Pattern Search for Mixed Variable Optimization Problems

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Outline

- Mixed variable problem motivation and formulation

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- GPS for linearly constrained MVP problems

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- Filter GPS for general constrained MVP problems

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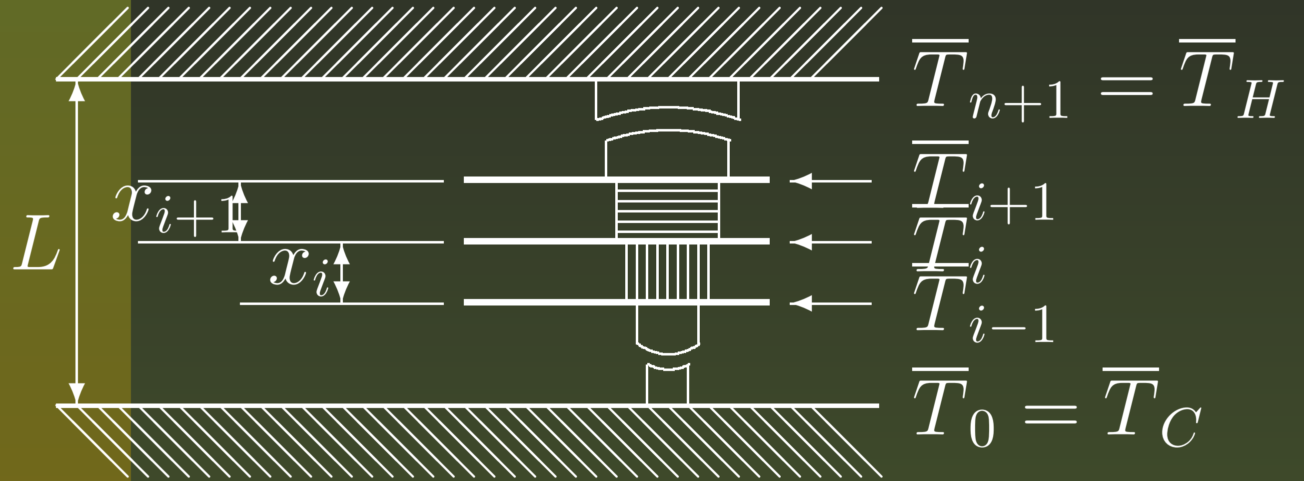
- Filter GPS for general constrained MVP problems
- Results for thermal insulation system design

Heat intercept insulation system



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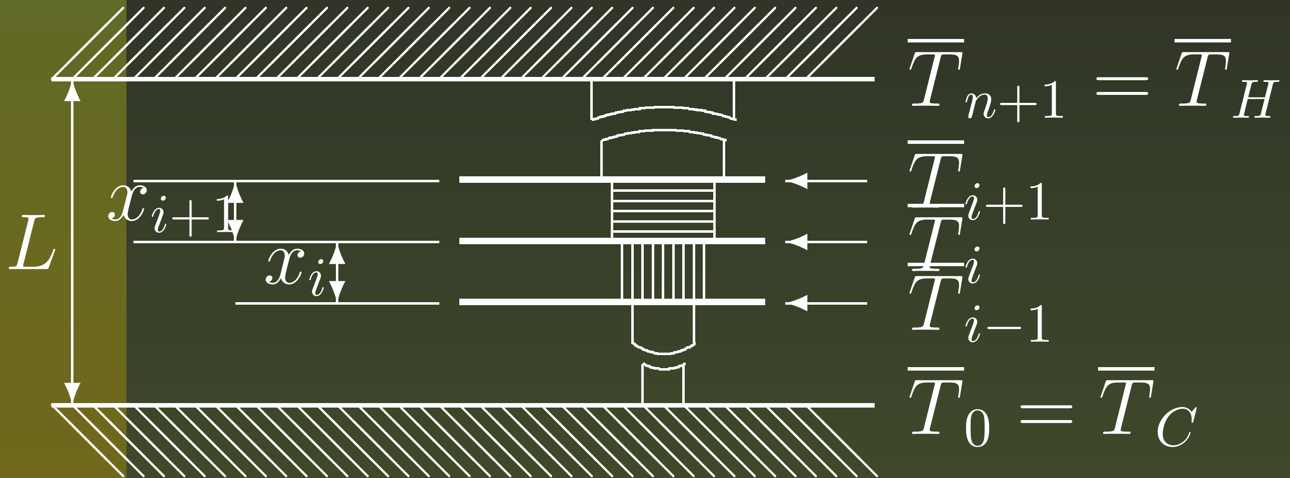

$$\min \quad power(n, I, x, \overline{T})$$

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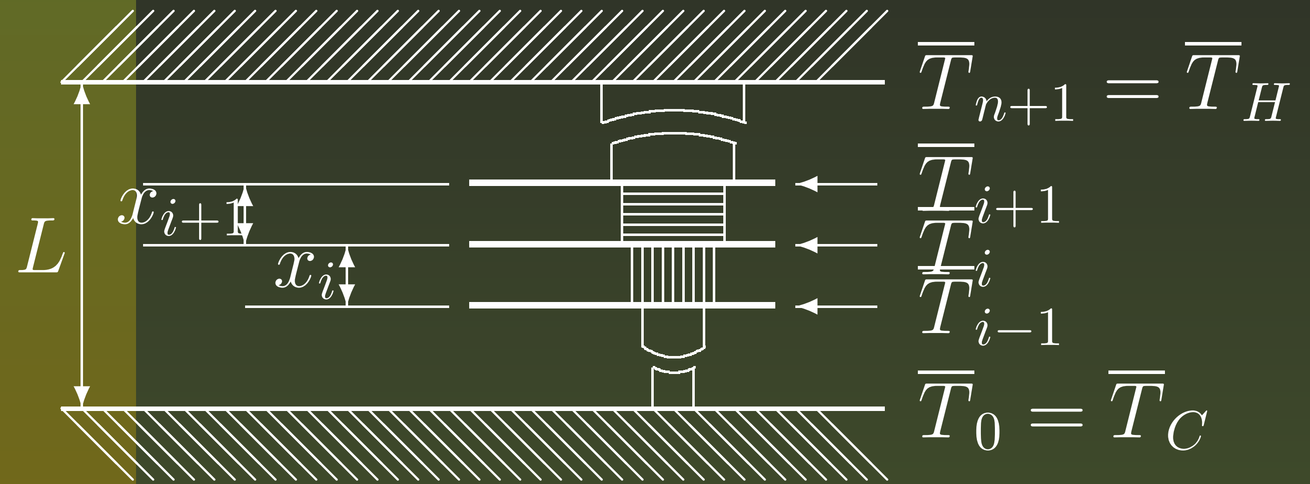

$$\begin{array}{ll} \min & power(n, I, x, \overline{T}) \\ \text{subject to} & n \in \{1, 2, \ldots, n_{\max}\}, \quad I \in \mathcal{I}^{n+1} \\ & \overline{T}_{i-1} \leq \overline{T}_i \leq \overline{T}_{i+1}, \; i = 1, 2, \ldots, n \\ & \sum_{i=1}^{n+1} x_i = L, \quad x_i \geq 0, \; i = 1, 2, \ldots, n+1 \end{array}$$

The general MVP problem

$$\min_{x \in X} f(x)$$
$$\text{subject to } C(x) \leq 0,$$

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Choices other than pattern search

Some methods that come to mind are:

- MINLP methods: cannot handle categorical variables

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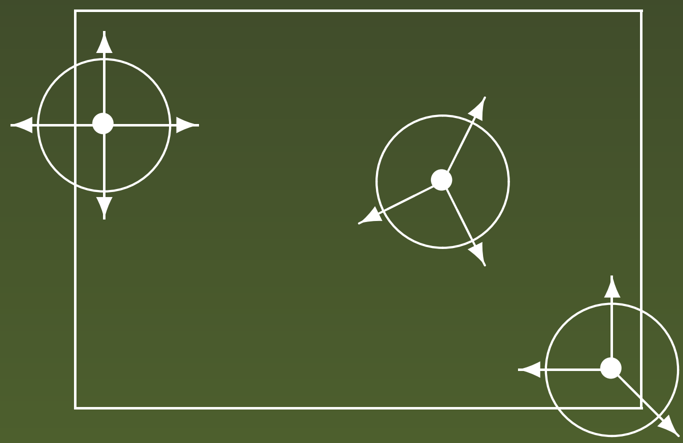
  - Evolutionary algorithms
- Other methods: SQP/direct search with 1 categorical variable

Generalized pattern searches

INITIALIZATION of directions and step size

For $k = 1, 2, \ldots$

- SEARCH a finite set of mesh points



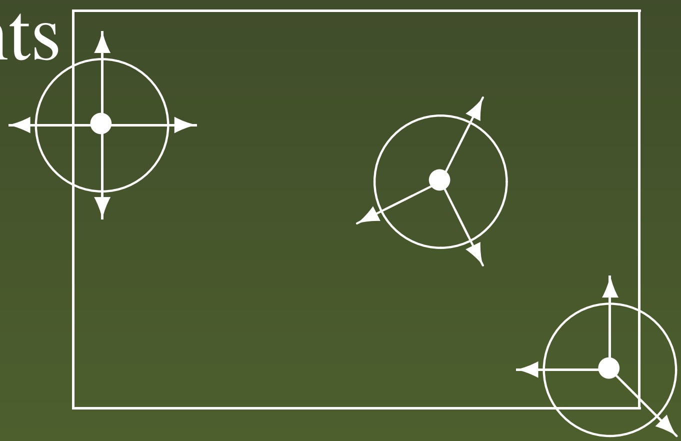
End

Generalized pattern searches

INITIALIZATION of directions and step size

For $k = 1, 2, \ldots$

- SEARCH a finite set of mesh points
- POLL neighboring mesh points



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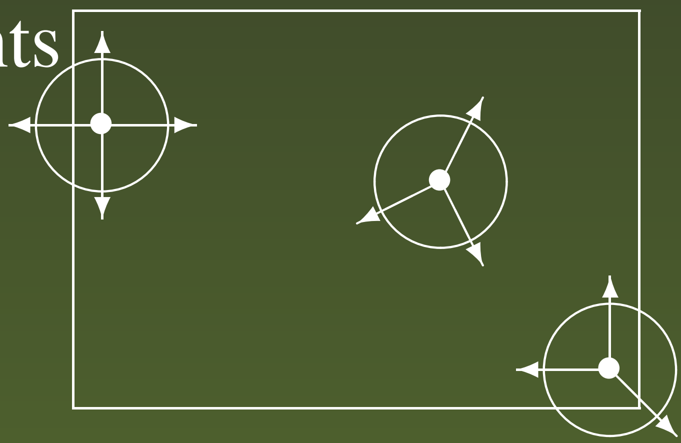

