Convergence Analysis of Genetic Algorithms for Topology Control in MANETs

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***Abstract*—We describe and verify convergence properties of our forced-based genetic algorithm (FGA) as a decentralized topology control mechanism distributed among software agents. FGA uses local information to guide autonomous mobile nodes over an unknown geographical terrain to obtain a uniform node distribution. Analyzing the convergence characteristics of FGA is difficult due to the stochastic nature of GA-based algorithms. Ergodic homogeneous Markov chains are used to describe the convergence characteristics of our FGA. In addition, simulation experiments verify the convergence of our GA-based algorithm.**

***Index Terms*—bio-inspired algorithms;Markov chains;genetic algorithms;MANETs;topology control.**

I. Introduction

The term *biologically inspired algorithm* represents a family of stochastic algorithms based on principals that are mimicked from nature. Bio-inspired algorithms are becoming a popular tool particularly in collobrative robotics, machine learning, artificial intelligence (AI), and pattern recognition techniques with direct applications in military and commercial tasks. GA is a particular set of bio-inspired algorithms that use techniques motivated by evolutionary biology such as tournament, selection, mutation, and elitism.

A Mobile ad-hoc network (MANET) is a network formed when wireless mobile nodes with limited communication capabilities come together and dynamically create a temporary network without any pre-existing network infrastructure. In many circumstances, mobile nodes are geographically dispersed so communication between far away mobile nodes can be established through a multi-hop routing. However, the network structure may rapidly and unpredictably change due to various reasons such as loss of mobile nodes, malfunctions, or mobile node movement. One way to maintain the network connection in MANETs is to provide the mobile nodes with ability to *self-organize*. Self-organization, known as *autonomic computing*, is composed of self-management, self-optimization, self-adaptation, self-healing, and self-regulating [1]. To provide a self-organization ability to the mobile nodes, they must be capable of deciding their speed and movement directions. The results from our earlier research show that GA-based algorithms are among the better approaches to provide self-organization capability of the mobile nodes in an unknown geographical terrain. Using our forced-based GA (FGA) as distributed software agents in mobile nodes protects the connectivity in a MANET using local information without any centralized controller [2], [3].

We study on the convergence of our FGA in the problem of covering maximum area and providing a fully connected network in a MANET with unknown terrain while only using local neighborhood knowledge. As in any problem solving paradigm using a stochastic algorithm, the main issue is to find whether our algorithm provides an optimal or close to optimal solution (GAs are stochastic in nature, they are not guaranteed to find the exact optimal solution). We use an *ergodic homogeneous finite Markov chain (MC)* to show the convergence of our FGA. Since the population of the FGA like all GA-based algorithms only depends on the population of the previous generation in a probabilistic manner, MC is an appropriate method to analyze the convergence of our FGA.

GAs are among the most popular control tools in MANETs particularly focusing on the topology control and routing: to overcome the battery limitation problem for routing selection in MANETS [4], to create clusters in MANETs [5], [6], and the network parameter selections [7]. In our FGA approach, there are no cluster heads in a MANET. Each mobile node runs the same software agent to decide its next movement. Our approach results in immunity from losing vital assets since there is no difference between the mobile nodes.

Schema theory [8] and *Markov chains* are widely used to provide a formal structure for analyzing GAs. For example, a modified elitist strategy is used to generate the current population from the reserved highest fitness valued individual and the rest are from the previous generation in [9]. [10] explains various Markov chain Monte Carlo methods, including the Gibb's sampler and the Metropolis sampler. [11] proposes an algorithm for control of autonomous swarms using the Gibbs sampler simulated annealing process. [12] discusses fundamental properties of finite Markov chain including graph theoretic considerations for transient and non-transient, recurrent and non-recurrent cases.

Our paper is organized as follows. Section II presents details about our FGA. The finite Markov chain model of our FGA and theoretical details of convergence analysis for the ergodic

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14. ABSTRACT
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homogeneous Markov chain are explained in Section III. Convergence analysis of our FGA is provided in Section IV.

II. Our Distributed Forced-Based Genetic Algorithm

A mobile node gathers information about its neighboring nodes' speed, direction, and location. Then it runs our GA to optimize its speed and direction [2], [3]. In our earlier works we introduced the FGA implementation [13] that is inspired by molecular forced-based distributions found in physics [14]. Each mobile node in the MANET is applied a force by its near neighbors (i.e., the neighbors are the nodes located within the communication range). The total force sums up to zero at equilibrium. The summed force is used as a fitness by FGA to calculate the next speed and location of the node such that the summed force on the mobile node is minimized. We can calculate the total force applied to a mobile node n as follows:

$$F(n) = \sum_{i=0}^{k} \sum_{j=0}^{k} \overline{N} \cdot (R_{com} - \mid ((x - x_i) + (y - y_j)) \mid) \quad (1)$$

where k is the total number of neighbors, (x, y) is the current coordinate value for node n, (x_i, y_j) is the location of a neighbor node, and $\overline{N}$ is the expected number of neighbors to maximize the coverage [2].

