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14. ABSTRACT
The Sonar Simulation Toolset (SST) software is widely used in the Navy high-frequency sonar community to generate simulated sound for use in sonar development, performance prediction, and other applications. Our goal is to extend to the low-frequency sonar community the benefits of such a detailed, engineering-level simulation system.

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Low Frequency Sonar Signal Simulation
Final Report

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LONG-TERM GOALS

The Sonar Simulation Toolset (SST) software is widely used in the Navy high-frequency sonar community to generate simulated sound for use in sonar development, performance prediction, and other applications. Our goal is to extend to the low-frequency sonar community the benefits of such a detailed, engineering-level simulation system.

OBJECTIVES

From the Proposal:
The Sonar Simulation Toolset (SST)[1,2] has become a *de facto* standard for detailed signal simulation of underwater sound at medium to high frequencies (a few kHz and up). We want to create a low-frequency sonar signal simulation system having a level of detail comparable to that of SST. The requirements shared with SST include:

- *Signal simulation*: Produce signals suitable for listening or for input to a signal processor. The primary output is a digital representation of sound, as heard by a multi-channel sonar system. (This is in contrast to less detailed outputs like intensity or power spectrum.)
- *Detailed sonar specification*: Support multiple elements or beams, realistic sensitivity patterns, broadband signals, active and passive processing, etc.
- *Detailed environment*: Support variable bathymetry, bottom types, sound speed, etc.
- *Detailed scenario*: Support multiple maneuvering vehicles, long listen times, multistatic geometry, etc.
- *High fidelity*, constrained by reasonably fast, but not real-time, performance.
- *Portable*: Runs on a variety of widely available computer systems.

- *Standard*: Most of the underlying models used by SST (e.g., propagation, surface, and bottom models) are well-tested and widely-accepted standard models from the literature. We will continue this tradition by bringing in standard models from the low-frequency sonar community.
- *Innovative*: Standard models are balanced and extended where necessary by new models, including those for reverberation and rough-boundary coherence loss, and knit together by innovative object-oriented software architecture.
- *Usable*: Ease of use is commensurate with the subject matter. Unnecessary barriers to productive simulation are minimized.
- *Documented*: The level and quality of SST's documentation is the best in the field. We are committed to maintaining that standard and improving on it.
- *Supported*: Direct support to SST users by APL-UW personnel has proven valuable and cost-effective in the past, and should continue.

SST is already useful for some purposes at low frequencies, but it falls short of quantitative realism, especially in shallow-water or long-range scenarios. Existing low-frequency codes do a better job than SST of modeling the physics of low-frequency sound propagation, but they all fall down on one or more of the other requirements listed above. Our goal is to bridge that gap.

Our approach to this goal is incremental and iterative. We will address the high-priority needs of developers and users of low-frequency sonar systems through a series of quick releases of enhanced versions of SST. At the same time, the sub-team led by Eric Thorsos will pursue a longer-term program to design a system optimized for low-frequency sonar systems.

APPROACH

From the Proposal:
Task 1, Improve SST Incrementally: Select a series of SST enhancements based on priorities and technology gaps identified by sponsors and by the simulation users identified for support. These will include new low-frequency environmental models, plus some improvements in usability and user interface. Adapt and install these enhancements into SST. Release new SST versions to the community at least twice each year.

Task 2, Support Navy Simulation Community: Support SST users in the Navy sonar development community by providing advice, developing simulation scripts, and participating in workshops.

Task 3, Assess SST Propagation Accuracy at Low Frequencies: Determine accuracy domain of SST eigenray-based propagation model at low frequencies for scenarios and environments of tactical interest.

Task 4, Assess SST Reverberation Accuracy at Low Frequencies: Use existing wave-based reverberation models to verify SST reverberation accuracy in the domain identified in Task 3.

Task 5, Synthesis: If it is deemed necessary, specify and design a signal simulation approach that covers the full domain of scenarios of Navy interest. Implement enough of this design to demonstrate that the concept is appropriate.

WORK COMPLETED AND RESULTS

GABIM Bottom Model (Task 1): The Geophysical-Acoustical Bottom Interaction Model (GABIM)[3] is a new model, recently submitted for inclusion in the Oceanographic-Atmospheric Master Library (OAML)[4]. GABIM predicts backscattering strength and forward loss for given geoacoustic parameters and profiles, including both scattering from rough interfaces and scattering from the sediment volume. This work is based in part on work done by P. D. Mourad and D. R. Jackson[5,6,7]. Whereas the older models are specialized for either low frequencies (100-1000 Hz) or high frequencies (10-100 kHz), GABIM covers the whole range of 100 Hz to 100 kHz, including intermediate frequencies (1-10 kHz). GABIM also extends the earlier models in that it includes shear waves and multiple bottom layers.

We integrated GABIM into SST; it is included in SST Release 4.8. GABIM is run as a sub-process to create tables of backscattering strength and reflection coefficient. During the simulation, SST uses the tables, which are fast.

