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A NUMERICAL INVESTIGATION OF THE NON-LINEAR MECHANICS OF WAVE DISTURBANCES IN PLANE POISEUILLE FLOWS

by

T. H. Gawain and W. H. Clark

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral A. S. Goodfellow, Jr.
Superintendent

M. U. Clauser
Academic Dean

ABSTRACT:

The response of a plane Poiseuille flow to disturbances of various initial wavenumbers and amplitudes is investigated by numerically integrating the equation of motion. It is shown that for very low amplitude disturbances the numerical integration scheme yields results that are consistent with those predictable from linear theory. It is also shown that because of non-linear interactions a growing unstable disturbance excites higher wavenumber modes which have the same frequency, or phase velocity, as the primary mode. For very low amplitude disturbances these spontaneously generated higher wavenumber modes have a strong resemblance to certain modes computed from the linear Orr-Sommerfeld equation.

In general it is found that the disturbance is dominated for a long time by the primary mode and that there is little alteration of the original parabolic mean velocity profile. There is evidence of the existence of an energy equilibrium state which is common to all finite-amplitude disturbances despite their initial wavenumbers. This equilibrium energy level is roughly 3-5% of the energy in the mean flow which is an order of magnitude higher than the equilibrium value predicted by existing non-linear theories.

T. H. Gawain
Professor of Aeronautics

W. H. Clark
Visiting Assistant Professor

Approved by:

R. W. Bell
Chairman
Department of Aeronautics

Released by:

C. E. Menneken
Dean of Research Administration

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1. Introduction

In this paper we present some results from a numerical investigation of the phenomena of stability, transition and turbulence in incompressible channel flows. Ideally, the goal of such an investigation would be to numerically integrate the Navier-Stokes equations and thereby be able to follow the growth of some unstable disturbance which perturbs an initial laminar Poiseuille flow. It would then be possible to observe in detail how this growing disturbance eventually produces a fully developed, stationary turbulent flow. Presumably, if one were to repeat such a calculation for a large number of initial disturbances then the final turbulent flow could be described by an ensemble average over all the initial conditions.

The results of such a computation would represent an exact (to within the limits of the numerical model) solution to the turbulence problem. Since turbulence is an inherently three-dimensional phenomenon, these computations would have to be carried out on a three-dimensional grid; unfortunately, the computational grid would have to be large enough to contain the large "eddies" representing the initial disturbance and still be able to resolve the smallest energy dissipation length scales of the final turbulence. Even at low Reynolds numbers the ratio of the larger length scale to the smallest scale is several orders of magnitude. The number of grid points is proportional to the cube of this ratio; hence, a complete three-dimensional solution is impractical, if not impossible, despite the large storage capacity of modern computers. (Emmons 1970) To overcome this impractically large storage requirement we have drastically simplified the problem by treating the flow on a

two-dimensional basis. It is recognized that a two-dimensional treatment can not adequately represent true physical turbulence. Never-the-less the two-dimensional problem can be handled without further approximation and, as will be pointed out in the next section, two-dimensional solutions are of interest in their own right.

2. Background

During the past decade considerable effort was devoted to developing theories for the response of plane Poiseuille flows to finite-amplitude disturbances. The most significant contribution of these theories (in particular, those of Stuart and Watson (1960) and Reynolds and Potter (1967)) was in showing that the square of the amplitude ($|A_1|^2$) of an initially infinitesimal unstable disturbance is governed by an equation of the form

$$\frac{d\,|A_1|^2}{dt} = k_1\,|A_1|^2 + k_2\,|A_1|^4 \qquad (1)$$

Equation 1, which can be derived from the Navier-Stokes equations, is an approximation valid in a region in the (α-R_e)-plane which is close to the "neutral curve". Here R_e denotes the Reynolds number and α the wave number of the disturbance. For disturbances with very small amplitudes the second term on the right of equation 1 must be neglible; hence, the square of the disturbance amplitude has an exponential growth and the constant, k_1, is related to the exponential amplification factor of the linear theory (i.e., $k_1 = -2\,\beta_{11}$). Of particular interest is the case of $k_1 > 0$ for which the flow is unstable to small disturbances.

If $k_2 < 0$ then it is possible that the higher order terms will eventually balance the leading term and the amplitude growth is limited such that

$$\frac{d\,|A_1|^2}{dt} \to 0$$

$$|A_1|^2 \to -k_1/k_2 \tag{1b}$$

as

$$t \to +\infty$$

Under such conditions a "supercritical equilibrium state" is said to exist. On the other hand, if $k_1 > 0$ and $k_2 > 0$ then no positive limiting value for $|A_1|^2$ is possible according to equation 1. It is possible to evaluate k_2 based on Stuart's (1960) theory and the necessary calculations have been carried out by Pekeris and Shkoller (1967) and Reynolds and Potter (1967). These authors have found that under certain conditions $k_2 < 0$ for some unstable disturbances. Both Pekeris and Shkoller and Reynolds and Potter have found that for regions near the lower branch of the "neutral curve" $k_2 < 0$ while $k_2 > 0$ near the upper branch. Hence, one can conclude that disturbances whose wavelength corresponds to a point near the lower branch can grow according to the non-linear theory and reach a supercritical equilibrium state. On the other hand, for unstable disturbances whose wavelength corresponds to a point in the (α, R_e) plane near the upper branch the non-linear theory does not seem to be applicable. At least the existing theories give us no information about the growth of such a disturbance beyond the linear range of amplitudes.

Such "supercritical equilibrium states" are not observed experimentally in channel flows; instead the initial instabilities lead immediately to three dimensional turbulent flows. It seems, that no experimental verification of this aspect of finite-amplitude theory is possible. A computer simulation of a two dimensional channel flow is, perhaps, the best way of investigating the two dimensional response of a plane Poiseuille flow to an initial unstable disturbance. A computer simulation offers, in addition to its two dimensionality, the advantage of providing an "exact" solution to the equations of motion ("exact" to within the limits of a finite difference representation) without introducing any approximations concerning the relative magnitudes of the various modes of which the disturbance is composed.

The present paper presents the results of a numerical investigation of the response of a plane Poiseuille flow to various initial disturbances. An attempt has been made to present the results in a manner that will facilitate comparison with the existing non-linear theory. The difficulty in making such comparisons is due to the fact that the theories mentioned concern the growth of an unstable disturbance with zero initial amplitude (at time $t = -\infty$) which grows at first according to linear theory and finally reaches an amplitude of such magnitude that the non-linear equations are appropriate. In a numerical simulation, with limited computation time available, the initial disturbance must have some finite amplitude. The growth rates predicted by the finite-amplitude theories are very small and the time required to follow the growth of a very small disturbance to the non-linear range consumes enormous amounts of computer time. For this reason the numerical

investigator is tempted to start his calculations by using a disturbance whose amplitude is of such magnitude that the entire linear range can be bypassed. It will be pointed out in Section 6 that there seem to be significant qualitative differences in the flow when large initial amplitudes are used as compared with those for which very small initial amplitudes are used in the calculations.

Another aspect of the present investigation was to determine under what circumstances, if any, a two dimensional representation of the flow could be made to resemble in some respects three dimensional turbulent flows.

It is clear that the "supercritical equilibrium states" in which the unstable disturbance mode remains dominant do not resemble turbulence in which the disturbance energy is distributed over a wide spectral range. Also in a "supercritical equilibrium state" the resultant mean velocity profile differs only slightly from the parabolic laminar profile while for turbulent flows the mean velocity profiles bear little resemblance to the original laminar one. It is recognized that there are significant differences between two dimensional and true three dimensional turbulence. This is particularly evident in the energy spectrum and mechanism for the transfer of energy and vorticity between the various spectral components (Lilly (1968) and Kraichnan (1967)).

However, could it be possible that for some choice of initial disturbance mode shapes and amplitudes the structure of the resultant two dimensional flow could lie somewhere between the (relative)

simplicity of a supercritical equilibrium state and the "complete chaos" characterizing a true turbulent flow? This aspect of the present paper will be discussed in Section 6.

3. Basic Equations

Consider the two dimensional flow of an incompressible fluid between two parallel planes. The governing equations of motion are

$$\frac{\partial \zeta}{\partial t} - \frac{\partial \Psi}{\partial y}\frac{\partial \zeta}{\partial x} + \frac{\partial \Psi}{\partial x}\frac{\partial \zeta}{\partial y} = \frac{1}{R_e}\nabla^2 \zeta \tag{2a}$$

and

$$\zeta = \nabla^2 \Psi \tag{2b}$$

where ζ and Ψ are the vorticity and stream function respectively with ζ and Ψ defined by

$$u = -\frac{\partial \Psi}{\partial y}\ ; \quad v = \frac{\partial \Psi}{\partial x} \tag{3a}$$

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\ . \tag{3b}$$

The variables are assumed to be normalized on suitable characteristic lengths and velocities which are shown in Figure 1 along with other parameters of interest. The velocity components can be expressed as

$$\begin{aligned} u &= U(y) + u'\ ; \quad v = v'\ ; \quad \zeta = -\frac{dU}{dy} + \zeta' \\ \Psi &= 3/2\ \left(\frac{1}{3}y^3 - y\right) + \Psi' \end{aligned} \tag{4}$$

where the primed quantities which depend on x, y, t represent the instantaneous departure of the flow from the original laminar flow. Substituting equations 4 into equations 2 results in

$$\frac{\partial \zeta'}{\partial t} + U \frac{\partial \zeta'}{\partial x} - 3 \frac{\partial \psi'}{\partial x} - \frac{\partial \psi'}{\partial y} \frac{\partial \zeta'}{\partial x} + \frac{\partial \psi'}{\partial x} \frac{\partial \zeta'}{\partial y} = \frac{1}{R_e} \nabla^2 \zeta' \tag{5a}$$

$$\zeta' = \nabla^2 \psi' \tag{5b}$$

The conditions of zero slip at the wall and constant mass flow rate imply that the stream function, ψ', satisfy the boundary conditions

$$\psi' = \partial \psi' / \partial y = 0 \quad \text{at} \quad y = \pm 1. \tag{6}$$

By considering periodic solutions to equation 2 the boundary conditions in the streamwise direction are fixed by

$$\psi' (x,y,t) = \psi' (x \pm 2nL, y, t) \tag{7a}$$

$$\zeta' (x,y,t) = \zeta' (x \pm 2nL, y, t) \tag{7b}$$

where the basic period is 2L. Equations 5a and 5b with the boundary conditions (6) and (7) are to be solved for $\psi'(x,y,t)$ for a given initial value of $\psi'(x,y,0)$.

At this point it is convenient to define some parameters that are useful in describing the flow structure. The assumed periodicity implies homogeneity in the streamwise direction; hence, all averaging is done with respect to the streamwise coordinate, x. The average value of any quantity, $q(x,y,t)$, is denoted by an overbar and obtained by integrating over one period of the basic wavelength.

$$\overline{q} (y,t) = \frac{1}{2L} \int_{-L}^{L} q(x,y,t) \, dx \tag{8}$$

It should again be emphasized that in our representation the flow is arbitrarily divided into a laminar and a disturbance part. Hence, the total streamwise velocity component is given by

$$u(x,y,t) = U(y) + u'(x,y,t) \tag{9}$$

and the mean velocity is

$$\overline{u}(y,t) = U(y) + \overline{u'}\,(y,t) \tag{10}$$

The difference between the mean and local values of any quantity is referred to as the turbulent part of that quantity and is denoted by a circumflex. Thus we have

$$\hat{u}\,(x,y,t) = u(x,y,t) - \overline{u}(y,t) = u' - \overline{u'} \tag{11a}$$

$$\hat{v}\,(x,y,t) = v'\,(x,y,t) \tag{11b}$$

$$\hat{\Psi}\,(x,y,t) = \Psi'\,(x,y,t) - \overline{\Psi'}\,(y,t) \tag{11c}$$

$$\hat{\zeta}\,(x,y,t) = \zeta'\,(x,y,t) - \overline{\zeta'}\,(y,t) \tag{11d}$$

and

$$\hat{u} = -\frac{\partial\hat{\Psi}}{\partial y} \tag{11e}$$

$$\overline{u'} = -\frac{\partial\overline{\Psi}'}{\partial y} \tag{11f}$$

$$\hat{v} = \frac{\partial\hat{\Psi}}{\partial x} \tag{11g}$$

$$\overline{v'} = \frac{\partial\overline{\Psi}'}{\partial x} = 0 \tag{11h}$$

The turbulent kinetic energy is

$$\hat{E}\,(x,y,t) = \tfrac{1}{2}\left[\left(-\frac{\partial\hat{\Psi}}{\partial y}\right)^2 + \left(\frac{\partial\hat{\Psi}}{\partial x}\right)^2\right] \tag{12a}$$

