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Binomial Distribution: Hypothesis Testing, Confidence Intervals (CI), and Reliability with Implementation in S-PLUS

by Joseph C. Collins

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Summary

The purpose of this report is to provide statisticians with the tools to solve certain reliability problems that arise in consultations with clients.

Statisticians often encounter clients seeking answers to questions such as, “I did an experiment 6 times with 3 successes and 3 failures, so the success rate is 50%, right?” A reasonable, useful, and informative response from the statistician would be, “yes, the estimate of success is 50%, but you can only state with 90% confidence that the true success rate lies between 15% and 85%. If you want to tighten that up, you need a larger sample.” The client asks, “how large?” and the conversation proceeds from there.

A client may pose a problem such as, “I need 90% confidence on 95% reliability. Can I do this with 10 runs?” The statistician can truthfully answer, no. You need 29 with no failures or 46 with 1 failure to assert 95% reliability with 90% confidence." The client may respond, “well, what can I get with 1 failure in 20?” and so the consultation continues.

In fact, the solutions to both of these problems exploit properties of the binomial distribution. System requirements can be expressed in terms of an upper (lower) limit on the probability of failure (success). Statistical analysis of such binary (0/1, success/failure, go/no-go) data typically requires point and interval estimation and inference or hypothesis tests on the associated event probability. For identical independent trials, the proportion observed serves as an estimate of the event rate. Based on the method of Clopper-Pearson (CP) and the likelihood ratio (LR) technique properties of the binomial distribution are used to derive interval estimates, which are in turn used in inference. An application is the determination of sample size and maximum permissible number of failures (nf) required to establish a specific reliability (probability of success) with given probability (confidence).

The statistician needs to address such issues and answer these questions in real time. This report provides the necessary technology expressed in the S-PLUS / S / R statistical computing environment language family, implementations of which are available as both commercial and free open-source software.

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1. Binomial Distribution

1.1 Definitions and Properties

The random variable X is Bernoulli(p), or B_p, if $X \in \{0,1\}$ and $\Pr[X = 1] = p = 1 - \Pr[X = 0]$. The probability mass function, or discrete probability density function (pdf), is

$$f_p(x) = p^x(1-p)^{1-x}. \tag{1}$$

The sum X of n independent and identically distributed (iid) B_p is Bernoulli(n, p), or $B_{n,p}$. Note that $B_p = B_{1,p}$. The binomial coefficient is $(n, k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$. Standard functions are defined for $k \in \{0,1,2, \dots, n\}$. The pdf is

$$f_{n,p}(x) = C(n,k)p^x(1-p)^{1-x}. \tag{2}$$

The cumulative distribution function (cdf) is

$$F_{n,p}(k) = \Pr[X \le k] = \sum_{j=0}^{k} f_{n,p}(j) \tag{3}$$

with $F_{n,p}(x) = 0$ if $x \le 0$, and $F_{n,p}(x) = 1$ if $x \ge n$, and right-continuous extension to $\mathbb{R}$, as is customary for discrete distributions. The quantile function (qf) is the usual unique left-continuous pseudo-inverse of the cdf,

$$Q_{n,p}(u) = \inf\{\, k: F_{n,p}(k) \ge u \,\} \tag{4}$$

It is also useful to have an upper-probability version of the cdf,

$$G_{n,p}(k) = \Pr[X \ge k] = \textstyle\sum_{j=k}^{n} f_{n,p}(j). \tag{5}$$

So $G_{n,p}(k) = 1 - F_{n,p}(k)$ and $F_{n,p}(k) = 1 - G_{n,p}(k+1)$. Note that $F_{n,p}(k)$ increases with k and decreases with n or p, and $G_{n,p}(k)$ decreases with k and increases with n or p.

Moments are

$$\mathrm{E}[X^k] = \sum_{j=0}^{n} j^k f_{n,p}(j) \tag{6}$$

So $\mathrm{E}[X] = np$, and $\mathrm{E}[X^2] = np(1-p) + n^2p^2$, and $\mathrm{Var[X]} = np(1-p)$.

The log-likelihood function is

$$\begin{aligned}\mathcal{L} &= \sum_{j=0}^{n}\left(\log C(n,j) + X_j \log p + \left(n - X_j\right)\log(1-p)\right) \\ &= \left(\sum_{j=0}^{n}\log C(n,j)\right) + k\log p + (n-k)\log(1-p)\,.\end{aligned} \tag{7}$$

The maximum likelihood estimator (MLE) of p is k/n.

1.2 Implementation Issue

Reasonable algorithms for $F_{n,p}(k)$ return the appropriate values for large *n* and small *k* but succumb to numeric underflow for large *k*.

For example, since $F_{n,p}(0) = (1-p)^n$ and $F_{n,p}(n-1) = 1 - p^n$, set $n = 100$ and $p = 0.6$ to get

$$F_{100,0.6}(0) = (1-0.6)^{100} = 0.4^{100} \cong 1.6 \times 10^{-40} \tag{8}$$

and

$$F_{100,0.6}(99) = 1 - 0.6^{100} \cong 1 - 6.5 \times 10^{-23}. \tag{9}$$

But with standard double precision resolution of about 16 decimal places, the result is $F_{100,0.6}(99) = 1$ exactly. The cure for lower tail probabilities too close to 1 is to use upper tail probabilities, which will be close to 0, and work with *G* instead of *F*, since $G_{n,p}(n) = p^n = 1 - F_{n,p}(n-1)$, and so

$$G_{100,0.6}(100) = 0.6^{100} = 1 - F_{100,0.6}(99) = 1 - (1-0.6)^{100} \cong 6.5 \times 10^{-23}. \tag{10}$$

But naïve use of $G_{n,p}(k) = 1 - F_{n,p}(k-1)$ directly will of course give $G_{100,0.6}(100) = 0$. To obtain a useful representation of $G(k)$ with *k* large in terms of an accurate algorithm for $F(k)$ with *k* small, let $X \sim B_{n,p}$ and $W = n - X$. Note that $F_W(k) = \Pr[\,W \le k] = \Pr[\,n - X \le k] = \Pr[X \ge n-k] = 1 - \Pr[X < n-k] = 1 - \Pr[X \le n-k-1] = 1 - F_X(n-k-1)$. Then $G_X(k) = 1 - F_X(k-1) = 1 - F_X(n-(n-k)-1) = F_W(n-k)$, so

$$G_{n,p}(k) = Pr\left[B_{n,p} \le k\right] = \Pr\left[B_{n,1-p} \le n-k\right] = F_{n,1-p}(n-k). \tag{11}$$

This gives $G_{100,0.6}(100) = F_{100,0.4}(0) = (1-0.4)^{100} = 0.6^{100} \cong 6.5 \times 10^{-23}$ as required.

1.3 Beta Distribution

The Gamma function, which has $\Gamma(n) = (n-1)!$ for n = 1,2,3,…, is given by

$$\Gamma(z) = \int_0^\infty u^{z-1} e^{-u} du. \tag{12}$$

The Beta function, which has $B(n,k) = (n-1)!\,(k-1)!/(n+k-1)!$ on positive integers, is

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \int_0^1 x^{a-1}(1-x)^{b-1} dx. \tag{13}$$

The Beta (a, b) distribution on [0,1], with $a > 0$ and $b > 0$, has pdf and cdf

$$\begin{aligned} f_{\text{Beta }(a,b)}(x) &= \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1} \\ F_{\text{Beta }(a,b)}(x) &= \int_0^x f_{\text{Beta }(a,b)}(u)\, du. \end{aligned} \tag{14}$$

The Binomial and Beta cdfs are related by $F_{n,p}(k-1) = F_{\text{Beta }(n-k+1,k)}(1-p)$, so

$$\begin{aligned} F_{n,p}(k) &= F_{\text{Beta }(n-k,k+1)}(1-p), \qquad \text{or} \\ F_{n-k+1,1-p}(k-1) &= F_{\text{Beta }(n,k)}(p) \end{aligned} \tag{15}$$

and also

$$\begin{aligned} G_{n,p}(k) &= F_{\text{Beta }(k,n-k+1)}(p), \qquad \text{or} \\ G_{n-k+1,p}(k) &= F_{\text{Beta }(n,k)}(p). \end{aligned} \tag{16}$$

To show $F_{n,p}(k-1) = F_{\text{Beta }(n-k+1,k)}(1-p)$, first note that $B(n-k+1,k) = (n-k)!\,(k-1)!/n!$, and also $F_{\text{Beta }(n,1)}\,(1-p) = n\int_0^1 x^{n-1} dx = (1-p)^n = f_{n,p}(0)$. Now, integrating by parts,

$$F_{\text{Beta }(n-k+1,k)}(1-p) = \frac{n!}{(n-k)!\,(k-n)!}\int_0^{1-p} x^{n-k}(1-x)^{k-1}dx$$

$$= \frac{n!}{(n-k)!\,(k-n)!}\left[\frac{x^{n-k}(1-x)^{k-1}}{n-k+1}\Bigg|_0^{1-p} + \int_0^{1-p}\frac{(k-1)x^{n-k+1}(1-x)^{k-2}}{n-k+1}\right]$$

$$= \frac{n!\,p^{k-1}(1-p)^{n-k+1}}{(n-k+1)!\,(k-1)!} + \frac{n!}{(k-2)!\,(n-k+1)!}\int_0^{1-p} x^{n-k+1}(1-x)^{k-2}dx$$

$$= f_{n,p}(k-1) + \frac{1}{B(n-k+2,k-1)}\int_0^{1-p} x^{n-k+1}(1-x)^{k-2}dx$$

$$= f_{n,p}(k-1) + F_{\text{Beta }(n-k+2,k-1)}(1-p)$$

$$= f_{n,p}(k-1) + \cdots + f_{n,p}(1) + F_{\text{Beta }(n,1)}(1-p)$$

$$= f_{n,p}(k-1) + \cdots + f_{n,p}(1) + f_{n,p}(0)$$

$$= F_{n,p}(k-1). \tag{17}$$

See, for example, Stuart (*1*).

In fact, the function

$$F(x) = \begin{cases} 0, & x \le -1 \\ F_{\text{Beta }(n-x+1,x)}(1-p), & -1 < x < n \\ 1, & n \le x \end{cases} \tag{18}$$

is a continuous cdf on $[-1, n]$, and $F(k) = F_{n,p}(k)$ for $k = 0,1,2,\dots,n$. So F serves as a continuous version, or interpolation, of $F_{n,p}$, which is sometimes useful. The corresponding quantile function $Q(u) = F^{-1}(u)$ is not related to Q_{Beta}, but can be obtained by solving $F(x) = u$ numerically to get $x = Q(u)$. See figure 1.

Note that Q_{Beta} does provide the solution p of $u = F_{n,p}(k)$. Since $u = F_{n,p}(k) = F_{\text{Beta }(n-k,k+1)}(1-p)$, and so $Q_{\text{Beta }(n-k,k+1)}(u) = 1-p$, observe that

$$p = 1 - Q_{\text{Beta }(n-k,k+1)}(u). \tag{19}$$

Likewise for $u = G_{(n,p)(k)}$. Since $u = G_{n,p}(k) = F_{\text{Beta }(k,n-k+1)}(p)$ and so $Q_{\text{Beta }(k,n-k+1)}(u) = p$, it follows that

$$p = Q_{\text{Beta }(k,n-k+1)}(u). \tag{20}$$

Library implementations of Q_{Beta} provide for efficient and accurate evaluation of p in these situations. This will be useful later.

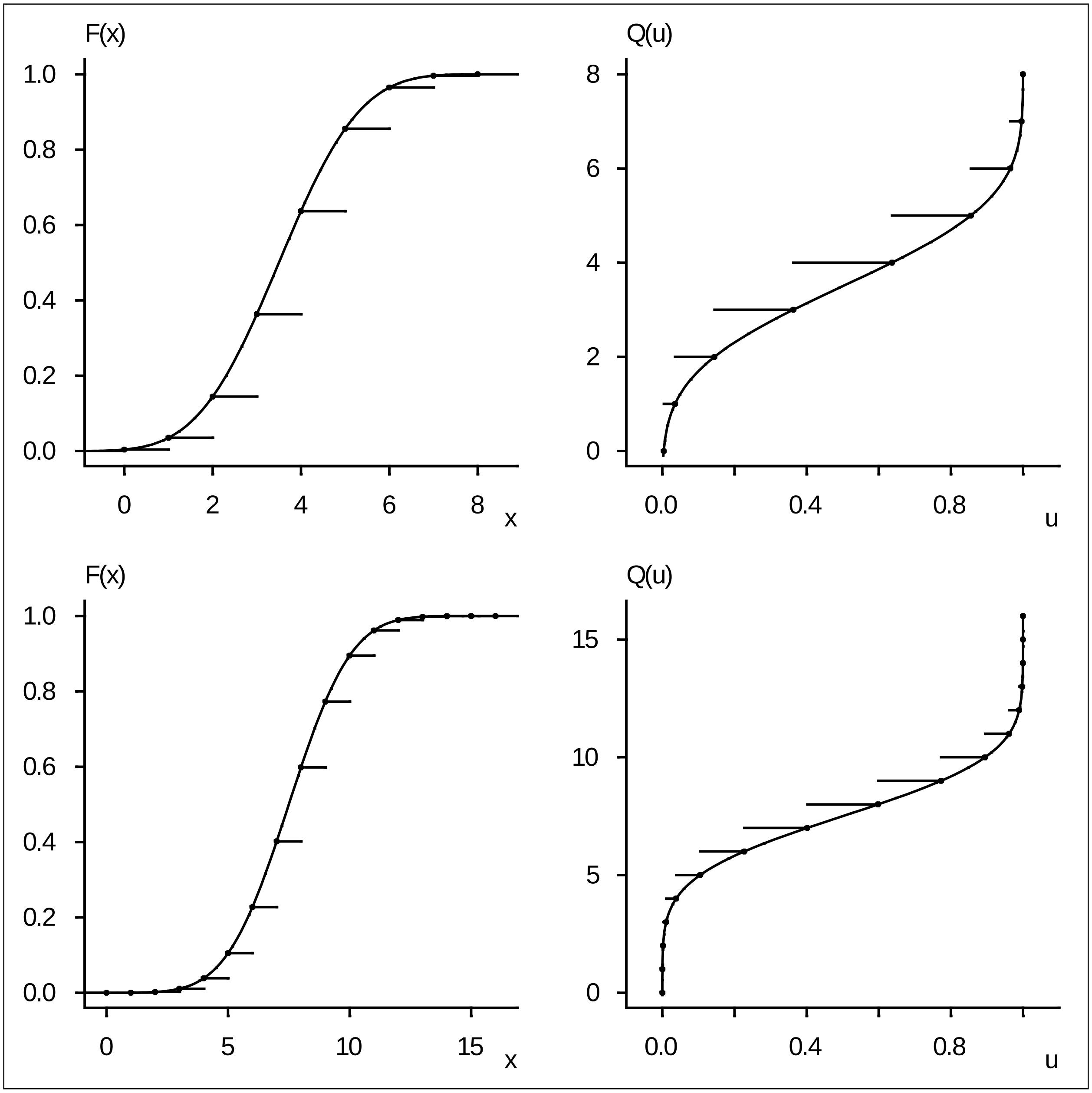

Figure 1. Binomial cdf and quantile function (qf) examples with continuous interpolation.

2. Hypothesis Testing

Tests of size γ and Type I error probability $\alpha = 1 - \gamma$ for

$$H_0: p = p_o \tag{21}$$

are based on data X. Here α is the null rejection probability, or probability of rejecting H_0 when it is true. See Mood (*2*) or Stuart (*3*) for an explanation.

2.1 One Sided Upper

The one-sided alternative

$$H_1: p < p_o \tag{22}$$

provides an upper limit for p upon rejection of H_0. The critical value is k_U, and the decision rule is to reject H_0 if $X \leq k_U$ where $\Pr[X \leq k_U \mid H_0] = \alpha$. Since X is discrete, take

$$k_U = \sup \{ k \mid \Pr[X \leq k \mid H_0] \leq \alpha \} = \sup \left\{ k \,\middle|\, F_{n,p_o}(k) \leq \alpha \right\}. \tag{23}$$

The rejection region is $I_1 = \{ 0, \dots, k_U \}$, and the non-rejection region is $I_0 = \{ k_U + 1, \dots, n \}$. Equivalently, the p-value for the test is $p_X = F_{n,p_o}(X)$, and the rule is to reject H_0 if $p_X \leq \alpha$. For example, let $p_o = 0.3$ and $\alpha = 0.1$ with $n = 100$. Since $F_{100,0.3}(23) = 0.0755$ and $F_{100,0.3}(24) = 0.114$, it follows that $k_U = 23$. These are also the p-values, $p_{23} = 0.0755$ and $p_{24} = 0.114$. The true value of α for this test is 0.0755.

For example, it is possible for the test to degenerate due to insufficient sample size. Consider $p_o = 0.03$ and $\alpha = 0.01$ with $n = 100$. Since $F_{100,0.03}(0) = 0.0476$, it follows that $F_{100,0.03}(k) > \alpha$ for all k, and so $I_1 = \emptyset$, and $I_0 = \{0, \dots, n\}$, and rejection never occurs. The true value of α for this test is 0.