For simple, single-layer bottoms, the most important difference between GABIM and SST's older bottom model, called JacksonBottom, is GABIM's support of shear waves in the sediment. **Figure 1** shows this difference for a very hard bottom (limestone) for which shear waves are very important.

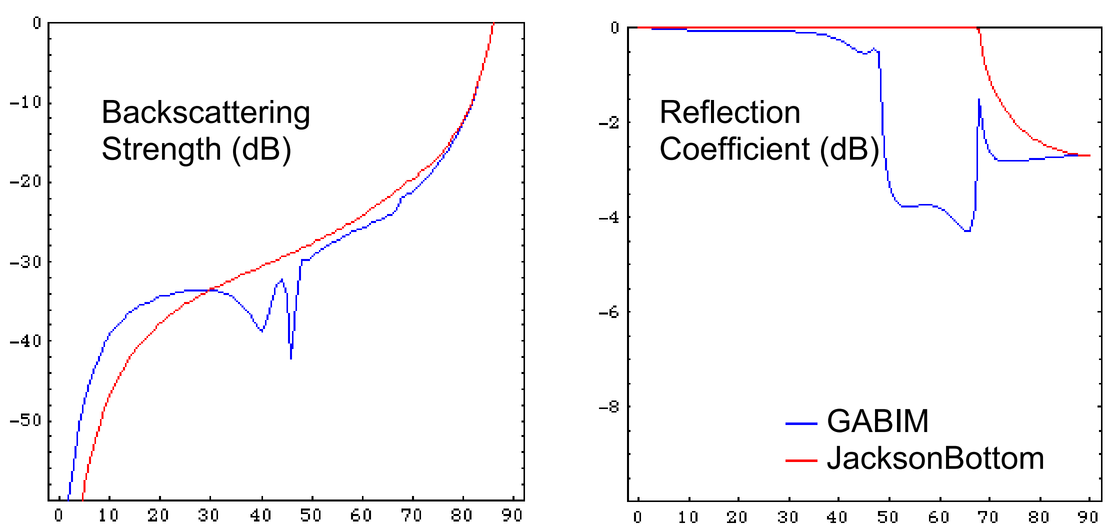

Figure 1. GABIM (with shear) vs. JacksonBottom (no shear) vs. grazing angle for a limestone bottom at a frequency of 3000 Hz. Backscattering strength (left) and reflection coefficient (right) for the new GABIM (blue) are compared with the older JacksonBottom model (red).

Coherent Bottom Reverberation (Task 1): We added to SST a new bottom reverberation model that includes the effects of interference between different two-way ray paths (eigenrays). The new algorithm, which is conceptually similar to the “point scatterer” approach, generates uncorrelated random Gaussian scattering amplitudes for many cells (smaller than the sonar resolution) covering the bottom. These are combined with eigenray pairs and weighted by the angle-dependent scattering strength to generate stochastic amplitude versus direction at the receiver. The result is held in a sparse structure holding complex amplitudes indexed by direction, Doppler, and frequency for each value of two-way travel time. Existing SST components are then used to combine with receiver beam patterns, integrate over direction, and combine with Doppler-shifted pulse replicas to generate bottom reverberation. The only new user-visible control is a Boolean *coherentBottom* attribute in the existing DirectionalDopplerDensity class, which the user sets to *true* to choose the new coherent algorithm or to *false* to choose the older incoherent algorithm.

This new model is included in SST 4.8. It is slower than SST’s other reverberation algorithms because we haven’t started optimizing it yet. Also, it is not yet described in the SST Science and Math report.

Figure 2 shows 20-ping average intensities for monostatic coherent reverberation for a very simple shallow-water scenario, with a center frequency of 3500 Hz. There are two one-way paths, with and without a surface reflection (no bottom reflections), for a total of four two-way paths. As expected, increasing the pulse length tends to smooth out the interference pattern. Increasing the pulse bandwidth (not shown) similarly smoothes out the pattern.

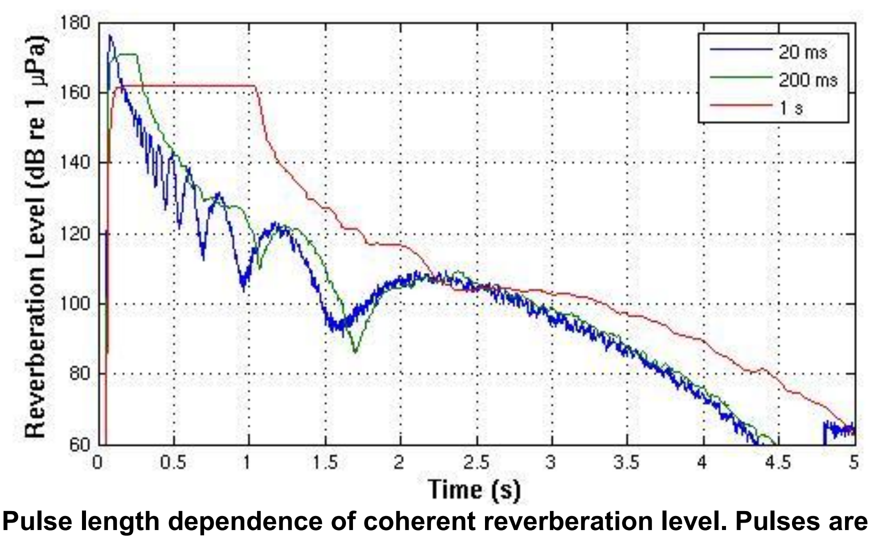

Figure 2. Pulse length dependence of coherent reverberation level. Pulses are pure tones.