Our FGA aims to provide each node with : *(i)* a near-optimal number of neighbors, *(ii)* self-spreading capability, *(iii)* a fully connected network, and *(iv)* minimizes the intersection between mobile nodes' communication coverage.

III. Finite Markov Chains

A discrete stochastic system can be characterized by a set ϑ of *states* and *transitions* between these states. The transitions occur at each discrete time beat so that the state of the system at time t is a value $X_t \epsilon \vartheta$, $t = (0, 1, 2, 3, \cdots)$. If the transition randomly occurs based on a Markov kernel, this discrete stochastic system is named a discrete stochastic process. A Markov chain is *memoryless* if the probability of a transition between states does not depend on the previous states.

$$\begin{aligned} &P(X_{t+1} = j | X_t = i, X_{t-1} = \nu_{t-1}, \cdots, X_0 = \nu_0) \\ &\quad = P(X_{t+1} = j | X_t = i), \forall i, j, \nu_0, \cdots, \nu_{t-1} \epsilon \vartheta \end{aligned} \quad (2)$$

A *finite Markov chain* is defined as a memoryless discrete stochastic process with a finite number of states.

A simplified behavior of a node running our FGA can be characterized by a finite Markov chain using the node's speed (mobile or immobile), fitness (good or bad), and direction (up, up-right, down-right, down, down-left, or up-left based on the hexagonal lattice) as seen in Fig. 1. The fitness value for each mobile node is calculated by using Eq. 1. Various fitness values of each mobile node are merged into two distinct values as either good or bad (shown as *1* or *0* in Fig. 1, respectively). The number of neighbors is another variable used in our simplified Markov model state. For $d(i)$ is the number of neighbors for a mobile agent i and $\bar{N}$ is the ideal number of neighbors [2] to maximize the area coverage. The state where $\bar{N} - 1 \leq d(i) \leq \bar{N} + 1$ is modeled as *ideal* in the Markov chain; otherwise, it is denoted as *non-ideal*.

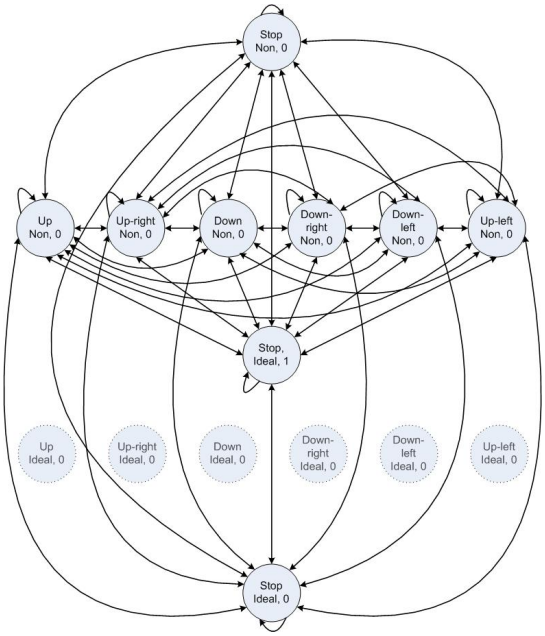

Fig. 1. A Simplified Markov chain model for our FGA (each state is connected to each of the states in dotted lines, which are not shown for simplicity)

The simplified Markov chain model of our FGA has 15 states as seen in Fig. 1. If a mobile agent is moving on one of the six directions, its state must be at the one of the 12 states based on its number of neighbors and fitness value: six directions with the ideal number of neighbors and the fitness value of 0, and six directions with non-ideal number of neighbors and fitness value of 0. Speed is inherently covered by including direction into the state information.The remaining three states are (stop, non, 0) state where the node is immobile due to the non-ideal number of neighbors and zero fitness, (stop, ideal, 0) state where the mobile node is immobile, the fitness is 0 in spite of the ideal number of neighbors, and (stop, ideal, 1) state where the mobile node does not move because of having ideal number of neighbors with fitness 1. The state (stop, ideal, 1) is the desired (final) state in our simplified Markov chain model. If a mobile node reaches this state, it has the desired number of neighbors at the correct locations and stops moving (perhaps until another node comes and disrupts its equilibrium).