But $F_{151,0.03}(0) = 0.0101$ and $F_{152,0.03}(0) = 0.00976$, so $n > 151$ is required for a non-degenerate test. With $n = 152$, since $F_{152,0.03}(1) = 0.0556$, it follows that $k_U = 0$. Then if $X = 0$ the decision is to reject H_0 and conclude that $p < 0.03$ with probability ≥ 0.99. If $X > 0$ there is no indication that $p < 0.03$. The true value of α for this test is 0.00976.

Let $p_o = 1/2$ and $\alpha = 7/128$ with $n = 10$. Note that $F_{10,1/2}(2) = 7/128 = \alpha$. It follows that $k_U = 2$ and $p_X \leq \alpha$ when $X \leq 2$, in which case H_0 is rejected. The true value of α for this test is exactly $7/128$.

2.2 One Sided Lower

The one-sided alternative

$$H_1 : p > p_o \tag{24}$$

provides a lower limit for p upon rejection of H_0. The critical value is k_L, and one rejects H_0 if $X \geq k_L$ where $\Pr[X \geq k_L | H_0] = \alpha$. Since X is discrete, take

$$k_L = inf\{k | Pr[X \geq k \mid H_0] \leq \alpha\} = \inf \left\{ k \;\middle|\; G_{n,p_o}(k) \leq \alpha \right\}. \tag{25}$$

The rejection region is $I_1 = \{k_L, \dots, n\}$, and the non-rejection region is $I_0 = \{0, \dots, k_L - 1\}$. Equivalently, the p-value for the test is $p_X = G_{n,p_o}(X)$, and one rejects H_0 if $p_X \leq \alpha$.

For example, let $p_o = 0.3$ and $\alpha = 0.1$ with $n = 100$. Since $G_{100,0.3}(36) = 0.116$ and $G_{100,0.3}(37) = 0.0799$, it follows that $k_L = 37$. These are also the p-values, $p_{36} = 0.116$ and $p_{37} = 0.0799$.

2.3 Two Sided

The two-sided alternative

$$H_1 : p \neq p_o \tag{26}$$

provides upper and lower limits for p upon rejection of H_0. The critical values are k_U and k_L, and one rejects H_0 if $X \leq k_U$ or $X \geq k_U$ where $\Pr[X \leq k_U \; or \; X \geq k_L \mid H_0] = \alpha$. For symmetry, set $\Pr[X \leq k_U \mid H_0] = \Pr[X \geq k_L | \; H_0] = \alpha/2$. Since X is discrete, take

$$\begin{aligned} k_U &= \sup\{k \mid \Pr[X \leq k \mid H_0] \leq \alpha/2\} = \sup\left\{k \;\middle|\; F_{n,p_o}(k) \leq \alpha/2\right\} \\ k_L &= \inf\{k | \Pr[X \geq k \mid H_0] \leq \alpha/2\} = \inf\left\{k \;\middle|\; G_{n,p_o}(k) \leq \alpha/2\right\} \end{aligned} \tag{27}$$

The rejection region is $I_1 = \{0, \dots, k_U\} \cup \{k_L, \dots, n\}$, and the non-rejection region is $I_0 = \{k_U + 1, \dots, k_L - 1\}$. Equivalently, the p-value for the test is $p_X = 2 \min\left(F_{n,p_o}(X), G_{n,p_o}(X)\right)$. One rejects H_0 if $p_X \leq \alpha$.

For example, let $p_o = 0.3$ and $\alpha = 0.1$ with $n = 100$. Since $F_{100,0.3}(22) = 0.0479$ and $F_{100,0.3}(23) = 0.0755$, it follows that $k_U = 22$. Since $G_{100,0.3}(38) = 0.0530$ and $G_{100,0.3}(39) = 0.0340$, it follows that $k_L = 39$. Note that $G_{100,0.3}(22) = 0.971$, $G_{100,0.3}(23) = 0.952$, $F_{100,0.3}(38) = 0.966$, and $F_{100,0.3}(39) = 0.979$. So the p-values are $p_{22} = 0.0957$, $p_{23} = 0.151$, $p_{38} = 0.106$, and $p_{39} = 0.0680$.

3. Confidence Intervals (CI)

The point estimate of p is of course X/n. CIs on p are obtained by pivoting or inverting hypothesis test critical regions as follows. See Clopper and Pearson (*4*).

3.1 Upper

Let I_1 be the set of p_o for which H_0 would be rejected in favor of $H_1\!: p < p_o$ with Type I error α, so $I_1 = \{\, p \mid F_{n,p}(X) \leq \alpha \,\} = [p_U, 1]$ where

$$\alpha = F_{n,p_U}(X) = F_{\text{Beta }(n-X,X+1)}(1 - p_U), \tag{28}$$

so p_U can be expressed by equation 19 as a Beta distribution quantile

$$p_U = 1 - Q_{\text{Beta }(n-X,X+1)}(\alpha). \tag{29}$$

Null rejection occurs with probability α, so the non-rejection region

$$I_0 = [0, p_U) = \{\, p \mid F_{n,p}(X) > \alpha \,\} \tag{30}$$

is a $100\gamma\%$ upper CI on p, and p_U is an upper confidence limit on p. To reject H_0 when $p \in I_1 = [p_U, 1]$ is precisely the p-value decision rule in section 0.

3.2 Lower

Let I_1 be the set of p_o for which H_0 would be rejected in favor of $H_1\!: p > p_o$ with Type I error α, so $I_1 = \{\, p \mid G_{n,p}(X) \leq \alpha \,\} = [0, p_L]$ where

$$\alpha = G_{n,p_L}(X) = F_{\text{Beta }(X,n-X+1)}(p_L), \tag{31}$$

so p_L can be expressed by equation (20) as a Beta distribution quantile

$$p_L = Q_{\text{Beta }(X,n-X+1)}(\alpha).. \tag{32}$$

Null rejection occurs with probability α, so the non-rejection region

$$I_0 = (p_L, 1] = \{\, p \mid G_{n,p}(X) > \alpha \,\} \tag{33}$$

is a $100\gamma\%$ lower CI on p, and p_L is a lower confidence limit on p. To reject H_0 when $p \in I_1 = [1, p_L]$ is precisely the p-value decision rule in section 2.2.

3.3 Two Sided

Let I_1 be the set of p_o for which H_0 would be rejected in favor of $H_1\!: p \neq p_o$ with Type I error α, so $I_1 = \{\, p \mid G_{n,p}(X) \leq \alpha/2\} \cup \{\, p \mid F_{n,p}(X) \leq \alpha/2\,\} = [0, p_L] \cup [p_U, 1]$ where

$$G_{n,p_L}(X) = F_{n,p_U}(X) = \alpha/2. \tag{34}$$

Then p_L and p_U can be expressed by equations 19 and 20 as Beta distributions quantiles

$$\begin{aligned} p_L &= Q_{\text{Beta}\,(X,n-X+1)}\,(\alpha/2) \\ p_U &= 1 - Q_{\text{Beta}\,(n-X,X+1)}\,(\alpha/2). \end{aligned} \tag{35}$$

Null rejection occurs with probability α, so the non-rejection region

$$I_0 = (p_L, p_U) = \{\, p \mid G_{n,p}(X) > \alpha/2\} \cap \{\, p \mid F_{n,p}(X) > \alpha/2\} \tag{36}$$

is a $100\gamma\%$ two-sided CI on p. To reject H_0 when $p_o \in [0, p_L] \cup [p_U, 1]$ is precisely the p-value decision rule of section 2.3.

3.4 Implementation

Code listings are in appendix A.

The function `bino.ci(n,r,g,type)` uses equations 29, 32, or 35 to give the required CI. The arguments of `bino.ci` are the number of trials `n`, the number of successes `r`, confidence interval size `g` which should be between 0.5 and 1.0, and an integer `type` which can be 0, 1, or 2 to request a lower CI $(L, 1]$, upper CI $[0, U)$, or 2-sided CI (L, U), respectively.

The return value from `bino.ci` is a named list holding the number of trials `$n`, number of successes `$r`, lower limit `$L`, parameter estimate `$p.hat`, upper limit `$U`, and confidence interval vector `$I`. For example,

```
bino.ci(n=100, r=30, g=0.9, type=2)
```

returns

```
$n:        100
$r:        30
$L:        0.22492
$p.hat:    0.3
$U:        0.38422
$I:        0.22492 0.38422
```

since $\Pr\left[B_{100,0.22492} \geq 30\right] = \Pr\left[B_{100,0.38422} \leq 30\right] = 0.05$.

3.5 Conservative Coverage

Coverage is the probability that a confidence interval captures the true parameter value.

Considering that $p_U(X)$ is random, and $[0, p_U(X))$ is a $100\gamma\%$ CI on p, one expects that $C_U(p) = \Pr[p < p_U(X)] = \mathrm{E}\left[I_{[0,p_U(X))}(p)\right] \geq \gamma$. This coverage probability is

$$C_U(p) = \sum_{k=0}^{n} Pr[X = k] \cdot I_{[0,p_U(X))}(p) = \sum_{p<p_U(k)} f_{n,p}(k). \qquad (37)$$

Figure 2 illustrates C_U for $n = 10$ and $\gamma = 0.9$.

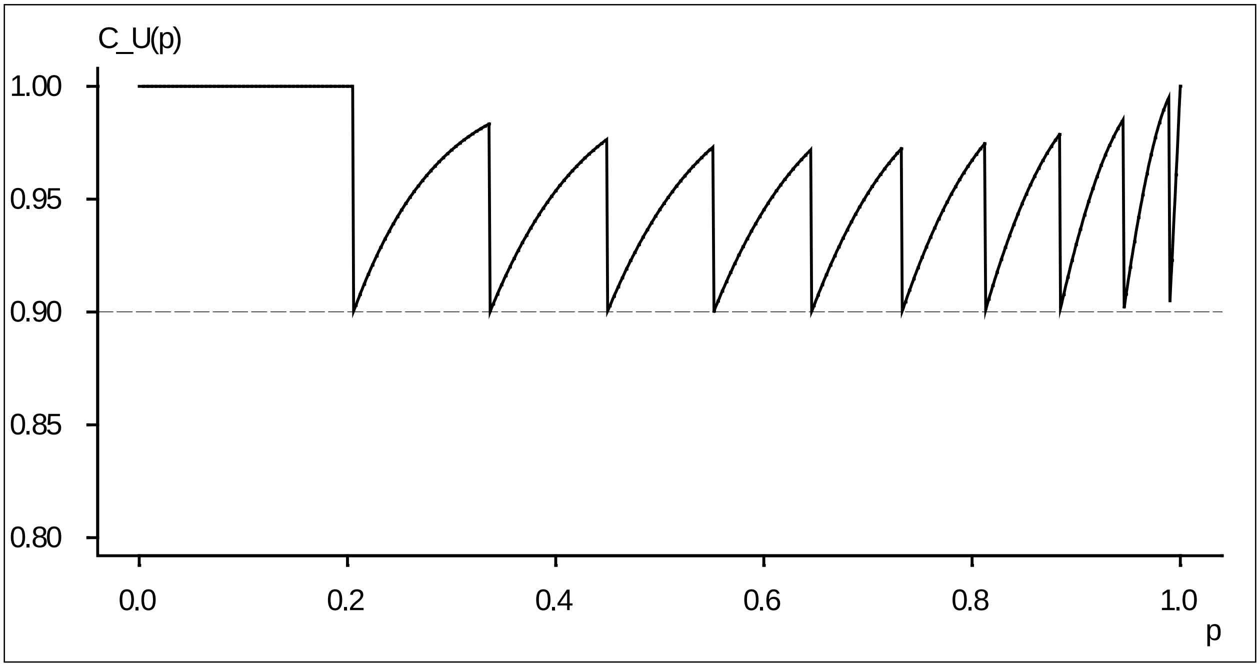

Figure 2. Upper CI coverage.

Likewise, $p_L(X)$ is random, and $(p_L(X), 1]$ is a $100\gamma\%$ CI on p. One expects that $C_L(p) = \Pr[p_L(X) < p] = \mathrm{E}\left[I_{(p_L(X),1]}(p)\right] \geq \gamma$. This coverage probability is

$$C_L(p) = \sum_{k=0}^{n} \Pr[X = k] \cdot I_{(p_L(X),1]}(p) = \sum_{p_L(k)<p} f_{n,p}(k). \qquad (38)$$

Figure 3 illustrates C_L for $n = 10$ and $\gamma = 0.9$.

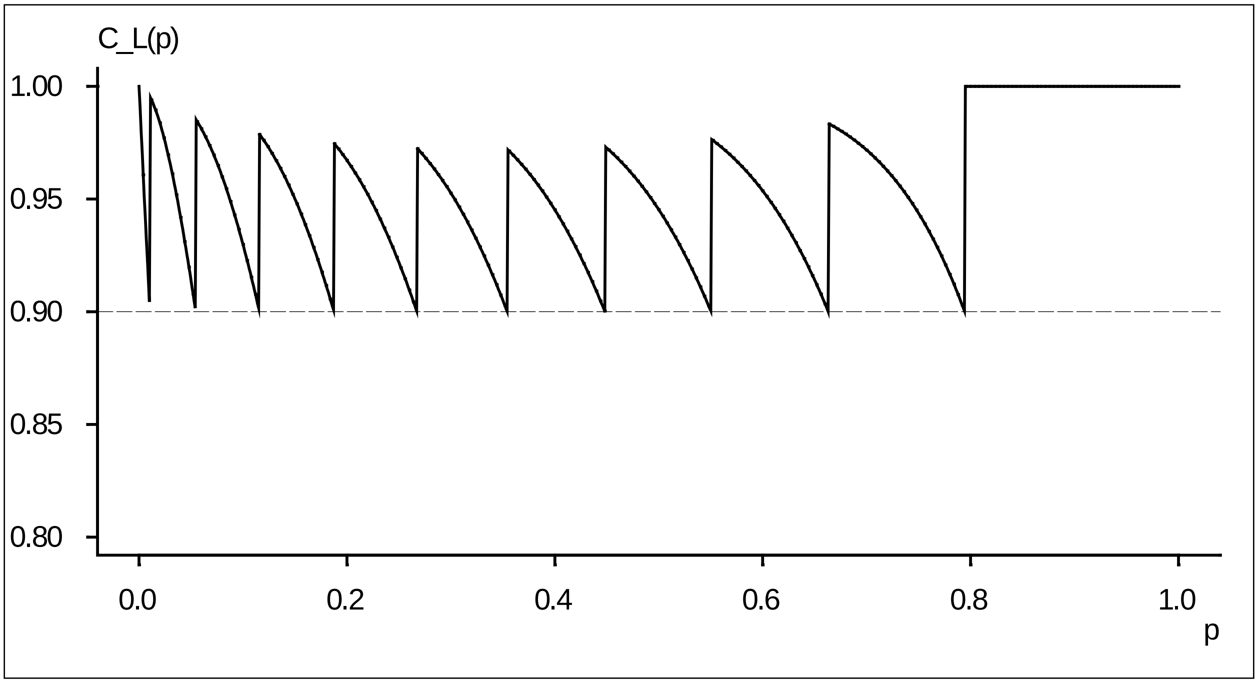

Figure 3. Lower CI coverage.

For the 2-sided CI, $(p_L(X), p_U(X))$ is a $100\gamma\%$ CI on p, and one expects that $C_2(p) = \Pr[p_L(X) < p < p_U(X)] = \mathrm{E}[I_{(p_L(X),p_U(X))}(p)] \geq \gamma$. This coverage probability is

$$C_2(p) = \sum_{k=0}^{n} Pr[X = k] \cdot I_{(p_L(X),p_U(X))}(p) = \sum_{p_L(k)<p<p_U(k)} f_{n,p}(k). \tag{39}$$

Figure 4 illustrates C_2 for $n = 10$ and $\gamma = 0.9$.

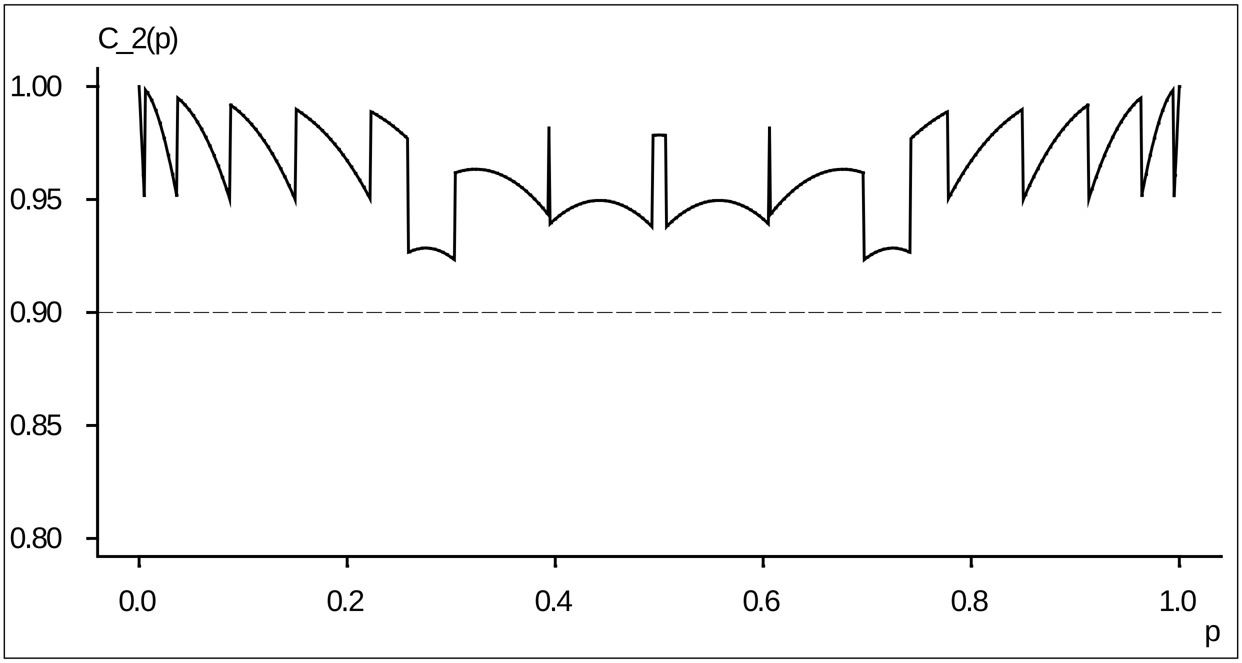

Figure 4. Two-sided CP CI coverage.