Figure 3 shows spectrogram of one ping for an FM pulse with 1600 Hz bandwidth (on the left). The interference appears as diagonal stripes, which remove the pattern only when integrated across frequency. The key observation is that the interference is still there in the simulated sound, even though it is absent in the reverberation level (on the right).

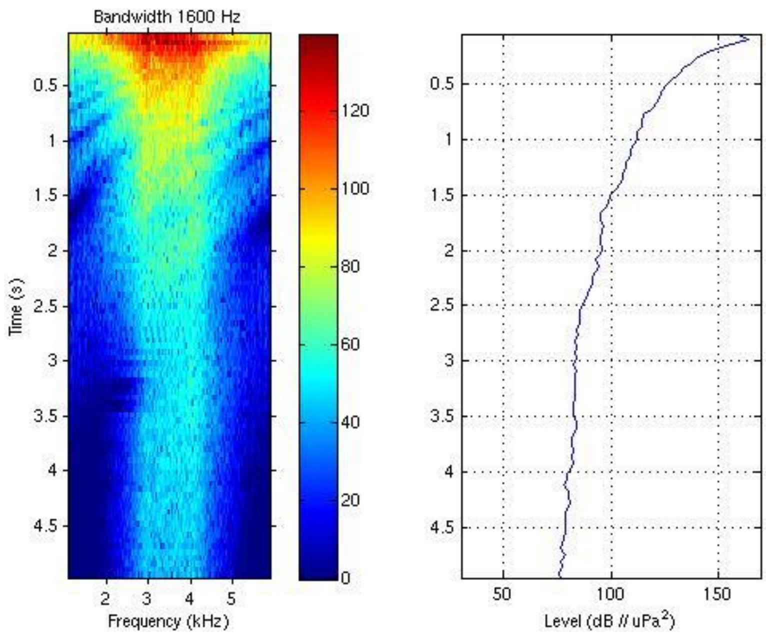

Figure 3. Spectrogram and level for coherent bottom reverberation, for pulse bandwidth of 1600 Hz.

SST User Support (Task 2): We provided direct support to SST users supported by ONR 321US. Here we cite just one example of many (with permission): Geoff Edelson and his team at BAE used SST extensively in his study of "Physics Based Depth Discrimination for Multi-Static Active Sonar". We provided advice and counseling in setting up simulations involving multiple transmitters and receivers distributed over a large collection of sonobuoys. The BAE team found SST simulation useful in developing strategies and algorithms for discriminating between submarines and surface contacts.

GRAB/PE Comparisons, Isovelocity (Task 3): The primary underlying propagation model used by SST is the Gaussian Ray Bundle (GRAB)[8] model. For this project, we performed detailed comparisons of GRAB results the Parabolic Equation (PE) model as implemented in the Range-dependent Acoustic Model (RAM)[9], which is well accepted as fast and accurate by both research and operational communities.

Our earliest PE/GRAB comparisons involved shallow, isovelocity environments, for acoustic frequencies between 50 and 1500 Hz. Agreement at 1500 Hz was nearly perfect, as expected. The first indication of fundamental limitations of GRAB for isovelocity conditions came at 50 Hz for a 100 m waveguide, where the ray model's approximate treatment of sediment attenuation leads to excess intensity that grows with range.

As the frequency was lowered from 1500 to 100 Hz, modest PE/GRAB differences appeared. We traced this difference to a deficiency with the PE starting field. The GRAB fields are confined to the water column, while the corresponding PE fields extend into the sediment. The use of the GRAB field, which is truncated at the water/sediment interface, for the PE starting field leads to diffraction effects due to the discontinuity of the field at the interface. This is a problem with the PE starting field, and does not indicate any problem with GRAB.

GRAB/PE Comparisons, Non-Isovelocity (Task 3): Later comparisons involved scenarios with depth-dependent sound speed, producing caustics. Our qualitative conclusion is unsurprising: A lower frequency limit to ray-based acoustics does, indeed, exist, for all but the simplest environments. Partial fixes, such bottom models with multiple layers and shear, can improve the realism of ray-based models and push their lower frequency limit down somewhat. Ray acoustics can provide usefully realistic results well into the "low frequency" range (below 1 kHz), but the low-frequency limit depends on the details of the environment, the ranges of interest, and the purpose of the simulation. In particular, ray acoustics can work well in deep water in the absence of a surface duct, and in shallow water (down to 100 wavelengths or so) under well-mixed (nearly isovelocity) conditions. Surface ducts, caustics, bottom penetration, internal waves, and very long ranges give ray-based models more trouble. Thus, important use cases exist for which the quantitative errors introduced by the ray approach are tactically significant.