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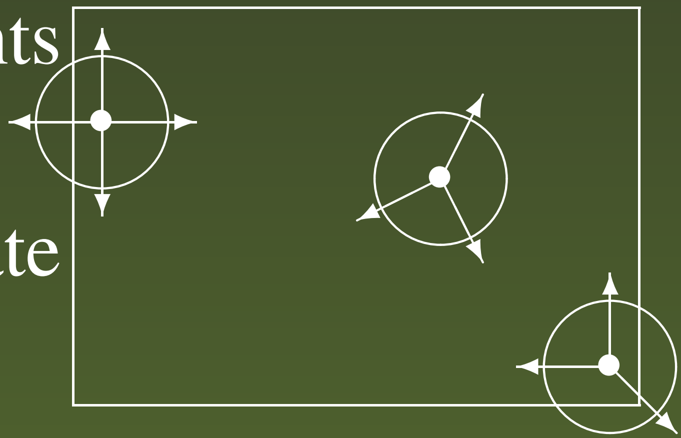

End

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For $k = 1, 2, \ldots$

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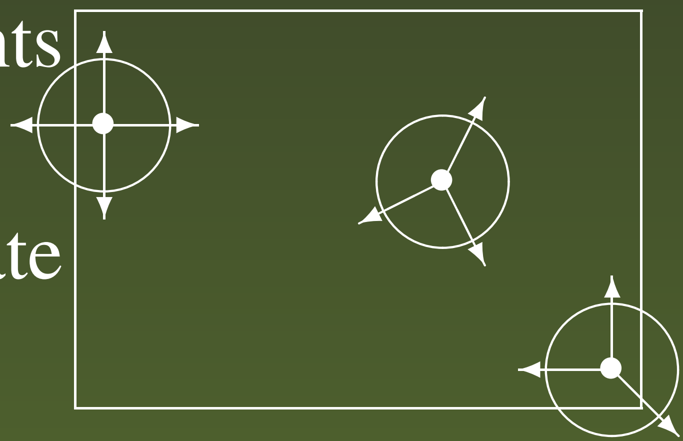
  - Success: Accept new iterate
  - Failure: Refine mesh

End



Details of kth POLL step

Mesh: $M_k = \{p_k + \Delta_k Dz : z \in \mathcal{Z}_+^{|D|}\}$,
Poll set: $P_k = \{p_k + \Delta_k d : d \in D_k \subseteq D\}$,

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One of these directions should be a descent direction.

Derivative information can reduce poll set to a singleton

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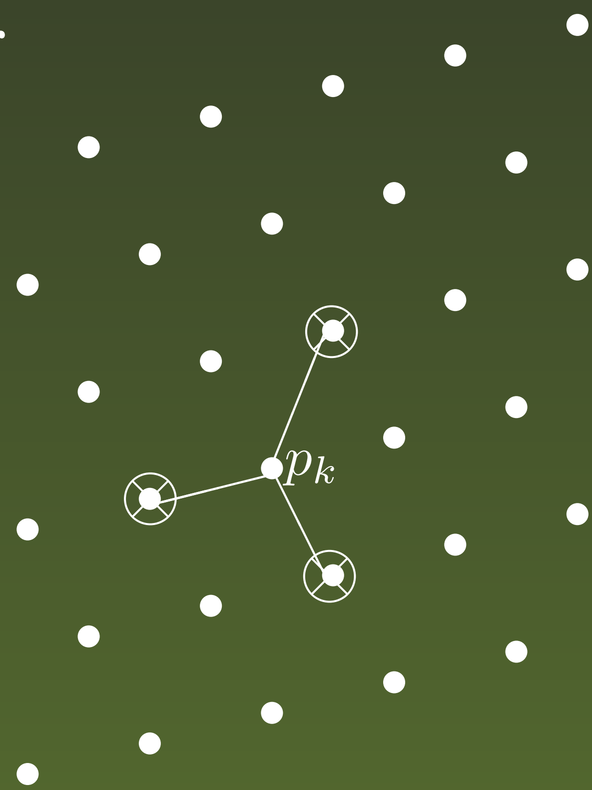

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Derivative information can reduce poll set to a singleton

Definition of local optimality

$x = (x^c, x^d) \in X$ is a *local minimizer of* f *with respect to neighbors* $\mathcal{N}(x) \subset X$ if $\exists \epsilon > 0$ such that $f(x) \leq f(v)$

$$\forall v \in X \cap \bigcup_{y \in \mathcal{N}(x)} \left(B(y^c, \epsilon) \times y^d\right).$$

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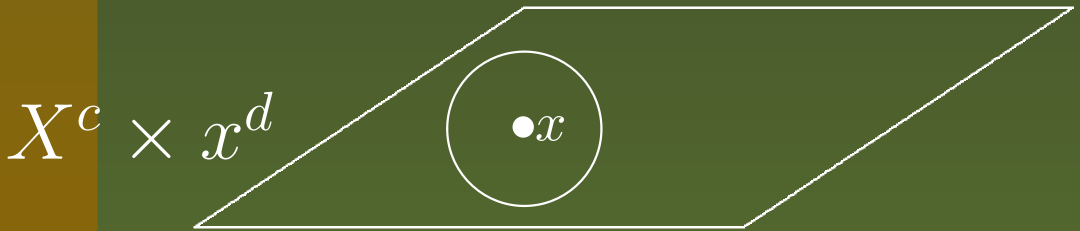

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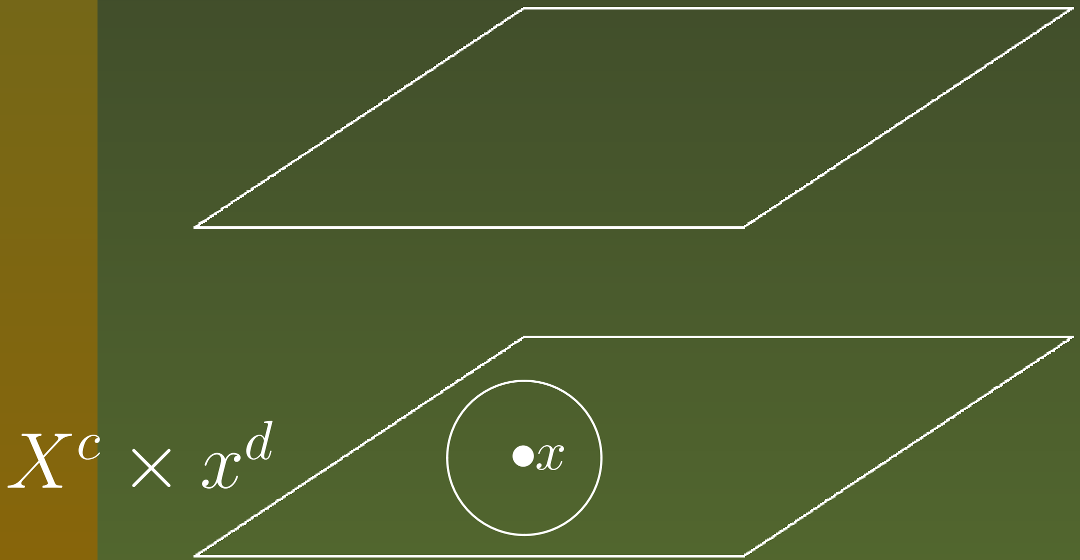

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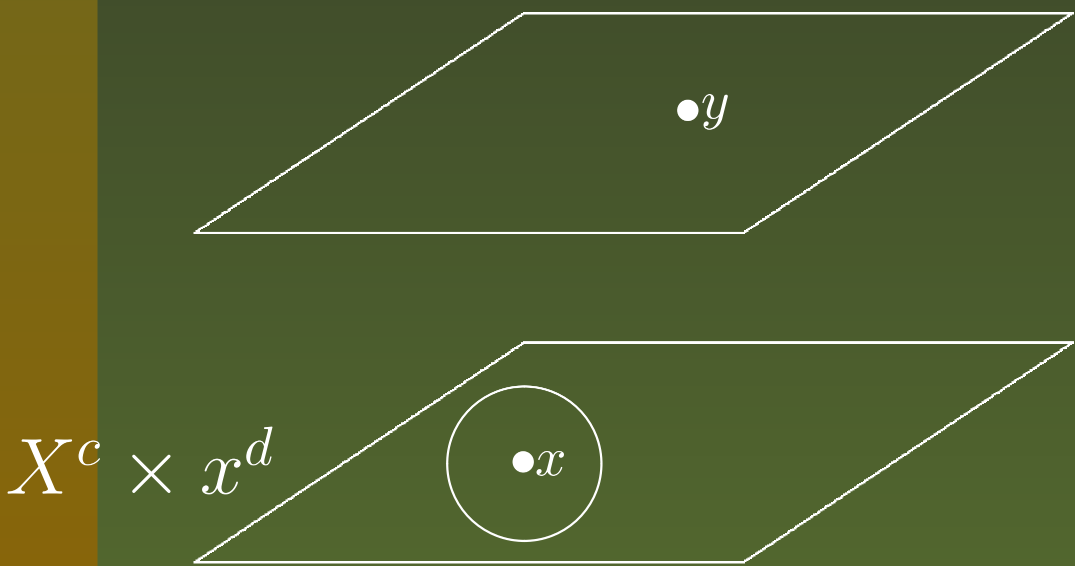