---

4. The LR Approach

For the parameter θ in a space Θ with some subset $\Theta_o \subseteq \Theta$, a test of

$$H_0\colon \theta \in \Theta_o$$
$$H_1\colon \theta \notin \Theta_o \tag{40}$$

can be conducted using the generalized LR

$$\Lambda = \frac{sup\,\{\,\mathcal{L} \mid \theta \in \Theta_o\}}{\sup\,\{\,\mathcal{L} \mid \theta \in \Theta\}}. \tag{41}$$

for the binomial distribution, the likelihood is

$$\mathcal{L} = C(n,k)p^k(1-p)^{n-k} \tag{42}$$

and, based on the maximum likelihood estimate $p = k/n$, the unconstrained supremum is

$$\sup\,\{\,\mathcal{L} \mid \theta \in \Theta\} = C(n,k)\left(\frac{k}{n}\right)^k\left(1-\frac{k}{n}\right)^{n-k}. \tag{43}$$

For the specific null

$$H_0 : p = p_o \tag{44}$$

the conditional supremum is

$$\sup \{ \mathcal{L} \mid \theta \in \Theta_o \} = \mathcal{L} = C(n,k)\, p_o{}^k (1-p_o)^{n-k}. \tag{45}$$

So the LR is

$$\Lambda = \left(\frac{p_o}{k/n}\right)^k \left(\frac{1-p_o}{1-k/n}\right)^{n-k} = \left(\frac{np_o}{k}\right)^k \left(\frac{n-np_o}{n-k}\right)^{n-k}. \tag{46}$$

For data $X \sim B_{n,p}$, the LR is

$$\Lambda = \left(\frac{np_o}{X}\right)^X \left(\frac{n-np_o}{n-X}\right)^{n-X}. \tag{47}$$

4.1 Hypothesis Tests

Consider the two-sided test

$$\begin{aligned} H_0 &: p = p_o \\ H_1 &: p \neq p_o. \end{aligned} \tag{48}$$

Since Λ has small values significant, one rejects H_0 if $\Lambda \leq \Lambda_o$ for the appropriate critical value. Note that the null probabilities are $\Pr[\Lambda = \Lambda(k)] = \Pr[X = k] = f_{n,p_o}(k)$ and the $\Lambda(k)$ can be sorted in ascending order and the probabilities summed to obtain the cdf F_Λ

$$F_\Lambda(t) = \sum_{\Lambda(k) \leq t} f_{n,p_o}(k) = \sum_{k=0}^{n} I_{[\Lambda(k),1]}(t) \cdot f_{n,p_o}(k). \tag{49}$$

In particular with $n = 10$ and $p_o = 0.33$, for $\alpha = 0.1$, the critical region $\Lambda \leq \Lambda(6)$ corresponds to $k \in \{0,6,7,8,9,10\}$ and the non-rejection region is $k \in \{1,2,3,4,5\}$. See table 1.

Table 1. LR cdf.

k	$f_{n,p_o}(k)$	$t = \Lambda(k)$	$F_\Lambda(t)$
10	0.01823	0.00002	0.00002
9	0.08978	0.00080	0.00033
8	0.19899	0.00941	0.00317
0	0.26136	0.01823	0.02140
7	0.22528	0.05765	0.03678
6	**0.13315**	**0.21789**	**0.09143**
1	**0.05465**	**0.23174**	**0.18121**
5	0.01538	0.54106	0.31436
2	0.00284	0.65894	0.51335
4	0.00031	0.89817	0.73864
3	0.00002	0.97952	1.00000

Now, based on the usual CP procedure $\alpha/2$ upper and lower tails of the B_{n,p_o} test statistic, the critical region is $\{0,7,8,9,10\}$ and the non-rejection region is $\{1,2,3,4,5,6\}$. See table 2. Note that $X = 6$ is critical for the LR test but not for the CP test.

Table 2. CP cdf.

k	$F_{n,p_o}(k)$	$G_{n,p_o}(k)$
0	**0.01823**	1.00000
1	**0.10801**	0.98177
2	0.30700	0.89199
3	0.56837	0.69300
4	0.79365	0.43163
5	0.92680	0.20635
6	0.98145	**0.07320**
7	0.99683	**0.01855**
8	0.99967	0.00317
9	0.99998	0.00033
10	1.00000	0.00002

4.2 Confidence Intervals

As usual, confidence intervals are obtained by inverting non-rejection regions. One-sided LR CIs coincide with CP CIs. Coverage for the two-sided LR CIs is illustrated in figure 5. Two-sided LR CIs are in general not as conservative as CP CIs. This is regarded as an advantage of the LR method, as the intervals are “tighter”.

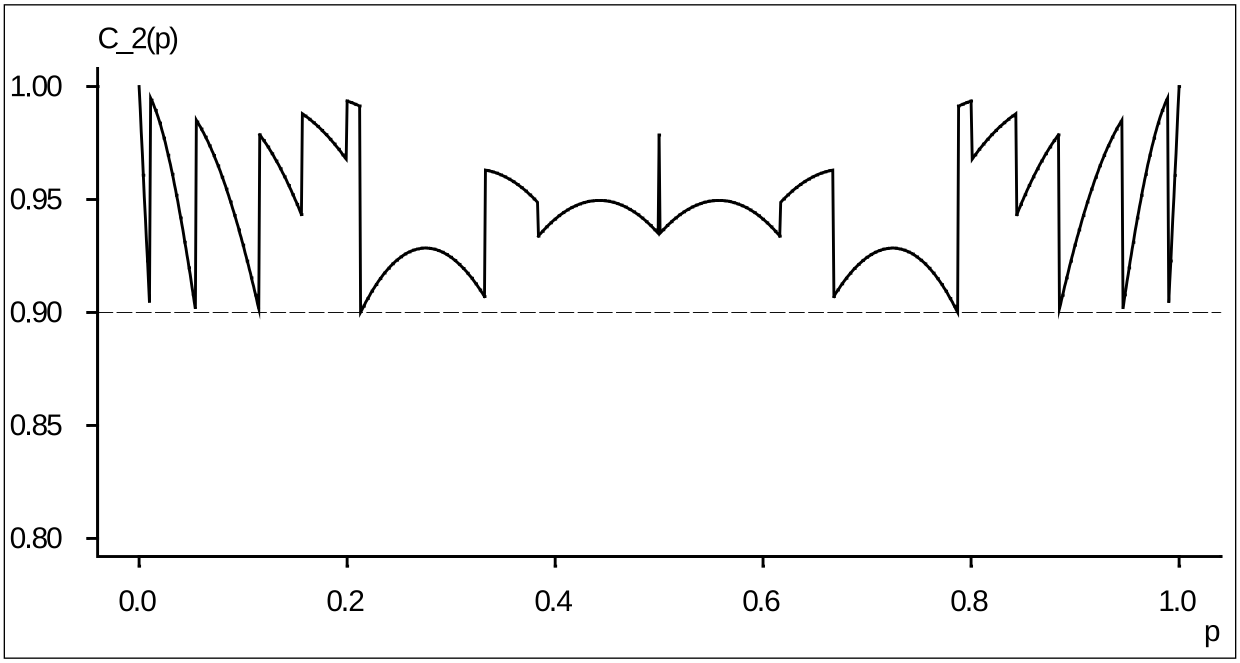

Figure 5. Two-sided LR CI coverage.

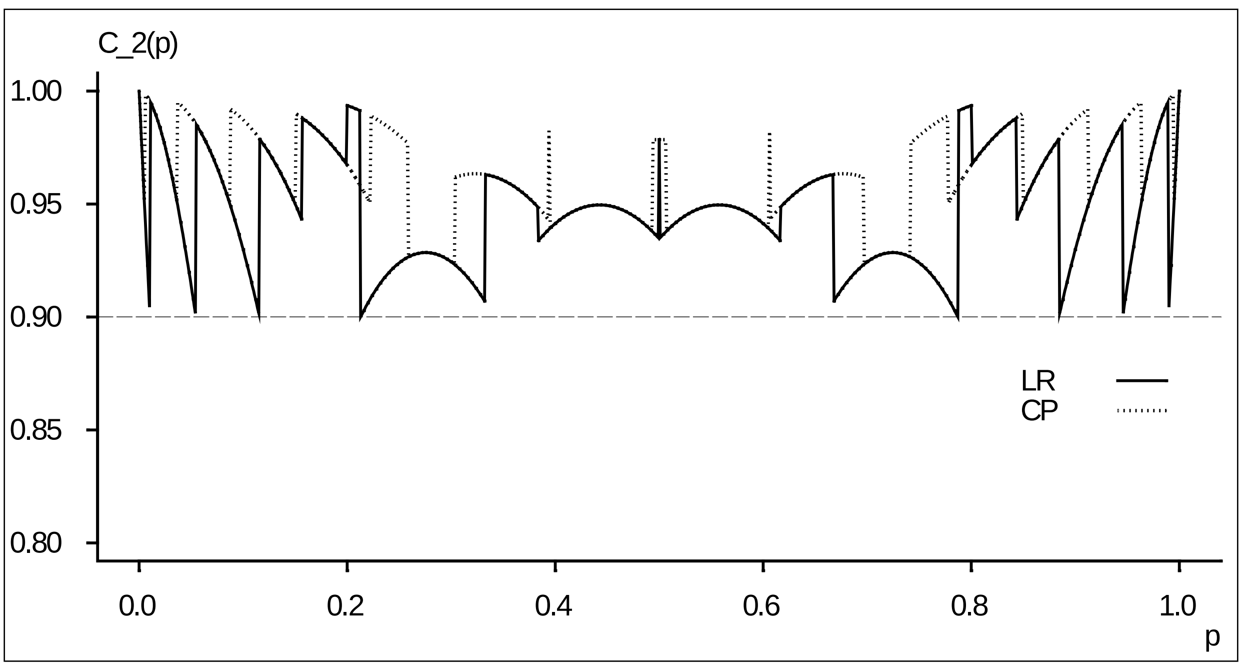

Figure 6. Two-sided CI coverage comparison.

4.3 Implementation

Code listings are in appendix A.

The function `bino.LR` evaluates the LR of equation 42 given n, k, and p. The function `bino.LR.F.x` evaluates the cdf components as in table 1, given n, p, and a technical parameter `up` which sets the orientation for breaking ties. The function `bino.LR.F.z` evaluates $F_\Lambda\big(\Lambda(k)\big)$. The hypothesis test of equation 48 is computed with `bino.LR.ht`, which presents the critical and non-rejection regions for the given values of n, p, and α. The function `bino.LR.ci` inverts the cdf to find CIs.

---

5. Reliability

5.1 Reliability Tables

Cooke (*5*) presents a tabulation of lower confidence limits p_L for various numbers of trials *n* and successes *r* and confidence levels γ.

This standard language may be misleading. The binomial parameter p is a reliability, which is a probability of success. The endpoints p_L and p_U of CIs $[p_L, p_U]$ on p are reliability limits, or limits of reliability, at the given confidence level γ. These “confidence limits” are not “limits of confidence.”

Code listings are in appendix A. The functions `pbinom` and `gbinom` evaluate $F_{n,p}(k)$ and $G_{n,p}(k)$, respectively.

The function `bino.rel.tab(n,r,g,type)` uses `bino.ci` to evaluate p_L and arranges the results in the proper format to reproduce Cooke’s tables, adding a column p for the parameter point estimate. The arguments of `bino.rel.tab` are the number of trials `n`, the number of successes `r`, confidence interval size `g` which should be between 0.5 and 1.0, and an integer type which can be 0, 1, or 2 to request a lower CI $(L, 1]$, upper CI $[0, U)$, or 2-sided CI (L, U), respectively. The arguments `r` and `g` can be vectors, in which case the function fills out the desired table.

For example, the first table, with $n = 4$, is given by

```
bino.rel.tab(4,4:2)
```

```
n r    p     80%     90%      95%    97.5%      99%    99.5%
4 4 1.00 0.66874 0.56234 0.472871 0.397635 0.316228 0.265915
4 3 0.75 0.41755 0.32046 0.248605 0.194120 0.140868 0.110885
4 2 0.50 0.21232 0.14256 0.097611 0.067586 0.041999 0.029445
```

For $n = 18$,

```
bino.rel.tab(18,18:9)
```

```
n   r       p     80%     90%     95%   97.5%     99%   99.5%
18 18 1.00000 0.91447 0.87992 0.84668 0.81470 0.77426 0.74501
18 17 0.94444 0.84262 0.80053 0.76234 0.72706 0.68398 0.65365
18 16 0.88889 0.77700 0.73058 0.68974 0.65288 0.60881 0.57833
18 15 0.83333 0.71472 0.66559 0.62332 0.58582 0.54170 0.51159
18 14 0.77778 0.65469 0.60398 0.56112 0.52363 0.48011 0.45076
```

18	13	0.72222	0.59642	0.54498	0.50217	0.46520	0.42280	0.39452
18	12	0.66667	0.53962	0.48816	0.44595	0.40993	0.36909	0.34214
18	11	0.61111	0.48412	0.43328	0.39216	0.35745	0.31858	0.29318
18	10	0.55556	0.42982	0.38020	0.34060	0.30757	0.27101	0.24739
18	9	0.50000	0.37669	0.32885	0.29120	0.26019	0.22630	0.20465

Tables for $n = 4 - 50$ are in appendix B. Compare to Cooke.

For $n = 100$,

```
bino.rel.tab(100,c(100:90, 60:50))
```

n	r	p	80%	90%	95%	97.5%	99%	99.5%
100	100	1.00	0.98403	0.97724	0.97049	0.96378	0.95499	0.94840
100	99	0.99	0.97035	0.96166	0.95344	0.94554	0.93546	0.92804
100	98	0.98	0.95770	0.94765	0.93838	0.92962	0.91859	0.91057
100	97	0.97	0.94554	0.93441	0.92429	0.91482	0.90303	0.89452
100	96	0.96	0.93370	0.92165	0.91080	0.90074	0.88830	0.87937
100	95	0.95	0.92209	0.90923	0.89775	0.88717	0.87415	0.86486
100	94	0.94	0.91064	0.89706	0.88501	0.87397	0.86045	0.85083
100	93	0.93	0.89933	0.88510	0.87254	0.86108	0.84710	0.83720
100	92	0.92	0.88814	0.87330	0.86028	0.84844	0.83405	0.82389
100	91	0.91	0.87703	0.86165	0.84820	0.83602	0.82125	0.81085
100	90	0.90	0.86601	0.85012	0.83628	0.82378	0.80867	0.79805
...								
100	60	0.60	0.55309	0.53115	0.51298	0.49721	0.47890	0.46647
100	59	0.59	0.54302	0.52106	0.50289	0.48714	0.46888	0.45648
100	58	0.58	0.53297	0.51100	0.49284	0.47712	0.45890	0.44655
100	57	0.57	0.52293	0.50096	0.48282	0.46713	0.44897	0.43666
100	56	0.56	0.51291	0.49095	0.47284	0.45719	0.43908	0.42682
100	55	0.55	0.50291	0.48096	0.46289	0.44728	0.42924	0.41704
100	54	0.54	0.49292	0.47100	0.45297	0.43741	0.41944	0.40730
100	53	0.53	0.48295	0.46107	0.44309	0.42758	0.40969	0.39762
100	52	0.52	0.47300	0.45116	0.43323	0.41779	0.39999	0.38798
100	51	0.51	0.46306	0.44128	0.42341	0.40804	0.39033	0.37840
100	50	0.50	0.45314	0.43142	0.41362	0.39832	0.38072	0.36886

For $n = 500$,

```
bino.rel.tab(500,c(500:490, 260:250))
```

n	r	p	80%	90%	95%	97.5%	99%	99.5%
500	500	1.000	0.99679	0.99541	0.99403	0.99265	0.99083	0.98946
500	499	0.998	0.99402	0.99224	0.99055	0.98891	0.98680	0.98523
500	498	0.996	0.99146	0.98939	0.98746	0.98563	0.98330	0.98159
500	497	0.994	0.98900	0.98669	0.98457	0.98257	0.98005	0.97822
500	496	0.992	0.98659	0.98408	0.98179	0.97964	0.97697	0.97503
500	495	0.990	0.98423	0.98153	0.97909	0.97682	0.97399	0.97196
500	494	0.988	0.98191	0.97903	0.97645	0.97407	0.97111	0.96898
500	493	0.986	0.97960	0.97657	0.97387	0.97137	0.96829	0.96608
500	492	0.984	0.97732	0.97414	0.97132	0.96872	0.96552	0.96324
500	491	0.982	0.97505	0.97174	0.96880	0.96611	0.96280	0.96044
500	490	0.980	0.97280	0.96935	0.96631	0.96353	0.96012	0.95769
...								
500	260	0.520	0.50018	0.49035	0.48223	0.47520	0.46704	0.46148
500	259	0.518	0.49818	0.48835	0.48024	0.47321	0.46504	0.45949

```
500 258 0.516 0.49618 0.48635 0.47824 0.47121 0.46305 0.45751
500 257 0.514 0.49418 0.48435 0.47624 0.46922 0.46106 0.45552
500 256 0.512 0.49218 0.48236 0.47425 0.46723 0.45907 0.45353
500 255 0.510 0.49019 0.48036 0.47226 0.46523 0.45709 0.45155
500 254 0.508 0.48819 0.47836 0.47026 0.46324 0.45510 0.44956
500 253 0.506 0.48619 0.47637 0.46827 0.46125 0.45311 0.44758
500 252 0.504 0.48419 0.47437 0.46628 0.45926 0.45112 0.44559
500 251 0.502 0.48220 0.47238 0.46428 0.45727 0.44914 0.44361
500 250 0.500 0.48020 0.47038 0.46229 0.45529 0.44716 0.44163
```

5.2 Sample Size

One application of this is to find the minimal sample size N to obtain a lower reliability limit at least p_L with confidence γ for a certain $n_f = n - r$, so $r = n - n_f$.