The mean value is

$$\bar{\hat{E}}\,(y,t) = \frac{1}{2L} \int_{-L}^{L} \hat{E}\,(x,y,t)\,dx \tag{12b}$$

and the normalized sum of turbulent kinetic energy over a domain extending from the upper to the lower wall and for one period of the basic disturbance is

$$\hat{E}(t) = \tfrac{1}{2} \int_{-1}^{1} dy \left\{ \frac{1}{2L} \int_{-L}^{L} \hat{E}\,(x,y,t)\,dx \right\} \tag{12c}$$

Similarly, the kinetic energy in the mean flow is

$$E\,(y,t) = \tfrac{1}{2} \left[U(y) - \frac{\partial \bar{\Psi}'}{\partial y} \right]^2 \tag{13a}$$

and the normalized sum over the domain is

$$E\,(t) = \tfrac{1}{2} \int_{-1}^{1} E\,(y,t)\,dy \tag{13b}$$

The normalized energy in the initial laminar flow is

$$E_{\ell} = \tfrac{1}{2} \int_{-1}^{1} \tfrac{1}{2} U^2\,dy = .600\ . \tag{13c}$$

We can represent the periodic disturbance by means of a Fourier series as follows:

$$\Psi'\,(x,y,t) = \sum_{n=-\infty}^{\infty} f_n\,(y,t)\,e^{-i\,n\alpha x} \tag{14a}$$

$$\zeta'\,(x,y,t) = \sum_{n=-\infty}^{\infty} g_n\,(y,t)\,e^{-in\alpha x} \tag{14b}$$

with

$$f_n(y,t) = \frac{1}{2L} \int_{-L}^{L} \Psi'(x,y,t)\, e^{in\alpha x}\, dx \tag{15a}$$

$$f_{-n} = f_n^*$$

$$g_n(y,t) = \frac{1}{2L} \int_{-L}^{L} \zeta'(x,y,t) e^{in\alpha x}\, dx \tag{15b}$$

$$g_{-n} = g_n^*$$

where $\alpha = \pi/L$, and superscript * denotes a complex conjugate. From 5b

$$g_n = f_n'' - (n\alpha)^2 f_n \tag{15c}$$

Notice that the mean value of the disturbance streamfunction and vorticity is simply

$$\overline{\Psi'}(y,t) = f_o(y,t) \tag{16}$$

$$\overline{\zeta'}(y,t) = g_o(y,t) \tag{17}$$

The mean value of the turbulent kinetic energy is given by

$$\overline{\hat{E}}(y,t) = \sum_{n=-\infty}^{\infty} E_n(y,t) - E_o(y,t) \tag{18}$$

where each of the components of the energy spectrum function, $E_n(y,t)$, is given by

$$E_n(y,t) = \tfrac{1}{2}\left(|f_n'|^2 + \left(\frac{n\pi}{L}\right)^2 |f_n|^2 \right) \tag{19}$$

The normalized total turbulent kinetic energy over the domain is

$$\hat{E}(t) = \sum_{n=-\infty}^{\infty} E_n(t) - E_o(t) \tag{19a}$$

where

$$E_n(t) = \tfrac{1}{2} \int_{-1}^{1} E_n(y,t)dy \tag{19b}$$

4. The Numerical Model

A rectangular grid of dimensions 2 x 2L is used with M x N grid points located as shown in Figure 2. For the calculations presented herein 64 x 201 grid points were used. The indices k, j, and ℓ denote stations along x, y, and t respectively and δx, δy, δt are the corresponding intervals. The value of the disturbance streamfunction $\psi'(x,y,t)$ is then denoted by $\psi^{\ell}_{K,J}$. The use of the primes will be discarded from this point unless necessary for clarity. Equations 5a and 5b are expressed in finite difference form as follows

$$\frac{\zeta^{\ell+1}_{K,J} - \zeta^{\ell-1}_{K,J}}{2\delta t} = 3\,\frac{\psi^{\ell}_{K+1,J} - \psi^{\ell}_{K-1,J}}{2\delta x} - U_J\,\frac{\zeta^{\ell}_{K+1,J} - \zeta^{\ell}_{K-1,J}}{2\delta x} + J^{\ell}_{K,J} + \frac{1}{R_e}\,\overline{\Delta^2}\,\zeta^{\ell}_{KJ} \tag{20a}$$

$$\zeta^{\ell}_{K,J} = \Delta^2\,\psi^{\ell}_{K,J} = \frac{1}{\delta x^2}\left(\psi^{\ell}_{K+1,J} - 2\psi^{\ell}_{K,J} + \psi^{\ell}_{K-1,J}\right) + \frac{1}{\delta y^2}\left(\psi^{\ell}_{K,J+1} - 2\psi^{\ell}_{K,J} + \psi^{\ell}_{K,J-1}\right) \tag{20b}$$

where $\overline{\Delta^2}$ represents the modified form of the Laplacian differential operator of DuFort and Frankel (1958) defined as

$$\overline{\Delta^2}\,\zeta^{\ell}_{K,J} = \frac{1}{\delta x^2}\left[\zeta^{\ell}_{K+1,J} - \left(\zeta^{\ell+1}_{K,J} + \zeta^{\ell-1}_{K,J}\right) + \zeta^{\ell}_{K-1,J}\right] + \frac{1}{\delta y^2}\left[\zeta^{\ell}_{K,J+1} - \left(\zeta^{\ell+1}_{K,J} + \zeta^{\ell-1}_{K,J}\right) + \zeta^{\ell}_{K,J-1}\right] \tag{21}$$

and $J^{\ell}_{K,J}$ is the total energy and mean square vorticity conservation form of the non-linear advection terms introduced by Arakawa (1966). i.e.,

$$J^{\ell}_{K,J} = \frac{1}{3}\left\{\Delta_y \Psi^{\ell}_{K,J}\,\Delta_x \zeta^{\ell}_{K,J} - \Delta_x \Psi^{\ell}_{K,J}\,\Delta_y \zeta^{\ell}_{K,J}\right\} + \frac{1}{3}\left\{\Delta_y(\Psi\,\Delta_x\zeta)^{\ell}_{K,J} - \Delta_x(\Psi\,\Delta_y\zeta)^{\ell}_{K,J}\right\} + \frac{1}{3}\left\{\Delta_x(\zeta\,\Delta_y\Psi)^{\ell}_{K,J} - \Delta_y(\zeta\,\Delta_x\Psi)^{\ell}_{K,J}\right\} \tag{23}$$

where Δ_x and Δ_y denote ordinary first central differences.

Equation 20a is solved explicity for $\zeta^{\ell+1}_{K,J}$ at each time step from the known values of $\Psi^{\ell}_{K,J}$, $\zeta^{\ell}_{K,J}$, and $\zeta^{\ell-1}_{K,J}$. The corresponding values of the $\Psi^{\ell+1}_{K,J}$ are obtained from equation 20b. Equation 20b represents a set of simultaneous algebraic equations to be solved for the $\Psi^{\ell}_{K,J}$ $K = 1,2,\ldots M$; $J = 1,2,\ldots N$ at each time step. Because of the large number of mesh points used in this investigation, conventional techniques are not suitable and an alternative method is used. We express the variables $\zeta^{\ell}_{K,J}$ and $\Psi^{\ell}_{K,J}$ in terms of their discrete Fourier transforms as follows:

$$\Psi_{K,J}^{\ell} = \sum_{n=1}^{M} f_{n,J}^{\ell} e^{-i\,\theta_{n,K}} \tag{24a}$$

$$\zeta_{K,J}^{\ell} = \sum_{n=1}^{M} g_{n,J}^{\ell} e^{-i\,\theta_{n,K}} \tag{24b}$$

with

$$f_{n,J}^{\ell} = \frac{1}{M} \sum_{k=1}^{M} \Psi_{K,J}^{\ell} e^{i\,\theta_{n,K}} \tag{24c}$$

$$g_{n,J}^{\ell} = \frac{1}{M} \sum_{k=1}^{M} \zeta_{K,J}^{\ell} e^{i\,\theta_{n,K}} \tag{24d}$$

where

$$\theta_{n,K} = \frac{2\pi(n-1)(k-1)}{M} \qquad k,n = 1,2,\ldots M$$

Substituting equations 24 into equation 20b yields the following equation for the Fourier components of $\Psi_{K,J}^{\ell}$:

$$f_{n,J+1} - \alpha_n^* f_{n,J} + f_{n,J-1} = \delta_y^2 g_{n,J} \tag{25}$$

$$n = 1,2,\ldots M$$

$$J = 1,2,\ldots N$$

where

$$\alpha_n^* = 2\left[1 - (\delta y/\delta x)^2 (\cos \theta_{n,2} - 1)\right] .$$

The transforms and inverse transforms defined by equations 24 are computed by the fast Fourier algorithm of Cooley and Tukey (1965) while equations 25 are tri-diagonal and are easily solved for the $f_{n,J}$'s by Gauss elimination.

The assumed periodicity in x is automatically satisfied through equations 24. The boundary conditions at the walls (equation 6) are solved by setting $\Psi^{\ell}_{K,1} = \Psi^{\ell}_{K,N} = 0$ for all times and the no slip condition is satisfied in the following way. We express $\Psi^{\ell}_{K,J}$ in terms of $\Psi^{\ell}_{K,1}$ through a Taylor series expansion about a point on the wall.

$$\begin{aligned}
\Psi^{\ell}_{K,2} &= \Psi^{\ell}_{K,1} + A_1\,\delta y + A_2\,\delta y^2 + A_3\,\delta y^3 + \ldots \\
\Psi^{\ell}_{K,3} &= \Psi^{\ell}_{K,1} + 2A_1\,\delta y + 4A_2\,\delta y^2 + 8A_3\,\delta y^3 + \ldots \\
\Psi^{\ell}_{K,4} &= \Psi^{\ell}_{K,1} + 2A_1\,\delta y + 9A_2\,\delta y^2 + 27A_3\,\delta y^3 + \ldots \\
&\vdots \\
&\text{etc.}
\end{aligned} \tag{26}$$

The no slip condition ($\partial\Psi'/\partial y = 0$) is satisfied if we require that $A_1 = 0$ which implies that (since $\Psi^{\ell}_{K,1} = 0$)

$$\Psi^{\ell}_{K,2} = \tfrac{1}{2}\,\Psi^{\ell}_{K,3} - \tfrac{1}{9}\,\Psi^{\ell}_{K,4} \tag{27a}$$

and

$$\Psi^{\ell}_{K,N-1} = \tfrac{1}{2}\,\Psi^{\ell}_{K,N-2} - \tfrac{1}{9}\,\Psi^{\ell}_{K,N-3} \tag{27b}$$

for all k and ℓ. The Poisson equation (5b) is satisfied (to second order) at the walls by

$$\zeta^{\ell}_{K,1} = 2A_2 = \frac{1}{\delta y^2}\left\{3/2\;\Psi^{\ell}_{K,3} - 4/9\;\Psi^{\ell}_{K,4}\right\} \tag{28a}$$

$$\zeta^{\ell}_{K,N} = \frac{1}{\delta y^2}\left\{3/2\;\Psi^{\ell}_{K,N-2} - 4/9\;\Psi^{\ell}_{K,N-3}\right\} \tag{28b}$$

The length of the time increment, δt, is fixed by the semi-empirical stability limit

$$\delta t = f \, \delta x \, \delta y \Big/ \left[(U + u)_o \quad \delta y + v_o' \quad \delta x \right] \tag{29}$$

where the velocity components $(U+u')_o$ to v_o' are the maximum absolute magnitudes of velocity selected from a large sample of grid points. Numerical experimentation has shown that for values of $f \leq .6$ consistent results are obtained. Values of f near unity may lead to catastrophic instability in this explicit technique.

Since equation 20a is not "self starting" (it requires three time levels), the computation procedure starts with the forward difference representation

$$\begin{aligned} \frac{\zeta_{K,J}^{\ell+1} - \zeta_{K,J}^{\ell}}{\delta t} = \tfrac{1}{2} \Bigg\{ 3 \, & \frac{\psi_{K+1,J}^{\ell+1} - \psi_{K-1,J}^{\ell+1}}{2\delta x} - U_J \frac{\zeta_{K+1,J}^{\ell+1} - \zeta_{K-1,J}^{\ell+1}}{2\delta x} \\ & + J_{K,J}^{\ell+1} + \frac{1}{R_e} \Delta^2 \zeta_{K,J}^{\ell+1} \Bigg\} \\ & + \tfrac{1}{2} \Bigg\{ 3 \, \frac{\psi_{K+1,J}^{\ell} - \psi_{K-1,J}^{\ell}}{2\delta x} - U_J \frac{\zeta_{K+1,J}^{\ell} - \zeta_{K-1,J}^{\ell}}{2\delta x} \\ & + J_{K,J}^{\ell} + \frac{1}{R_e} \Delta^2 \zeta_{K,J}^{\ell} \Bigg\} . \end{aligned} \tag{30}$$

Equation 30 is also used periodically during the calculations in order to suppress any instabilities associated with the central time differencing used in equation 20a.