A *transition (stochastic) matrix* is used to specify the probabilities that the mobile agents move from one state to another in unit time (i.e., the behavior of a Markov chain depends on the values in the transition matrix). Due to the stochastic nature of the GAs, the probability values of the transition matrix cannot be calculated analytically. These values must be determined experimentally by using our simulation software [2], [3]. We set up a set of experiments and collect data concerning the observed state of the nodes at each time unit. When our FGA decides the mobile node's next speed and direction using the neighborhood information, the mobile

node moves from one state to another in our simplified Markov chain shown in Fig. 1. Performing more experiments increases the accuracy of the probability values in the transition matrix since noise in the data is eliminated. After running enough experiments, the transition matrix can be formed using the data showing the observed state of each node at each time unit. The resulting transition matrix from the simulation experiments is a *right* transition matrix with nonnegative elements and entries in each row adding to one.

A homogeneous Markov chain is defined by having a Markov kernel (i.e., transition matrix) that is equivalent for every time step (i.e., $P_t = P$, where $t = 1, 2, \cdots$) on a finite space X, with some initial distribution ν. The distribution of states $x \epsilon X$ at times $t \geq 0$ is given by $P^{(t)}(x_0, \cdots, x_t) = \nu(x_0)P_1(x_0, x_1) \cdots P_t(x_{t-1}, x_t)$. An *ergodic* (metrically transitive) Markov chain is defined by being both *irreducible* and *aperiodic* as explained below. (The proofs for the following lemmas are skipped for succinctness, but can be found in [15])

Lemma 1. *The Markov chain representation of our* FGA *is irreducible if and only if $P(\tau_y < \infty | x_0 = x) > 0$ for all $x, y \epsilon X$ assuming $P^0(x_0 = x)$ and τ_y is the minimum number of steps to traverse from state x to y (i.e., the probability of moving from one state to another in a finite number of steps is positive).*

Definition 1. *The periodicity (dx) of a state x in a Markov chain is defined by the expression $dx = gcd\{n : Pr(x_n = i | x_0 = i) > 0\}$ where gcd means "greatest common divisor" (i.e., the periodicity is obtained by finding the number of steps for all possible paths to exit and return to a given state, then finding the largest number that can be divided by all of them).*

Lemma 2. *A Markov chain is aperiodic when $dx = 1$ in Def. 1 for all states (i.e., every state contains a self-loop transition). (Proof is in [15])*

Lemma 3. *The Markov chain representation of our* FGA *is irreducible and aperiodic, hence ergodic. (Proof is in [15])*

A. Convergence Analysis of a Metrically Transitive Homogeneous Finite Markov Chain

To prove the convergence of a homogeneous Markov chain, the following measures are presented. The total variation between distributions in a given set is represented by $\|\mu - \nu\| = \sum_n |\mu(x) - \nu(x)|$, for a finite set X with distributions μ and ν on X. This measure is the basis for Dobrushin's contraction coefficient [16] that gives a rough measure of the orthogonality between the distributions of a transition matrix. This is expressed as $c(P) = \frac{1}{2} \cdot \underbrace{max}_{x,y} |P(x, \cdot) - P(y, \cdot)|$, where c is the contraction coefficient and P is a Markov kernel. Simply stated, the contraction coefficient represents $\frac{1}{2}$ the greatest total variation of all combinations of rows in the Markov kernel. When any two distributions of the Markov kernel are completely disjoint $c(P) = 1$. $c(P) = 0$ when every row in the Markov kernel $P(x, \cdot)$ is equal. These measures are used to make the following statements:

Lemma 4. *Let P and Q be transition matrices and let μ and ν be probability distributions : $|\mu P - \nu P| \leq c(P)\,|\mu - \nu|$, $c(PQ) \leq c(P)c(Q) \Rightarrow |\mu P - \nu P| \leq |\mu - \nu|$, $|\mu P - \nu P| \leq 2 \cdot c(P)$.*

Proof: Proof is given by Winkler in [10]. ■

Here, Winkler verifies that as distributions transition through an ergodic system (Markov kernel) the orthoganality between them decreases (assuming the distributions are not disjoint).