For example, with $n_f = 0$ failures ($r = n$) and 90% reliability limit $p_L = 0.9$ with 95% confidence $\gamma = 0.95$, one can wade through Cooke's phone-book-sized report, or run `bino.rel.tab` to find suitable n, which bound the desired reliability limit,

`bino.rel.tab(n=28, r=28, g=0.95)` gives $p_L = 0.89853$ and
`bino.rel.tab(n=29, r=29, g=0.95)` gives $p_L = 0.90186$, so $N \geq 29$.

For $n_f = 1$ failure, $r = n - 1$

`bino.rel.tab(n=45, r=44, g=0.95)` gives $p_L = 0.89887$ and
`bino.rel.tab(n=46, r=45, g=0.95)` gives $p_L = 0.90098$, so $N \geq 46$.

By equation 31, this amounts to finding

$$N = \inf\{ n \,| G_{n,p_L}(n - n_f) \leq 1 - \gamma \}. \tag{50}$$

For $n_f = 0$ failures, $p_L = 0.9$, and $\gamma = 0.95$,

$G_{28,0.9}(28) =$ `gbinom(28,28,0.9)`$= 0.052335$ and
$G_{29,0.9}(29) =$ `gbinom(29,29,0.9)`$= 0.047101$, so $N \geq 29$.

For $n_f = 1$ failures, $p_L = 0.9$, and $\gamma = 0.95$,

$G_{45,0.9}(45) =$ `gbinom(45,45,0.9)`$= 0.052368$ and
$G_{46,0.9}(46) =$ `gbinom(46,46,0.9)`$= 0.048004$, so $N \geq 46$.

By equation 11, this is equivalent to

$$N = \inf\{ n \,| F_{n,1-p_L}(n_f) \leq 1 - \gamma \}. \tag{51}$$

For $n_f = 0$ failures, $p_L = 0.9$, and $\gamma = 0.95$,

$F_{28,0.9}(0) =$ `pbinom(0,28,0.1)`$= 0.052335$ and
$F_{29,0.9}(0) =$ `pbinom(0,29,0.1)`$= 0.047101$, so $N \geq 29$.

For $n_f = 1$ failures, $p_L = 0.9$, and $\gamma = 0.95$,

$F_{45,0.9}(1) =$ `pbinom(1,45,0.1)`$= 0.052368$ and
$F_{46,0.9}(1) =$ `pbinom(1,46,0.1)`$= 0.048004$, so $N \geq 46$.

The function `bino.rel.size(nf,p,g,x)` takes arguments representing the `nf`, desired lower reliability limit `p`, confidence `g`, and an optional scale factor `x` for the numerical solution routine. It calculates the required sample size N and returns a named list holding

`$par`:	parameters `nf`, `p`, and `g`
`$n`:	required minimum sample size, N
`$p`:	reliability limit for confidence `g` and sample size N
`$g`:	confidence for reliability limit `p` and sample size N
`$n1`:	the next lower sample size, $N - 1$
`$p1`:	reliability limit for confidence `g` and sample size $N - 1$
`$g1`:	confidence for reliability limit `p` and sample size $N - 1$

One expects that `$p1` ‹ `p` ≤ `$p` and `$g1` ‹ `g` ≤ `$g`.

For example,

```
bino.rel.size(nf = 1, p = 0.9, g = 0.95)
```

returns

```
$par$nf:    1
$par$p:     0.9
$par$g:     0.95
$n:         46
$p:         0.90098
$g:         0.952
$n1:        45
$p1:        0.89887
$g1:        0.94763
```

Consulting the tables for the sample sizes `$n` = 46 and `$n1` = 45 and confidence `g` = 0.95 for the number of successes `r` = `$n-nf` = 45 and `r1` = `$n1-nf` = 44, one sees the bounding reliability limits `$p` = 0.90098, and `$p1` = 0.89887.

6. References

1. Stuart, A.; Ord, J. K. *Kendall's Advanced Theory of Statistics: Distribution Theory;* Oxford University Press: New York, 1994.
2. Mood, A.; Graybill, F. A.; Boes, D. C**.** *Introduction to the Theory of Statistics. 3*; McGraw-Hill, New York, 1974.
3. Stuart, A.; Ord, J. K. *Kendall's Advanced Theory of Statistics: Classical Inference and Relationship*; Oxford University Press, New York, 1991.
4. Clopper, C. J.; Pearson, E. S. *The Use of Confidence or Fiducial Limits Illustrated in the Case of the Binomial.* 1934, 4, Biometrika, Vol 26, pp 404–413.
5. Cooke, J. R.; Lee, M. T.; Vanderbeck, J. P. *Binomial Reliability Table (Lower Confidence Limits for the Binomial Distribution).* China Lake, CA, Bureau of Naval Weapons, 1964.

Appendix A. S-PLUS Code

A.1 Library Functions

```
pbinom(q,
       size = stop("no size arg"),
       prob = stop("no prob arg"))
```

is the S-PLUS library implementation of the Binomial distribution cdf.

In the notation of this report,

$$\texttt{pbinom(k,n,p)} = F_{n,p}(k) = \Pr\left[B_{n,p} \leq k\right]$$

where, k is number of successes, n is the number of trials, and p is the probability of success.

```
qbeta(p,
      shape1 = stop("no shape1 arg"),
      shape2 = stop("no shape2 arg"))
```

is the S-PLUS library implementation of the Beta distribution qf.

In the notation of this report,

$$\texttt{qbeta(u,a,b)} = Q_{\mathrm{Beta}\,(a,b)}(u) = \inf\left\{x \mid F_{\mathrm{Beta}\,(a,b)}(x) \geq u\right\}$$

where, u is the desired probability, and a and b are the usual Beta distribution parameters.

A.2 Upper-Tail Binomial Probabilities

gbinom calculates upper tail binomial probabilities.

```
gbinom <- function(q,

                   size = stop("no size arg"),
                   prob = stop("no prob arg"))
{
    ##
    ## Pr [ Binomial(size,prob) >= q ]
    ##
    ## underflow error
    ## 1-pbinom(q-1,size,prob)
    ##
    ## no underflow
    ##
    pbinom ( size - q , size , 1 - prob )
}
```

In the notation of this report,

$$\texttt{gbinom(k,n,p)} = G_{n,p}(k) = \texttt{pbinom(n-k,n,1-p)}$$

where, k is number of successes, n is the number of trials, and p is the probability of success.

A.3 Binomial Parameter Confidence Interval: CP

`bino.ci` calculates a CP confidence interval on a binomial parameter estimate.

```
bino.ci <- function(n          ,        # sample size
                    r          ,        # number of successes
                    g    = 0.8 ,        # CI size
                    type = 0   ,        # 0,1,2 for lower, upper, 2-sided
                    a    = 1-g )        # 1 - (CI size)
{
    ## 100g% Confidence Interval [ L , U ] on Binomial Parameter p
    ##
    ## type 0 : lower CI = ( L , 1 ] : Pr [ B(n,L) >= r ] = 1-g
    ## type 1 : upper CI = [ 0 , U ) : Pr [ B(n,U) <= r ] = 1-g
    ## type 2 : two-sided CI = (L,U) : Pr[B(n,L)>=r] = Pr[B(n,U)<=r]=(1-g)/2

    if(type==2) a <- a/2

    if ( type==1 || r==0 )
      L <- 0
    else
      L <- qbeta ( a , r , n-r+1 )

    if(type==0 || r==n)
      U <- 1
    else
      U <- 1 - qbeta ( a , n-r , r+1 )

    list(n=n, r=r, L=L, p.hat=r/n, U=U, I=c(L,U))
}
```

A.4 Binomial Reliability Table

`bino.rel.tab` tabulates binomial confidence limits.

```
bino.rel.tab <- function(n = 100 ,                      # number of trials
                         r = n   ,                      # number of successes
                         g = c(.8,.9,.95,.975,.99,.995),# confidence levels
                         type = 0    )                  # 0,1 = lower, upper
{
    nr <- length(r)                     # success vector length
    ng <- length(g)                     # number of CI levels
    z <- matrix(NA, nc=ng, nr=nr)       # reliability limits

    for( i in 1:nr ) for(j in 1:ng )    # compute reliabilities
      z[i,j] <- bino.ci(n,r[i],g[j],type)$I[type+1]

    z <- cbind(n, r, r/n, z)            # augmented table
    dimnames(z) <-                      # name the table columns
      list(NULL, c("n","r","p",paste(100*g,"%", sep="")))

    z                                   # return table
}
```

A.5 Binomial Reliability Sample Size

`bino.rel.size` calculates sample size for reliability with a given confidence limit.

```
bino.rel.size <- function(nf = 0    ,   # number of failures
                          p  = 0.9  ,   # reliability limit
                          g  = 0.95 ,   # confidence level
                          x  = 6    )   # log_2 ( search delta )
{
    ## N = inf { n : pbinom(nf, n, 1-p) } <= 1-g }

    a <- 1-g                            # confidence
    q <- 1-p                            # reliability

    f <- function(nf, n, q) list(n=n, a=pbinom(nf, n, q))   # ( n , a )

    rtn <- function(nf, p, g, z1, z)    # return list
      list(par=list(nf=nf, p=p, g=g),
           n=z$n, p=bino.ci(z$n,  z$n-nf, g, 0)$L, g=1-z$a ,
           n1=z1$n, p1=bino.ci(z1$n, z1$n-nf, g, 0)$L, g1=1-z1$a)

    z <- f(nf, 2^x, q)                  # initial ( n , a ) guess
    while ( z$a < a ) z <- f(nf, z$n/2, q)
    while ( z$a > a ) z <- f(nf, z$n*2, q)

    z0 <- z                             # upper end at n
    z1 <- f(nf, z0$n/2, q)              # lower end at n/2
    dn <- (z0$n-z1$n)/2                 # interval radius
    while ( dn >= 1 ) {
        z <- f(nf, z1$n+dn, q)          # midpoint
        if ( z$a < a )                  # one midpoint becomes new endoint
          z0 <- z else z1 <- z
        dn <- dn/2                      # new radius
    }

    rtn(nf, p, g, z1, z0)
}
```

The algorithm finds sample sizes $n_1 = 2^{k-1}$ and $n = 2^k$ that bound the confidence for the desired reliability limit. Then it repeatedly bisects the interval $[n_1, n]$ and replaces one endpoint with the midpoint, maintaining the bound, until $n - n_1 = 1$. Reliability limits at the desired confidence level should now bound the desired reliability limit. Using powers of 2 ensures that the sample sizes are integers.

A.6 Binomial LR

```
bino.LR <- function(k ,                 # successes
                    n ,                 # trials
                    p )                 # parameter
{
    ## binomial LR
    ( n*p/k ) ^ k * ( n*(1-p)/(n-k) ) ^ (n-k)
}
```

A.7 Binomial LR cdf Table

```
bino.LR.F.x <- function(n        ,      # trials
                        p        ,      # null parameter
                        up  =  T )      # tie-breaker direction
{
```

```
    ## binomial(n,p) LR cdf: complete table
    k <- 0:n                                # successes
    f <- dbinom(k,n,p)                      # probabilities
    L <- bino.LR(k,n,p)                     # LR values

    if(up) I <- order(L,k)                  # order LR values, tiebreaker
    else I <- order(L,-k)
    L <- L[I]                               # sort LR values
    k <- k[I]                               # order successes by LR value
    F.L <- cumsum(f[I])                     # LR cdf
    cbind(k, I, f, L, F.L)
}
```

A.8 Binomial LR cdf Evaluation

```
bino.LR.F.z <- function(k   ,           # trials
                        n   ,           # successes
                        p   ,           # null parameter
                        x=F )           # extended return
{
    ## binomial(n,p) LR cdf (k)
    z <- bino.LR.F.x(n=n, p=p, k>n*p)
    u <- order(z[,"I"])
    if(x) z[u[k+1],]
    else  z[u[k+1],"F.L"]
}
```

A.9 Binomial LR Hypothesis Test

```
bino.LR.ht <- function(n       ,        # trials
                       p       ,        # null parameter
                       a = 0.1 )        # significance
{
    ## binomial LR hypothesis test
    z <- bino.LR.F.x(n=n, p=p)

    I.rej <- z[,"F.L"] <= a
    k.rej <- sort(z[,"k"][I.rej])
    k.acc <- sort(z[,"k"][!I.rej])
    list(z=z, k.acc=k.acc, k.rej=k.rej)
}
```

A.10 Binomial Parameter Confidence Interval: LR

```
bino.LR.ci <- function(n = 100 ,        # sample size
                       r = 50  ,        # number of successes
                       g = 0.9 )        # CI size
{
    h <- r/n
    q <- 1-g

    ## [0,h] : L is increasing
    if(r == 0)
      p0 <- list(neg=0, pos=0, f0=q, f1=q)
    else
      p0 <- uniroot(function(x, n., r., q.) bino.LR.F.z(k=r., n=n., p=x)-q.,
                     lower=0, upper=h, q.=q, n.=n, r.=r)
```

```
    ## [h,1] : L is decreasing
    if(r == n)
      p1 <- list(neg=1, pos=1, f0=q, f1=q)
    else
      p1 <- uniroot(function(x, n., r., q.)  bino.LR.F.z(k=r., n=n., p=x)-q.,
                    lower=h, upper=1, q.=q, n.=n, r.=r)

    list(n=n, r=r, L=p0$pos, p.hat=h, U=p1$neg, I=c(p0$pos, p1$neg))
}
```