The computation procedure can now be outlined step-by-step.

1. An initial distribution of $\psi_{K,J}^{o}$ is chosen and the corresponding $\zeta_{K,J}^{o}$ are computed from equation 20b.

2. The time increment, δt, is computed from equation 29.

3. The values of $\zeta^{1}_{K,J}$ are computed for all the interior points ($J = 3,4,\ldots N-2$) by iteratively solving equation 30.

4. The Fourier coefficients, $g^{1}_{n,J}$ are computed from equation 24d, equation 25 is solved for the $f^{1}_{n,J}$ and the corresponding values for the $\Psi^{1}_{K,J}$ found from equation 24a for the interior points ($J = 3,4,\ldots N-2$).

5. The remaining grid points for $\Psi^{1}_{K,J}$, $\zeta^{1}_{K,J}$; $J = 1,2,N-1,N$) are computed from equations 27a, 27b, 28a,b and 20b.

6. For each time step the procedure in steps 3, 4, and 5 is repeated except the explicit central difference equation 20a is used in step 3 rather than equation 30.

7. Periodically, say every 50 time steps, a new time increment, δt, is computed (step 2) and a single time step using the implicit forward difference equation 30 is used.

For all of the computations presented in this paper a grid with dimensions $M = 64$, $N = 201$ was used which required a computation time of approximately 17 seconds per time step.

5. Linear Calculations

In an investigation such as this it is desirable to make some comparison between the results of our numerical computations and some known solution to the Navier Stokes equations. The imposed restraint of two dimensionality precludes any comparison with experimental data and, of course, there are no exact solutions to the non-linear equations of motion. The only alternative is to consider the limiting case of infinitesimal disturbances for which the results from linear theory

are available. For this reason, a rather thorough investigation of the linear solutions to equations 5a and 5b was undertaken. For small amplitude disturbances the non-linear terms in the vorticity transport equation are neglected and the equations of motion become

$$\frac{\partial \zeta'}{\partial t} + U \frac{\partial \zeta'}{\partial x} - 3 \frac{\partial \psi'}{\partial x} = \frac{1}{R_e} \nabla^2 \zeta' \tag{31a}$$

$$\zeta' = \nabla^2 \psi' \tag{31b}$$

The general solution to equations 31 can be expressed in the form

$$\psi'(x,y,t) = \sum_{n=-\infty}^{\infty} \sum_{m=1}^{\infty} \Phi_{nm}(y)\, e^{-i(n\alpha x - \beta_{nm} t)} \tag{32a}$$

$$\zeta'(x,y,t) = \sum_{n=-\infty}^{\infty} \sum_{m=1}^{\infty} \theta_{nm}(y)\, e^{-i(n\alpha x - \beta_{nm} t)} \tag{32b}$$

with

$$\theta_{nm} = \Phi''_{nm} - (n\alpha)^2 \Phi_{nm} \tag{32c}$$

$$\alpha = 2\pi/\lambda$$

where λ is the wavelength of the disturbance.

Substituting equation 32 into 31 results in the well known Orr-Sommerfeld equation for $\Phi_{nm}(y)$; i.e.,

$$\Phi^{IV}_{nm} - 2(n\alpha)^2 \Phi''_{nm} + (n\alpha)^4 \Phi_{nm} + i\, n\alpha R_e \left[\left(U - \frac{\beta_{nm}}{n\alpha}\right)\left(\Phi''_{nm} - (n\alpha)^2 \Phi_{nm}\right) + U'' \Phi_{nm}\right] = 0 \tag{33a}$$

with boundary conditions

$$\Phi_{nm} = \Phi'_{nm} = 0 \qquad \text{at} \qquad y = \pm 1 \ . \tag{33b}$$

Solutions of 33a for $\Phi_{nm}(y)$ for given α, n, and R_e can be made to satisfy the homogeneous boundary conditions (33b) only for the discrete eigenvalues, β_{nm}, $1 \leq m \leq \infty$. For the given α, n, and R_e we are usually interested in determining if there exist corresponding β_{nm}'s which have negative imaginary parts. The corresponding disturbance, $\Phi_{nm}(y)$, then grows exponentially with time and the flow is unstable to disturbances at the specified α, n, and R_e.

For the present analysis we are interested in solutions to equation 20a; hence, we must consider the finite difference representation of the linearized equation. For small disturbances equation 20a becomes

$$\frac{\zeta^{\ell+1}_{K,J} - \zeta^{\ell-1}_{K,J}}{2\delta t} = 3\,\frac{\psi^{\ell}_{K+1,J} - \psi^{\ell}_{K-1,J}}{2\delta x} - U_J\,\frac{\zeta^{\ell}_{K+1,J} - \zeta^{\ell}_{K-1,J}}{2\delta x} + 1/R_e\,\overline{\Delta^2}\,\zeta^{\ell}_{K,J} \tag{34}$$

Again we will consider solutions of the form (32); hence, we have

$$\psi^{\ell}_{K,J} = (\Phi_J)_{nm}\, e^{-i(n\alpha(k-1)\delta x - \beta_{nm}\,\ell\,\delta t)}$$

$$\zeta^{\ell}_{K,J} = (\theta_J)_{nm}\, e^{-i(n\alpha(k-1)\delta x - \beta_{nm}\,\ell\,\delta t)} \tag{35}$$

Substituting into 34 and 20b results in

$$\beta'_{nm}(\theta_J)_{nm} = in\alpha'\left[U_J(\theta_J)_{nm} + 3(\Phi_J)_{nm}\right] + \frac{1}{R_e\delta y^2}\left[(\theta_{J+1})_{nm} + \omega(\theta_J)_{nm} + (\theta_{J-1})_{nm}\right] \tag{36a}$$

$$(\theta_J)_{nm} = \frac{1}{\delta y^2}\left[(\Phi_{J+1})_{nm} + \omega' (\Phi_J)_{nm} + (\Phi_{J-1})_{nm}\right] \tag{36b}$$

where

$$\omega = 2\left((\delta y/\delta x)^2 \cos n\alpha\, \delta x\right) \tag{37a}$$

$$\omega' = -2\left[1 + \left(\frac{\delta y}{\delta x}\right)^2 (1 - \cos n\alpha\, \delta x)\right] \tag{37b}$$

$$\alpha' = \sin n\alpha\, \delta x / n\delta x \tag{37c}$$

and

$$\beta'_{nm} = 2\left[1 + 1/R_e \delta x^2\right] \cos \beta_{nm}\, \delta t + i\, \frac{\sin \beta_{nm} \delta t}{\delta t} \tag{37d}$$

$$i = \sqrt{-1}$$

When expression 36b for $(\theta_J)_{nm}$ is substituted into equation 36a there results a homogeneous set of linear algebraic equations for the $(\Phi_J)_{nm}$. For given values of α, n, R_e, δx, δy, and δt we wish to determine the eigenvalues and eigenvectors of the matrix of coefficients of the $(\Phi_J)_{nm}$. In particular, we are interested in those eigenvalues β_{nm} for which the imaginary part of β_{nm} is negative and in the associated eigenvectors $(\Phi_J)_{nm}$. The effects of δx, δy, and δt on the linear solution are especially important for the present application.

The eigenvalues and eigenvectors have been computed for a number of Reynolds numbers and for a wide range of α's using the QR algorithm (Wilkinson 1965 or Fair 1971). For more details on these calculations the reader is referred to O'Brien (1970) or to the report by Gawain and Clark (1971). In the latter report some of the eigenvalues (β_{nm}) and

eigenfunctions ($\Phi_{nm}(y)$) are tabulated for the case $\alpha = 1.0$, $R_e = 6667$. In the limit as δx, $\delta t \to 0$, the problem is the same as that solved by Thomas (1953) although the methods used are quite different from those employed by Thomas. For the case $\alpha = 1.0$, $R_e = 6667$, and $\delta y = .01$ (in Thomas' notation this corresponds to $\alpha = 1.0$, $R_e = 10{,}000$, $\delta y = .01$) we have solved equations 36 for n = 1, 2, 3, and 4. Our solutions show that there is only one unstable eigenvalue and this unstable solution occurs for n = 1. We shall order the eigenvalues, β_{nm}, so that for a given n, the m's are arranged according to the stability of the computed eigenvalues. Hence, for the case $\alpha = 1.0$, $R_e = 6667$, β_{11} is the unstable eigenvalue with the corresponding eigenfunction $\Phi_{11}(y)$. Thomas gives a value of $\beta_{11} = .3653 - i\ .0055$ and the corresponding mode shape, $\Phi_{11}(y)$ is tabulated in his Table V. (Using our non-dimensionalization, β_{11} is exactly 1.5 times the value given by Thomas.) Our calculations yield the value $\beta_{11} = .3593 - i\ .0041$ and the corresponding mode shape, $\Phi_{11}(y)$, is shown in Figure 3 for comparison with that of Thomas.

Now consider the influence of the finite difference parameters δx, δy, and δt on the solutions to 36. The effect of δy on the growth rate ($-\beta_{11}$) is illustrated in Figure 4. As δy increases the flow becomes more and more stable and when $\delta y = .05$, the phenomenon of linear instability has been completely obscured by the crude finite difference representation of the equations of motion. Indeed, calculations with $\delta y = .05$ for a wide range of Reynolds numbers and α's have shown no indication of linear instability.

The effect of varying δx was investigated at $R_e = 6667$, $\alpha = 1.0$, $\delta y = .01$ and $\delta t < .02$. Figure 5 shows the variation with δx of the growth

rate ($\mathcal{J}\beta_{11}$) and phase velocity ($\alpha\beta_{11}$) for the unstable mode. For $\delta x < .8$ the changes in the disturbance profile ($\phi_{11}(y)$) where insignificant.

Perhaps the most surprising discovery in this investigation of the linear behavior of the discrete representation of the equations of motion was the significance of the time interval, δt, on the disturbance growth rate ($\mathcal{J}\beta_{11}$). The effects of δt on this parameter are shown in Figure 6. There are significant departures from the "exact" value of $\mathcal{J}\beta_{11}$ for $\delta t > .04$ while the phase velocity and mode shape are not significantly changed. Hence, in addition to the usual stability constraint (29) on the time interval, δt, it is necessary that $\delta t < .04$ in the numerical integration scheme described in section 4.

We can now summarize the results presented in the preceeding discussion. Increasing the y increment, δy, tends to make the flow more stable and for coase grids ($\delta y > .05$) the phenomenon of linear instability is completely obscured. The linear solution is comparatively insensitive to changes in δx, at least, until the x-increment is approximately the same length as the channel half-width. The growth rate of the disturbance is strongly influenced by the δt increment (due to the DuFort-Frankel representation of the diffusion term) and for large δt the growth rate is much higher than that predicted by the "exact" linear solution.

6. Numerical Integration of the Vorticity Equation

(a) Small Amplitude Disturbances

We now return to the numerical integration of the vorticity equation using the techniques described in Section 4. As a check on the validity of the solution we first consider the limiting case of

small amplitude disturbances and compare the results obtained with the known results from the linear solution as described in the preceeding section. To be more explicit, for a given Reynolds number and initial disturbance streamfunction, $\Psi'(x,y,0)$, are the results from the numerical integration consistent with the values of β_{nm} and $\Phi_{nm}(y)$ predicted by linear theory?