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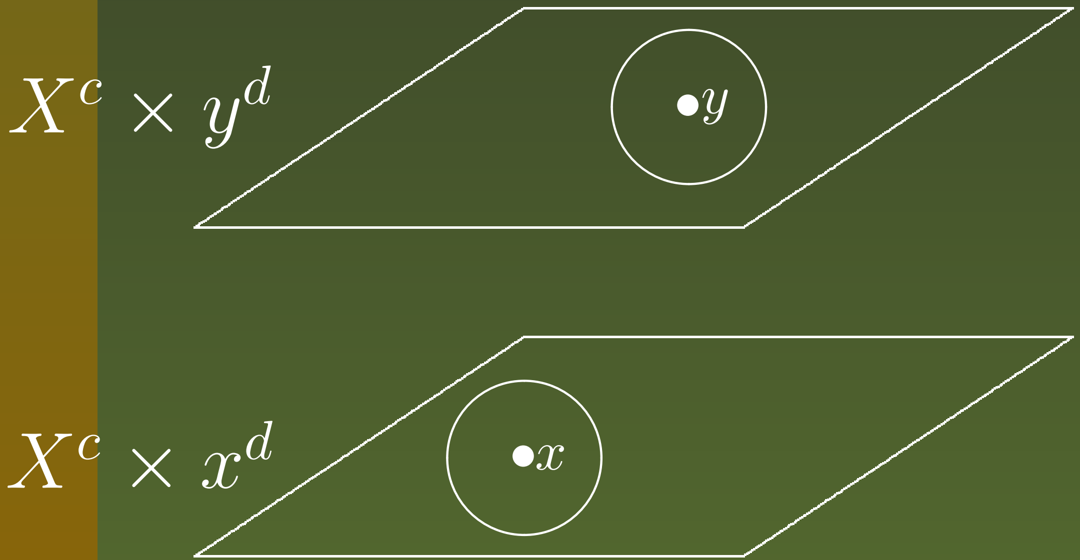

Heatshield discrete neighbors

- Replace any single insulator with a different type

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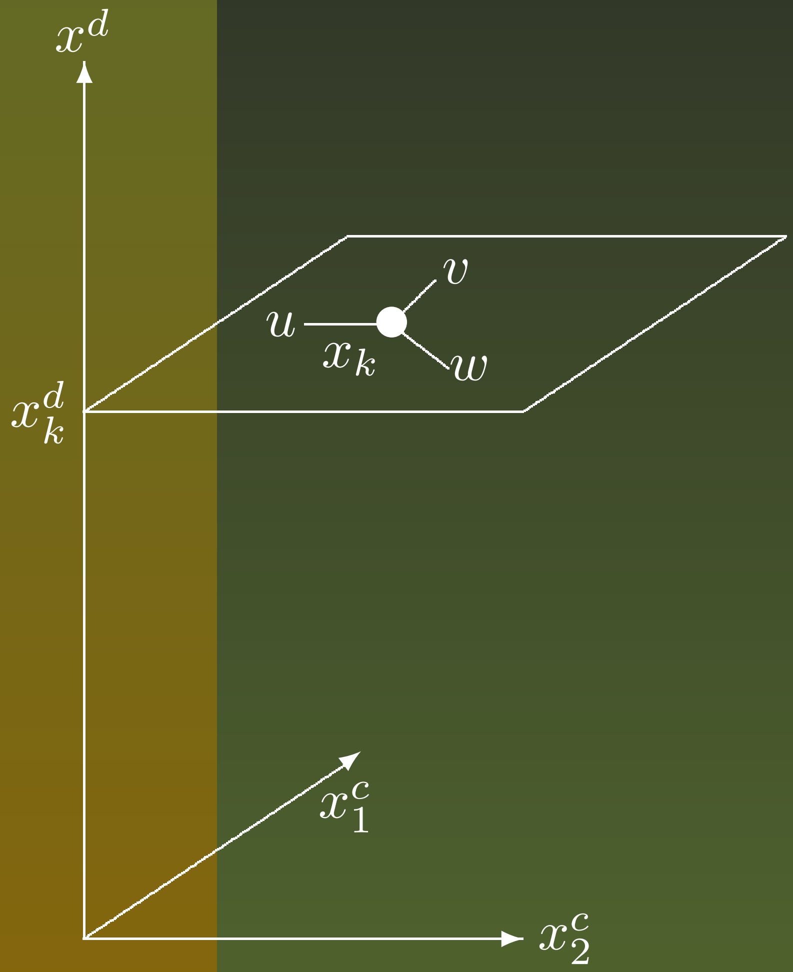
  - The cooling temperature is set to the average of the two intercepts adjacent to it (rounded to the mesh)

Construction of the poll set



$$P_k = \{u, v, w\}$$

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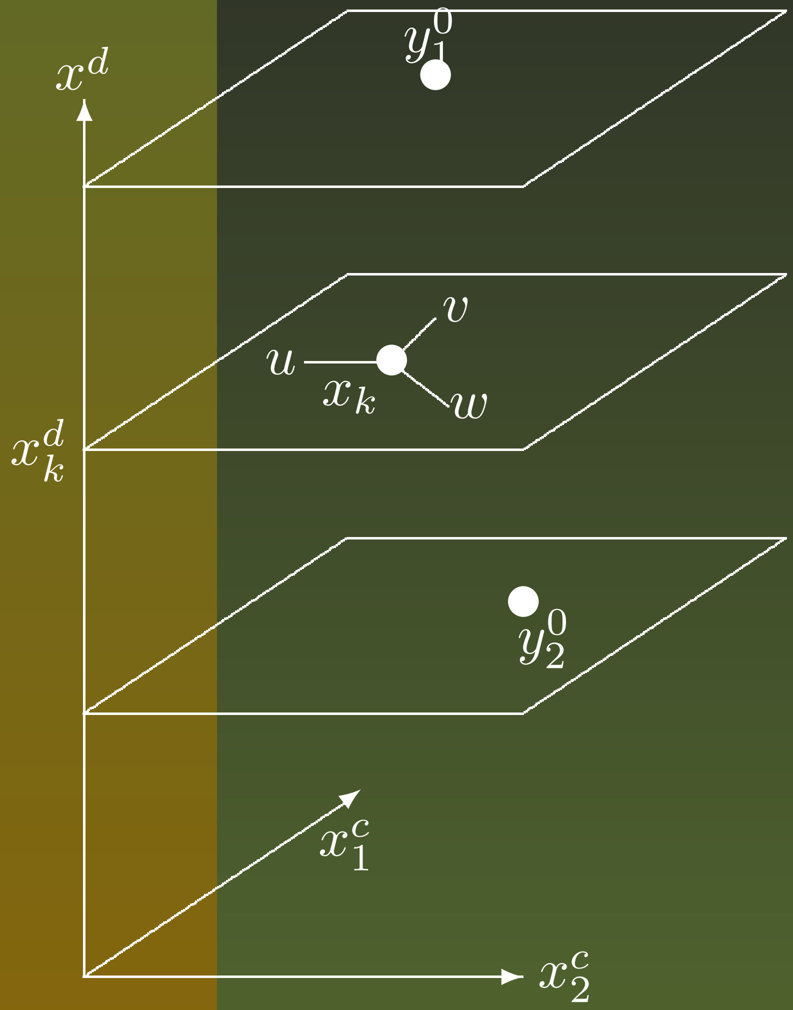

$$P_k = \{u, v, w\}$$
$$\mathcal{N}(x_k) = \{x_k, y_1^0, y_2^0\}$$

$y_1^0 \in \mathcal{N}(x_k)$ satisfies
$$f(x_k) < f(y_1^0) < f(x_k) + \xi$$

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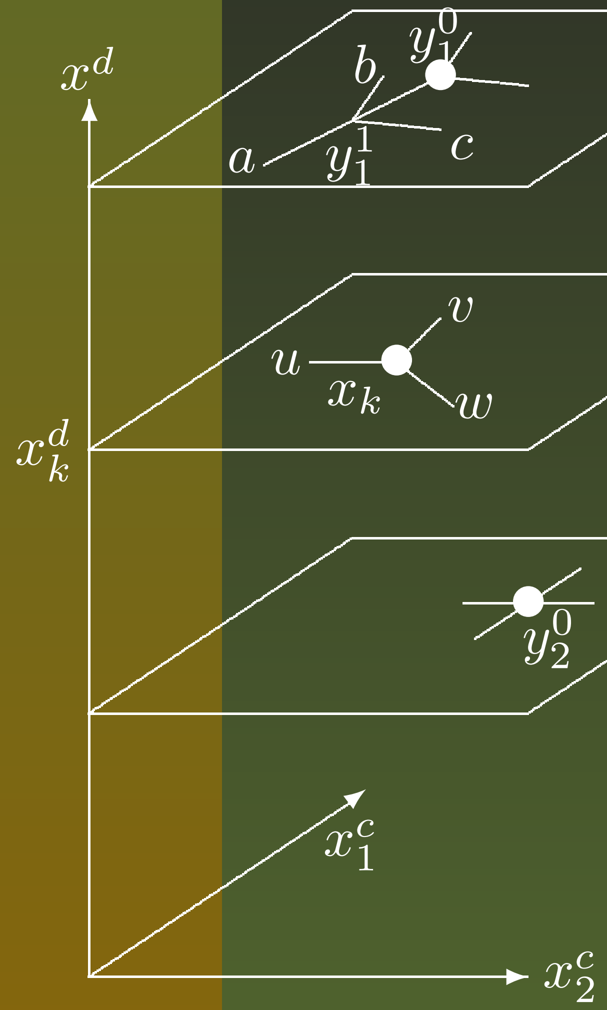