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Appendix B. Reliability Table

n	r	p	80%	90%	95%	97.5%	99%	99.5%
1	1	1.00000	0.20000	0.10000	0.05000	0.02500	0.01000	0.00500
1	0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

n	r	p	80%	90%	95%	97.5%	99%	99.5%
2	2	1.00000	0.44721	0.31623	0.22361	0.15811	0.10000	0.07071
2	1	0.50000	0.10557	0.05132	0.02532	0.01258	0.00501	0.00250

n	r	p	80%	90%	95%	97.5%	99%	99.5%
3	3	1.00000	0.58480	0.46416	0.36840	0.29240	0.21544	0.17100
3	2	0.66667	0.28714	0.19580	0.13535	0.09430	0.05890	0.04140
3	1	0.33333	0.07168	0.03451	0.01695	0.00840	0.00334	0.00167

n	r	p	80%	90%	95%	97.5%	99%	99.5%
4	4	1.00000	0.66874	0.56234	0.47287	0.39764	0.31623	0.26591
4	3	0.75000	0.41755	0.32046	0.24860	0.19412	0.14087	0.11088
4	2	0.50000	0.21232	0.14256	0.09761	0.06759	0.04200	0.02944

n	r	p	80%	90%	95%	97.5%	99%	99.5%
5	5	1.00000	0.72478	0.63096	0.54928	0.47818	0.39811	0.34657
5	4	0.80000	0.50981	0.41611	0.34259	0.28358	0.22207	0.18510
5	3	0.60000	0.32660	0.24664	0.18926	0.14663	0.10564	0.08283
5	2	0.40000	0.16861	0.11223	0.07644	0.05274	0.03268	0.02288

n	r	p	80%	90%	95%	97.5%	99%	99.5%
6	6	1.00000	0.76472	0.68129	0.60696	0.54074	0.46416	0.41352
6	5	0.83333	0.57755	0.48968	0.41820	0.35877	0.29431	0.25399
6	4	0.66667	0.41461	0.33319	0.27134	0.22278	0.17307	0.14360
6	3	0.50000	0.26865	0.20091	0.15316	0.11812	0.08473	0.06628

n	r	p	80%	90%	95%	97.5%	99%	99.5%
7	7	1.00000	0.79460	0.71969	0.65184	0.59038	0.51795	0.46912
7	6	0.85714	0.62914	0.54744	0.47930	0.42128	0.35664	0.31509
7	5	0.71429	0.48324	0.40382	0.34126	0.29042	0.23632	0.20297
7	4	0.57143	0.35009	0.27860	0.22532	0.18405	0.14227	0.11770
7	3	0.42857	0.22833	0.16964	0.12876	0.09899	0.07080	0.05530

n	r	p	80%	90%	95%	97.5%	99%	99.5%
8	8	1.00000	0.81777	0.74989	0.68766	0.63058	0.56234	0.51567
8	7	0.87500	0.66963	0.59375	0.52932	0.47349	0.41006	0.36848
8	6	0.75000	0.53790	0.46178	0.40031	0.34914	0.29323	0.25783
8	5	0.62500	0.41634	0.34462	0.28924	0.24486	0.19820	0.16970
8	4	0.50000	0.30323	0.23966	0.19290	0.15701	0.12095	0.09987

n	r	p	80%	90%	95%	97.5%	99%	99.5%
9	9	1.00000	0.83625	0.77426	0.71687	0.66373	0.59948	0.55505
9	8	0.88889	0.70223	0.63164	0.57086	0.51750	0.45597	0.41503
9	7	0.77778	0.58232	0.50992	0.45036	0.39991	0.34369	0.30739
9	6	0.66667	0.47086	0.40058	0.34494	0.29930	0.25003	0.21914
9	5	0.55556	0.36609	0.30097	0.25137	0.21201	0.17097	0.14606
9	4	0.44444	0.26755	0.21040	0.16875	0.13700	0.10526	0.08679

n	r	p	80%	90%	95%	97.5%	99%	99.5%
10	10	1.00000	0.85134	0.79433	0.74113	0.69150	0.63096	0.58870
10	9	0.90000	0.72901	0.66315	0.60584	0.55498	0.49565	0.45571
10	8	0.80000	0.61906	0.55040	0.49310	0.44390	0.38826	0.35180
10	7	0.70000	0.51634	0.44827	0.39338	0.34755	0.29712	0.26489
10	6	0.60000	0.41913	0.35422	0.30354	0.26238	0.21834	0.19092
10	5	0.50000	0.32683	0.26732	0.22244	0.18709	0.15044	0.12831

n	r	p	80%	90%	95%	97.5%	99%	99.5%
11	11	1.00000	0.86389	0.81113	0.76160	0.71509	0.65793	0.61775
11	10	0.90909	0.75140	0.68976	0.63564	0.58722	0.53018	0.49144
11	9	0.81818	0.64993	0.58484	0.52991	0.48224	0.42768	0.39151
11	8	0.72727	0.55478	0.48924	0.43563	0.39026	0.33958	0.30672
11	7	0.63636	0.46431	0.40053	0.34981	0.30790	0.26220	0.23320
11	6	0.54545	0.37787	0.31772	0.27125	0.23379	0.19398	0.16931
11	5	0.45455	0.29526	0.24053	0.19958	0.16749	0.13439	0.11447

n	r	p	80%	90%	95%	97.5%	99%	99.5%
12	12	1.00000	0.87449	0.82540	0.77908	0.73535	0.68129	0.64305
12	11	0.91667	0.77038	0.71250	0.66132	0.61520	0.56046	0.52297
12	10	0.83333	0.67622	0.61448	0.56189	0.51586	0.46266	0.42705
12	9	0.75000	0.58765	0.52473	0.47267	0.42814	0.37781	0.34478
12	8	0.66667	0.50315	0.44100	0.39086	0.34888	0.30240	0.27248

n	r	p	80%	90%	95%	97.5%	99%	99.5%
12	7	0.58333	0.42206	0.36228	0.31524	0.27667	0.23489	0.20853
12	6	0.50000	0.34411	0.28817	0.24530	0.21094	0.17461	0.15219

n	r	p	80%	90%	95%	97.5%	99%	99.5%
13	13	1.00000	0.88355	0.83768	0.79418	0.75295	0.70170	0.66527
13	12	0.92308	0.78668	0.73216	0.68366	0.63970	0.58717	0.55098
13	11	0.84615	0.69886	0.64022	0.58990	0.54553	0.49383	0.45896
13	10	0.76923	0.61606	0.55574	0.50535	0.46187	0.41224	0.37936
13	9	0.69231	0.53686	0.47657	0.42738	0.38574	0.33910	0.30872
13	8	0.61538	0.46061	0.40176	0.35480	0.31578	0.27289	0.24543
13	7	0.53846	0.38700	0.33086	0.28705	0.25135	0.21288	0.18870
13	6	0.46154	0.31596	0.26373	0.22396	0.19223	0.15882	0.13827

n	r	p	80%	90%	95%	97.5%	99%	99.5%
14	14	1.00000	0.89140	0.84834	0.80736	0.76836	0.71969	0.68492
14	13	0.92857	0.80083	0.74933	0.70327	0.66132	0.61090	0.57597
14	12	0.85714	0.71856	0.66279	0.61461	0.57187	0.52174	0.48769
14	11	0.78571	0.64085	0.58302	0.53434	0.49202	0.44333	0.41082
14	10	0.71429	0.56636	0.50803	0.45999	0.41896	0.37257	0.34206
14	9	0.64286	0.49446	0.43689	0.39041	0.35138	0.30797	0.27986
14	8	0.57143	0.42486	0.36913	0.32503	0.28861	0.24880	0.22343
14	7	0.50000	0.35741	0.30455	0.26358	0.23036	0.19472	0.17240

n	r	p	80%	90%	95%	97.5%	99%	99.5%
15	15	1.00000	0.89826	0.85770	0.81896	0.78198	0.73564	0.70242
15	14	0.93333	0.81322	0.76443	0.72060	0.68052	0.63211	0.59841
15	13	0.86667	0.73585	0.68271	0.63656	0.59540	0.54683	0.51367
15	12	0.80000	0.66265	0.60721	0.56022	0.51911	0.47149	0.43947
15	11	0.73333	0.59237	0.53603	0.48925	0.44900	0.40311	0.37269
15	10	0.66667	0.52441	0.46829	0.42256	0.38380	0.34029	0.31184
15	9	0.60000	0.45846	0.40353	0.35957	0.32287	0.28229	0.25613
15	8	0.53333	0.39436	0.34152	0.29999	0.26586	0.22873	0.20514
15	7	0.46667	0.33207	0.28218	0.24373	0.21267	0.17946	0.15873

n	r	p	80%	90%	95%	97.5%	99%	99.5%
16	16	1.00000	0.90430	0.86596	0.82925	0.79409	0.74989	0.71810
16	15	0.93750	0.82417	0.77783	0.73604	0.69768	0.65116	0.61864
16	14	0.87500	0.75115	0.70044	0.65617	0.61652	0.56951	0.53724
16	13	0.81250	0.68198	0.62878	0.58343	0.54354	0.49706	0.46564
16	12	0.75000	0.61548	0.56108	0.51560	0.47623	0.43103	0.40087
16	11	0.68750	0.55107	0.49649	0.45165	0.41338	0.37005	0.34151
16	10	0.62500	0.48844	0.43456	0.39101	0.35435	0.31341	0.28677
16	9	0.56250	0.42745	0.37504	0.33337	0.29878	0.26069	0.23623
16	8	0.50000	0.36801	0.31783	0.27860	0.24651	0.21172	0.18969

n	r	p	80%	90%	95%	97.5%	99%	99.5%
17	17	1.00000	0.90967	0.87333	0.83843	0.80494	0.76270	0.73223
17	16	0.94118	0.83390	0.78979	0.74988	0.71311	0.66837	0.63697
17	15	0.88235	0.76478	0.71630	0.67381	0.63559	0.59008	0.55871
17	14	0.82353	0.69923	0.64813	0.60436	0.56568	0.52038	0.48960
17	13	0.76471	0.63613	0.58361	0.53945	0.50101	0.45661	0.42682
17	12	0.70588	0.57493	0.52193	0.47808	0.44042	0.39749	0.36901
17	11	0.64706	0.51535	0.46265	0.41971	0.38328	0.34229	0.31541
17	10	0.58824	0.45722	0.40551	0.36401	0.32925	0.29062	0.26558
17	9	0.52941	0.40044	0.35039	0.31083	0.27812	0.24225	0.21928
17	8	0.47059	0.34500	0.29726	0.26011	0.22983	0.19711	0.17644

n	r	p	80%	90%	95%	97.5%	99%	99.5%
18	18	1.00000	0.91447	0.87992	0.84668	0.81470	0.77426	0.74501
18	17	0.94444	0.84262	0.80053	0.76234	0.72706	0.68398	0.65365
18	16	0.88889	0.77700	0.73058	0.68974	0.65288	0.60881	0.57833
18	15	0.83333	0.71472	0.66559	0.62332	0.58582	0.54170	0.51159
18	14	0.77778	0.65469	0.60398	0.56112	0.52363	0.48011	0.45076
18	13	0.72222	0.59642	0.54498	0.50217	0.46520	0.42280	0.39452
18	12	0.66667	0.53962	0.48816	0.44595	0.40993	0.36909	0.34214
18	11	0.61111	0.48412	0.43328	0.39216	0.35745	0.31858	0.29318
18	10	0.55556	0.42982	0.38020	0.34060	0.30757	0.27101	0.24739
18	9	0.50000	0.37669	0.32885	0.29120	0.26019	0.22630	0.20465

n	r	p	80%	90%	95%	97.5%	99%	99.5%
19	19	1.00000	0.91878	0.88587	0.85413	0.82353	0.78476	0.75665
19	18	0.94737	0.85046	0.81023	0.77363	0.73972	0.69820	0.66889
19	17	0.89474	0.78802	0.74349	0.70420	0.66862	0.62595	0.59632
19	16	0.84211	0.72869	0.68141	0.64057	0.60422	0.56127	0.53184
19	15	0.78947	0.67147	0.62247	0.58088	0.54435	0.50175	0.47289
19	14	0.73684	0.61587	0.56595	0.52420	0.48797	0.44621	0.41821
19	13	0.68421	0.56161	0.51144	0.47003	0.43450	0.39399	0.36709
19	12	0.63158	0.50854	0.45868	0.41806	0.38358	0.34468	0.31910

n	r	p	80%	90%	95%	97.5%	99%	99.5%
19	11	0.57895	0.45654	0.40754	0.36812	0.33500	0.29805	0.27399
19	10	0.52632	0.40558	0.35793	0.32009	0.28864	0.25395	0.23160
19	9	0.47368	0.35564	0.30983	0.27395	0.24447	0.21235	0.19189

n	r	p	80%	90%	95%	97.5%	99%	99.5%
20	20	1.00000	0.92268	0.89125	0.86089	0.83157	0.79433	0.76727
20	19	0.95000	0.85757	0.81904	0.78389	0.75127	0.71121	0.68286
20	18	0.90000	0.79800	0.75523	0.71738	0.68302	0.64166	0.61287
20	17	0.85000	0.74136	0.69581	0.65634	0.62107	0.57927	0.55053
20	16	0.80000	0.68670	0.63934	0.59897	0.56339	0.52172	0.49339
20	15	0.75000	0.63354	0.58511	0.54442	0.50895	0.46789	0.44024
20	14	0.70000	0.58162	0.53273	0.49218	0.45721	0.41714	0.39039
20	13	0.65000	0.53078	0.48197	0.44197	0.40781	0.36906	0.34343
20	12	0.60000	0.48093	0.43267	0.39358	0.36054	0.32342	0.29909
20	11	0.55000	0.43200	0.38475	0.34693	0.31528	0.28008	0.25723
20	10	0.50000	0.38397	0.33817	0.30195	0.27196	0.23896	0.21775

n	r	p	80%	90%	95%	97.5%	99%	99.5%
21	21	1.00000	0.92622	0.89615	0.86705	0.83890	0.80309	0.77701
21	20	0.95238	0.86403	0.82706	0.79327	0.76184	0.72316	0.69571
21	19	0.90476	0.80709	0.76595	0.72945	0.69623	0.65614	0.62815
21	18	0.85714	0.75291	0.70898	0.67079	0.63658	0.59589	0.56784
21	17	0.80952	0.70059	0.65478	0.61559	0.58093	0.54021	0.51242
21	16	0.76190	0.64967	0.60267	0.56302	0.52834	0.48802	0.46076
21	15	0.71429	0.59991	0.55229	0.51261	0.47825	0.43869	0.41217
21	14	0.66667	0.55114	0.50339	0.46406	0.43032	0.39185	0.36627
21	13	0.61905	0.50327	0.45584	0.41720	0.38435	0.34724	0.32277
21	12	0.57143	0.45623	0.40954	0.37190	0.34021	0.30472	0.28153
21	11	0.52381	0.41000	0.36443	0.32811	0.29781	0.26421	0.24245
21	10	0.47619	0.36457	0.32051	0.28580	0.25713	0.22567	0.20549

n	r	p	80%	90%	95%	97.5%	99%	99.5%
22	22	1.00000	0.92946	0.90063	0.87269	0.84563	0.81113	0.78597
22	21	0.95455	0.86993	0.83441	0.80188	0.77156	0.73416	0.70757
22	20	0.90909	0.81539	0.77578	0.74053	0.70839	0.66950	0.64229
22	19	0.86364	0.76348	0.72106	0.68409	0.65088	0.61127	0.58389
22	18	0.81818	0.71331	0.66896	0.63091	0.59715	0.55737	0.53013
22	17	0.77273	0.66445	0.61883	0.58020	0.54630	0.50674	0.47990
22	16	0.72727	0.61667	0.57030	0.53151	0.49778	0.45879	0.43256
22	15	0.68182	0.56982	0.52316	0.48454	0.45128	0.41316	0.38771
22	14	0.63636	0.52379	0.47725	0.43913	0.40658	0.36960	0.34510
22	13	0.59091	0.47853	0.43248	0.39516	0.36355	0.32795	0.30456
22	12	0.54545	0.43399	0.38881	0.35254	0.32210	0.28813	0.26598
22	11	0.50000	0.39017	0.34619	0.31126	0.28221	0.25008	0.22932

n	r	p	80%	90%	95%	97.5%	99%	99.5%
23	23	1.00000	0.93242	0.90474	0.87788	0.85181	0.81855	0.79425
23	22	0.95652	0.87534	0.84116	0.80980	0.78051	0.74433	0.71856
23	21	0.91304	0.82302	0.78481	0.75075	0.71962	0.68188	0.65541
23	20	0.86957	0.77318	0.73219	0.69636	0.66411	0.62555	0.59882
23	19	0.82609	0.72499	0.68203	0.64507	0.61219	0.57332	0.54663
23	18	0.78261	0.67804	0.63374	0.59610	0.56297	0.52419	0.49779
23	17	0.73913	0.63210	0.58695	0.54902	0.51595	0.47758	0.45166
23	16	0.69565	0.58703	0.54144	0.50356	0.47081	0.43313	0.40787
23	15	0.65217	0.54271	0.49709	0.45954	0.42734	0.39060	0.36615
23	14	0.60870	0.49910	0.45378	0.41685	0.38542	0.34985	0.32635
23	13	0.56522	0.45616	0.41147	0.37539	0.34495	0.31076	0.28835
23	12	0.52174	0.41386	0.37012	0.33515	0.30588	0.27329	0.25210
23	11	0.47826	0.37219	0.32971	0.29609	0.26820	0.23742	0.21756

n	r	p	80%	90%	95%	97.5%	99%	99.5%
24	24	1.00000	0.93514	0.90852	0.88265	0.85753	0.82540	0.80191
24	23	0.95833	0.88032	0.84738	0.81711	0.78880	0.75375	0.72875
24	22	0.91667	0.83003	0.79315	0.76020	0.73003	0.69337	0.66761
24	21	0.87500	0.78212	0.74246	0.70773	0.67639	0.63883	0.61274
24	20	0.83333	0.73576	0.69412	0.65819	0.62616	0.58819	0.56205
24	19	0.79167	0.69058	0.64754	0.61086	0.57849	0.54048	0.51454
24	18	0.75000	0.64635	0.60237	0.56531	0.53289	0.49515	0.46959
24	17	0.70833	0.60292	0.55840	0.52127	0.48905	0.45185	0.42683
24	16	0.66667	0.56021	0.51551	0.47858	0.44678	0.41035	0.38601
24	15	0.62500	0.51814	0.47359	0.43711	0.40594	0.37049	0.34698
24	14	0.58333	0.47669	0.43258	0.39679	0.36643	0.33218	0.30961
24	13	0.54167	0.43582	0.39245	0.35756	0.32821	0.29534	0.27383
24	12	0.50000	0.39553	0.35317	0.31942	0.29124	0.25994	0.23962

n	r	p	80%	90%	95%	97.5%	99%	99.5%
25	25	1.00000	0.93765	0.91201	0.88707	0.86281	0.83176	0.80902
25	24	0.96000	0.88491	0.85313	0.82388	0.79648	0.76251	0.73824
25	23	0.92000	0.83652	0.80086	0.76896	0.73969	0.70407	0.67899