Lemma 5. *For each step with the transition matrix P the sequence $(c(P^t))t \geq 0$ decreases.*

Proof: Using Lemma 4 as shown in [10]:

$$c(P^{t+1}) \leq c(P)c(P^t) \leq c(P^t) \tag{3}$$

■

Lemma 6. *If the transition matrix, P is primitive, the sequence decreases to 0.*

Proof: Using Lemma 5 as shown in [10]:

$$c(P^t) \leq (Q^k P^{t-\tau \cdot k}) \leq c(Q)^k \cdot c(P^{t-\tau \cdot k}) \leq c(Q)^k \tag{4}$$

where $Q = P^\tau$ and τ is the minimum number of steps for any state to reach any other state. Hence, Q is a simplified Markov kernel where every state can reach every other state in a single *large* step. k is the number of *large* steps such that $\tau \cdot k \geq t$. If P is primitive and hence Q is strictly positive (i.e., all states are reachable) then $c(Q) < 1$ without equality and $c(P^t)$ must go to zero as t goes to infinity [10]. ■

Theorem 1. *For a primitive transition matrix P on a finite space with a stationary distribution μ, starting from any distribution ν, $\nu P^t \to \mu$ as $t \to \infty$.*

Proof: By Lemma 5, the sequence $lim_{t \to \infty} c(P^t) = 0$ and by Lemma 4

$$\left|\nu P^t - \mu\right| = \left|\nu P^t - \mu P^t\right| \leq |\nu - \mu|\, c(P^t) \leq 2 \cdot c(P^t) \tag{5}$$

■

Lemmas 5 and 6 demonstrate that as an ergodic Markov chain progresses through each generation, it converges towards a stationary distribution. This result is generalized for any initial distribution in Theorem 1. Based on Theorem 1 we assert that our FGA will converge to a stationary behavior:

Theorem 2. *The transition matrix for the simplified* FGA *is ergodic and therefore it will converge to a stationary distribution.*

Proof: (*sketch*) It is shown in [15] that the Markov kernel for our FGA is irreducible, aperiodic and ergodic. Therefore, using Theorem 1, it will converge to a stationary distribution. ■

Using this theorem we can state that our FGA will converge to a stationary behavior. Also, experimentally we can find a

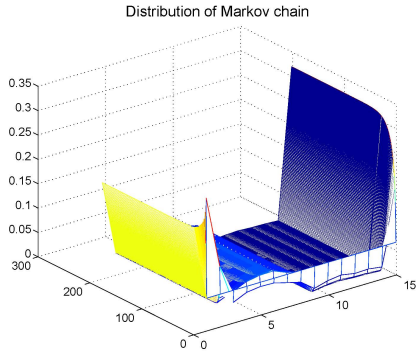

Fig. 2. Distribution of Markov chain for $t \to \infty$ ($N = 100$)

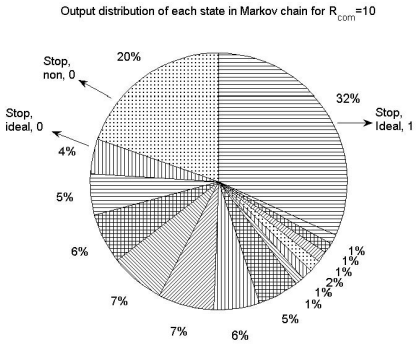

Fig. 3. Final Distribution of Markov chain for $t \to \infty$ ($N = 100$)

close approximation of the final stationary distribution of the Markov chain for our distributed FGA. With this we can compare the rate of convergence for various initial distributions of the mobile nodes using our FGA as a software agent.

IV. Convergence Experiments for Our FGA

We implemented simulation software in Java to study the effectiveness of our FGA for a uniform distribution of knowledge sharing mobile nodes [3]. Eclipse SDK version 3.2.0 was used as the development environment, and Mason, a fast discrete-event multi-agent simulation library core developed by George Mason Universitys ECJLab, was used for the GUI interface. The simulation software implementation has nearly 4,000 lines of algorithmic Java code. The user is able to assign different values for the following parameters: *(i)* total number of mobile nodes(N), *(ii)* communication range (R_{com}), *(iii)* maximum number of iterations (T_{max}), *(iv)* mean number of available links ($\bar{N}$), *(v)* size of the geographical terrain (d_{max}), *vi* initial node distribution, and *(vii)* number and position of obstacles. We consider an experimental scenario where the nodes enter an unknown geographical area without any prior information and without a central control unit. Without loss of generality, each node has the same limited communication range (R_{com}), and, hence, can only be aware of its neighbors and runs its own GA-based software application.

For each experiment, the area of deployment is set to be 100x100 units with all nodes initially placed randomly in the north-east corner of the terrain. We ran experiments for networks with $N = 100$, $N = 125$ and $N = 150$ nodes and all nodes with $R_{com} = 10$. To reduce the noise in the outcomes, each simulation experiment is repeated for 50 times with the same initial values for node speed, R_{com}, and direction, and with the same initial node deployment.