A more surprising conclusion came from our detailed comparison between PE and GRAB in the presence of caustics. Figure 4 compares propagation losses from PE and GRAB for a simple down-bending sound speed profile. The GRAB run uses its "coherent superposition" option. GRAB predicted extremely high (and obviously incorrect) amplitudes near the caustic in this case, especially with fine ray spacing between GRAB's test rays (0.01 degree in this case) – which is normally expected to improve GRAB's accuracy. This problem appeared at 3 kHz, where we normally expect ray acoustics to perform well.

The context for this comparison includes the observation that detailed signal generation systems like SST require coherent superposition of sound that travels via multiple eigenrays (ray paths) in order to reproduce interference effects seen in PE results and experiments. With GRAB, our choices are to choose incoherent superposition and give up on seeing the expected multipath interference, or to choose coherent superposition and risk wildly incorrect results in a small portion of the environment (as in Figure 4).

Henry Weinberg, the author of GRAB, developed a simple "fix" that eliminated the main effects of the problem, but no one but Weinberg fully understands it. Even if these issues could be avoided by choosing or developing an alternative ray code, there would remain fundamental limitations with the ray method for cases with complex caustics that could still lead to erroneous results. As a consequence, we have concluded that a wave-based propagation model should be developed and integrated into SST (or a successor) as an option to SST's current ray-based models. This will require development and integration of wave-based models for sources, receivers, targets, and reverberation.

We have submitted a proposal for follow-on work, to produce a version of SST (or a successor) based on PE instead of ray acoustics.

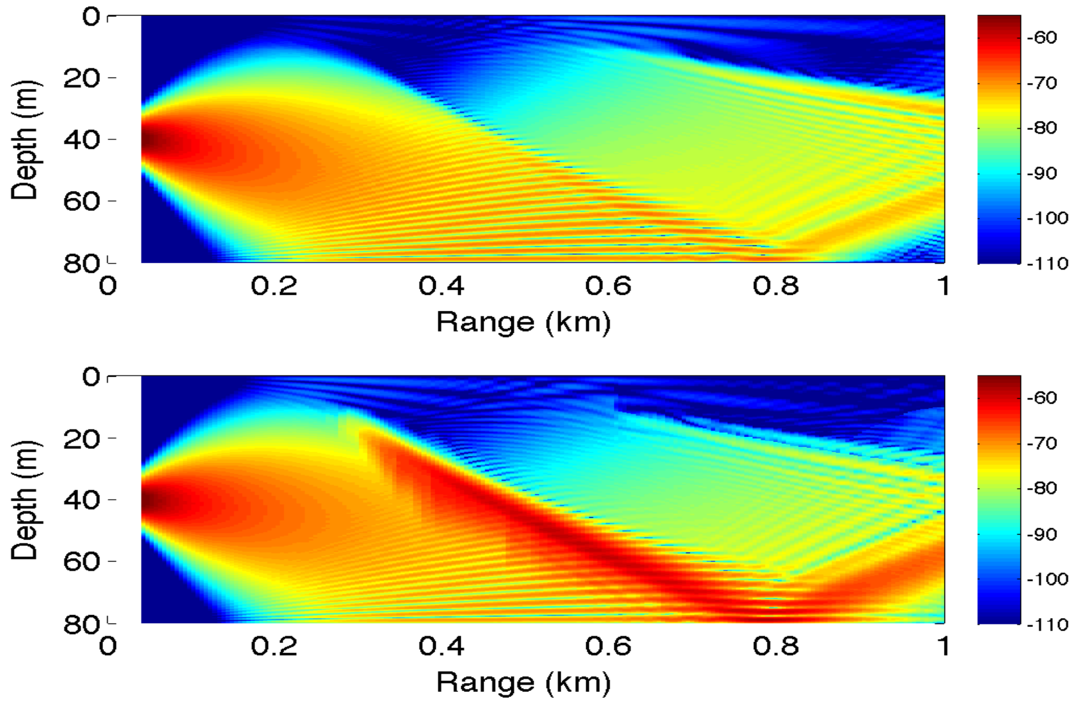

Figure 4. Propagation loss at 3 kHz from PE (top) and GRAB (bottom) for a simple down-bending sound speed profile. The GRAB "coherent superposition" option was used.

Reverberation Modeling Workshops (Task 4): Project personnel participated in the ONR Reverberation Modeling Workshops held in November 2006 and March 2008. One important contribution of these workshops is a standard set of problem scenarios. Many different reverberation modeling algorithms were compared with each other on the same set of problems. Algorithms based on GRAB, PE, normal modes, transport theory, and other propagation models were included. Since those results bear directly on this Task, we did not repeat them for this project. E. Thorsos of APL-UW was one of the organizers of the Workshops, and other APL-UW personnel were intimately involved, so we are very familiar with the results, and we will find them useful in future SST improvements.