n	r	p	80%	90%	95%	97.5%	99%	99.5%
25	22	0.88000	0.79038	0.75198	0.71828	0.68781	0.65122	0.62574
25	21	0.84000	0.74573	0.70533	0.67039	0.63917	0.60208	0.57649
25	20	0.80000	0.70219	0.66034	0.62459	0.59296	0.55573	0.53024
25	19	0.76000	0.65954	0.61669	0.58048	0.54871	0.51163	0.48643
25	18	0.72000	0.61765	0.57418	0.53779	0.50612	0.46944	0.44468
25	17	0.68000	0.57643	0.53266	0.49636	0.46500	0.42894	0.40476
25	16	0.64000	0.53581	0.49205	0.45607	0.42521	0.38997	0.36651
25	15	0.60000	0.49576	0.45228	0.41684	0.38665	0.35244	0.32979
25	14	0.56000	0.45625	0.41332	0.37862	0.34928	0.31626	0.29455
25	13	0.52000	0.41725	0.37514	0.34139	0.31306	0.28141	0.26074
25	12	0.48000	0.37878	0.33774	0.30513	0.27797	0.24786	0.22835

n	r	p	80%	90%	95%	97.5%	99%	99.5%
26	26	1.00000	0.93998	0.91525	0.89117	0.86773	0.83768	0.81564
26	25	0.96154	0.88917	0.85847	0.83017	0.80363	0.77068	0.74709
26	24	0.92308	0.84253	0.80803	0.77711	0.74870	0.71406	0.68962
26	23	0.88462	0.79804	0.76083	0.72810	0.69846	0.66279	0.63791
26	22	0.84615	0.75497	0.71575	0.68176	0.65132	0.61508	0.59002
26	21	0.80769	0.71296	0.67226	0.63741	0.60649	0.57002	0.54500
26	20	0.76923	0.67179	0.63003	0.59465	0.56352	0.52710	0.50228
26	19	0.73077	0.63134	0.58888	0.55323	0.52213	0.48598	0.46152
26	18	0.69231	0.59151	0.54866	0.51300	0.48210	0.44646	0.42248
26	17	0.65385	0.55225	0.50929	0.47384	0.44333	0.40837	0.38500
26	16	0.61538	0.51351	0.47071	0.43566	0.40571	0.37161	0.34896
26	15	0.57692	0.47527	0.43286	0.39842	0.36918	0.33612	0.31429
26	14	0.53846	0.43751	0.39574	0.36209	0.33371	0.30184	0.28094
26	13	0.50000	0.40022	0.35932	0.32664	0.29927	0.26876	0.24888

n	r	p	80%	90%	95%	97.5%	99%	99.5%
27	27	1.00000	0.94213	0.91825	0.89498	0.87230	0.84319	0.82182
27	26	0.96296	0.89312	0.86343	0.83603	0.81029	0.77830	0.75536
27	25	0.92593	0.84811	0.81470	0.78470	0.75710	0.72339	0.69958
27	24	0.88889	0.80516	0.76906	0.73726	0.70841	0.67363	0.64933
27	23	0.85185	0.76357	0.72546	0.69237	0.66269	0.62727	0.60273
27	22	0.81481	0.72298	0.68337	0.64938	0.61917	0.58344	0.55888
27	21	0.77778	0.68319	0.64248	0.60790	0.57742	0.54164	0.51722
27	20	0.74074	0.64408	0.60261	0.56770	0.53715	0.50156	0.47742
27	19	0.70370	0.60556	0.56362	0.52861	0.49819	0.46298	0.43924
27	18	0.66667	0.56757	0.52542	0.49052	0.46039	0.42575	0.40253
27	17	0.62963	0.53008	0.48796	0.45336	0.42368	0.38977	0.36717
27	16	0.59259	0.49304	0.45119	0.41707	0.38798	0.35497	0.33309
27	15	0.55556	0.45644	0.41509	0.38161	0.35326	0.32129	0.30022
27	14	0.51852	0.42028	0.37963	0.34697	0.31950	0.28871	0.26855
27	13	0.48148	0.38454	0.34481	0.31314	0.28667	0.25723	0.23806

n	r	p	80%	90%	95%	97.5%	99%	99.5%
28	28	1.00000	0.94414	0.92106	0.89853	0.87656	0.84834	0.82760
28	27	0.96429	0.89680	0.86806	0.84149	0.81652	0.78543	0.76312
28	26	0.92857	0.85331	0.82092	0.79180	0.76497	0.73215	0.70893
28	25	0.89286	0.81180	0.77675	0.74583	0.71774	0.68381	0.66006
28	24	0.85714	0.77158	0.73454	0.70231	0.67335	0.63873	0.61470
28	23	0.82143	0.73232	0.69376	0.66060	0.63107	0.59607	0.57196
28	22	0.78571	0.69383	0.65413	0.62033	0.59047	0.55535	0.53132
28	21	0.75000	0.65598	0.61546	0.58127	0.55128	0.51626	0.49244
28	20	0.71429	0.61869	0.57763	0.54327	0.51333	0.47859	0.45510
28	19	0.67857	0.58190	0.54055	0.50621	0.47648	0.44220	0.41915
28	18	0.64286	0.54556	0.50416	0.47002	0.44065	0.40699	0.38447
28	17	0.60714	0.50966	0.46841	0.43464	0.40577	0.37287	0.35099
28	16	0.57143	0.47417	0.43328	0.40004	0.37179	0.33980	0.31864
28	15	0.53571	0.43907	0.39874	0.36620	0.33870	0.30775	0.28740
28	14	0.50000	0.40437	0.36479	0.33309	0.30647	0.27670	0.25724

n	r	p	80%	90%	95%	97.5%	99%	99.5%
29	29	1.00000	0.94601	0.92367	0.90186	0.88056	0.85317	0.83302
29	28	0.96552	0.90024	0.87238	0.84661	0.82236	0.79212	0.77040
29	27	0.93103	0.85817	0.82673	0.79844	0.77234	0.74037	0.71772
29	26	0.89655	0.81800	0.78395	0.75386	0.72648	0.69337	0.67016
29	25	0.86207	0.77907	0.74304	0.71163	0.68336	0.64951	0.62598
29	24	0.82759	0.74106	0.70350	0.67113	0.64225	0.60797	0.58430
29	23	0.79310	0.70379	0.66505	0.63200	0.60275	0.56828	0.54464
29	22	0.75862	0.66712	0.62752	0.59403	0.56460	0.53014	0.50666
29	21	0.72414	0.63098	0.59079	0.55706	0.52762	0.49336	0.47014
29	20	0.68966	0.59531	0.55476	0.52099	0.49168	0.45778	0.43493
29	19	0.65517	0.56007	0.51938	0.48573	0.45669	0.42332	0.40093
29	18	0.62069	0.52524	0.48461	0.45123	0.42260	0.38988	0.36804
29	17	0.58621	0.49079	0.45041	0.41746	0.38936	0.35743	0.33623
29	16	0.55172	0.45671	0.41676	0.38439	0.35694	0.32592	0.30544
29	15	0.51724	0.42299	0.38366	0.35200	0.32531	0.29534	0.27565
29	14	0.48276	0.38963	0.35109	0.32030	0.29449	0.26567	0.24686

n	r	p	80%	90%	95%	97.5%	99%	99.5%
30	30	1.00000	0.94777	0.92612	0.90497	0.88430	0.85770	0.83811
30	29	0.96667	0.90345	0.87643	0.85140	0.82783	0.79841	0.77725
30	28	0.93333	0.86271	0.83219	0.80467	0.77926	0.74810	0.72600
30	27	0.90000	0.82380	0.79070	0.76140	0.73471	0.70238	0.67969
30	26	0.86667	0.78609	0.75101	0.72038	0.69278	0.65967	0.63662
30	25	0.83333	0.74925	0.71264	0.68103	0.65279	0.61920	0.59597
30	24	0.80000	0.71311	0.67531	0.64299	0.61433	0.58049	0.55724

n	r	p	80%	90%	95%	97.5%	99%	99.5%
34	29	0.85294	0.77742	0.74420	0.71535	0.68943	0.65842	0.63686
34	28	0.82353	0.74523	0.71078	0.68113	0.65468	0.62325	0.60152
34	27	0.79412	0.71353	0.67809	0.64784	0.62102	0.58934	0.56755
34	26	0.76471	0.68224	0.64604	0.61534	0.58829	0.55651	0.53476
34	25	0.73529	0.65133	0.61454	0.58355	0.55638	0.52463	0.50300
34	24	0.70588	0.62075	0.58354	0.55239	0.52522	0.49362	0.47218
34	23	0.67647	0.59047	0.55300	0.52181	0.49473	0.46339	0.44222

30	23	0.76667	0.67756	0.63886	0.60605	0.57716	0.54327	0.52012
30	22	0.73333	0.64250	0.60316	0.57007	0.54111	0.50734	0.48440
30	21	0.70000	0.60790	0.56813	0.53493	0.50604	0.47256	0.44992
30	20	0.66667	0.57370	0.53372	0.50056	0.47188	0.43882	0.41658
30	19	0.63333	0.53988	0.49987	0.46691	0.43856	0.40605	0.38430
30	18	0.60000	0.50642	0.46657	0.43395	0.40603	0.37421	0.35302
30	17	0.56667	0.47330	0.43378	0.40163	0.37427	0.34325	0.32270
30	16	0.53333	0.44052	0.40149	0.36995	0.34326	0.31315	0.29331
30	15	0.50000	0.40807	0.36970	0.33889	0.31297	0.28390	0.26485

n	r	p	80%	90%	95%	97.5%	99%	99.5%
31	31	1.00000	0.94941	0.92841	0.90789	0.88781	0.86195	0.84289
31	30	0.96774	0.90647	0.88023	0.85591	0.83298	0.80433	0.78370
31	29	0.93548	0.86698	0.83731	0.81054	0.78578	0.75539	0.73381
31	28	0.90323	0.82925	0.79704	0.76850	0.74246	0.71088	0.68868
31	27	0.87097	0.79267	0.75850	0.72863	0.70166	0.66927	0.64669
31	26	0.83871	0.75694	0.72123	0.69036	0.66273	0.62981	0.60702
31	25	0.80645	0.72188	0.68496	0.65335	0.62527	0.59205	0.56919
31	24	0.77419	0.68737	0.64953	0.61739	0.58904	0.55571	0.53290
31	23	0.74194	0.65334	0.61481	0.58234	0.55387	0.52060	0.49795
31	22	0.70968	0.61973	0.58074	0.54810	0.51964	0.48658	0.46418
31	21	0.67742	0.58651	0.54724	0.51458	0.48627	0.45355	0.43149
31	20	0.64516	0.55365	0.51428	0.48175	0.45370	0.42144	0.39980
31	19	0.61290	0.52113	0.48183	0.44956	0.42187	0.39021	0.36906
31	18	0.58065	0.48892	0.44986	0.41797	0.39076	0.35980	0.33922
31	17	0.54839	0.45703	0.41835	0.38698	0.36034	0.33019	0.31025
31	16	0.51613	0.42545	0.38731	0.35657	0.33061	0.30138	0.28214
31	15	0.48387	0.39417	0.35673	0.32674	0.30155	0.27334	0.25488

n	r	p	80%	90%	95%	97.5%	99%	99.5%
32	32	1.00000	0.95095	0.93057	0.91063	0.89112	0.86596	0.84741
32	31	0.96875	0.90930	0.88380	0.86015	0.83783	0.80991	0.78980
32	30	0.93750	0.87098	0.84213	0.81606	0.79193	0.76227	0.74119
32	29	0.90625	0.83437	0.80301	0.77518	0.74977	0.71891	0.69719
32	28	0.87500	0.79887	0.76556	0.73640	0.71005	0.67835	0.65622
32	27	0.84375	0.76417	0.72933	0.69916	0.67212	0.63986	0.61749
32	26	0.81250	0.73012	0.69406	0.66313	0.63561	0.60300	0.58052
32	25	0.78125	0.69660	0.65959	0.62810	0.60027	0.56750	0.54504
32	24	0.75000	0.66354	0.62581	0.59394	0.56595	0.53318	0.51083
32	23	0.71875	0.63088	0.59263	0.56055	0.53253	0.49990	0.47775
32	22	0.68750	0.59858	0.56001	0.52786	0.49992	0.46756	0.44570
32	21	0.65625	0.56663	0.52790	0.49581	0.46807	0.43610	0.41460
32	20	0.62500	0.53499	0.49626	0.46436	0.43692	0.40546	0.38439
32	19	0.59375	0.50366	0.46508	0.43349	0.40645	0.37559	0.35503
32	18	0.56250	0.47262	0.43433	0.40317	0.37663	0.34649	0.32649
32	17	0.53125	0.44186	0.40401	0.37339	0.34744	0.31812	0.29875
32	16	0.50000	0.41139	0.37412	0.34415	0.31887	0.29047	0.27180

n	r	p	80%	90%	95%	97.5%	99%	99.5%
33	33	1.00000	0.95240	0.93260	0.91322	0.89424	0.86975	0.85167
33	32	0.96970	0.91196	0.88717	0.86415	0.84241	0.81519	0.79556
33	31	0.93939	0.87476	0.84667	0.82127	0.79774	0.76878	0.74817
33	30	0.90909	0.83919	0.80864	0.78150	0.75668	0.72651	0.70526
33	29	0.87879	0.80470	0.77222	0.74375	0.71798	0.68695	0.66526
33	28	0.84848	0.77099	0.73697	0.70748	0.68101	0.64938	0.62742
33	27	0.81818	0.73789	0.70265	0.67237	0.64540	0.61339	0.59129
33	26	0.78788	0.70530	0.66910	0.63824	0.61092	0.57870	0.55657
33	25	0.75758	0.67315	0.63620	0.60493	0.57741	0.54513	0.52308
33	24	0.72727	0.64139	0.60388	0.57235	0.54476	0.51257	0.49068
33	23	0.69697	0.60998	0.57209	0.54044	0.51289	0.48090	0.45925
33	22	0.66667	0.57889	0.54078	0.50914	0.48173	0.45007	0.42872
33	21	0.63636	0.54809	0.50993	0.47842	0.45125	0.42001	0.39904
33	20	0.60606	0.51759	0.47950	0.44823	0.42139	0.39069	0.37017
33	19	0.57576	0.48735	0.44948	0.41856	0.39215	0.36208	0.34206
33	18	0.54545	0.45739	0.41986	0.38940	0.36351	0.33416	0.31471
33	17	0.51515	0.42768	0.39064	0.36074	0.33544	0.30691	0.28809
33	16	0.48485	0.39824	0.36181	0.33258	0.30796	0.28034	0.26221

n	r	p	80%	90%	95%	97.5%	99%	99.5%
34	34	1.00000	0.95377	0.93452	0.91566	0.89718	0.87333	0.85570
34	33	0.97059	0.91448	0.89035	0.86793	0.84673	0.82018	0.80101
34	32	0.94118	0.87832	0.85095	0.82619	0.80323	0.77494	0.75479
34	31	0.91176	0.84374	0.81396	0.78747	0.76322	0.73372	0.71290
34	30	0.88235	0.81021	0.77851	0.75070	0.72550	0.69511	0.67384

