is clearly evident in the figure. For example, for a false alarm rate of 0.1, the detection probability is 0.99 for the AWGN channel and drops to 0.74 for the Rayleigh channel. This figure also illustrates that the approximation in (29) for the average detection probability is in good agreement with the simulation results.

Next, the average probability of detection as a function of SNR is shown in Fig. 3. The false-alarm rate is fixed at 0.01 in this case. It can be seen that for average detection rate of 0.9, the Rayleigh fading channel requires an increase of by 7.5 dB in the average SNR.
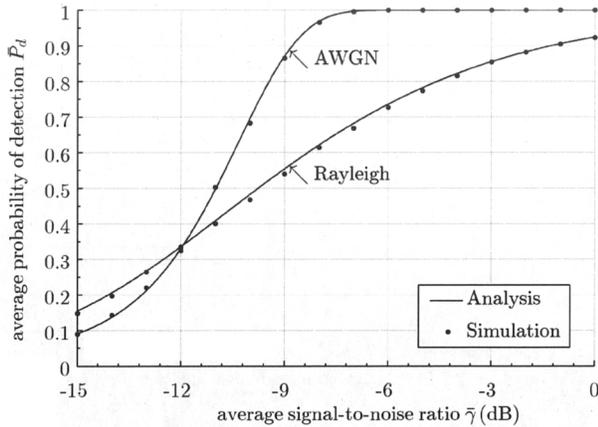
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Fig. 3. Detection probability of non-diversity ($K = 1$) spectrum-sensing algorithm as a function of SNR with fixed false-alarm rate ($P_{fa} = 0.01$, $N = 1,000$, $L = 2$).

IV. DIVERSITY TECHNIQUES FOR SPECTRUM SENSING

It is clear from illustrations in Section III-D that it is essential to improve the robustness of the spectrum-sensing algorithm against channel fading effects. Toward this end we consider three receiver diversity techniques. The first two approaches, namely equal gain combining and selection combining, are based on well-known diversity combining schemes. The third method is an equal gain combining of the autocorrelations of the received signals.

In the case of AWGN channels we can evaluate the performance of the three combining methods using the approximation techniques based on the results presented in the previous sections. For the case of Rayleigh fading channels, however, we have only presented simulation results in the following.

To evaluate the performance of the diversity combining techniques, we need to approximate the false-alarm rate P_{fa} and the detection rate P_d given in (11) and (13), respectively. To that end we argue that given the hypothesis H_η, the decision statistics $T_1, T_2, \cdots, T_K$ are independent and their distribution functions are asymptotically Gaussian. Then to evaluate P_{fa} and P_d we only need the conditional means and variances of T_k's. In particular we can show that $E(T_k|H_0) = 0$ and

$$\text{var}(T_k|H_0) \approx \hat{\sigma}_0^2 = \frac{\Sigma}{N}. \tag{33}$$

Consequently,

$$P_{fa} \approx Q\left(\frac{\lambda}{\sqrt{\hat{\sigma}_0^2}}\right) \tag{34}$$

In addition, $E(T_k|H_1) = \mu_1$ given by (14) and

$$\text{var}(T_k|H_1) \approx \hat{\sigma}_1^2 = \frac{(\Sigma + q\gamma)(\gamma + 1)^2 - p^2\gamma^2}{N(\gamma + 1)^4}. \tag{35}$$

Consequently,

$$P_d \approx Q\left(\frac{\lambda - \mu_1}{\sqrt{\hat{\sigma}_1^2}}\right) \tag{36}$$

A. Equal Gain Combining

With equal gain combining, we compute the decision statistics T_k in (10) for $k = 1, 2, \cdots, K$ and then add the resulting statistics to get

$$T_{EGC} \triangleq \sum_{k=1}^{K} T_k \tag{37}$$

The false-alarm and detection probabilities for the AWGN channel are found to be

$$P_{fa,EGC} = Q\left(\frac{\lambda}{\sqrt{K\hat{\sigma}_0^2}}\right) \tag{38}$$

and

$$P_{d,EGC} = Q\left(\frac{\lambda - K\mu_1}{\sqrt{K\hat{\sigma}_1^2}}\right). \tag{39}$$

B. Selective Combining

In this method, the largest of the K decision statistics is chosen as the combined statistics, i.e.,

$$T_{SC} \triangleq \max(T_1, ..., T_K). \tag{40}$$

With the i.i.d. Gaussian assumption of the decision statistics $T_1, T_2, \cdots, T_K$, the false-alarm and detection probabilities are simply given by

$$P_{fa,SC} = 1 - \left[1 - Q\left(\frac{\lambda}{\sqrt{\hat{\sigma}_0^2}}\right)\right]^K \tag{41}$$

and

$$P_{d,SC} = 1 - \left[1 - Q\left(\frac{\lambda - \mu_1}{\sqrt{\hat{\sigma}_1^2}}\right)\right]^K. \tag{42}$$