A series of calculations were made at $R_e = 6667$ and the initial disturbance given by

$$\Psi'(x,y,0) = \epsilon \left\{ f_1(y,0)\, e^{-i\alpha x} + f_1^*(y,0)\, e^{+i\alpha x} \right\} \tag{38}$$

where ϵ is an amplitude factor. For ϵ sufficiently small the non-linear terms in equation 5a should be neglible and the numerical integration should give results which are in agreement with the linear theory. As a first test we choose $\alpha = 1.0$, $R_e = 6667$, $\epsilon = .005$ and the initial mode shape identical to the unstable mode computed from linear theory; i.e., $f_1(y,0) = \Phi_{11}(y)$, where $\Phi_{11}(y)$ was shown in Figure 3. The growth rate of the disturbance kinetic energy, $\hat{E}(t)$, as defined by equation 12c is shown in Figure 7. The computed growth rate gives a value of $\beta_{11} = .0050$ which is in good agreement with the value given by Thomas. From equation 14 we can determine the phase, $\varphi_n(y,t)$, of any of the spectral components of $\Psi'(x,y,t)$ by

$$\varphi_n(y,t) = \tan^{-1}\left(\Im f_n / \Re f_n \right) \tag{39}$$

The phase velocity, c_n, or rate of propagation, of any of the spectral components is then

$$c_n = \frac{1}{n\alpha} \partial\varphi_n/\partial t \tag{40}$$

For these computations the grid length, 2L, was set equal to the wavelength, λ, of the disturbance; so for the disturbance (primary) mode, n = 1, and in the linear range we can identify c_1 with $\mathcal{R}\,\beta_{11}$ from the linear theory. Figure 10 shows that c_1 is, indeed, a constant and the slope of this phase versus time plot shows that $c_1 = .363$ which is in excellent agreement with Thomas' value of $\mathcal{R}\,\beta_{11} = .3653$ and with our value of $\mathcal{R}\,\beta_{11} = 3593$.

Other short computations were carried out with, again, $\alpha = 1.0$, $R_e = 6667$, $\epsilon = .005$, but with the initial mode shape, $f_1(y,0)$, chosen to correspond to one of the stable modes computed from linear theory. These calculations were carried out by arbitrarily setting $f_1(y,0)$ equal to the mode Φ_{17}, Φ_{28}, and Φ_{22}* which are shown in Figure 9. The energy decay rates for these three cases are shown in Figure 10. From the energy decay rates and from the computed phase velocities (not shown here) the numerical integration technique yields the following values for the appropriate eigenvalues

$$\beta_{17} = 1.351 + i\ .138, \quad \beta_{28} = 1.425 + i\ .207$$

$$\beta_{22} = 1.456 + i\ .0764.$$

The linear matrix solution described in the preceeding section gave the values

$$\beta_{17} = 1.363 + i\ .136; \quad \beta_{28} = 1.403 + i\ .192;$$

$$\beta_{22} = 1.462 + i\ .0742.$$

---

* Since equation 33a is symmetrical in y it is possible to divide the solutions for $\Phi_{nm}(y)$ into even and odd solutions. The boundary conditions are $\Phi_{nm}(\pm 1) = \Phi'_{nm}(\pm 1) = 0$, with $\Phi'_{nm}(0) = \Phi'''(0) = 0$ for even solutions and $\Phi_{nm}(0) = \Phi''_{nm}(0) = 0$ for odd solutions. For our numerical model with N lateral stations there are N-2 unknown $(\Phi_J)_{nm}$'s; hence, N-2 eigenvalues are computed for each n, α, R_e. Of these N-2 eigenvalues, $\frac{N-1}{2}$ correspond to even solutions and $\frac{N-3}{2}$ correspond to odd solutions.

A more critical test of the numerical integration is the ability to predict the correct mode shape, $\Phi_{nm}(y)$, for a disturbance of known wavelength and at a fixed R_e. Suppose in equation 38 we choose $\alpha = 1.0$ but arbitrarily select the initial disturbance mode shape $f_1(y,0)$. The arbitrary function $f_1(y,0)$ can be expressed in terms of the eigenfunctions, $\Phi_{1m}(y)$, of the Orr-Sommerfeld equation (33a) for the chosen α and R_e (Schensted, 1961). i.e.,

$$f_1(y,0) = \sum_{m=1}^{\infty} A_m \Phi_{1m}(y) \tag{41a}$$

with

$$A_m = \int_{-1}^{1} f_1(y,0)\, \chi_m(y)\, dy \tag{41b}$$

and

$$\chi_m(y) = \left(\frac{d^2}{dy^2} - \alpha^2\right) \tilde{\Phi}_{1m}$$

where $\tilde{\Phi}_{1m}(y)$ is the solution to the adjoint of equation 33a. However, the linear solution tells us that <u>only one</u> of the Φ_{1m} in equation 41a is unstable; all other modes will decay with time. Hence, in the linear range only one mode, $\Phi_{11}(y)$, grows and will eventually dominate all the other terms in the expansion (41a). Therefore, the mode shape

$$f_1(y,t) \to \Phi_{11}(y) \quad \text{as} \quad t \to \infty$$

where $\Phi_{11}(y)$ is as shown in Figure 3 for the case $\alpha = 1.0$, $R_e = 6667$. The initial shape chosen for $f_1(y,0)$ is shown in Figure 11. The disturbance kinetic energy, $\hat{E}(t)$, is given in Figure 12 and the mode shapes, $f_1(y,t)$, are shown for various times during the calculations.

As expected, the disturbance energy at first rapidly decreases then grows according to linear theory while the computed mode shape is clearly tending towards the predicted shape of $\Phi_{11}(y)$.

From the results discussed in this section and from calculations at other wavenumbers, α, we have found that the numerical integration scheme consistently gives results that are in excellent agreement with the linear theory. The agreement between the results of the numerical model and the predicted results for small amplitude disturbances is gratifying and gives some confidence in the non-linear calculations now to be discussed.

b) Finite Amplitude Disturbances

Before discussing the results of the non-linear aspects of this investigation it is appropriate to give a very brief outline of the features of the existing non-linear theories which predict equations of the form of equation 1. The following discussion is from the paper by Pekeris and Shkoller (1967) which in turn outlines Stuart's (1960) theory for finite amplitude disturbances. If the Fourier representations for the disturbance streamfunction, Ψ', and vorticity, ζ'; (equations 14 and 15) are substituted into the equation of motion (5a) the following equation for the modes results:

$$\frac{\partial g_n}{\partial t} = \frac{1}{R_e}\left(g_n'' - (n\alpha)^2 g_n\right) + i\, n\, \alpha \left[(U - f_o')g_n - (U'' - f_o''')\, fn\right] \qquad (42)$$

$$- i\alpha H_n \qquad\qquad n = 0,1,2,\ldots$$

where

$$g_n = f_n'' - (n\alpha)^2\, fn \qquad (15c)$$

and

$$H_n = \sum_{s=1}^{\infty} \left\{(n-s)(f_s' g_{n-s} - g_s' f_{n-s}) + (n+s)(f_s^{*\prime} g_{n+s} - g_s^{*\prime} f_{n+s})\right\} \tag{42a}$$

If the parameter, $\epsilon_o^2 \equiv |\beta_{11}|$, is small (implying proximity to the neutral curve) then Stuart showed that in an approximation in which terms of order $\epsilon_o^{3/2}$ are retained, only the f_0, f_1, and f_2 in (14a) are significant. Now expand $f_1(y,t)$ in terms of the eigenfunctions, Φ_{1m}, of the Orr-Sommerfeld equation as was done in equation (41a), so that

$$f_1(y,t) = \epsilon_o \sum_{m=1}^{\infty} A_m(t)\, \Phi_{1m} \tag{43}$$

Stuart's asymptotic analysis for small ϵ_o shows that only the first term in (43) is important, i.e.,

$$f_1(y,t) \approx \epsilon_o A_1(t)\, \Phi_{11}(y) + O(\epsilon_o^3) \tag{44}$$

and that the terms f_o and f_2 are of smaller order so that

$$f_o(y,t) = \epsilon_o^2 |A_1(t)|^2 G_o(y) + O(\epsilon_o^4) \tag{45}$$

$$f_2(y,t) = \epsilon_o^2 A_1^2(t)\, G_2(y)\, O(\epsilon_o)^4 \tag{46}$$

The functions G_o and G_2 are determined from Pekeris and Shkoller's equations (22), (23), and (24). By substituting equations (44), (45), and (46) into (42) one can obtain an expression of the form (1) for the amplitude parameter, $|A_1(t)|^2$, i.e.,

$$\frac{d|A_1|^2}{dt} = k_1 |A_1|^2 + k_2 |A_1|^4 \qquad (1)$$

The important point from our viewpoint is that according to the above analysis, the disturbance energy should be contained primarily in the $f_1(y,t)$ mode and that the growth of this energy should behave according to equation (1). Assuming that the disturbance streamfunction can be adequately represented by f_1 then we have from (44)

$$\Psi'(x,y,t) = \epsilon_o\left(A_1(t)\ \Phi_{11}(y)e^{-i\alpha x} + A_1^*(t)\ \Phi_{11}^*(y)e^{+i\alpha x}\right) \qquad (47)$$

The disturbance kinetic energy as defined by equation (12c) (since we are neglecting the contribution from f_0 the "disturbance" and "turbulent" kinetic energies are the same) becomes

$$\hat{E}(t) = \epsilon_o^2\ I\ |A_1(t)|^2 \qquad (48)$$

where

$$I = \int_0^1 \left\{|\Phi_{11}'|^2 + \alpha^2\ |\Phi_{11}|^2\right\} dy \qquad (49)$$

It was pointed out in part 2 of this paper that the constant, k_1, in (1) is related to the amplification factor of linear theory ($k_1 = -2\beta_{11}$) and if $k_2 < 0$ then the growth of an unstable disturbance is limited so that

$$|A_1|^2 \to k_1/k_2 \qquad (50)$$

Pekeris and Shkoller found that these conditions are satisfied for α and R_e inside the lower portion of the neutral curve. For example, at $R_e = 6667$ and $\alpha = .90$ their calculations give

$$|A_1|^2 \to 1/9$$

Our solutions to the linear eigenvalue problem at $R_e = 6667$ and $\alpha = .875$ gives $\beta_{11} = -.00394$ and $I = 1.848$. Hence,

$$\hat{E}(t) \to (1.848)(.00394)/9.0 = 8.08 \times 10^{-4}$$

Figure 13 shows the energy growth for modes E_0, E_1, E_2, and E_3 for a run with the initial disturbance given by (38) with $\alpha = .875$, $R_e = 6667$, $\epsilon = .005$, and $f_1(y,0) = \Phi_{11}(y)$ where Φ_{11} in the unstable eigenfunction for $\alpha = .875$. The disturbance is, indeed dominated by $E_1(t)$ and the mode shape, $f_1(y,t)$, normalized so that $f_1(0,t) = 1.0$, never departed from the initial shape given by $\Phi_{11}(y)$. A longer calculation was made using the same initial conditions except in this case $\alpha = 1.0$. Again the energy remained predominately in the $n = 1$ mode and again the mode shape for $f_1(y,t)$ never departed from the initial $\Phi_{11}(y)$ shape indicating that (44) and (47) do indeed adequately represent the flow as far as the overall disturbance energy is concerned.

Although this latter calculation required 34.5 hours of computer time, we can see that the energy growth is still within the linear range. This demonstrates that, as was pointed out in part 2, it is impractical using the present techniques to follow the disturbance through the linear and into the non-linear range of amplitudes. For this reason we have been unable to establish whether or not the "supercritical" equilibrium states as predicted by (1) and (50) do, in fact, exist for our exact treatment. Our calculations, as shown in Figures 13 and 14, do indicate, however, that there is no significant

difference between cases for which $\alpha = .875$ or $\alpha = 1.0$ in equation (38). The calculation of Pekeris and Shkoller, however, show that "super-critical" equilibrium states are possible for $\alpha = .875$ but not for $\alpha = 1.0$. All our calculations for both low and high amplitude disturbances have shown no strong dependence on α as far as the final result is concerned.

Although this last run was entirely within the linear energy growth range we will see that there were very interesting non-linear features to the calculations. The appearance of energy in modes E_0, E_2, and E_3 as shown in Figure 14 is, of course, due to the presence of the non-linear terms in equation (5a). In identical calculations for which the non-linear terms in (5a) were deliberately omitted the primary (n=1) mode grew according to linear theory while the energy in the other modes remained constant or decreased and the energy levels (due to numerical "noise") were twelve to fifteen orders of magnitude lower than that of the primary mode. Figure 14 shows that the energy in the other modes (n = 0,2,3) grows very rapidly at first and then seems to follow an exponential growth rate, i.e.,

$$E_n(t) \sim e^{2\beta_n t} \qquad n = 0,1,2,3$$

for large t. From the data shown in Figure 14 the values for β_n are $\beta_0 = .0114$, $\beta_1 = .0050$, $\beta_2 = .0080$, and $\beta_3 = .0117$.

The most interesting feature is the shape of the modes, f_2, f_3, with respect to y^*. These shapes are shown in Figure 15 where the modes are normalized so that $f_n(0,t) = 1.0$ for even modes and for odd modes the maximum real and imaginary parts of $f_n(y)$ are unity. The phase velocities for these modes averaged between t = 65 and t = 71 were $c_2 = .362$ and $c_3 = .363$. These phase velocities are the same as the phase velocity of the primary mode predictable from linear theory (i.e. $\beta_{11} = .363$).