The reverberation level computed using SST agrees with other CASS-based models, as expected. SST's strengths, including signal generation and Doppler, were not exercised. Hence, SST results will not appear in the workshop reports. However, we learned a lot about possible future SST directions, notably from presentations by LePage (who described R-SNAP/BiStaR, based on normal modes), Thorsos and Tang (who found that integral equation approach using OASES works well, fairly fast), and Knobles (who advised modelers to take care with the definition of the scattering strength, whether it is normalized to the full field or the down-going field). In addition, the problems proved very helpful in our own studies, as illustrated in the following section.

Wave-based Reverberation Modeling (Task 5): This section describes work completed in the last few months of the project, after submission of the 2009 Annual Report. Because we have not reported on these results before, this section is more detailed than the others.

Our earlier work led us to the conclusion that ray-based modeling of sound propagation is inadequate for realistic simulations for some tactically important scenarios, and that wave-based models based on PE (Parabolic Equation) or normal mode propagation models should be made available in SST (or its successor) as an alternative to ray models. Here we describe steps taken along that path in preparation for a proposed future project to add wave-based modeling to SST. Work was done in two areas: (1) developing a mode-based simulation capability for 3-D geometries for comparison with PE-based results, and (2) beginning the development of a formalism for integrating scattering cross section concepts into a wave-based simulation approach.

Work in the first area was designed to take advantage of two important assets that could support PE-based reverberation modeling development. The first is a set of results from the recent Reverberation Modeling Workshops (RMW) that provide ideal test cases for certain reverberation problems with well-established solutions. (E. Thorsos has been co-chair of these workshops.) The second asset is a mode-based reverberation code being developed at APL-UW. J. Yang at APL-UW has been developing this code and is mainly responsible for the results summarized here. The goal of the mode-based reverberation development is to obtain a reliable reverberation simulation tool for detailed comparison with PE-based results, and the RMW problem solutions provide ideal benchmarks for validating the accuracy of the reverberation code based on modes. As it happened, the result of this work was more significant than expected; it helped clarify an important issue that had not been resolved in the RMW simulation work.

Before comparisons could be made between RMW solutions and the APL mode reverberation code, some development was required. The original version of the mode code had been formulated to solve some of the 2-D reverberation problems posed for RMW participants. The code was also formulated as a Monte Carlo simulation method using realizations of rough 1-D surfaces. For this work, the code was converted to use rough surface scattering cross sections instead of surface realizations, and it was converted to the 3-D geometry.

The APL code was first checked against previous solutions obtained for Problem V from RMW I (the first workshop) as shown in Figure 5, which gives results for 3.5 kHz. The depth of the waveguide is 100 m, the source is at 30 m depth, and the receiver is at 50 m depth. The reverberation in this isovelocity case is due to bottom scattering only, and the bottom roughness was considered “typical.” Problem V is actually two problems in one, because two versions of bottom reflection loss were suggested to participants. For the upper family of solutions in Figure 5, the bottom is taken as flat for the purpose of forward propagation, which means the reflection loss is for the flat bottom. For the lower family of solutions, the coherent reflection loss for bottom roughness was included in the bottom reflection loss. This turns out to be a relatively high bottom loss case. The physical solution for this problem should be close to the upper family, because it is unrealistic for this problem to model the bottom loss using the full coherent reflection loss due to roughness. The reason is that for the "typical" bottom roughness model most of the scattered energy stays in the waveguide and is not actually lost. Nevertheless, using the coherent loss makes a well-defined problem and brings up interesting modeling issues.