$P_k = \{u, v, w\}$
$\mathcal{N}(x_k) = \{x_k, y_1^0, y_2^0\}$
$\mathcal{X}_k = \{y_1^1\} \cup \{a, b, c\}$

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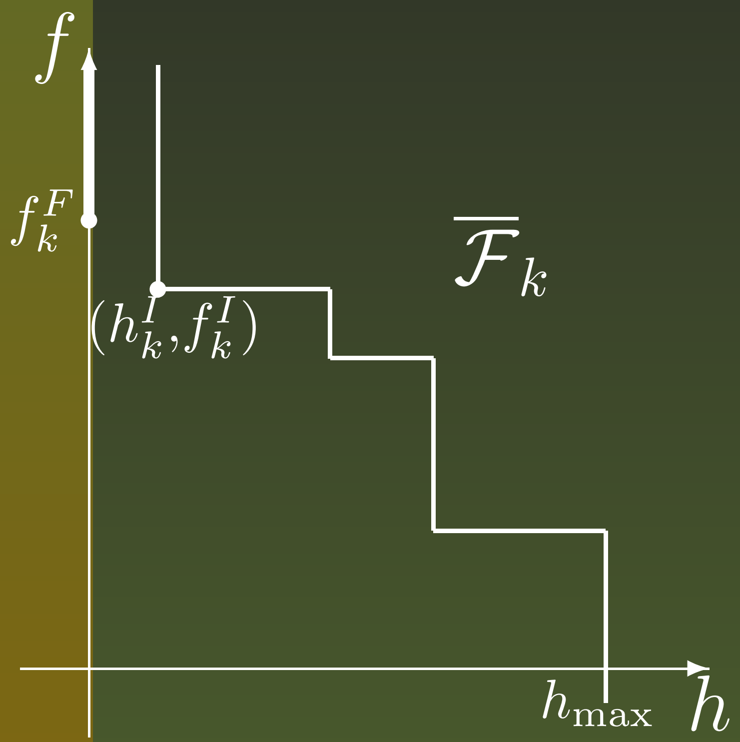
Poll Set: $P_k \cup \mathcal{N}(x_k) \cup \mathcal{X}_k$

Filter GPS for nonlinear constraints



$$h(x) = \|C(x)_+\|^2$$

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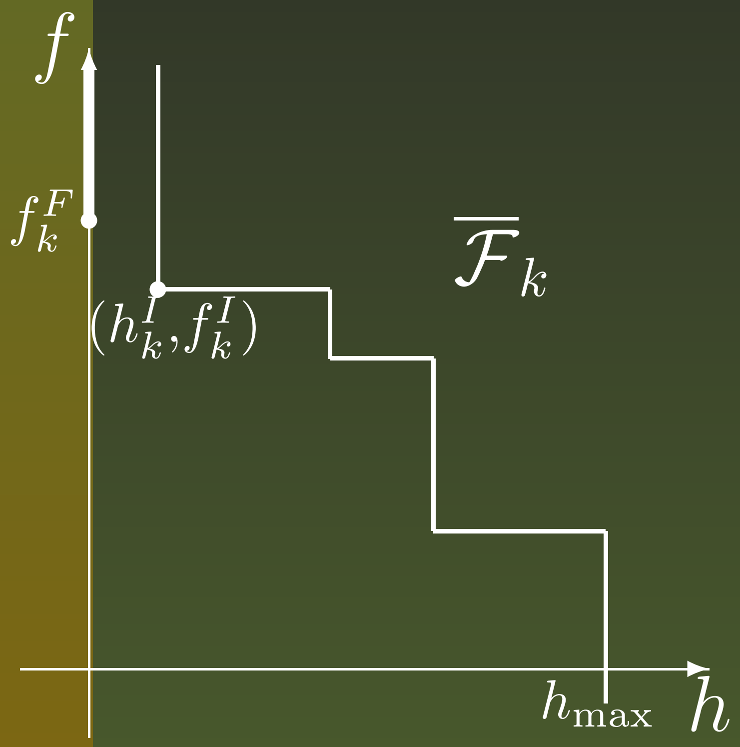

$$h(x) = \|C(x)_+\|^2$$

Poll center is either best feasible point or least infeasible point.

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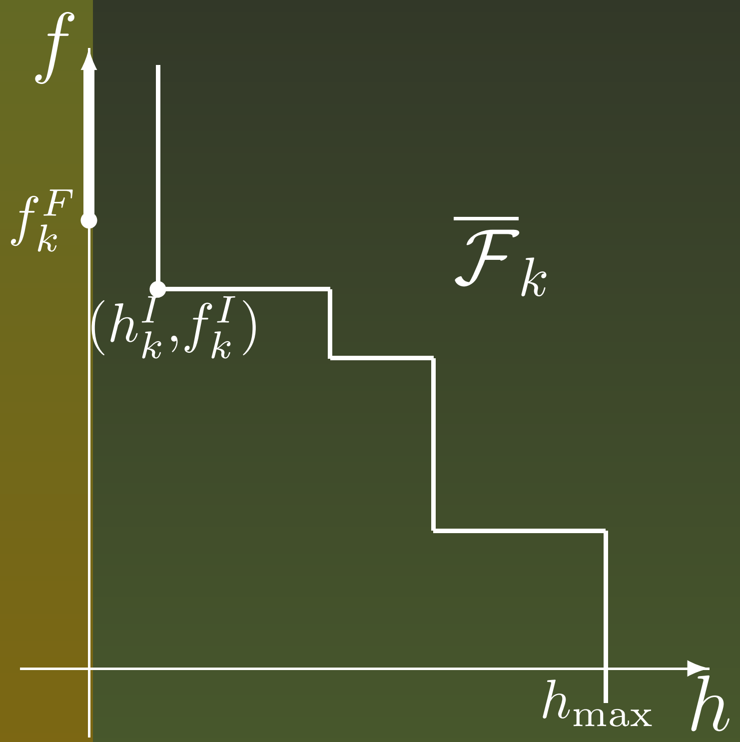

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Poll center is either best feasible point or least infeasible point.

For each trial point x, $h(x)$ and $f(x)$ are plotted on the bi-loss map.

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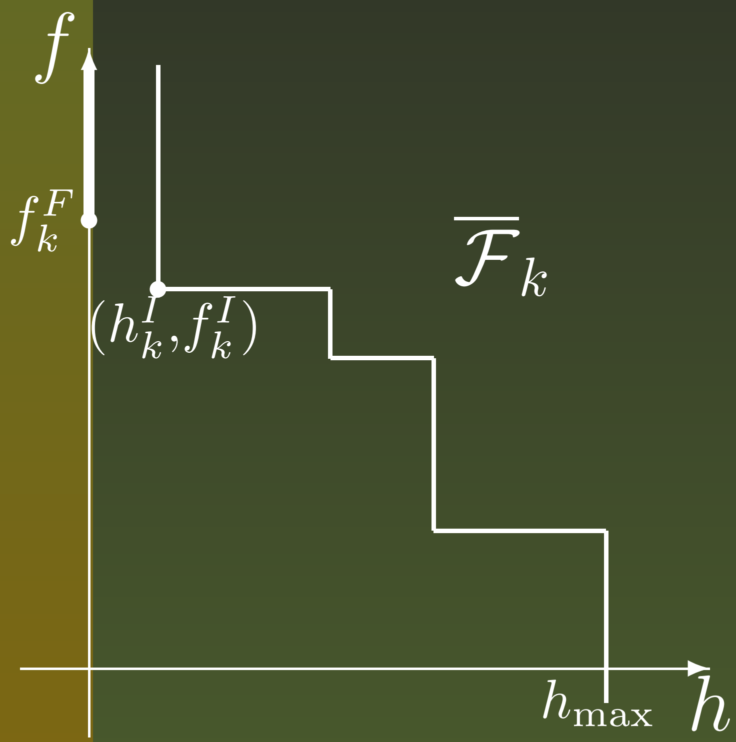

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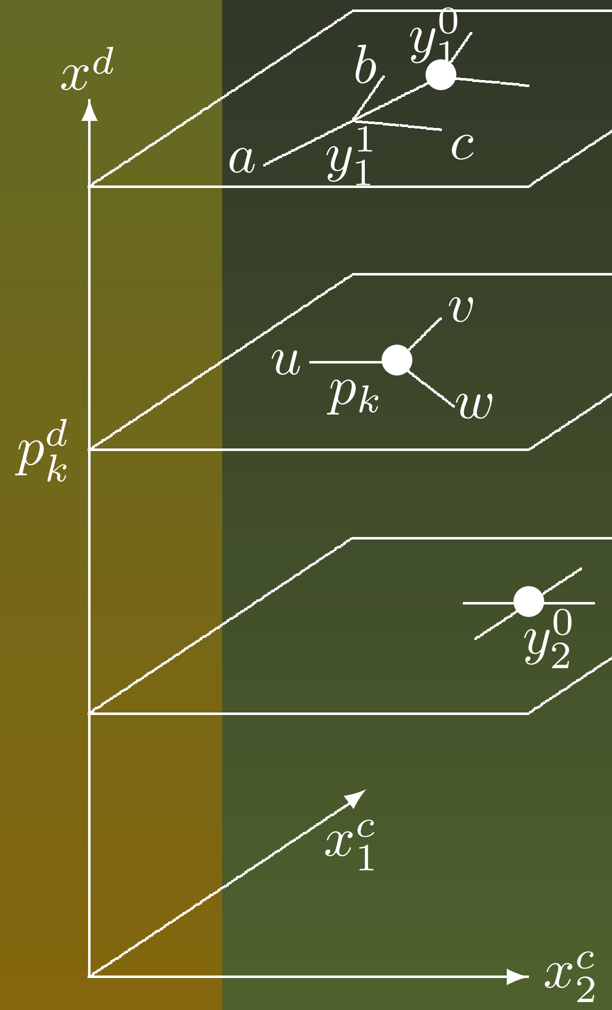
Poll center is either best feasible point or least infeasible point.