Figs. 2 and 3 display the convergence characteristics of the Markov kernel's states for the experiments (where $N = 100$) as the nodes perform our FGA. Fig. 2 shows that the system evolves to a stationary distribution as t goes to infinity. It is important to note that any initial distribution will converge to the same stationary distribution. The only difference in using varied initial distributions will be the number of steps that the system takes to reach the stationary distribution. This makes practical sense when considering the manner that the nodes are initially deployed. If the mobile nodes are initially dispersed such that they are close to uniform spatial distribution over the geographical area, then they will take very few movements achieve a uniform distribution. In the experiments, nodes are placed in the *worst case* scenario where all of the mobile agents are clustered in a single corner. In this case, many mobile agents will initially be trapped between other mobile agents and the boundaries of the geographical area. This will increase the time required to reach spatial uniformity. The importance of the relationship between initial distributions of the Markov chain and the initial dispersement of mobile nodes is that the Markovian representation of the FGA, accurately represents experimental behavior.

Fig. 3 represents the possible outcome percentages of each state in the Markov chain. The mobile agents in the ideal state have the highest probability of 32% when time approaches infinity. It proves that most of the mobile agents reach the state where they have the desired number of neighbors and location that results in minimal external force. The probability of reaching a stop state with poor fitness and non-ideal number of neighbors is 20%. The mobile nodes in this state may have more neighbors than $\bar{N}$. The remaining states that are not explicitly labeled with values ranging from 5% to 7% represent states where the node is moving and has a ideal number of neighbors. The final remaining states with values ranging from 1% to 2% represent the nodes that are moving with a non-ideal number of neighbors. The final stationary distribution verifies the experimental behavior of our FGA where mobile agents achieve a distribution that is close to the uniform distribution. Some nodes continue to move slightly, these nodes exert small external forces on neighbors who in turn readjust themselves to return to ideal fitness.

Fig. 4 shows Dobrushin's contraction coefficient when time goes to infinity for three different experiments with differing numbers of nodes ($N = 100$, 125, and 150). The graph is only based on the transition matrix for FGA and not on various initial distributions of the Markov chain. This allows direct comparisons to be made between experiments with varying parameters. As seen Fig. 4, experiments with larger numbers of nodes converge at a slower rate than experiments with smaller

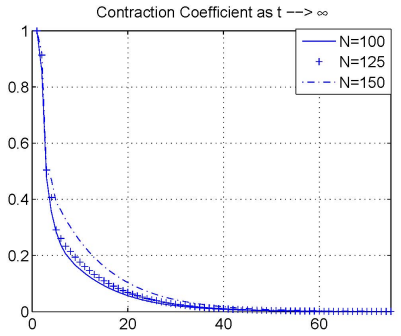

Fig. 4. Contraction coefficients when $t \to \infty$

numbers of nodes. This is due to the fact that more nodes are initially trapped with limited mobility. As nodes at the periphery of the cluster quickly begin to spread, the mobility of nodes in the center of the cluster also begins to increase. The Markov kernel for our FGA reaches the final distribution when $t \approx 50$ for all experiments. The convergence for varied initial distributions can be added to this graph by finding $|\nu P^t - \mu P^t|$ with respect to t for any of the experimental cases. It is important to note that the graph of this line with all initial distributions (ν) will fall below the corresponding graph of the contraction coefficient of the Markov kernel as in the proof of Theorem 1. This statement also makes practical sense since the convergence of the Markov kernel shows the upper bound on the slowest convergence of the FGA. Theoretically, this situation would be were all nodes are located in the exact same position at a singe corner of the geographic area. In practice, this scenario is impossible and the *worst case* scenario is the one used in the experiment where all nodes are randomly clustered in one corner of the geographical area, but where none of the mobile agents are occupying the same space. Conceptually, any scenario different than the one just described would converge to a uniform spatial distribution faster with an upper bound at the scenario where all mobile agents are initially dispersed such that they all have ideal fitness. This further verifies that the Markovian model described section III-A is appropriate.

V. Concluding Remarks

We investigate the convergence of our FGA for self-spreading nodes in MANETs. A Markov chain is introduced to study the convergence of the FGA using contraction coefficients. Using our FGA for MANET nodes to position themselves in an unknown geographical terrain to maximize the area coverage and to provide a fully connected network, we show that, our GA-based algorithm converges to a stationary behavior. The simulation experiment results in Sec. IV supports the convergence of the algorithm. The effects of different number of nodes on the convergence of our FGA are also discussed. We observed that experiments with larger numbers of nodes converge at a slower rate than experiments with smaller numbers of nodes. This is due to the fact that more nodes need more movement to reach the desired state.

As a next step of this research, we will include a detailed convergence analysis of our FGA and the effects of different network parameters including communication range and position of initial node deployment.

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