34	22	0.64706	0.56048	0.52289	0.49177	0.46489	0.43390	0.41306
34	21	0.61765	0.53076	0.49318	0.46225	0.43564	0.40512	0.38467
34	20	0.58824	0.50130	0.46386	0.43321	0.40697	0.37700	0.35700
34	19	0.55882	0.47209	0.43492	0.40466	0.37886	0.34953	0.33004
34	18	0.52941	0.44312	0.40635	0.37657	0.35129	0.32269	0.30377
34	17	0.50000	0.41440	0.37814	0.34894	0.32427	0.29649	0.27819

n	r	p	80%	90%	95%	97.5%	99%	99.5%
35	35	1.00000	0.95506	0.93633	0.91797	0.89997	0.87671	0.85952
35	34	0.97143	0.91685	0.89335	0.87150	0.85083	0.82491	0.80618
35	33	0.94286	0.88168	0.85501	0.83085	0.80843	0.78079	0.76108
35	32	0.91429	0.84805	0.81899	0.79312	0.76942	0.74055	0.72017
35	31	0.88571	0.81541	0.78447	0.75728	0.73262	0.70285	0.68200
35	30	0.85714	0.78350	0.75104	0.72282	0.69743	0.66701	0.64584
35	29	0.82857	0.75217	0.71847	0.68944	0.66350	0.63263	0.61127
35	28	0.80000	0.72130	0.68661	0.65695	0.63062	0.59947	0.57802
35	27	0.77143	0.69084	0.65536	0.62523	0.59864	0.56734	0.54589
35	26	0.74286	0.66073	0.62464	0.59418	0.56744	0.53613	0.51477
35	25	0.71429	0.63094	0.59439	0.56374	0.53696	0.50574	0.48453
35	24	0.68571	0.60144	0.56459	0.53385	0.50712	0.47611	0.45513
35	23	0.65714	0.57222	0.53519	0.50448	0.47789	0.44718	0.42648
35	22	0.62857	0.54324	0.50618	0.47560	0.44923	0.41892	0.39856
35	21	0.60000	0.51452	0.47753	0.44718	0.42112	0.39129	0.37133
35	20	0.57143	0.48602	0.44924	0.41920	0.39353	0.36427	0.34477
35	19	0.54286	0.45776	0.42130	0.39167	0.36646	0.33784	0.31886
35	18	0.51429	0.42973	0.39369	0.36457	0.33989	0.31201	0.29359
35	17	0.48571	0.40192	0.36643	0.33789	0.31383	0.28676	0.26895

n	r	p	80%	90%	95%	97.5%	99%	99.5%
36	36	1.00000	0.95628	0.93804	0.92015	0.90261	0.87992	0.86314
36	35	0.97222	0.91910	0.89620	0.87488	0.85471	0.82940	0.81109
36	34	0.94444	0.88486	0.85885	0.83527	0.81336	0.78633	0.76705
36	33	0.91667	0.85212	0.82375	0.79848	0.77531	0.74705	0.72708
36	32	0.88889	0.82034	0.79011	0.76352	0.73939	0.71022	0.68976
36	31	0.86111	0.78926	0.75753	0.72990	0.70503	0.67519	0.65439
36	30	0.83333	0.75874	0.72577	0.69732	0.67188	0.64157	0.62056
36	29	0.80556	0.72867	0.69470	0.66561	0.63975	0.60912	0.58800
36	28	0.77778	0.69899	0.66421	0.63463	0.60848	0.57767	0.55652
36	27	0.75000	0.66964	0.63423	0.60429	0.57797	0.54710	0.52600
36	26	0.72222	0.64061	0.60470	0.57454	0.54814	0.51732	0.49635
36	25	0.69444	0.61185	0.57560	0.54532	0.51893	0.48826	0.46748
36	24	0.66667	0.58335	0.54689	0.51658	0.49030	0.45988	0.43934
36	23	0.63889	0.55509	0.51854	0.48832	0.46221	0.43213	0.41189
36	22	0.61111	0.52706	0.49054	0.46049	0.43464	0.40497	0.38509
36	21	0.58333	0.49926	0.46288	0.43309	0.40757	0.37840	0.35893
36	20	0.55556	0.47167	0.43554	0.40609	0.38098	0.35239	0.33338
36	19	0.52778	0.44430	0.40852	0.37951	0.35486	0.32694	0.30843
36	18	0.50000	0.41713	0.38182	0.35332	0.32922	0.30203	0.28408

n	r	p	80%	90%	95%	97.5%	99%	99.5%
37	37	1.00000	0.95743	0.93966	0.92223	0.90511	0.88297	0.86658
37	36	0.97297	0.92122	0.89889	0.87810	0.85840	0.83366	0.81576
37	35	0.94595	0.88787	0.86249	0.83946	0.81805	0.79161	0.77273
37	34	0.91892	0.85597	0.82828	0.80357	0.78090	0.75323	0.73366
37	33	0.89189	0.82501	0.79547	0.76946	0.74582	0.71723	0.69716
37	32	0.86486	0.79472	0.76368	0.73663	0.71225	0.68297	0.66255
37	31	0.83784	0.76497	0.73270	0.70482	0.67986	0.65008	0.62942
37	30	0.81081	0.73566	0.70238	0.67384	0.64845	0.61832	0.59752
37	29	0.78378	0.70672	0.67261	0.64357	0.61786	0.58753	0.56668
37	28	0.75676	0.67810	0.64334	0.61392	0.58801	0.55758	0.53675
37	27	0.72973	0.64978	0.61451	0.58483	0.55881	0.52838	0.50765
37	26	0.70270	0.62173	0.58608	0.55624	0.53020	0.49989	0.47930
37	25	0.67568	0.59392	0.55802	0.52813	0.50215	0.47203	0.45165
37	24	0.64865	0.56634	0.53031	0.50045	0.47461	0.44478	0.42467
37	23	0.62162	0.53898	0.50293	0.47320	0.44757	0.41810	0.39830
37	22	0.59459	0.51184	0.47587	0.44634	0.42100	0.39196	0.37254
37	21	0.56757	0.48489	0.44911	0.41988	0.39488	0.36637	0.34735
37	20	0.54054	0.45815	0.42266	0.39380	0.36922	0.34129	0.32274
37	19	0.51351	0.43161	0.39650	0.36810	0.34400	0.31673	0.29868
37	18	0.48649	0.40526	0.37064	0.34277	0.31921	0.29268	0.27518

n	r	p	80%	90%	95%	97.5%	99%	99.5%
38	38	1.00000	0.95853	0.94120	0.92419	0.90749	0.88587	0.86985
38	37	0.97368	0.92324	0.90145	0.88115	0.86190	0.83772	0.82021

n	r	p	80%	90%	95%	97.5%	99%	99.5%
38	36	0.94737	0.89074	0.86595	0.84344	0.82251	0.79663	0.77814
38	35	0.92105	0.85964	0.83257	0.80841	0.78623	0.75912	0.73993
38	34	0.89474	0.82944	0.80056	0.77510	0.75195	0.72391	0.70421
38	33	0.86842	0.79991	0.76954	0.74304	0.71914	0.69040	0.67033
38	32	0.84211	0.77089	0.73929	0.71196	0.68747	0.65821	0.63789
38	31	0.81579	0.74230	0.70968	0.68168	0.65674	0.62711	0.60663
38	30	0.78947	0.71406	0.68061	0.65209	0.62681	0.59694	0.57639
38	29	0.76316	0.68614	0.65202	0.62309	0.59759	0.56759	0.54703
38	28	0.73684	0.65850	0.62385	0.59463	0.56899	0.53897	0.51847
38	27	0.71053	0.63112	0.59606	0.56666	0.54097	0.51101	0.49063
38	26	0.68421	0.60397	0.56863	0.53914	0.51347	0.48367	0.46347
38	25	0.65789	0.57704	0.54153	0.51204	0.48647	0.45690	0.43693
38	24	0.63158	0.55032	0.51474	0.48534	0.45994	0.43068	0.41099
38	23	0.60526	0.52381	0.48826	0.45902	0.43386	0.40499	0.38562
38	22	0.57895	0.49748	0.46207	0.43307	0.40821	0.37979	0.36080

n	r	p	80%	90%	95%	97.5%	99%	99.5%
41	25	0.60976	0.53185	0.49760	0.46938	0.44505	0.41706	0.39825
41	24	0.58537	0.50740	0.47323	0.44518	0.42110	0.39348	0.37498
41	23	0.56098	0.48310	0.44909	0.42129	0.39750	0.37031	0.35215
41	22	0.53659	0.45895	0.42519	0.39770	0.37425	0.34755	0.32977
41	21	0.51220	0.43496	0.40151	0.37440	0.35134	0.32519	0.30783
41	20	0.48780	0.41112	0.37807	0.35138	0.32878	0.30322	0.28632

n	r	p	80%	90%	95%	97.5%	99%	99.5%
42	42	1.00000	0.96240	0.94665	0.93116	0.91592	0.89615	0.88148
42	41	0.97619	0.93038	0.91052	0.89196	0.87434	0.85215	0.83604
42	40	0.95238	0.90086	0.87821	0.85759	0.83836	0.81453	0.79745
42	39	0.92857	0.87260	0.84780	0.82561	0.80517	0.78012	0.76233
42	38	0.90476	0.84514	0.81862	0.79517	0.77378	0.74778	0.72945
42	37	0.88095	0.81828	0.79032	0.76584	0.74368	0.71695	0.69821

38	21	0.55263	0.47135	0.43616	0.40748	0.38299	0.35510	0.33652
38	20	0.52632	0.44540	0.41054	0.38224	0.35818	0.33088	0.31277
38	19	0.50000	0.41964	0.38519	0.35736	0.33379	0.30715	0.28954

n	r	p	80%	90%	95%	97.5%	99%	99.5%
39	39	1.00000	0.95957	0.94267	0.92606	0.90975	0.88862	0.87297
39	38	0.97436	0.92516	0.90389	0.88405	0.86524	0.84158	0.82444
39	37	0.94872	0.89345	0.86924	0.84723	0.82676	0.80142	0.78330
39	36	0.92308	0.86312	0.83666	0.81302	0.79130	0.76473	0.74591
39	35	0.89744	0.83366	0.80540	0.78048	0.75779	0.73029	0.71095
39	34	0.87179	0.80484	0.77511	0.74915	0.72570	0.69749	0.67777
39	33	0.84615	0.77652	0.74557	0.71876	0.69472	0.66597	0.64598
39	32	0.82051	0.74861	0.71664	0.68916	0.66465	0.63551	0.61534
39	31	0.79487	0.72105	0.68823	0.66021	0.63536	0.60594	0.58568
39	30	0.76923	0.69379	0.66029	0.63185	0.60674	0.57717	0.55688
39	29	0.74359	0.66680	0.63274	0.60400	0.57873	0.54909	0.52884
39	28	0.71795	0.64006	0.60557	0.57661	0.55126	0.52166	0.50150
39	27	0.69231	0.61354	0.57874	0.54966	0.52431	0.49482	0.47481
39	26	0.66667	0.58723	0.55223	0.52311	0.49783	0.46853	0.44871
39	25	0.64103	0.56113	0.52602	0.49695	0.47180	0.44276	0.42319
39	24	0.61538	0.53521	0.50009	0.47114	0.44619	0.41749	0.39821
39	23	0.58974	0.50948	0.47445	0.44569	0.42100	0.39270	0.37375
39	22	0.56410	0.48393	0.44907	0.42058	0.39621	0.36838	0.34981
39	21	0.53846	0.45855	0.42395	0.39581	0.37181	0.34452	0.32636
39	20	0.51282	0.43335	0.39910	0.37136	0.34780	0.32111	0.30341
39	19	0.48718	0.40832	0.37451	0.34725	0.32418	0.29815	0.28095

n	r	p	80%	90%	95%	97.5%	99%	99.5%
40	40	1.00000	0.96056	0.94406	0.92784	0.91190	0.89125	0.87594
40	39	0.97500	0.92699	0.90620	0.88681	0.86841	0.84527	0.82848
40	38	0.95000	0.89604	0.87237	0.85085	0.83080	0.80599	0.78823
40	37	0.92500	0.86643	0.84055	0.81741	0.79614	0.77009	0.75163
40	36	0.90000	0.83767	0.81002	0.78560	0.76336	0.73638	0.71739
40	35	0.87500	0.80954	0.78042	0.75497	0.73197	0.70426	0.68488
40	34	0.85000	0.78188	0.75155	0.72526	0.70165	0.67339	0.65372
40	33	0.82500	0.75463	0.72327	0.69629	0.67221	0.64354	0.62368
40	32	0.80000	0.72770	0.69550	0.66797	0.64352	0.61456	0.59458
40	31	0.77500	0.70108	0.66817	0.64021	0.61549	0.58634	0.56631
40	30	0.75000	0.67471	0.64124	0.61294	0.58804	0.55880	0.53879
40	29	0.72500	0.64858	0.61465	0.58612	0.56112	0.53187	0.51193
40	28	0.70000	0.62266	0.58840	0.55972	0.53468	0.50552	0.48569
40	27	0.67500	0.59695	0.56245	0.53370	0.50871	0.47969	0.46003
40	26	0.65000	0.57143	0.53679	0.50805	0.48316	0.45436	0.43492
40	25	0.62500	0.54609	0.51140	0.48275	0.45801	0.42951	0.41032
40	24	0.60000	0.52093	0.48628	0.45778	0.43327	0.40512	0.38623
40	23	0.57500	0.49593	0.46141	0.43314	0.40890	0.38117	0.36262
40	22	0.55000	0.47110	0.43679	0.40881	0.38491	0.35765	0.33949
40	21	0.52500	0.44644	0.41242	0.38480	0.36128	0.33457	0.31682
40	20	0.50000	0.42194	0.38830	0.36109	0.33802	0.31190	0.29461

n	r	p	80%	90%	95%	97.5%	99%	99.5%
41	41	1.00000	0.96151	0.94539	0.92954	0.91396	0.89376	0.87877
41	40	0.97561	0.92872	0.90841	0.88945	0.87145	0.84878	0.83234
41	39	0.95122	0.89851	0.87536	0.85429	0.83467	0.81035	0.79294
41	38	0.92683	0.86959	0.84426	0.82160	0.80075	0.77522	0.75709
41	37	0.90244	0.84150	0.81442	0.79049	0.76869	0.74220	0.72355
41	36	0.87805	0.81401	0.78549	0.76053	0.73796	0.71074	0.69169
41	35	0.85366	0.78699	0.75726	0.73146	0.70827	0.68049	0.66114
41	34	0.82927	0.76036	0.72960	0.70311	0.67944	0.65123	0.63167
41	33	0.80488	0.73405	0.70244	0.67539	0.65133	0.62281	0.60312
41	32	0.78049	0.70802	0.67570	0.64820	0.62386	0.59513	0.57537
41	31	0.75610	0.68225	0.64935	0.62149	0.59695	0.56810	0.54833
41	30	0.73171	0.65670	0.62333	0.59522	0.57056	0.54167	0.52194
41	29	0.70732	0.63136	0.59763	0.56935	0.54463	0.51578	0.49615
41	28	0.68293	0.60622	0.57222	0.54385	0.51913	0.49041	0.47092
41	27	0.65854	0.58126	0.54709	0.51869	0.49405	0.46551	0.44620
41	26	0.63415	0.55647	0.52222	0.49388	0.46936	0.44107	0.42199

n	r	p	80%	90%	95%	97.5%	99%	99.5%
45	45	1.00000	0.96487	0.95012	0.93560	0.92129	0.90273	0.88893
45	44	0.97778	0.93492	0.91629	0.89887	0.88230	0.86139	0.84619
45	43	0.95556	0.90730	0.88602	0.86662	0.84851	0.82601	0.80986
45	42	0.93333	0.88085	0.85753	0.83661	0.81732	0.79362	0.77676
45	41	0.91111	0.85515	0.83016	0.80802	0.78779	0.76315	0.74574
45	40	0.88889	0.82999	0.80361	0.78046	0.75946	0.73407	0.71623
45	39	0.86667	0.80525	0.77769	0.75370	0.73208	0.70607	0.68790
45	38	0.84444	0.78085	0.75228	0.72759	0.70545	0.67897	0.66054
45	37	0.82222	0.75674	0.72731	0.70203	0.67947	0.65261	0.63399
45	36	0.80000	0.73287	0.70271	0.67694	0.65404	0.62690	0.60815
45	35	0.77778	0.70923	0.67845	0.65227	0.62911	0.60176	0.58294
45	34	0.75556	0.68579	0.65448	0.62798	0.60463	0.57715	0.55830
45	33	0.73333	0.66252	0.63079	0.60404	0.58055	0.55301	0.53418
45	32	0.71111	0.63943	0.60735	0.58042	0.55685	0.52931	0.51054
45	31	0.68889	0.61649	0.58414	0.55710	0.53351	0.50603	0.48734
45	30	0.66667	0.59370	0.56116	0.53406	0.51050	0.48313	0.46458
45	29	0.64444	0.57105	0.53839	0.51129	0.48780	0.46060	0.44221
45	28	0.62222	0.54853	0.51582	0.48878	0.46541	0.43843	0.42024
45	27	0.60000	0.52615	0.49345	0.46652	0.44331	0.41660	0.39864
45	26	0.57778	0.50389	0.47128	0.44451	0.42150	0.39511	0.37741
45	25	0.55556	0.48176	0.44929	0.42273	0.39997	0.37394	0.35653
45	24	0.53333	0.45975	0.42748	0.40118	0.37872	0.35310	0.33601
45	23	0.51111	0.43786	0.40586	0.37987	0.35774	0.33258	0.31584