For each trial point x, $h(x)$ and $f(x)$ are plotted on the bi-loss map.

If x is unfiltered, it is added to the filter; otherwise, the mesh is refined.

Construction of the poll set



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$\mathcal{N}(p_k) = \{p_k, y_1^0, y_2^0\}$
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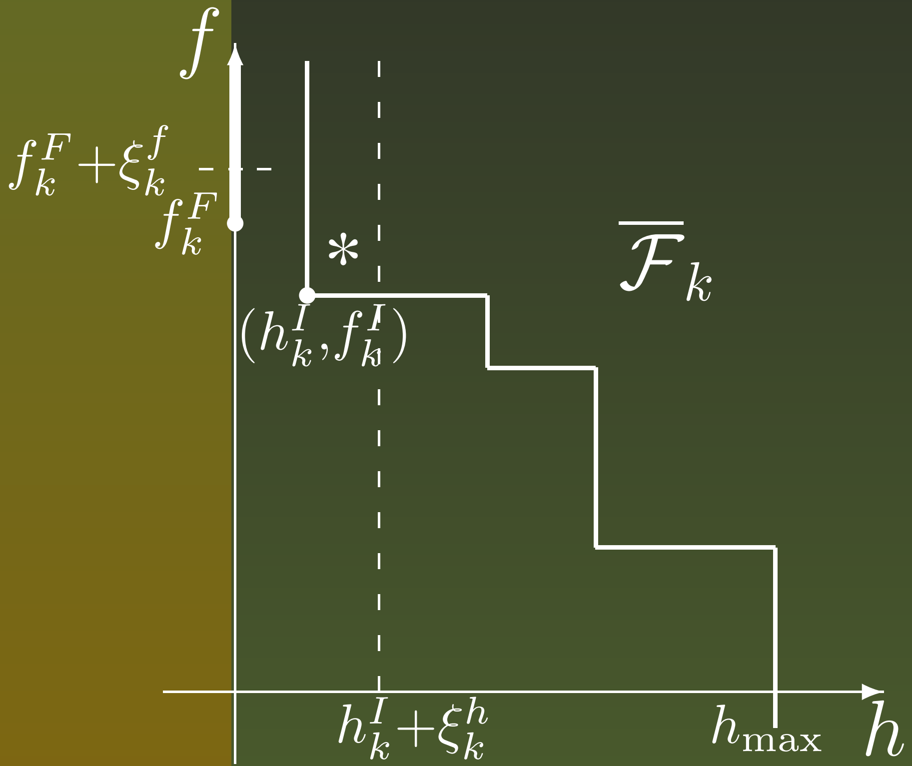
$y_1^0 \in \mathcal{N}(p_k)$ satisfies the extended poll criteria

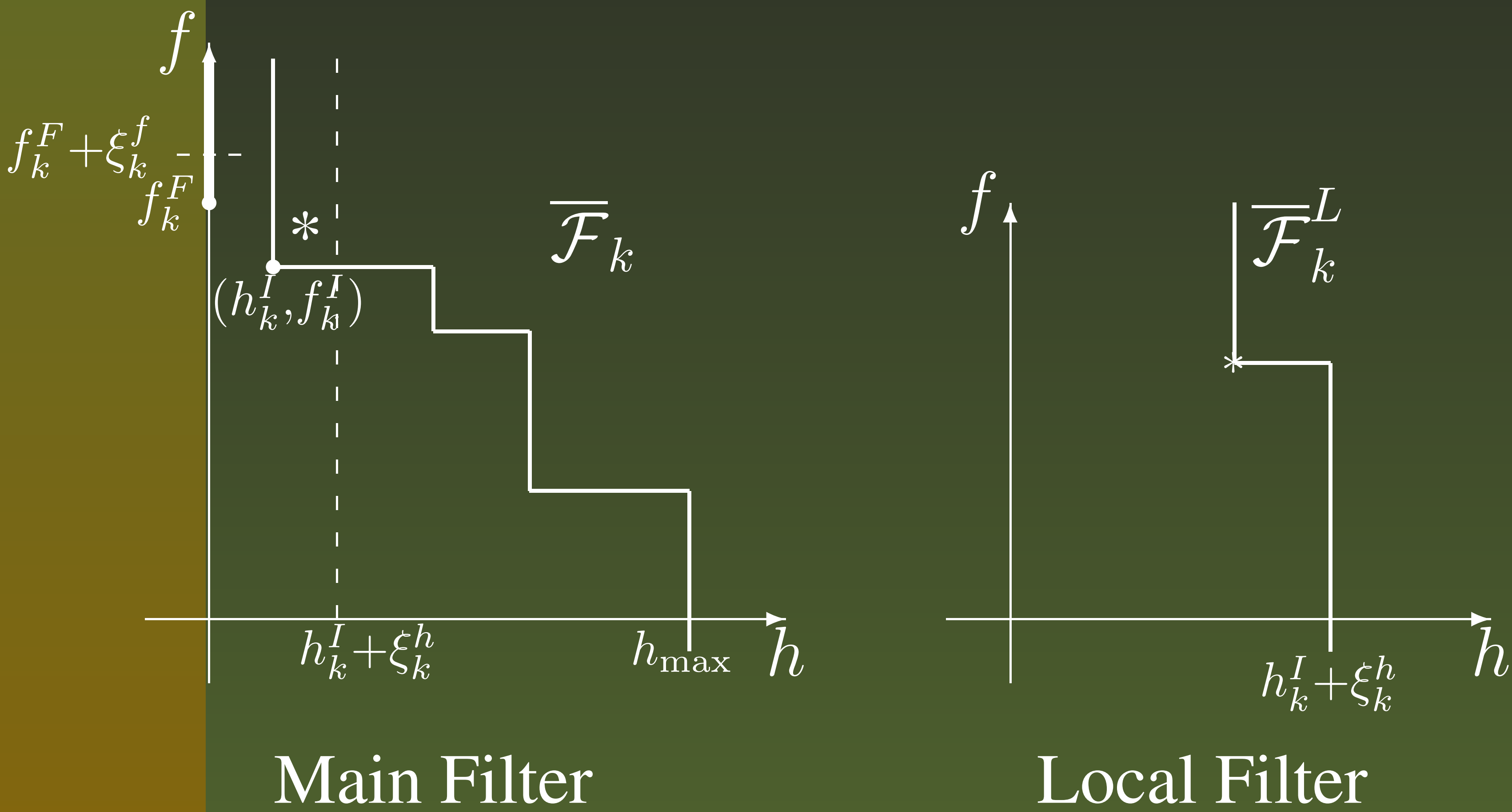
Poll Set: $P_k \cup \mathcal{N}(p_k) \cup \mathcal{X}_k$

Local filter for extended polling



Main Filter

Local filter for extended polling



Filter GPS algorithm for MVP

INITIALIZATION: Set $\Delta_0, \xi > 0$, and populate filter
For $k = 1, 2, \ldots,$ do

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  - If (not found), set $\Delta_{k+1} < \Delta_k$

Convergence theory assumptions

- All iterates lie in a compact set

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- The linear constraint matrix A is rational

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- All discrete neighbors lie on the current mesh
- The set-valued neighborhood function $\mathcal{N} : X \to 2^X$ satisfies a notion of continuity.

Limit points of the algorithm

$\exists$ subsequence K such that $\lim\limits_{k \in K} \Delta_k = 0$, with limit points:

1. $\hat{p} = \lim\limits_{k \in K} p_k$, where $p_k \in \{p_k^F, p_k^I\}$
2. $\hat{y} = \lim\limits_{k \in K} y_k$, where $y_k \in \mathcal{N}(p_k)$ and $\hat{y} \in \mathcal{N}(\hat{p})$.
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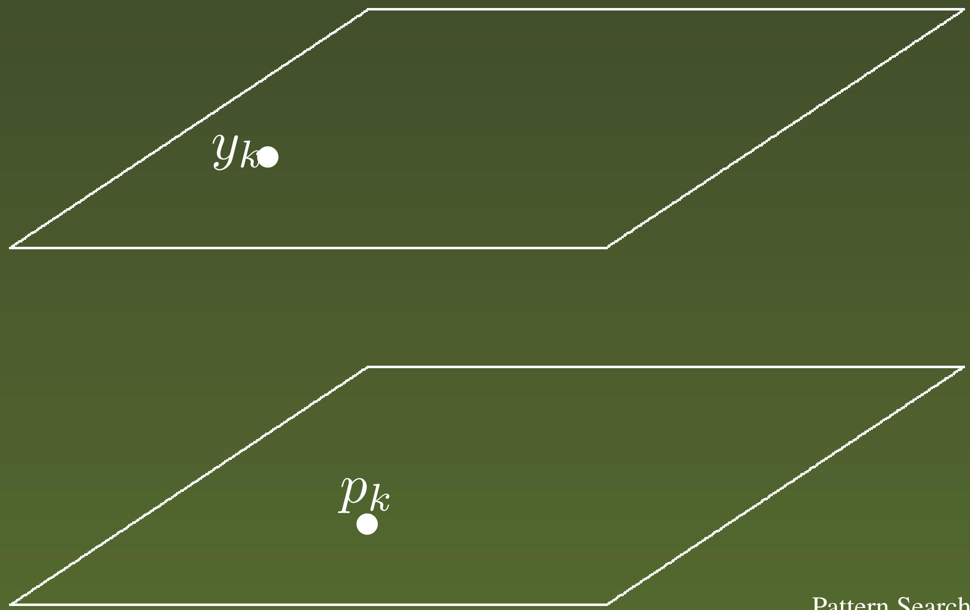