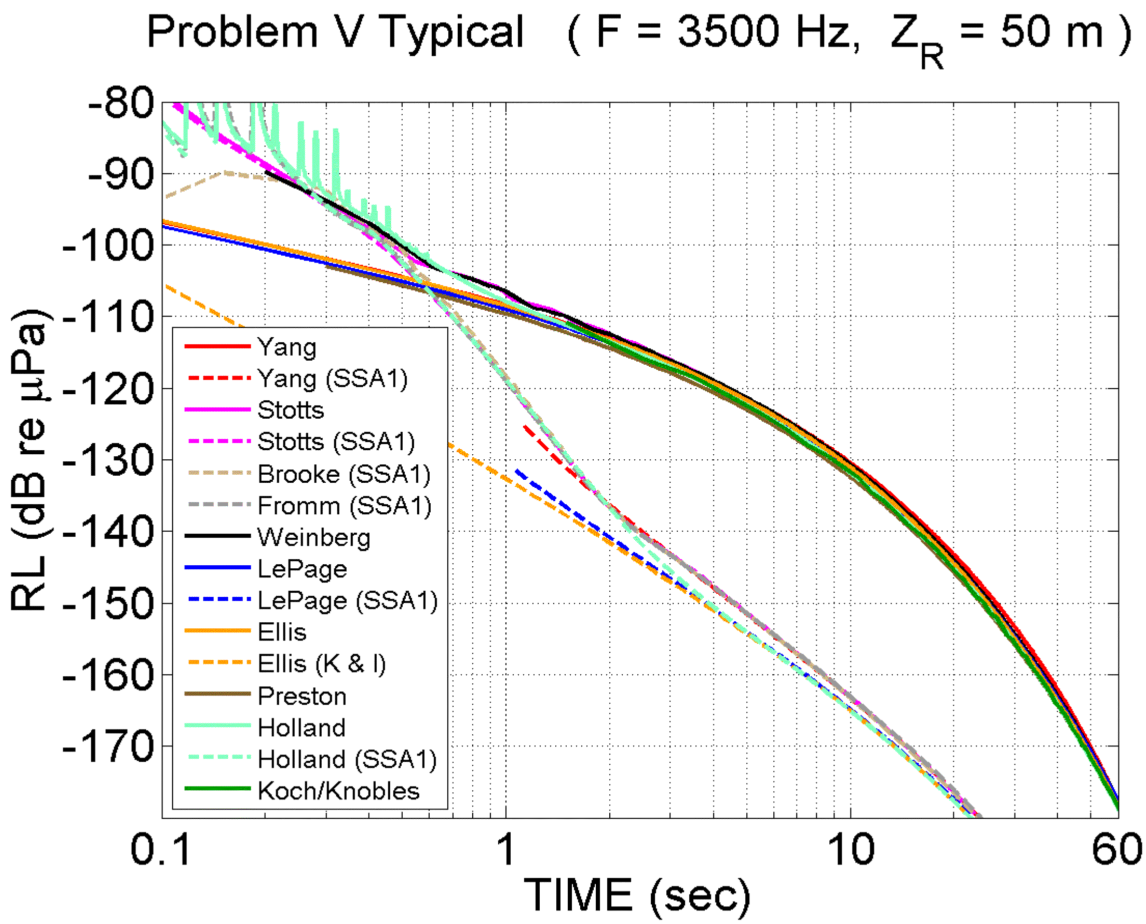

Figure 5. Comparison of solutions for Problem V from RMW I at 3.5 kHz. An isovelocity sound speed profile was used, and the reverberation was due to scattering from bottom roughness. There are two separate families of solutions: (1) For the upper family with solid lines, the bottom reflection loss assumed a flat bottom. (2) For the lower family with dashed lines, the bottom reflection loss included the coherent loss due to scattering from roughness. The APL solutions are those by Yang.

Note that at early times (< 1 s) the upper family itself breaks up into two subfamilies, which are the ray (above) and the mode (below) solutions. The mode solutions use only trapped modes, those corresponding to grazing angles below the critical angle of about 28 deg, whereas the ray solutions include backscattering at higher grazing angles at early times, leading to a higher reverberation level. The solution by Yang agrees well with the other upper family solutions and is with the mode group at early times, as expected, though it is essentially invisible because it lies underneath other curves in the figure.

The solutions for the lower family bring up interesting modeling issues. Note that the lower family of solutions again breaks up into two subfamilies. The LePage and Ellis curves in the lower subfamily are mode solutions, while the solutions slightly higher are ray solutions, at least prior to the Yang solution becoming available. (One exception is the Holland solution, which uses rays at short range with a transition to an energy flux method at long range where it agrees with the mode solutions.) This difference in the subfamily solutions becomes much greater for cases with a sound speed gradient, as will be discussed shortly, but is at least noticeable for this isovelocity case. Considerable effort has been expended to understand which of these subfamily solutions is correct for this problem. Rough bottom PE simulations for one-way propagation suggested that the ray solutions are more accurate for this case, but a full reverberation demonstration of this had not been done previously. Note that the Yang solution indeed agrees with the ray subfamily, which turned out to be a major step forward in understanding this difference. The reason for this improvement will be discussed shortly. (Both the Yang and LePage solutions were cut off at times less than 1 s because there is a known reason that they became inaccurate in that region for this particular problem.)

Problem VI differs from problem V only in that the sound speed has a down-refracting linear gradient, decreasing from 1530 m/s at the surface to 1500 m/s at the bottom. Solutions for Problem VI are shown in Figure 6 and Figure 7; in Figure 6 the solutions by Yang and LePage use incoherent superposition of modes, and in Figure 7 a coherent superposition of modes is used for both. All other solutions in both figures use incoherent superposition of modes or rays. Again there are effectively two separate problems for the two different models for bottom reflection loss, and for this case the difference is more dramatic. The focus here will be entirely on the lower family of solutions. For the lower family the reverberation level for the ray solutions suddenly drops at a time of about 2 s, the last time that a direct ray can reach the bottom. The initial mode solutions by Ellis and LePage were in agreement with each other and did not show the sudden drop nor even approximately the correct level coming up to the sudden drop. (Only the original Ellis solutions are shown in Figure 6 and Figure 7; the original LePage solution agreed closely with it.) These initial solutions used an incoherent superposition of modes for the propagation leading to very smooth curves for reverberation. The use of a coherent superposition of modes does lead to more structure in the reverberation curves, but originally did not come close to matching the ray solution levels at times leading up to 2 s.