The striking feature is the strong resemblence between these modes and the modes calculated from the Orr-Sommerfeld equation. Indeed, it appears that the non-linear solution has spontaneously generated modes that can be predicted from the linear theory even though the linear solution indicates that these are stable modes and would decay with time. This means that if we represent each mode by an expression of the form of (43) i.e.,

$$f_n(y,t) = \sum_{m=1}^{\infty} A_m(t)\ \Phi_{nm}(y) \tag{51}$$

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*From equation (15c) we see that g_n has the same symmetry or anti-symmetry as does f_n; i.e. if $f_n(y)$ is an even function of y then g_n is also even. Then from equation (42) we can establish that if $\Psi'(x,y,0)$ is given by (38) and $f_1(y,0)$ is even in y then $f_0(y,t)$ is odd, $f_2(y,t)$ is odd, $f_3(y,t)$ is even, $f_4(y,t)$ is odd, etc. Likewise if $f_1(y,0)$ is odd then $f_0(y,t)$ is odd, $f_2(y,t)$ even, $f_3(y,t)$ odd, $f_4(y,t)$ is even, etc. For example, let $f_1(y,0)$ be even. Then since $f_n(y,0) = 0 \quad n \neq 1$

$$\frac{\partial g_2}{\partial t}\Big|_{t=0} = -i\alpha H_2\Big|_{t=0} = -i\alpha\left[f_1'\, g_1 - g_1'\, f_1\right]$$

= odd function of y.

At the next time step, $t = \delta t$

$$g_2(y,\delta t) = \left(\partial g_2/\partial t\right)\delta t = \text{odd function of } y$$

then one mode seems to dominate the expansion. It is interesting to notice that in each case the dominant mode is one for which there exist a step gradient near the wall. Most of the stable modes that we have computed from linear theory are similar to the ones shown in Figure 9 which have a detailed structure near the center of the channel rather than near the wall.

A series of higher energy runs with various initial disturbance amplitudes was made at $R_e = 6667$ using as initial conditions a disturbance with wave number $\alpha = 1.0$ and that mode shape given by Thomas (see Figure 3). The total energy growth (or decay) is shown in Figure 16 for several different initial energy levels. A further energy breakdown is given in Figure 17 for the cases $\epsilon = (.05)\sqrt{5}$. For this case the total energy remains almost constant while there is a continual exchange of energy between the mean flow and the turbulence. The turbulent kinetic energy oscillates about a value of $\hat{E}(t) = .03$ which is roughly ten times the equilibrium value predicted from equation 50. From Figure 14a it seems that there is a tendency towards some common energy equilibrium state for the different initial conditions. A more detailed inspection of the flow structure shows that there are significant differences between the three flows. Starting with the high energy run we summarize the structure of the flow by presenting plots of the mean velocity profile, Reynolds stresses, and energy spectrum at various times throughout the run (Figures 18, 19, and 20). Also, in Figure 21 the shape of the primary ($n = 1$) mode at the end of the calculations is shown for each of the three energy levels. For the high energy run (although the time parameter is comparatively small) there are drastic

alterations of the mean velocity profile and primary mode shape and considerable energy transfer from the primary mode to the higher harmonics. For the two lower energy runs there is little alteration of the parabolic velocity profile and only small amounts of energy transfer to higher wave numbers. For the lowest initial energy level ($\epsilon = .05 / \sqrt{2}$) the primary mode shape remains essentially unchanged throughout the entire calculation. More details of this low energy run are shown in Figure 22 where the growth of the total turbulent energy in the first three spectral components (equation 19b) is shown. The phase velocities (equation 40) were computed near the end of the run and are summarized below:

$$c_1 = .492, \quad c_2 = .484, \quad c_3 = .473$$

The measured C_n were independent of y and all are roughly 40% higher than the wave velocity ($\mathcal{R}\beta_{11}$) of the primary unstable mode predicted by linear theory. This low energy run has some qualitative similarities to the predictions of the non-linear theory which was outlined at the beginning of this section. That is, there appears to be an asymptotic energy level, the primary mode shape is only slightly altered and the energy in the primary mode remains dominant.

In order to investigate the influence of initial wavenumber on the flow two runs were made at $R_e = 6667$ with $\alpha = .875$ (near the lower branch of the neutral curve) and with $\alpha = 1.05$ (near the upper branch). For both cases the initial mode shape was determined from the eigenvalue solution to the linear equations and the initial energy was chosen to be slightly lower than that of the run ($\epsilon = .05/\sqrt{5}$,

Figure 16) which seemed to remain in an energy equilibrium state. The energy variation for these cases is shown in Figure 23. There is no obvious qualitative difference between the three wavenumbers; the initial high energy level, however, apparently causes oscillations of the disturbance kinetic energy and makes any direct comparison with non-linear theory rather difficult.

Another important question which arises in the consideration of non-linear aspects in the stability of plane Poiseuille flows is whether or not a flow which is stable to infinitesimal disturbances might be unstable to disturbances of some finite amplitude. Such instabilities are generally referred to as "subcritical" instabilities. To investigate this aspect of the problem several runs were made using as initial conditions a disturbance that is known to be stable in the linear range. The case chosen was for $\alpha = .78$, $R_e = 6667$ for which $\beta_{11} = .249 + i\ .00124$ for the least stable eigenfunction determined from the linear solution described in Section 5. Again the initial disturbance was given by equation 38 with the function $f_1(y,0) = \Phi_{11}(y)$ determined from the linear solution. For a small amplitude disturbance ($\varepsilon = .005$) the disturbance energy decreased with time according to linear theory while for a larger amplitude ($\varepsilon = .0635$). The energy variation was as shown in Figure 24. The familiar oscillation of the turbulent kinetic energy which is associated with an initially high disturbance energy is present in this case; while the total kinetic energy appears to remain approximately constant after an initial decrease.

We now address ourselves to the question posed at the end of Section 2 concerning the character of a two dimensional "turbulence"

in channel flows. For all the runs discussed above the computational grid length (2L) was equal to the wavelength of the initial unstable disturbance (i.e. $2L = 2\pi/\alpha$). In any finite difference representation of the equations of motion there are only discrete wavelengths available to be excited by the non-linear interactions. Recall from equation 24a that the streamfunction is expressed as

$$\Psi_{K,J} = \sum_{n=1}^{M} f_{n,J}\, e^{-i\theta_{n,k}} \qquad (24a)$$

and since $\Psi_{K,J}$ is real

$$\mathcal{R}\, f_{n,J} = \mathcal{R}\, f_{M-n+2,J}$$

$$\mathcal{J}\, f_{n,J} = -\mathcal{J}\, f_{m-n+2,J}$$

i.e., $\mathcal{R}\, f_{n,J}$ is even and $\mathcal{J}\, f_{n,J}$ is odd over the interval n = 1,2,...M/2, ...M. Therefore, there are M/2 distinct Fourier modes "available" to the solution of the equations of motion. When the grid length equals the wavelength of the initial disturbance there is only one lower mode available to be excited; namely, the zero mode, $f_{0,J}$. It is known that two-dimensionality constrains the transfer of energy between the modes of which the disturbance is composed. For an isotropic two-dimensional turbulence there can be no systematic transfer of energy from lower to higher wave numbers without a corresponding transfer to still lower wavenumbers (Lilly (1968), Kraichnan (1967)). Hence, one may suspect that our discrete representation in which there are no available modes at lower wave numbers could force the disturbance energy to remain in the primary initial mode. The first

alternative to this dilemma would seem to be to increase the grid length, 2L, thereby allowing modes with both higher and lower wave numbers to fit into the available grid length. However, if the initial disturbance energy is contained in, say, wave number n then the non-linear interaction of this mode with itself will initially excite wave numbers 0 and 2n; later modes 0, 2n, 3n, 4n, etc. The modes that can be excited will be separated by a spacing of n in wave number space, resulting in a spectrum with "holes" or regions in which no energy can appear. Clearly, this is not a realistic case. The alternative to this situation is to initially perturb the flow with a disturbance in which at least two contiguous (in the discrete wave number space) modes are present. For example, if the initial disturbance contains modes n and n+1 then the non-linear interaction will, at first, excite modes 0, 1, 2n, 2n+1, 2n+2; these in turn will excite modes 0, 1, n-1, 2n-1, 4n, etc. until all possible wave numbers have been excited.

It seems reasonable to require that the wavelengths of these two initial modes should both fall within the unstable region in the $(\alpha\text{-}R_e)$ plane. If we set the grid length so that $2L=16\pi$ (or $\alpha = 1/8$) then modes with wavenumbers 7α and 8α will both fall within the unstable region. Hence

$$\Psi'(x,y,0) = \epsilon \left\{ f_7 e^{-i7\alpha x} + f_8 e^{-i8\alpha x} + f_7^* e^{+i7\alpha x} + f_8^* e^{+i8\alpha x} \right\}$$

A lengthy numerical integration of the vorticity equations using the initial conditions just described is summarized in Figures 25 and 26 which show the turbulent kinetic energy time variation and the energy spectrum at various times during the run. For this run the turbulent kinetic energy initially grows and then appears to vary randomly about a value of $\hat{E}(t) = .019$. The energy spectrum is quite different from the other cases presented in that there is a gradual transfer of significant amounts of energy to other wavenumbers. Although it is not shown here, the mean velocity profile again differed only slightly from the parabolic distribution. Although, there are significant differences between this run and the others presented, all the examples that have been discussed are qualitatively similar in that most of the energy remains in modes whose wavelength approximately equals that of the primary (unstable) mode.

7. Conclusion

Our numerical technique for integrating the vorticity equation has been shown to be stable and predicts results that are consistent with linear solutions to the vorticity equation.

The influence of the finite difference representation on the linear eigenvalue solution to the equations of motion was investigated and some interesting conclusions resulted. The size of the δy increment significantly influences the stability of the equations and for grids with $\delta y > .05$ (approximately) the equations are stable to all disturbance. Because of the DuFort-Frankel representation of the diffusion term, the growth rate of an unstable disturbance is strongly influenced by the δt increment. The growth rate increases

rapidly with δt even though the time step increment may still be small enough to satisfy ordinary numerical stability requirements. The δx increment has the smallest effect on the linear solution and seems to be significant only when δx is approximately equal to the channel half-width.

Because of the long computing time required we were unable to follow the growth of an unstable disturbance through the linear and into the non-linear range of disturbance amplitudes. This precluded any direct comparison of our results with those predicted by the non-linear theories of Stuart and Watson and Reynolds and Potter. We can, however, make some general comparisons. We find qualitative agreement in the sense that the unstable primary mode dominates the disturbance and there is little distortion of the mean flow. The calculations presented indicate the existence of a disturbance energy equilibrium state which seems to be common (for a given Reynolds number) to all unstable disturbances despite their initial energy or wavenumber. Indeed, we have found no outstanding differences in the behaviour of disturbances with varying initial wave numbers (at a constant R_e) which is not in agreement with the non-linear theories. The distortion of the primary mode shape is significant when the initial energy is high enough to be well beyond the linear range but for lower initial energies the distortion of the mode shape is less important.

The computed flows which have been discussed in this paper show little resemblance to actual turbulent shear flows. In a true three dimensional flow one would expect an initial unstable disturbance

with a length scale approximately 2π times the channel half width to grow in amplitude and through non-linear interactions the energy of the disturbance would be transferrred to higher wavenumbers with length scales comparable to the channel half width. The corresponding situation for a two dimensional turbulent channel flow is not clear. However, for two dimensional homogeneous turbulence the energy would cascade to the lower wavenumbers while mean square vorticity is transferred to higher wavenumbers when the turbulent energy is continually fed into the flow in a narrow band of spectral components. Neither of these processes was observed for the present calculations.

One can speculate that our calculated "quasi-equilibrium" states in which most of the energy remains in one or two spectral components with little alteration of the parabolic velocity profile represent two dimensional solutions to the equations of motion which are, in some sense, highly unstable flows and, for this reason, are not observed experimentally.