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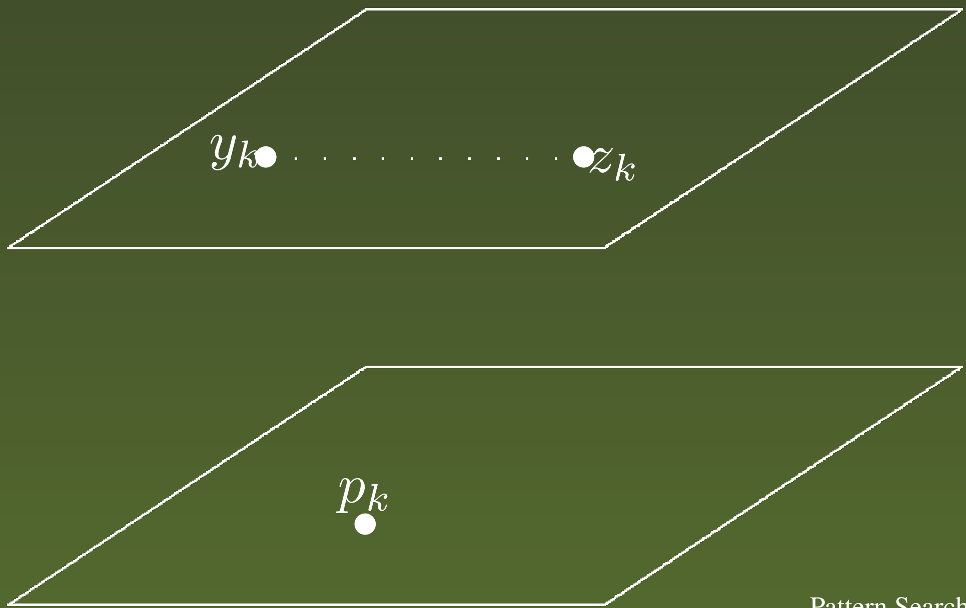

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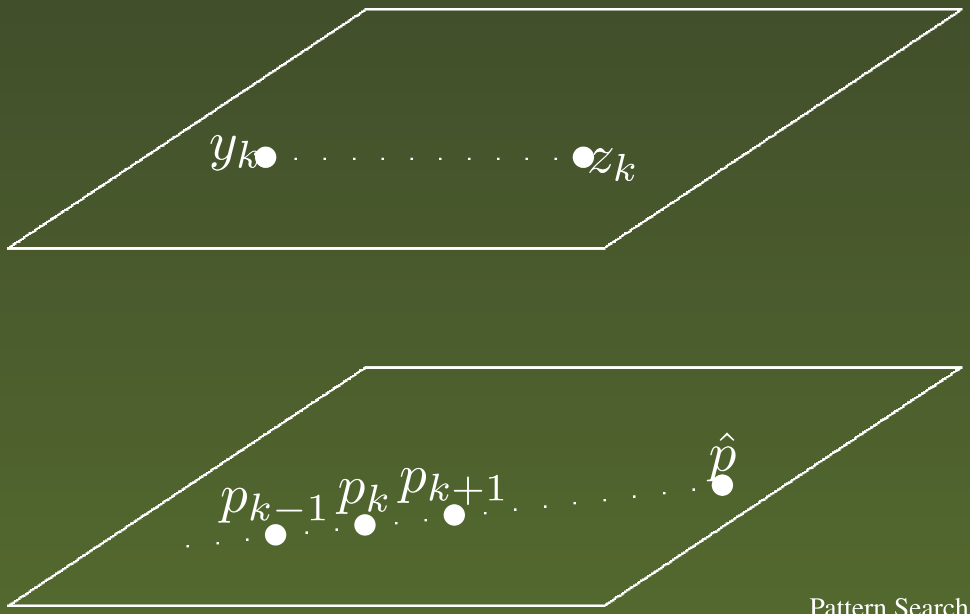

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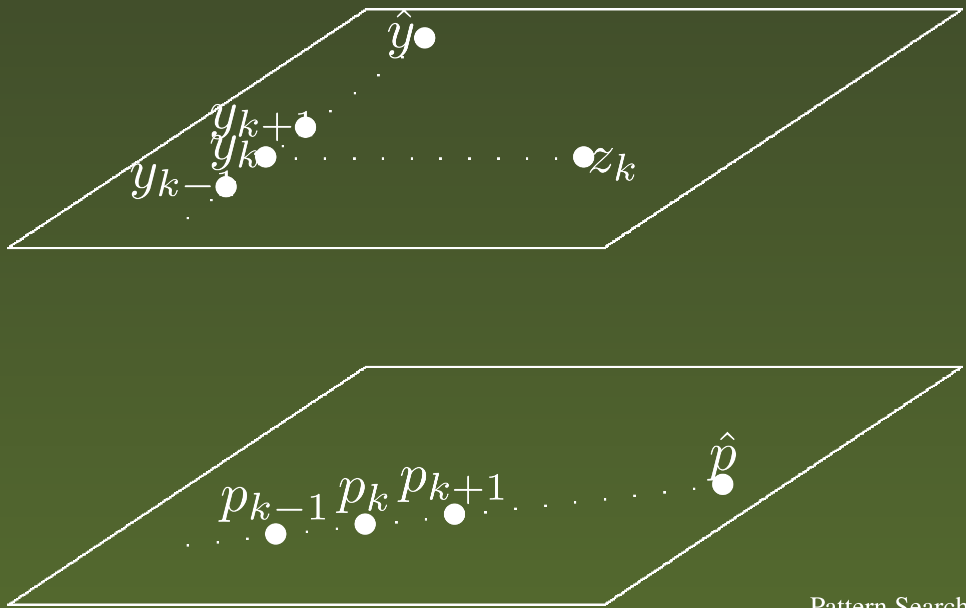

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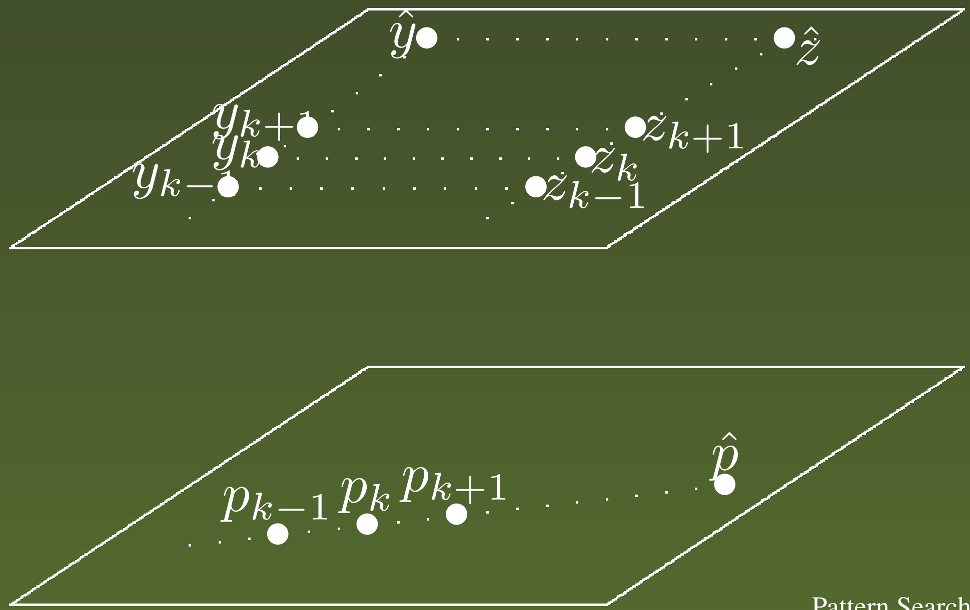

Filter convergence results

Let $D(\hat{p})$ be the set of polling directions used i.o.