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C. Equal Gain Correlation Combining

In this case we combine the autocorrelation estimates from the K diversity branches in order to obtain a more accurate estimate of the the autocorrelation function. We refer to this approach as the equal gain correlation combining. With the K branches experiencing independent fading, this approach, in effect, gives us a better estimate of the autocorrelation. The decision statistic is then computed as in (10) using the new autocorrelation estimate.

Since only the real part of the autocorrelation estimates is utilized in our spectrum sensing algorithm, the combining method computes

$$\alpha_l \triangleq \sum_{l=1}^{K} \text{Re}\{\hat{r}_{k,l}\}. \tag{43}$$

The decision statistic is then computed by

$$T_{EGCC} = \frac{\sum_{l=1}^{L} w_l \alpha_l}{\alpha_0} \tag{44}$$

It is easy to verify that for AWGN channel, false-alarm and detection probabilities for this combining method take the same expressions as in (11) and (13), with the number of samples (N) replaced by KN.

V. NUMERICAL RESULTS AND PERFORMANCE COMPARISONS

In this section, we evaluate the performance of the three diversity combining algorithms introduced in Section IV. In these experiments the number of samples is $N = 250$ and the number of branches is $K = 4$. All other system parameters are the same as those listed in Section III-D.

Fig. 4 shows the ROC curves obtained from simulation and analysis for AWGN channel. In addition to the three diversity combining techniques, the figure includes the analytical ROC curves of the single-branch algorithm with two different sample-sizes, namely 250 and 1000. These are denoted by $T_1^{(N)}$ and $T_1^{(KN)}$, respectively. Note that the single-branch algorithm using 1000 samples utilizes the same number of total samples as the diversity combining techniques. It can be seen that the performance of equal gain combining T_{EGC} and equal gain correlation combining T_{EGCC} is identical to that of the single-branch algorithm $T_1^{(KN)}$ with 1000 samples. In other words, as expected, for the AWGN channel, using N samples from K different branches is equivalent to using NK samples from a single branch. On the other hand, the selection combining T_{SC} method results
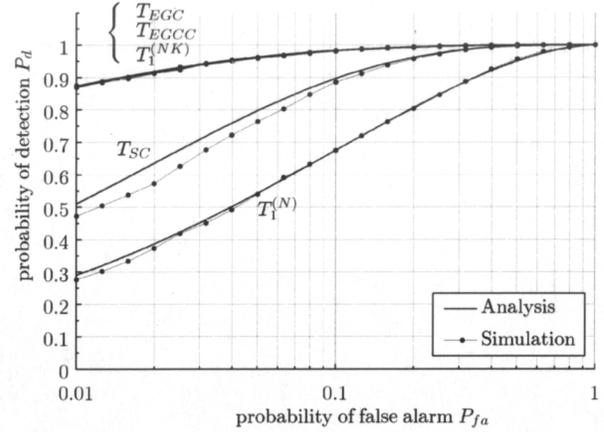

Fig. 4. ROC curves of diversity spectrum-sensing algorithms under AWGN channel ($K = 4$, $N = 250$, $L = 2$, and $\bar{\gamma} = -9$ dB).
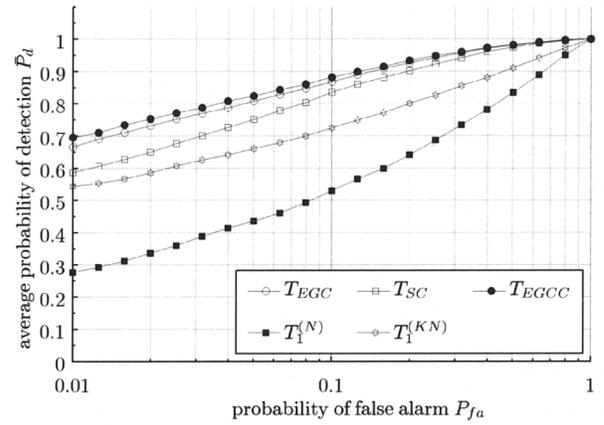

Fig. 5. ROC curves for diversity spectrum-sensing algorithms under Rayleigh fading channel ($K = 4$, $N = 250$, $L = 2$, and $\bar{\gamma} = -9$ dB).

in a poor performance compared to $T_1^{(KN)}$. Lastly, we observe that for high values of P_d, the results from analysis are very close to those obtained from simulation.