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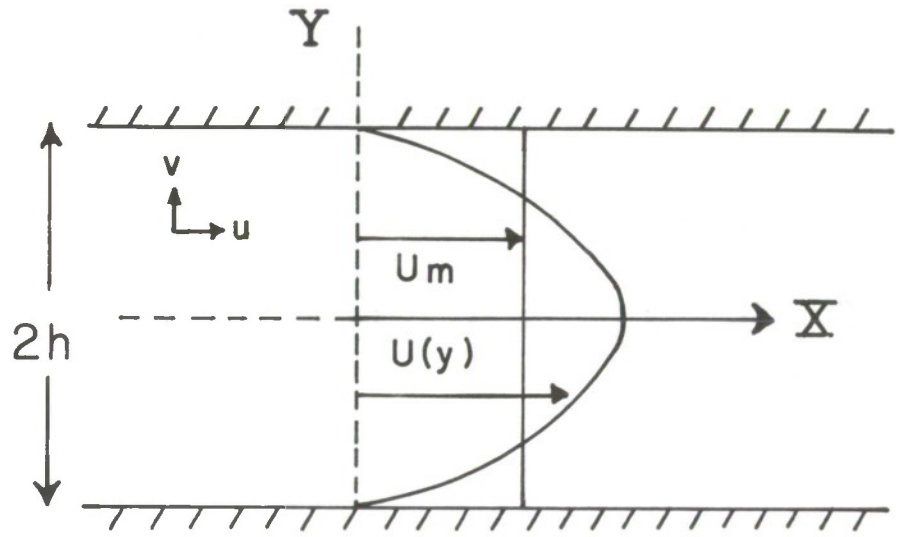


Figure 1. $R_e = U_m h / \nu$; $y = Y/h$, $x = X/h$

$U(y) = 3/2\ (1 - y^2)$

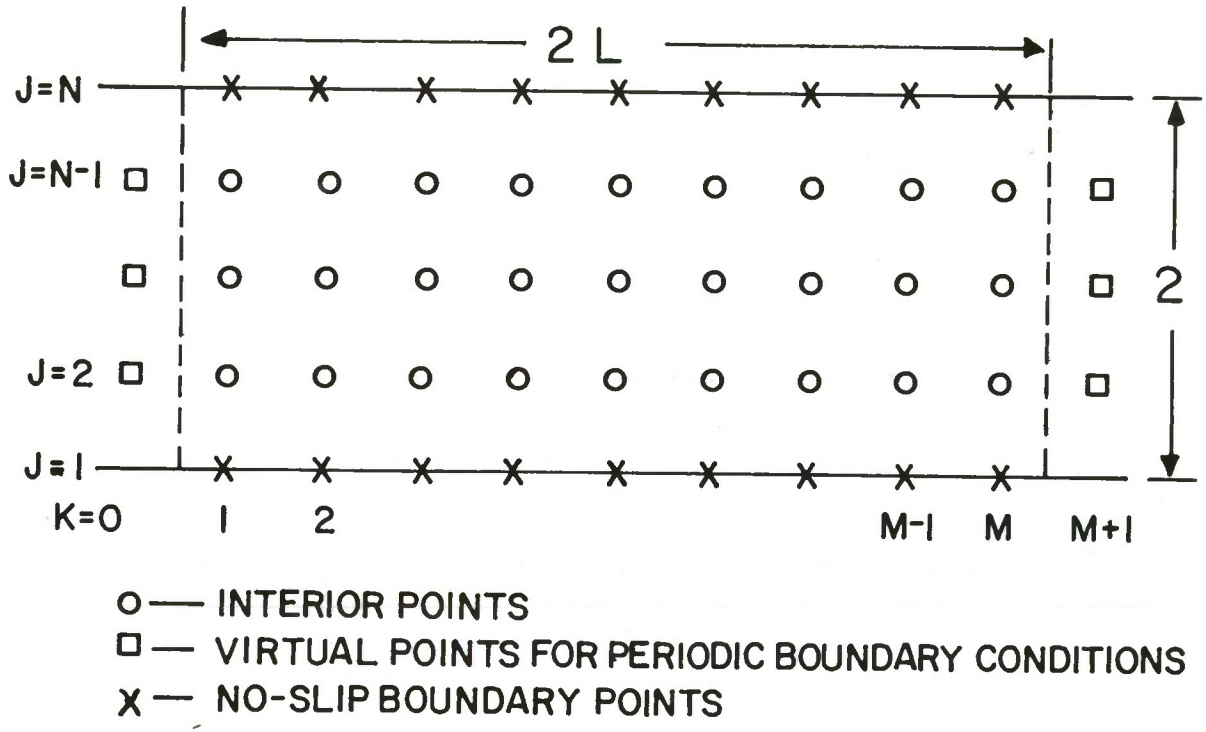


Figure 2. Finite Difference Mesh

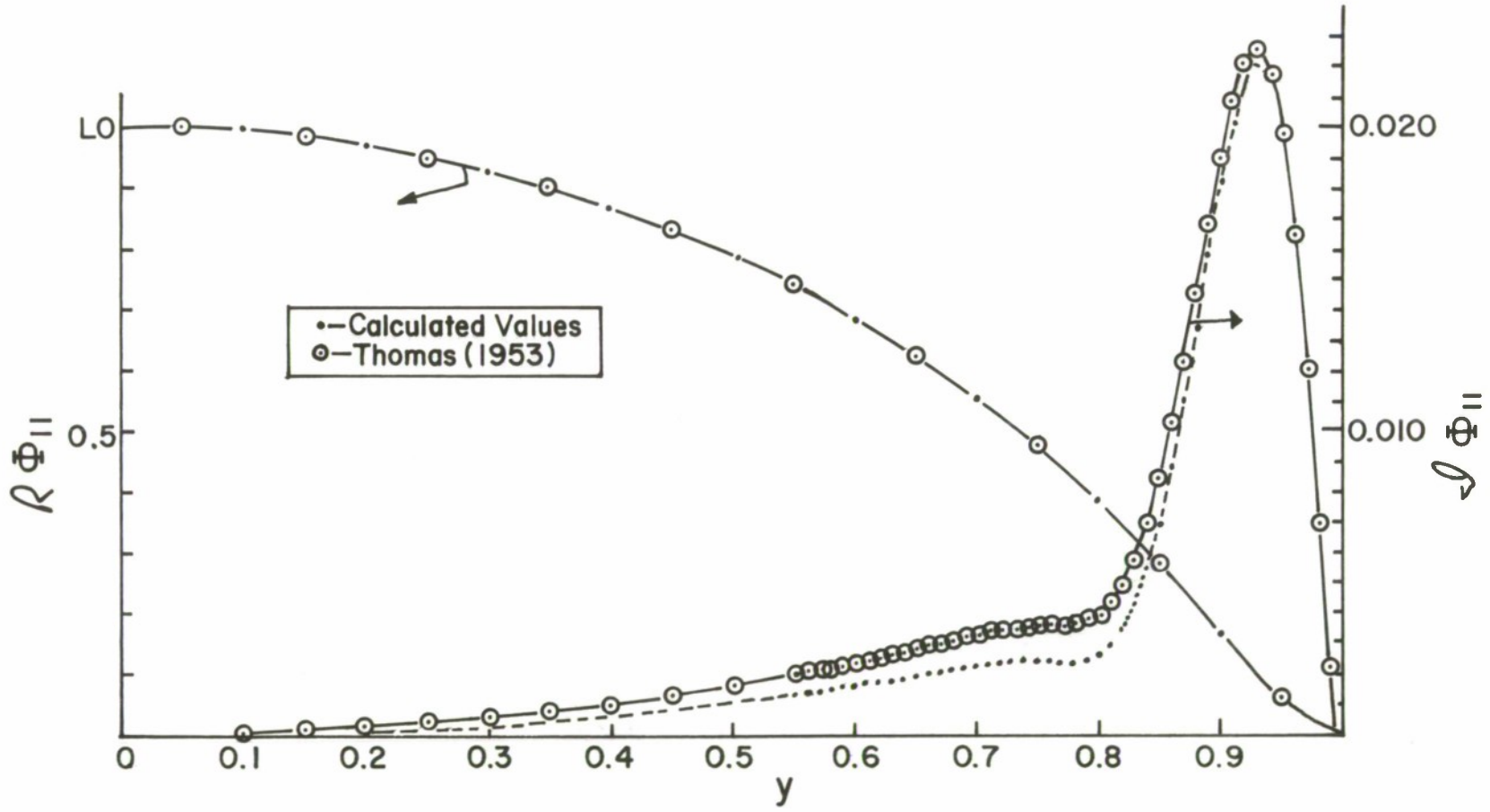


Figure 3. Comparison of the Eigenfunction, Φ_{11}, for R_e=6667, α= 1.0.