- h continuous* at $\hat{p}$ and $\hat{y} \;\Rightarrow\; h(\hat{p}) \leq h(\hat{y})$
- f continuous* at $\hat{p}$ and $\hat{y}$ and $p_k = p_k^F$ i. o. $\Rightarrow f(\hat{p}) \leq f(\hat{y})$
- h Lipschitz* near $\hat{p} \;\Rightarrow\; h^\circ(\hat{p};(d,0)) \geq 0 \;\forall d \in D(\hat{p})$
- f Lipschitz* near $\hat{p}$ and $p_k = p_k^F$ i. o. $\Rightarrow f^\circ(\hat{p};(d,0)) \geq 0 \;\forall d \in D(\hat{p})$
- h strictly differentiable* at $\hat{p}$ and $\;\Rightarrow\; \nabla h(\hat{p}) = 0$

Similar results hold for certain $\hat{z}$

Filter convergence results

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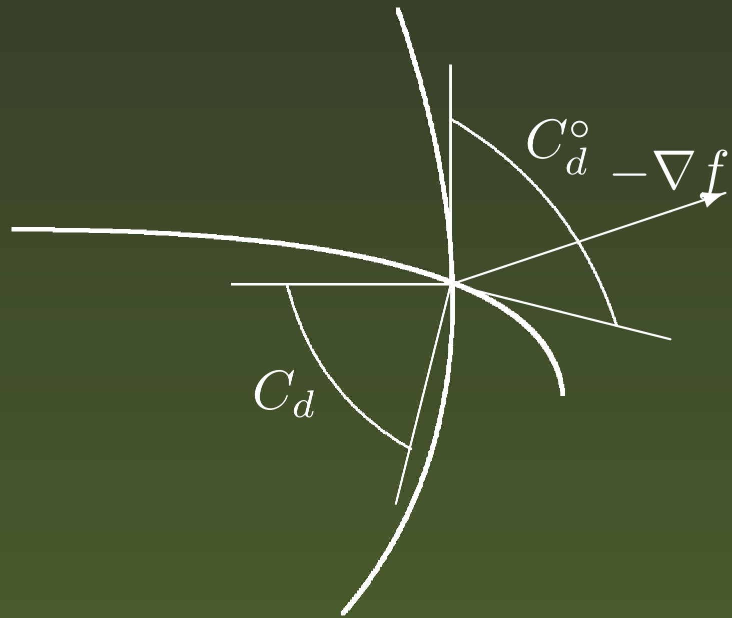
$\Rightarrow\ -\nabla^c f(\hat{p}) \in C_d^\circ$. Similar results hold for certain $\hat{z}$



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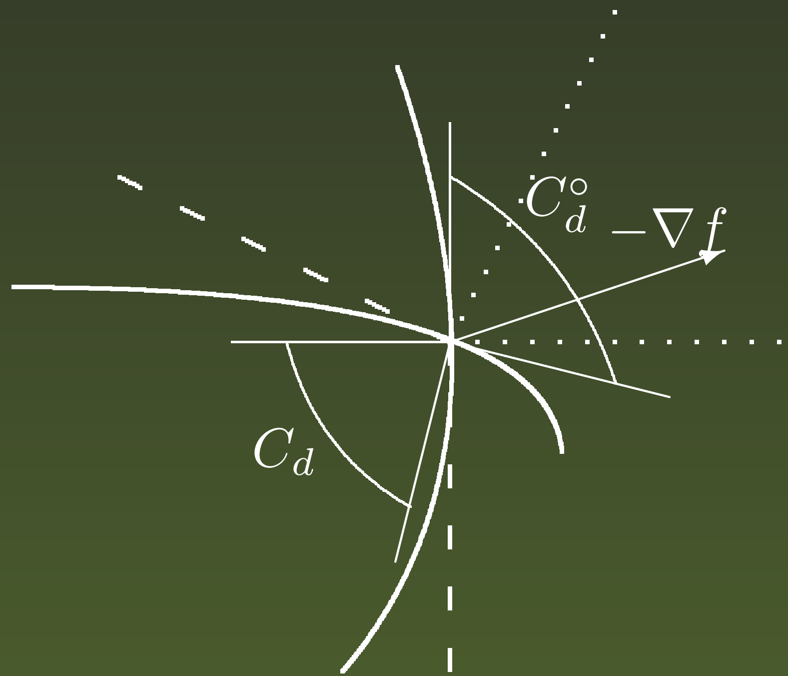

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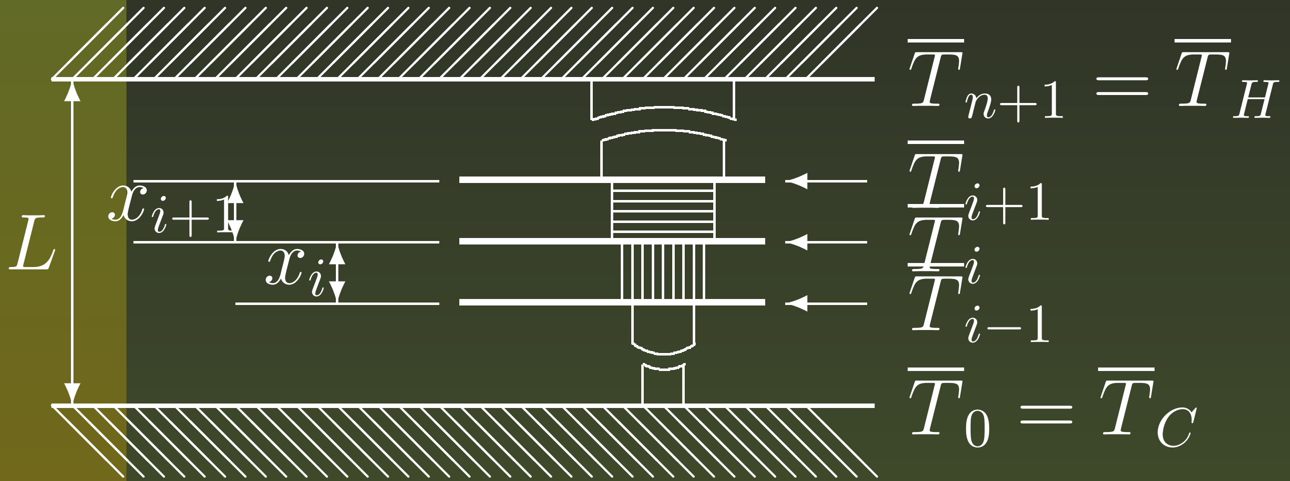
Heat intercept insulation system



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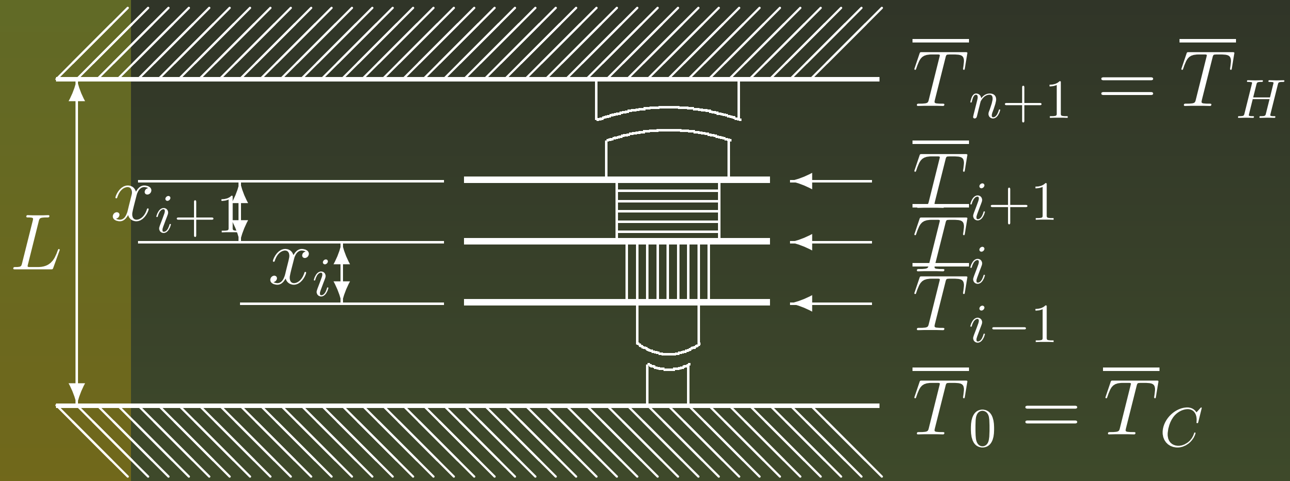

$$\min \quad power(n, I, x, \overline{T})$$

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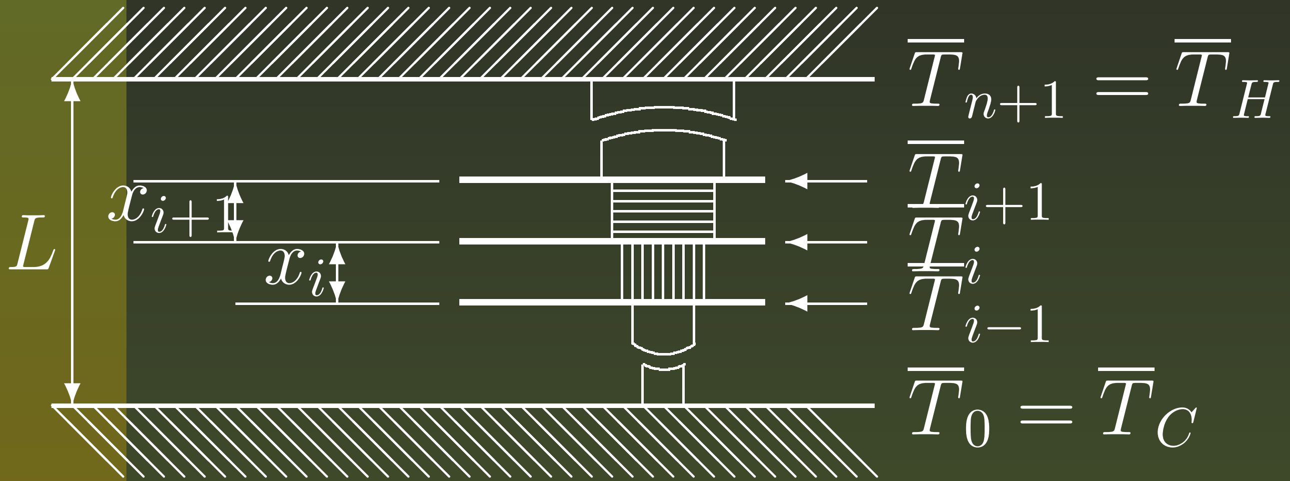

$$\begin{array}{rl} \min & power(n, I, x, \overline{T}) \\ \text{subject to} & n \in \{1, 2, \ldots, n_{\max}\}, \quad I \in \mathcal{I}^{n+1} \\ & \overline{T}_{i-1} \leq \overline{T}_i \leq \overline{T}_{i+1},\ i = 1, 2, \ldots, n \\ & \displaystyle\sum_{i=1}^{n+1} x_i = L, \quad x_i \geq 0,\ i = 1, 2, \ldots, n+1 \end{array}$$