Note that the APL (Yang) solution for the lower family in Figure 6, which used an incoherent superposition of modes, is well above the Ellis solution, and the solution using a coherent superposition of modes (Figure 7) is in quite good agreement with the ray solutions. Only after the APL solution became available was LePage able to match it with his smooth blue dashed line in Figure 6 (incoherent solution), and the more irregular blue dashed line in Figure 7 (coherent solution).

These comparisons showed that the approach used in the original mode solutions was not accurate for the high bottom loss case for Problem VI. That approach was based on the commonly used method of incorporating the bottom loss into a modification of the imaginary parts of the horizontal mode wave numbers, but to make no change in the mode functions themselves, i.e., to use unperturbed modes. The solutions by Yang (and later by LePage) incorporated the bottom loss into a modification of both the horizontal mode wave numbers and the mode functions. This leads to solutions for this case that are in much better agreement with rays solutions for direct path interaction with the bottom.

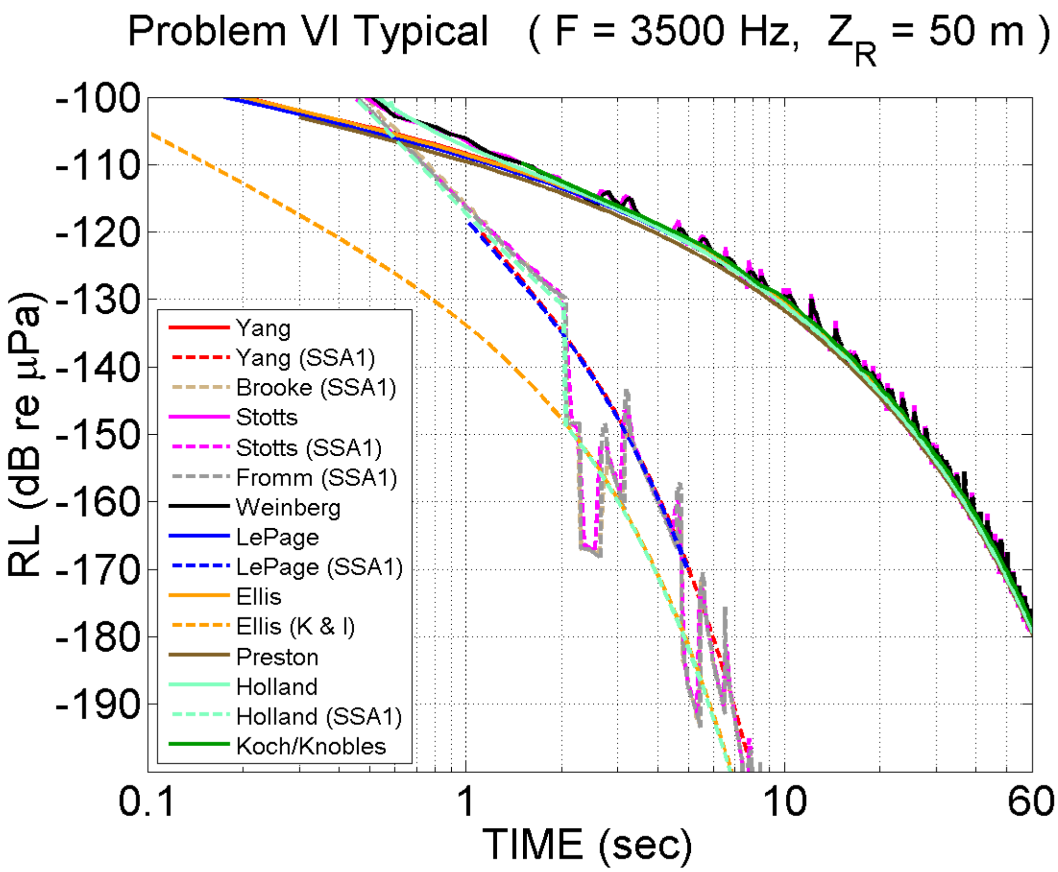

Figure 6. Comparison of solutions for Problem VI from RMW I at 3.5 kHz. A down-refracting sound speed profile was used, and the reverberation was due to scattering from bottom roughness. There are two separate families of solutions: (1) For the upper family with solid lines, the bottom reflection loss assumed a flat bottom. (2) For the lower family with dashed lines, the bottom reflection loss included the coherent loss due to scattering from roughness. The APL solutions are those by Yang. All solutions in this figure use incoherent superposition of modes or rays.

In summary for this component of work, a mode-based reverberation code has been modified to use scattering cross sections to model boundary scattering and was converted to a 3-D geometry. Comparisons with RMW problem solutions show good agreement and indicate that a reliable modeling tool is available for detailed comparisons with PE-based reverberation solutions.