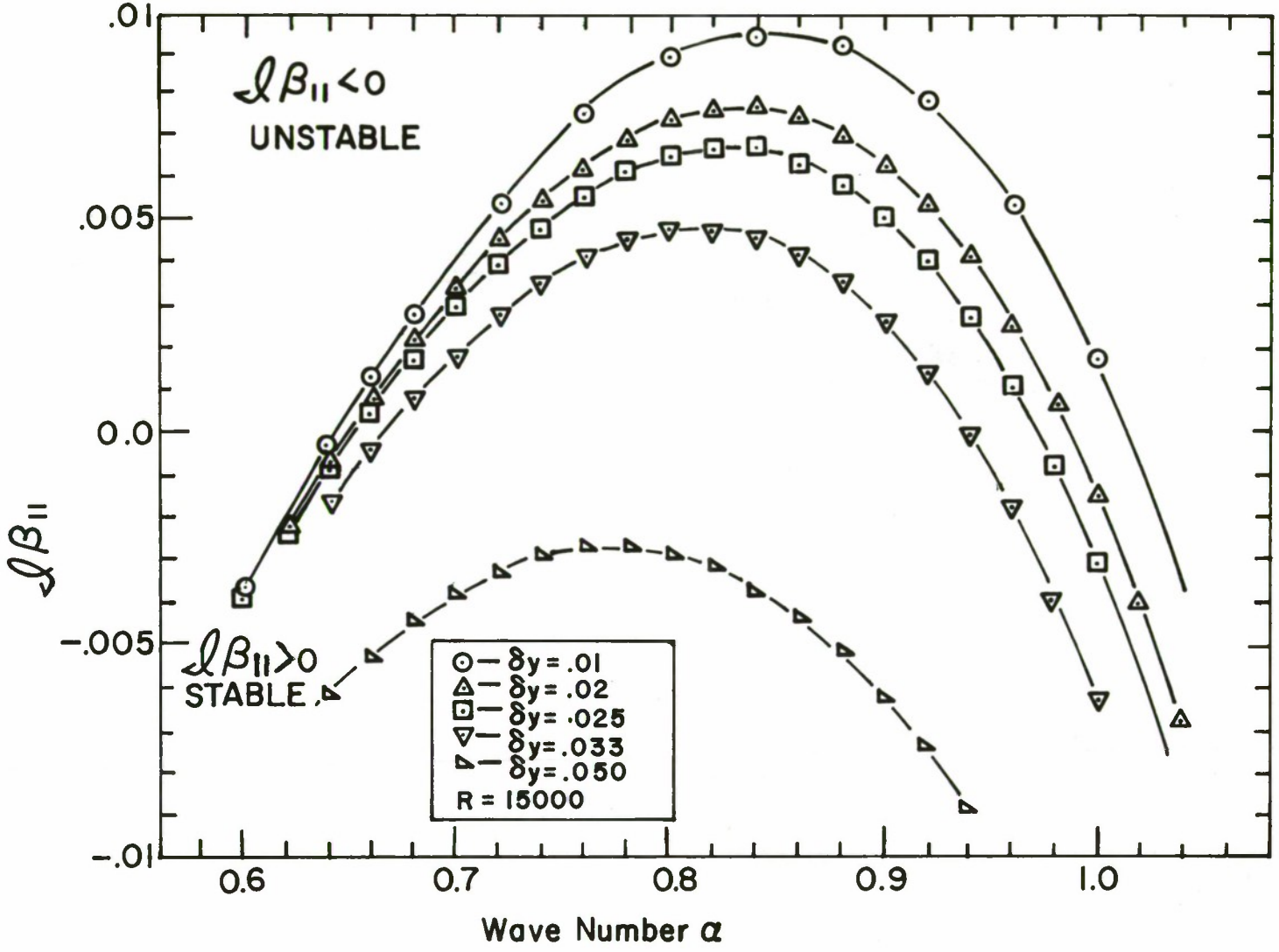


Figure 4. Effect of Grid Parameter, δy, on the Disturbance Growth Rate

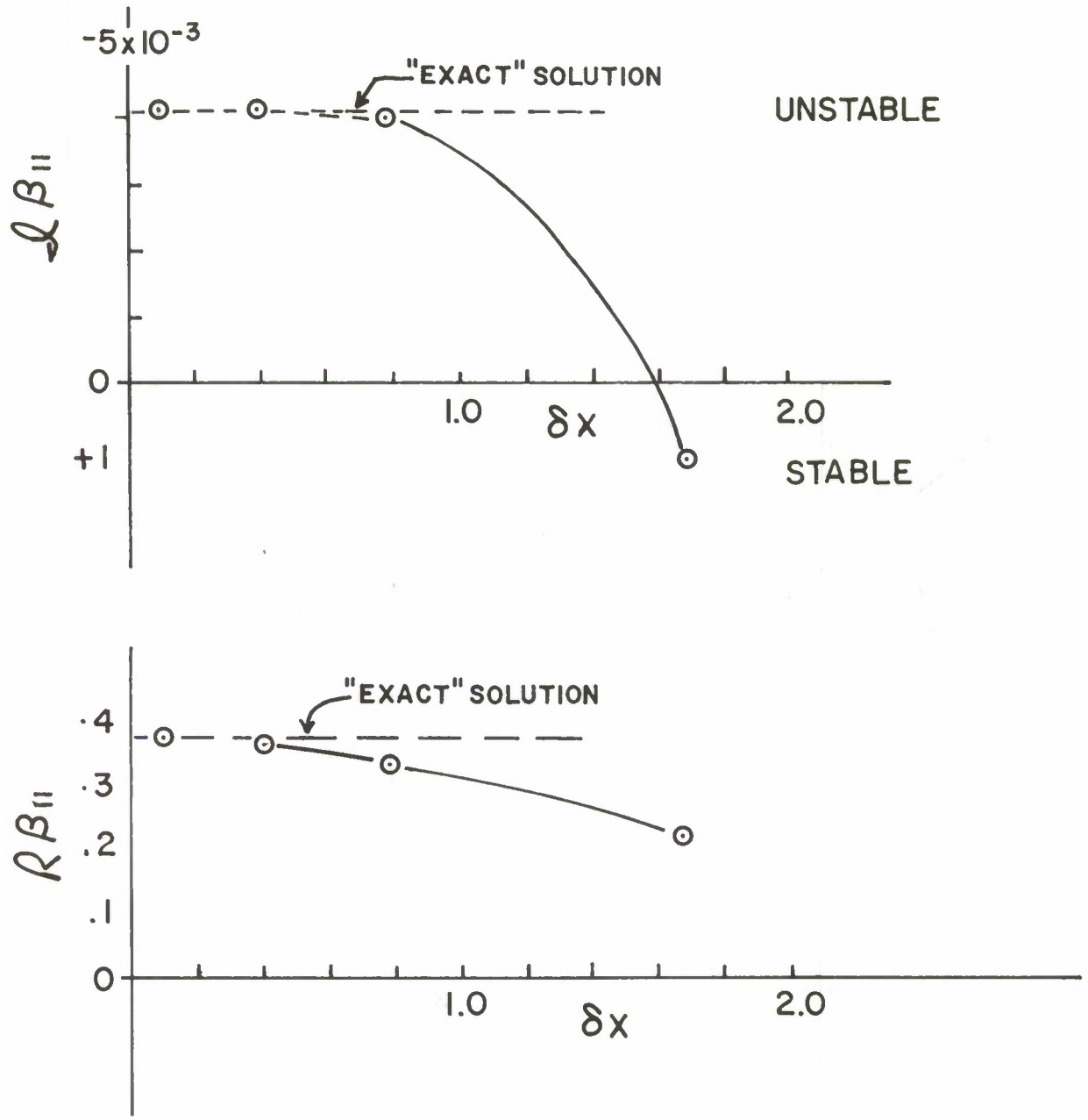


Figure 5. Effect of Grid Parameter, δx, on the Eigenvalue, β_{11}

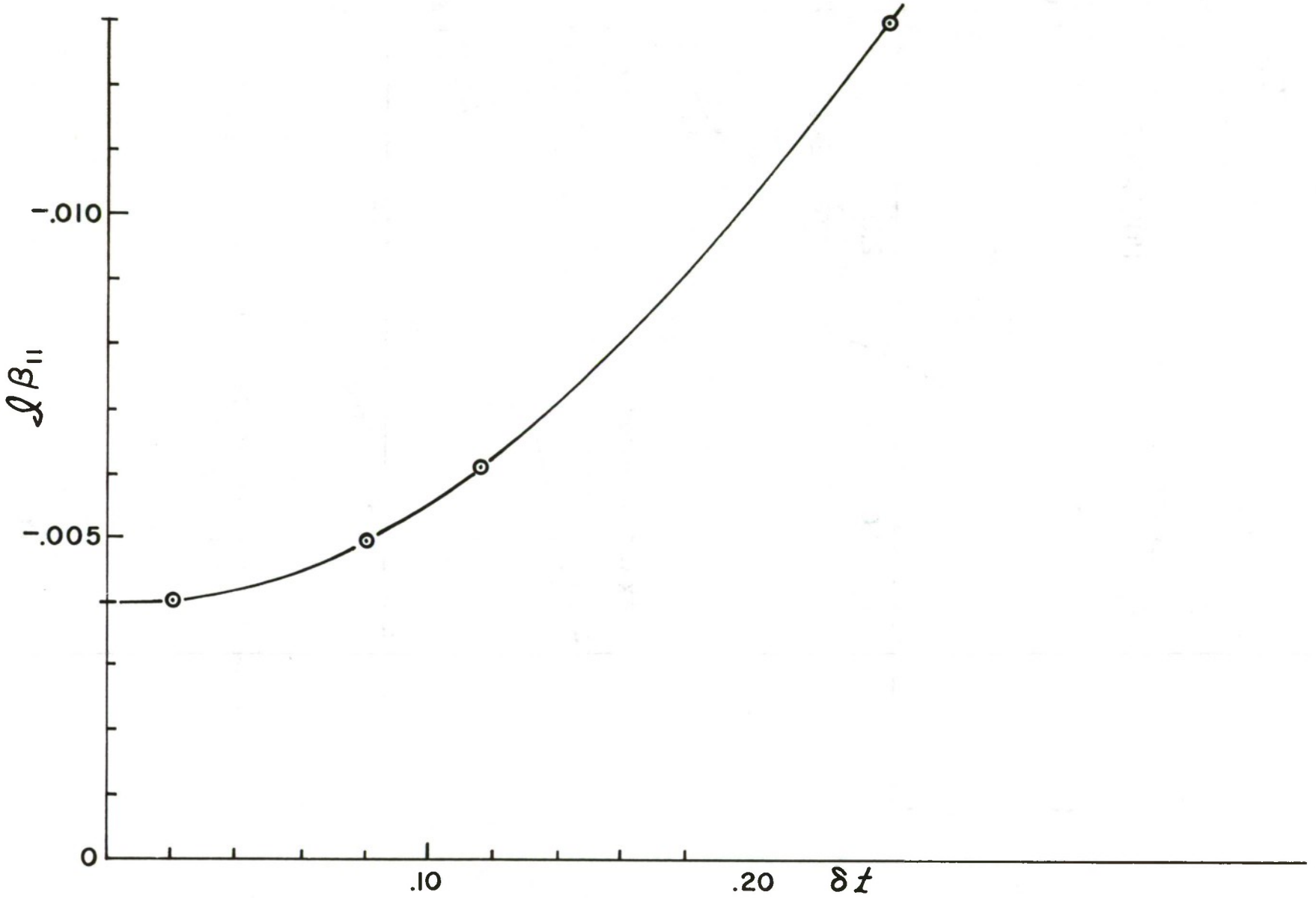


Figure 6. Effect of Grid Parameter, δt, on the Disturbance Growth Rate

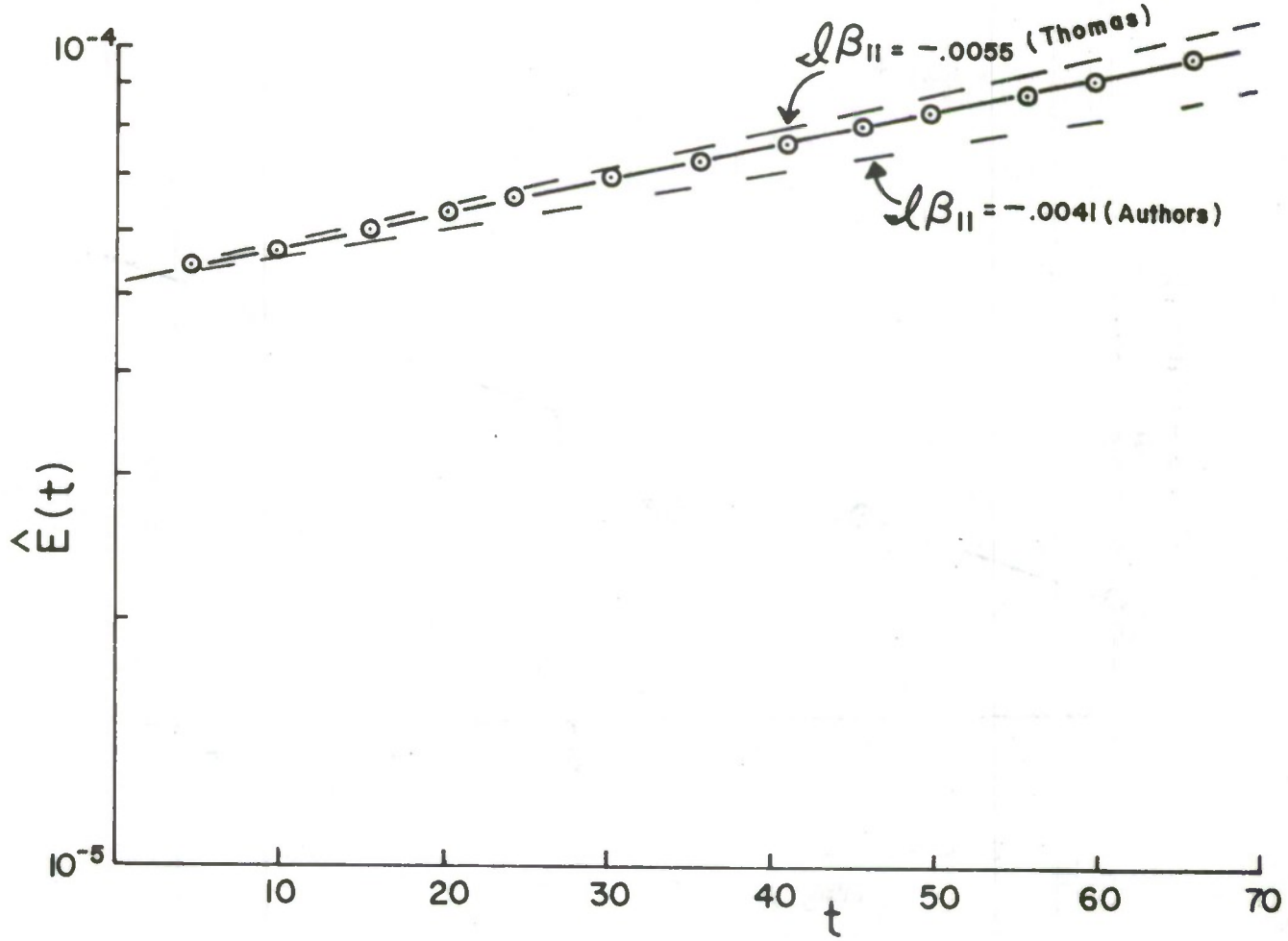


Figure 7. Energy Growth for a Low Amplitude Run.
$R_e = 6667$, $\alpha = 1.0$

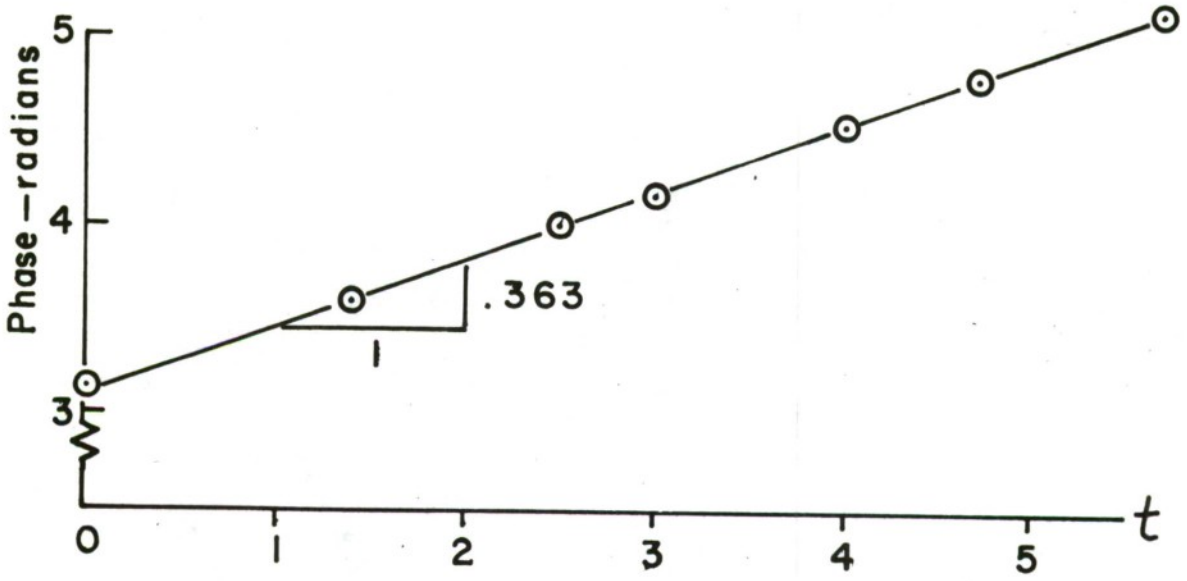


Figure 8. Phase, φ_1, Variation for a Low Amplitude Run. $R_e = 6667$, $\alpha = 1.0$