Previous heat shield studies

- **Hilal & Boom**: 1-3 intercepts, single insulator type, constant cross-sectional areas

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- **Current work**: Variable number of intercepts, multiple insulator types, variable cross-sectional areas, load-bearing nonlinear constraints

Nomenclature

$$
\begin{aligned}
k(T;I_i) &= \text{Thermal conductivity function for insulator } i\\
P_i &= \text{Power applied to intercept } i\\
C_i &= \text{Thermodynamic cycle coefficient at intercept } i\\
q_i &= \text{Heat flow from intercept } i \text{ to } i-1\\
A_i &= \text{Cross-sectional area of insulator } i\\
\sigma(T;I_i) &= \text{maximum allowable stress function}\\
e(T;I_i) &= \text{unit thermal expansion function}\\
\rho(I_i) &= \text{density of the insulator } i \text{ material}\\
F &= \text{load (force) to be placed on the system}\\
m_{\max} &= \text{maximum allowable mass of the insulators}\\
\delta &= \text{maximum allowable \% thermal contraction}
\end{aligned}
$$

Heat shield objective and nonlinear constraints

- Minimize **power**: $\sum_{i=1}^{n} P_i = \sum_{i=1}^{n} C_i \left(\frac{\overline{T}_H}{\overline{T}_i} - 1 \right) (q_i - q_{i-1})$

  By Fourier's law: $q_i = \frac{A_i}{x_i} \int_{\overline{T}_{i-1}}^{\overline{T}_i} k(T; I_i) dT, \quad i = 1, 2, \ldots, n+1$

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Heat shield implementation

- **Materials**:

Nylon	6063-T5 Aluminum
Teflon	Fiberglass Epoxy (normal cloth)
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Heat shield computational results

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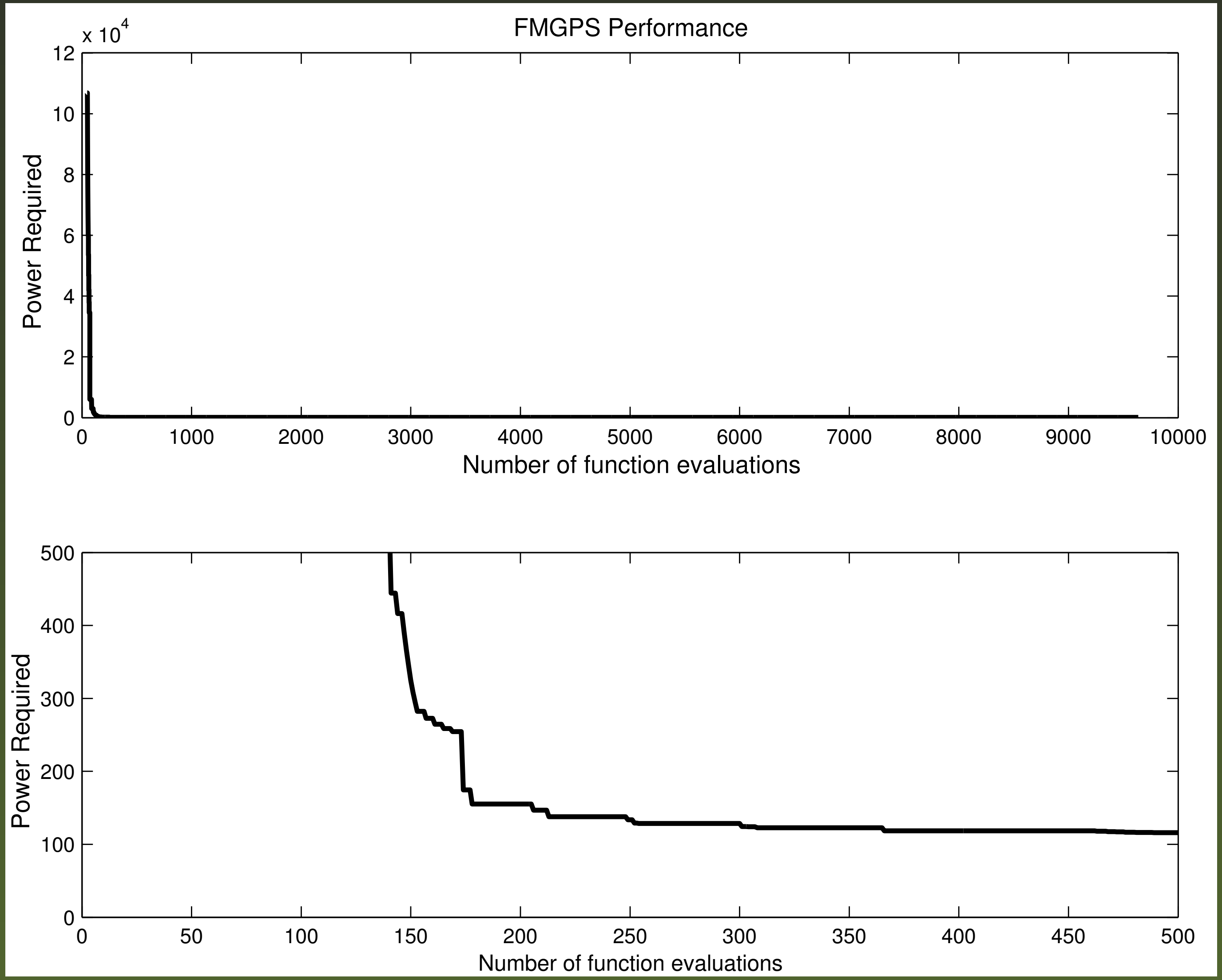
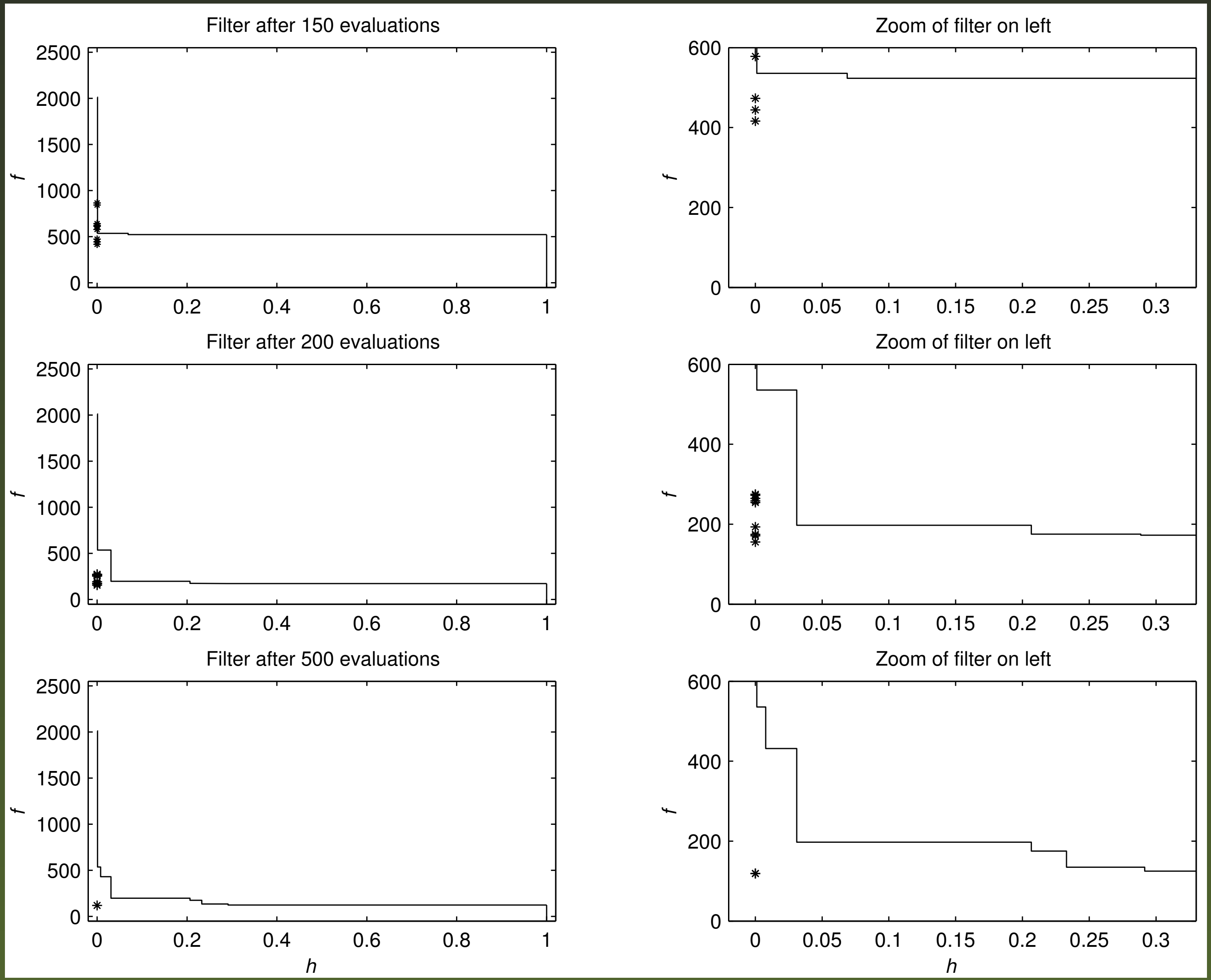
Profile of heat shield run



Heat shield filter progress



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