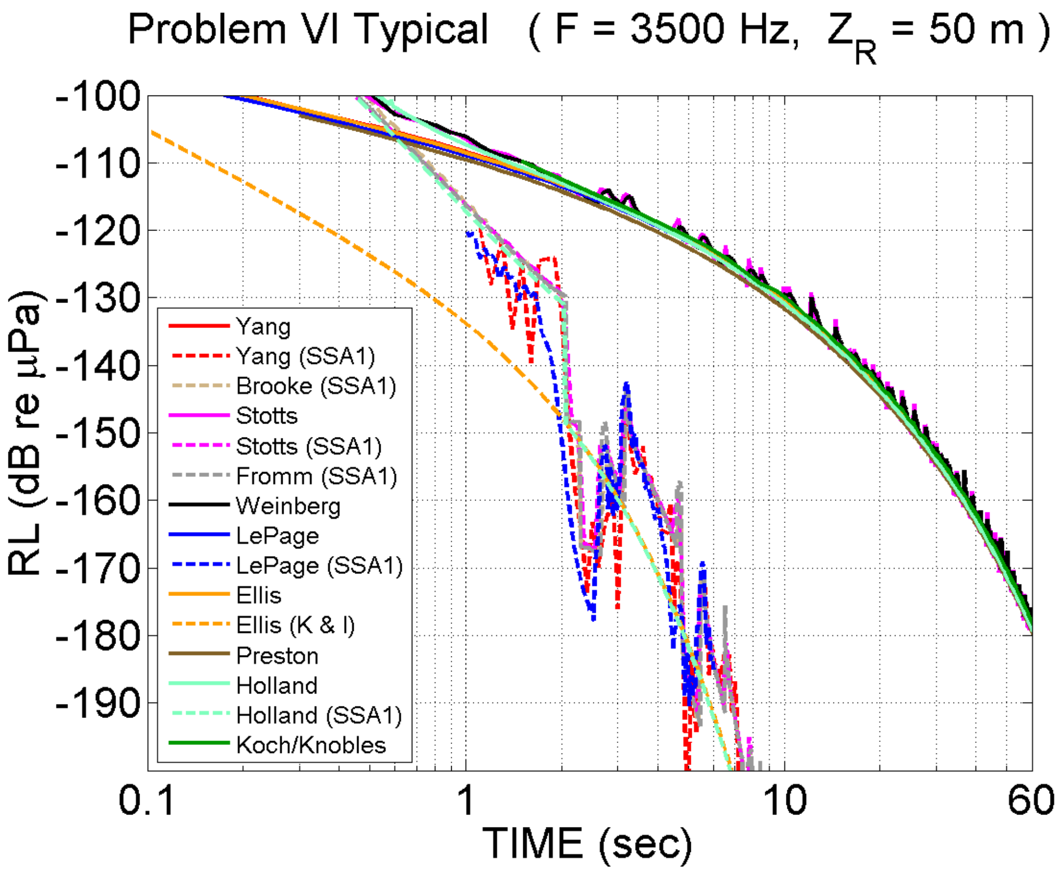

Figure 7. Comparison of solutions for Problem VI from RMW I at 3.5 kHz. All curves are the same as in Figure 6, except the lower family (dashed curves) by Yang and by LePage use a coherent superposition of modes.

A second component of work began development of a formalism for integrating scattering cross section concepts into a wave-based simulation approach such as PE. Ultimately, both k-space (i.e., using scattering cross sections) and coordinate space pictures will be examined to couple the forward and back-going fields, but the initial work has focused on the former. Only a very brief description of this work is given here. Typically, the reverberation at a given two-way travel time is evaluated as a product of the intensity from the source to the point(s) on the bottom corresponding to that time, a cross section at the bottom, and the reciprocal intensity from the bottom point to the receiver. For an ensemble-averaged return, that procedure is probably reliable, but for issues like false targets, it may not be. The issue is that the travel time to a single point is a broad-band concept, whereas the cross section, for example due to Bragg scattering, is a narrow band concept. We have started the reconciliation between these competing aspects for arbitrary bandwidths, using more complete expressions for the returned scattered wave.

IMPACT/APPLICATIONS

Our primary goal is to serve the scientists and engineers who are developing the next generations of low-frequency, long-range Navy sonar systems, primarily for antisubmarine warfare (ASW). A detailed, engineering-level sonar simulation system can provide those developers with simulated sound having enough realism to test algorithms, decide on engineering trade-offs, interpret experimental results, and develop a concept of operation and tactics to guide system development. This simulated sound can complement or augment measured sound, especially for systems that haven't been built yet or for environments where the systems haven't been used yet.

In addition, simulations can be useful for training sonar operators and tactical officers. A simulation-based training system gives the instructor the freedom to set up scenarios and problems for the students to solve, and provides students with feedback as to the consequences of their decisions, without risk to personnel, ships, or marine life. This project is not aimed directly at developing training systems, but it should help us design future training systems.

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