Fig. 5 shows the ROC curves of the diversity sensing algorithms for Rayleigh flat fading channel. All the curves are obtained from simulation. As shown in Fig. 2, Rayleigh channel severely degrades the performance of non-diversity detectors. Fig. 5 further illustrates that while increasing the number of samples from 250 ($T_1^{(N)}$) to 1000 ($T_1^{(KN)}$) improves the performance, all three diversity sensing methods demonstrate performance improvement over the non-diversity detector with the same number of samples ($T_1^{(KN)}$). the selective combiner.

Fig. 6 captures the CFAR detection performance of the diversity sensing techniques for Rayleigh fading channel. As in Fig. 3, the false-alarm rate is fixed to
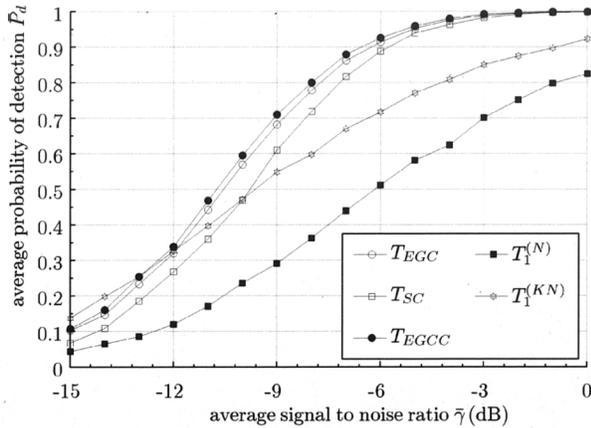
6

Fig. 6. CFAR detection probability vs. SNR of diversity spectrum-sensing algorithms for Rayleigh fading channel ($P_{fa} = 0.01$, $K = 4$, $N = 250$, and $L = 2$).

Fig. 7. CFAR detection probability vs. the number of branches K of diversity spectrum-sensing algorithms for Rayleigh fading channel ($P_{fa} = 0.01$, $\bar{\gamma} = -9$ dB, $N = 250$, and $L = 2$).

$P_{fa} = 0.01$. For detection probabilities of interest the diversity combining methods provide a clear advantage over $T_1^{(KN)}$. For example for a detection probability of 0.9 the required SNR is -6.7 dB for T_{EGCC}, -6.3 dB for T_{EGC}, and -5.7 dB for T_{SC} and -1.3 dB for T_1^{KN}. Hence, the equal gain correlation combining provides an SNR improvement of 5.4 dB over a single-branch system using the same number of samples.

In Fig. 7 we show the probability of detection vs. the number of diversity branches K. As expected the diversity gain improves as the number of branches, K, increases. Finally we would like to point out that the improvement of the diversity combining methods over $T_1^{(KN)}$ as shown in Figures 5-7 is purely due to the diversity gain of the three combining methods.

VI. CONCLUSION

In this paper, diversity techniques are applied to the autocorrelation-based spectrum sensing algorithm to improve the detection performance over Rayleigh fading channel. We have employed the commonly used diversity combining methods, namely equal gain combining and selective combining. In addition, we have introduced equal gain correlation combining method, in which the autocorrelation estimates are combined first and the autocorrelation-based decision statistic is computed from the combined estimate. Analysis and simulation results suggest that for detection probabilities of interests (e.g., $> .9$), a diversity gain of about 5 dB in SNR can be achieved over a non-diversity system using the same number of samples. Furthermore, equal gain correlation combining method has the best performance of the three combining techniques.
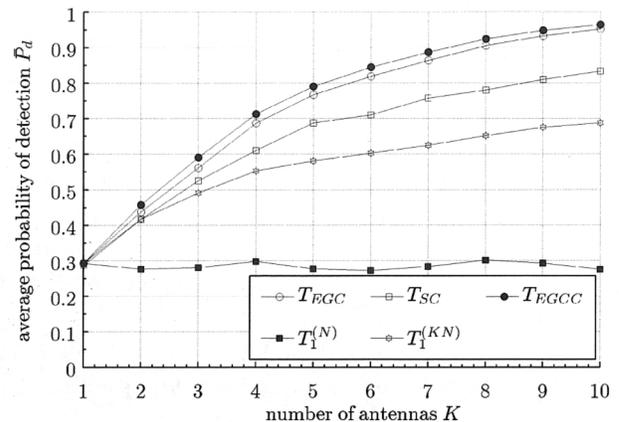
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