42	36	0.85714	0.79187	0.76271	0.73738	0.71461	0.68729	0.66824
42	35	0.83333	0.76583	0.73565	0.70963	0.68636	0.65860	0.63933
42	34	0.80952	0.74011	0.70907	0.68248	0.65882	0.63072	0.61131
42	33	0.78571	0.71466	0.68290	0.65585	0.63188	0.60356	0.58406
42	32	0.76190	0.68945	0.65710	0.62968	0.60550	0.57703	0.55750
42	31	0.73810	0.66446	0.63163	0.60393	0.57960	0.55107	0.53157
42	30	0.71429	0.63968	0.60646	0.57857	0.55416	0.52564	0.50621
42	29	0.69048	0.61508	0.58157	0.55356	0.52914	0.50070	0.48139
42	28	0.66667	0.59066	0.55695	0.52889	0.50451	0.47623	0.45707
42	27	0.64286	0.56640	0.53258	0.50454	0.48026	0.45218	0.43323
42	26	0.61905	0.54230	0.50845	0.48050	0.45637	0.42856	0.40984
42	25	0.59524	0.51836	0.48455	0.45674	0.43282	0.40534	0.38690
42	24	0.57143	0.49457	0.46088	0.43328	0.40961	0.38252	0.36439
42	23	0.54762	0.47092	0.43743	0.41009	0.38673	0.36008	0.34229
42	22	0.52381	0.44742	0.41419	0.38719	0.36418	0.33801	0.32061
42	21	0.50000	0.42406	0.39118	0.36455	0.34195	0.31633	0.29935

n	r	p	80%	90%	95%	97.5%	99%	99.5%
43	43	1.00000	0.96326	0.94786	0.93270	0.91779	0.89844	0.88407
43	42	0.97674	0.93196	0.91253	0.89437	0.87711	0.85536	0.83957
43	41	0.95349	0.90310	0.88093	0.86073	0.84189	0.81852	0.80176
43	40	0.93023	0.87547	0.85119	0.82944	0.80939	0.78481	0.76734
43	39	0.90698	0.84863	0.82264	0.79964	0.77865	0.75312	0.73511
43	38	0.88372	0.82236	0.79495	0.77093	0.74917	0.72289	0.70446
43	37	0.86047	0.79653	0.76792	0.74306	0.72068	0.69381	0.67506
43	36	0.83721	0.77106	0.74144	0.71587	0.69299	0.66566	0.64668
43	35	0.81395	0.74590	0.71542	0.68927	0.66599	0.63831	0.61917
43	34	0.79070	0.72100	0.68979	0.66318	0.63958	0.61165	0.59241
43	33	0.76744	0.69634	0.66452	0.63753	0.61369	0.58560	0.56631
43	32	0.74419	0.67188	0.63957	0.61228	0.58828	0.56011	0.54082
43	31	0.72093	0.64763	0.61491	0.58741	0.56331	0.53512	0.51589
43	30	0.69767	0.62355	0.59053	0.56288	0.53875	0.51061	0.49147
43	29	0.67442	0.59964	0.56639	0.53868	0.51456	0.48654	0.46753
43	28	0.65116	0.57589	0.54250	0.51478	0.49073	0.46289	0.44406
43	27	0.62791	0.55230	0.51884	0.49117	0.46725	0.43964	0.42102
43	26	0.60465	0.52885	0.49540	0.46785	0.44410	0.41677	0.39840
43	25	0.58140	0.50555	0.47218	0.44480	0.42127	0.39429	0.37620
43	24	0.55814	0.48238	0.44917	0.42201	0.39875	0.37217	0.35439
43	23	0.53488	0.45935	0.42636	0.39949	0.37655	0.35040	0.33298
43	22	0.51163	0.43646	0.40376	0.37723	0.35465	0.32900	0.31196
43	21	0.48837	0.41370	0.38136	0.35522	0.33305	0.30795	0.29133

n	r	p	80%	90%	95%	97.5%	99%	99.5%
44	44	1.00000	0.96408	0.94901	0.93418	0.91958	0.90063	0.88655
44	43	0.97727	0.93347	0.91445	0.89666	0.87976	0.85844	0.84295
44	42	0.95455	0.90525	0.88353	0.86374	0.84527	0.82234	0.80590
44	41	0.93182	0.87822	0.85443	0.83310	0.81344	0.78931	0.77215
44	40	0.90909	0.85196	0.82648	0.80392	0.78331	0.75824	0.74053
44	39	0.88636	0.82626	0.79937	0.77580	0.75442	0.72859	0.71047
44	38	0.86364	0.80098	0.77291	0.74849	0.72649	0.70007	0.68161
44	37	0.84091	0.77606	0.74698	0.72185	0.69935	0.67245	0.65375
44	36	0.81818	0.75143	0.72149	0.69578	0.67286	0.64560	0.62673
44	35	0.79545	0.72706	0.69639	0.67020	0.64695	0.61942	0.60043
44	34	0.77273	0.70292	0.67163	0.64505	0.62156	0.59384	0.57479
44	33	0.75000	0.67899	0.64718	0.62029	0.59662	0.56879	0.54973
44	32	0.72727	0.65524	0.62302	0.59590	0.57210	0.54424	0.52520
44	31	0.70455	0.63166	0.59911	0.57183	0.54798	0.52014	0.50118
44	30	0.68182	0.60825	0.57545	0.54807	0.52422	0.49647	0.47762
44	29	0.65909	0.58499	0.55202	0.52461	0.50081	0.47320	0.45450
44	28	0.63636	0.56187	0.52881	0.50143	0.47772	0.45031	0.43180
44	27	0.61364	0.53890	0.50582	0.47852	0.45496	0.42780	0.40951
44	26	0.59091	0.51606	0.48303	0.45587	0.43250	0.40564	0.38761
44	25	0.56818	0.49336	0.46044	0.43347	0.41034	0.38384	0.36609
44	24	0.54545	0.47079	0.43805	0.41133	0.38847	0.36237	0.34495
44	23	0.52273	0.44834	0.41585	0.38943	0.36690	0.34125	0.32418
44	22	0.50000	0.42603	0.39385	0.36777	0.34561	0.32047	0.30377

n	r	p	80%	90%	95%	97.5%	99%	99.5%
48	45	0.93750	0.88810	0.86609	0.84631	0.82804	0.80556	0.78955
48	44	0.91667	0.86394	0.84033	0.81936	0.80017	0.77676	0.76019
48	43	0.89583	0.84028	0.81532	0.79337	0.77342	0.74925	0.73224
48	42	0.87500	0.81701	0.79090	0.76812	0.74754	0.72275	0.70538
48	41	0.85417	0.79406	0.76695	0.74347	0.72236	0.69706	0.67942
48	40	0.83333	0.77137	0.74341	0.71932	0.69778	0.67207	0.65421
48	39	0.81250	0.74891	0.72020	0.69561	0.67371	0.64767	0.62965
48	38	0.79167	0.72665	0.69731	0.67228	0.65009	0.62381	0.60567
48	37	0.77083	0.70457	0.67468	0.64931	0.62688	0.60042	0.58221
48	36	0.75000	0.68266	0.65230	0.62664	0.60404	0.57746	0.55922
48	35	0.72917	0.66090	0.63016	0.60427	0.58154	0.55490	0.53667
48	34	0.70833	0.63927	0.60822	0.58217	0.55937	0.53271	0.51453
48	33	0.68750	0.61779	0.58649	0.56032	0.53749	0.51087	0.49276
48	32	0.66667	0.59643	0.56494	0.53871	0.51589	0.48937	0.47137
48	31	0.64583	0.57518	0.54358	0.51734	0.49457	0.46818	0.45031
48	30	0.62500	0.55406	0.52239	0.49618	0.47351	0.44729	0.42959
48	29	0.60417	0.53304	0.50137	0.47524	0.45270	0.42671	0.40920
48	28	0.58333	0.51214	0.48051	0.45451	0.43213	0.40640	0.38912
48	27	0.56250	0.49134	0.45982	0.43398	0.41181	0.38639	0.36934
48	26	0.54167	0.47064	0.43928	0.41366	0.39172	0.36664	0.34987
48	25	0.52083	0.45005	0.41890	0.39353	0.37187	0.34717	0.33070
48	24	0.50000	0.42957	0.39867	0.37360	0.35225	0.32798	0.31183

45	22	0.48889	0.41609	0.38442	0.35879	0.33703	0.31237	0.29601

n	r	p	80%	90%	95%	97.5%	99%	99.5%
46	46	1.00000	0.96562	0.95118	0.93695	0.92294	0.90474	0.89120
46	45	0.97826	0.93630	0.91805	0.90098	0.88473	0.86422	0.84931
46	44	0.95652	0.90926	0.88841	0.86939	0.85161	0.82953	0.81367
46	43	0.93478	0.88337	0.86050	0.83998	0.82104	0.79776	0.78119
46	42	0.91304	0.85820	0.83369	0.81196	0.79208	0.76786	0.75075
46	41	0.89130	0.83356	0.80768	0.78494	0.76430	0.73933	0.72178
46	40	0.86957	0.80933	0.78228	0.75870	0.73743	0.71185	0.69395
46	39	0.84783	0.78544	0.75737	0.73309	0.71131	0.68523	0.66707
46	38	0.82609	0.76182	0.73290	0.70802	0.68581	0.65934	0.64099
46	37	0.80435	0.73844	0.70878	0.68341	0.66085	0.63409	0.61559
46	36	0.78261	0.71528	0.68499	0.65921	0.63638	0.60939	0.59080
46	35	0.76087	0.69231	0.66148	0.63537	0.61233	0.58520	0.56656
46	34	0.73913	0.66951	0.63825	0.61187	0.58868	0.56146	0.54283
46	33	0.71739	0.64688	0.61525	0.58868	0.56540	0.53815	0.51956
46	32	0.69565	0.62440	0.59249	0.56578	0.54245	0.51524	0.49673
46	31	0.67391	0.60206	0.56993	0.54315	0.51983	0.49271	0.47430
46	30	0.65217	0.57985	0.54759	0.52078	0.49751	0.47053	0.45227
46	29	0.63043	0.55778	0.52544	0.49866	0.47548	0.44869	0.43060
46	28	0.60870	0.53583	0.50347	0.47678	0.45374	0.42719	0.40930
46	27	0.58696	0.51400	0.48169	0.45513	0.43227	0.40600	0.38835
46	26	0.56522	0.49229	0.46009	0.43371	0.41107	0.38513	0.36774
46	25	0.54348	0.47070	0.43867	0.41251	0.39013	0.36456	0.34747
46	24	0.52174	0.44922	0.41742	0.39154	0.36946	0.34430	0.32753
46	23	0.50000	0.42786	0.39634	0.37078	0.34904	0.32434	0.30792

n	r	p	80%	90%	95%	97.5%	99%	99.5%
47	47	1.00000	0.96634	0.95219	0.93825	0.92451	0.90666	0.89339
47	46	0.97872	0.93763	0.91974	0.90300	0.88706	0.86694	0.85230
47	45	0.95745	0.91115	0.89070	0.87204	0.85459	0.83291	0.81733
47	44	0.93617	0.88578	0.86335	0.84321	0.82461	0.80174	0.78545
47	43	0.91489	0.86113	0.83708	0.81574	0.79621	0.77240	0.75556
47	42	0.89362	0.83699	0.81158	0.78924	0.76895	0.74438	0.72711
47	41	0.87234	0.81325	0.78667	0.76350	0.74259	0.71740	0.69978
47	40	0.85106	0.78983	0.76226	0.73838	0.71694	0.69126	0.67336
47	39	0.82979	0.76669	0.73826	0.71378	0.69191	0.66583	0.64772
47	38	0.80851	0.74378	0.71461	0.68963	0.66740	0.64101	0.62275
47	37	0.78723	0.72108	0.69127	0.66587	0.64336	0.61673	0.59837
47	36	0.76596	0.69856	0.66821	0.64247	0.61974	0.59294	0.57453
47	35	0.74468	0.67622	0.64541	0.61940	0.59650	0.56960	0.55117
47	34	0.72340	0.65403	0.62285	0.59662	0.57362	0.54668	0.52827
47	33	0.70213	0.63198	0.60051	0.57413	0.55106	0.52413	0.50578
47	32	0.68085	0.61008	0.57837	0.55190	0.52882	0.50195	0.48369
47	31	0.65957	0.58830	0.55643	0.52992	0.50687	0.48011	0.46198
47	30	0.63830	0.56665	0.53468	0.50817	0.48520	0.45860	0.44062
47	29	0.61702	0.54512	0.51311	0.48666	0.46380	0.43741	0.41962
47	28	0.59574	0.52371	0.49172	0.46537	0.44266	0.41653	0.39895
47	27	0.57447	0.50241	0.47049	0.44430	0.42178	0.39594	0.37860
47	26	0.55319	0.48122	0.44944	0.42344	0.40116	0.37565	0.35858
47	25	0.53191	0.46014	0.42855	0.40279	0.38078	0.35565	0.33887
47	24	0.51064	0.43917	0.40783	0.38236	0.36064	0.33594	0.31948
47	23	0.48936	0.41831	0.38727	0.36212	0.34076	0.31651	0.30041

n	r	p	80%	90%	95%	97.5%	99%	99.5%
48	48	1.00000	0.96703	0.95316	0.93950	0.92603	0.90852	0.89549
48	47	0.97917	0.93890	0.92136	0.90494	0.88930	0.86955	0.85517
48	46	0.95833	0.91295	0.89290	0.87459	0.85746	0.83616	0.82085

n	r	p	80%	90%	95%	97.5%	99%	99.5%
49	49	1.00000	0.96769	0.95410	0.94069	0.92748	0.91030	0.89751
49	48	0.97959	0.94012	0.92292	0.90681	0.89146	0.87206	0.85794
49	47	0.95918	0.91469	0.89501	0.87703	0.86021	0.83929	0.82424
49	46	0.93878	0.89032	0.86872	0.84929	0.83134	0.80925	0.79349
49	45	0.91837	0.86664	0.84345	0.82285	0.80399	0.78096	0.76465
49	44	0.89796	0.84344	0.81892	0.79734	0.77772	0.75393	0.73718
49	43	0.87755	0.82062	0.79496	0.77256	0.75231	0.72789	0.71079
49	42	0.85714	0.79811	0.77147	0.74836	0.72758	0.70265	0.68526
49	41	0.83673	0.77586	0.74836	0.72465	0.70343	0.67809	0.66047
49	40	0.81633	0.75383	0.72559	0.70137	0.67978	0.65410	0.63631
49	39	0.79592	0.73200	0.70311	0.67846	0.65657	0.63063	0.61272
49	38	0.77551	0.71034	0.68090	0.65589	0.63376	0.60762	0.58963
49	37	0.75510	0.68885	0.65893	0.63362	0.61130	0.58503	0.56700
49	36	0.73469	0.66750	0.63719	0.61164	0.58918	0.56283	0.54479
49	35	0.71429	0.64628	0.61565	0.58991	0.56737	0.54099	0.52298
49	34	0.69388	0.62520	0.59430	0.56844	0.54585	0.51949	0.50154
49	33	0.67347	0.60423	0.57313	0.54720	0.52460	0.49831	0.48044
49	32	0.65306	0.58339	0.55215	0.52618	0.50361	0.47743	0.45969
49	31	0.63265	0.56265	0.53132	0.50537	0.48288	0.45685	0.43925
49	30	0.61224	0.54202	0.51067	0.48477	0.46239	0.43655	0.41913
49	29	0.59184	0.52150	0.49016	0.46437	0.44213	0.41653	0.39930
49	28	0.57143	0.50107	0.46982	0.44416	0.42210	0.39678	0.37978
49	27	0.55102	0.48075	0.44962	0.42415	0.40231	0.37729	0.36054
49	26	0.53061	0.46053	0.42958	0.40432	0.38273	0.35807	0.34159
49	25	0.51020	0.44040	0.40968	0.38469	0.36338	0.33910	0.32292
49	24	0.48980	0.42038	0.38993	0.36524	0.34425	0.32040	0.30454

n	r	p	80%	90%	95%	97.5%	99%	99.5%
50	50	1.00000	0.96832	0.95499	0.94184	0.92888	0.91201	0.89945
50	49	0.98000	0.94130	0.92442	0.90860	0.89353	0.87448	0.86060
50	48	0.96000	0.91635	0.89704	0.87939	0.86286	0.84230	0.82750
50	47	0.94000	0.89246	0.87124	0.85216	0.83452	0.81279	0.79729
50	46	0.92000	0.86923	0.84645	0.82621	0.80766	0.78500	0.76895
50	45	0.90000	0.84648	0.82238	0.80117	0.78186	0.75845	0.74195
50	44	0.88000	0.82410	0.79887	0.77683	0.75690	0.73285	0.71600
50	43	0.86000	0.80201	0.77581	0.75306	0.73260	0.70804	0.69089
50	42	0.84000	0.78018	0.75313	0.72978	0.70887	0.68389	0.66651
50	41	0.82000	0.75857	0.73077	0.70691	0.68563	0.66030	0.64274
50	40	0.80000	0.73715	0.70870	0.68440	0.66282	0.63721	0.61952
50	39	0.78000	0.71590	0.68689	0.66223	0.64039	0.61457	0.59679
50	38	0.76000	0.69480	0.66532	0.64034	0.61831	0.59235	0.57451
50	37	0.74000	0.67385	0.64396	0.61874	0.59655	0.57049	0.55264
50	36	0.72000	0.65303	0.62280	0.59738	0.57509	0.54899	0.53115
50	35	0.70000	0.63233	0.60182	0.57627	0.55392	0.52781	0.51002
50	34	0.68000	0.61175	0.58103	0.55538	0.53301	0.50695	0.48923
50	33	0.66000	0.59128	0.56040	0.53470	0.51235	0.48638	0.46876
50	32	0.64000	0.57092	0.53994	0.51423	0.49193	0.46609	0.44860
50	31	0.62000	0.55066	0.51963	0.49396	0.47175	0.44608	0.42874
50	30	0.60000	0.53051	0.49947	0.47388	0.45179	0.42633	0.40918
50	29	0.58000	0.51045	0.47946	0.45399	0.43206	0.40684	0.38989
50	28	0.56000	0.49049	0.45960	0.43428	0.41254	0.38761	0.37089
50	27	0.54000	0.47062	0.43988	0.41476	0.39324	0.36863	0.35216
50	26	0.52000	0.45084	0.42030	0.39541	0.37415	0.34989	0.33370
50	25	0.50000	0.43116	0.40086	0.37625	0.35527	0.33140	0.31551

List of Symbols, Abbreviations, and Acronyms

cdf	cumulative distribution function
CI	confidence interval
CP	Clopper-Pearson
iid	independent and identically distributed
LR	likelihood ratio
MLE	maximum likelihood estimator
nf	number of failures
pdf	probability density function
qf	quantile function

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1	US ARMY DEV TEST COM CSTE DTC TT T 314 LONGS CORNER RD APG MD 21005-5055
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