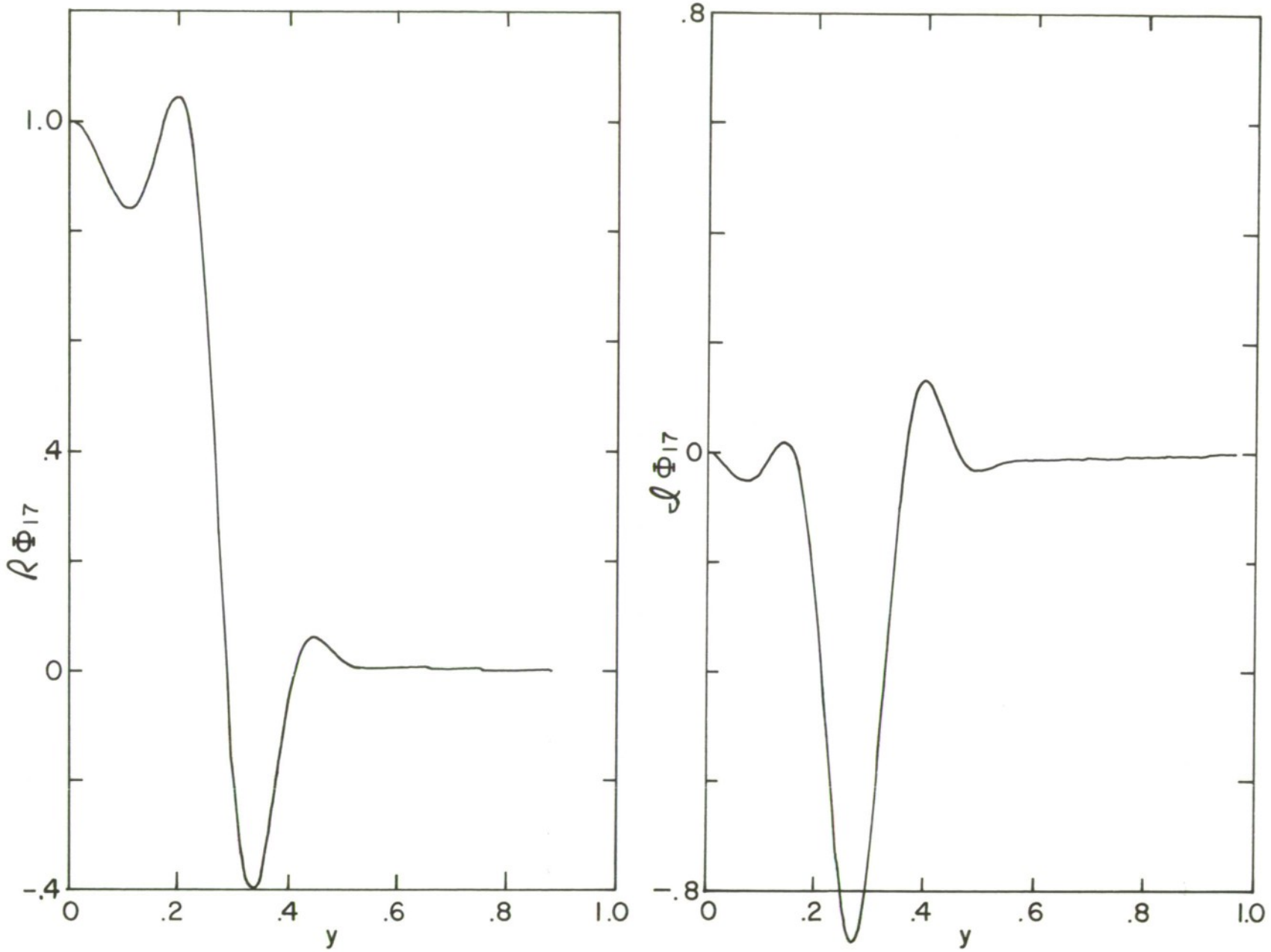


Figure 9a. Eigenfunction, Φ_{17}(even), for R_e = 6667, α = 1.0

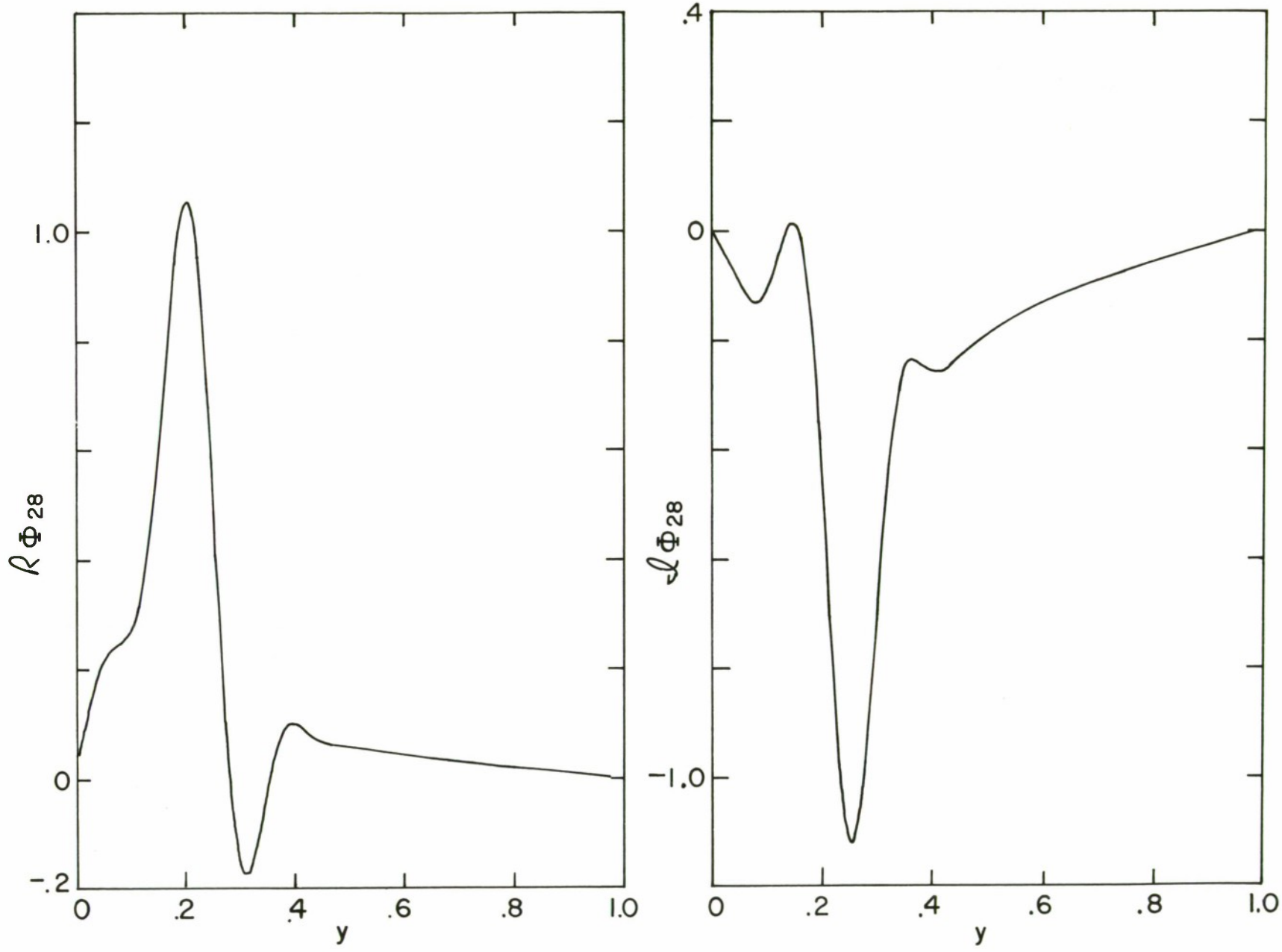


Figure 9b. Eigenfunction, ϕ_{28}(odd), for R_e = 6667, α = 1.0

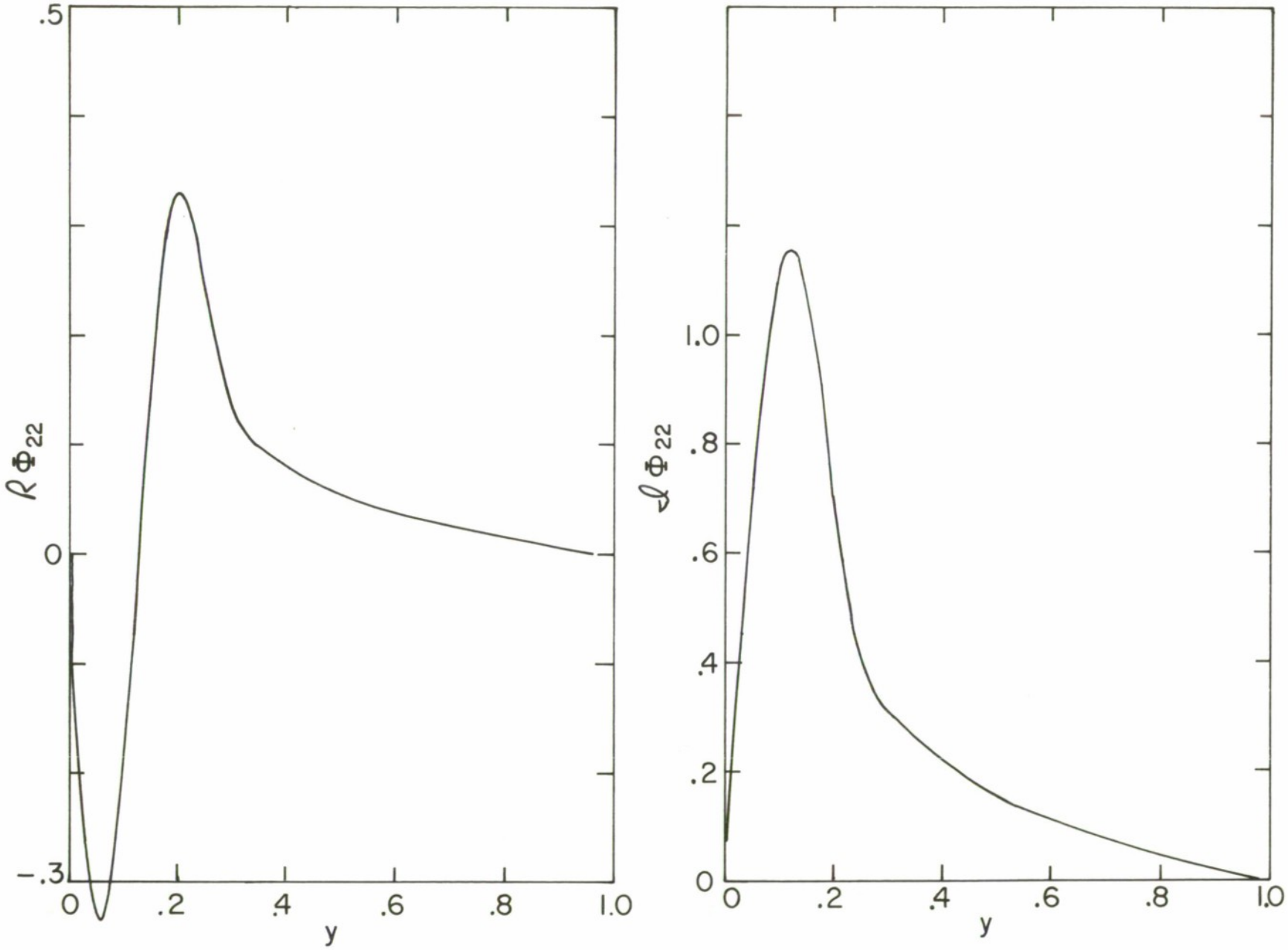


Figure 9c. Eigenfunction, ϕ_{22}(odd), for R_e = 6667, α = 1.0

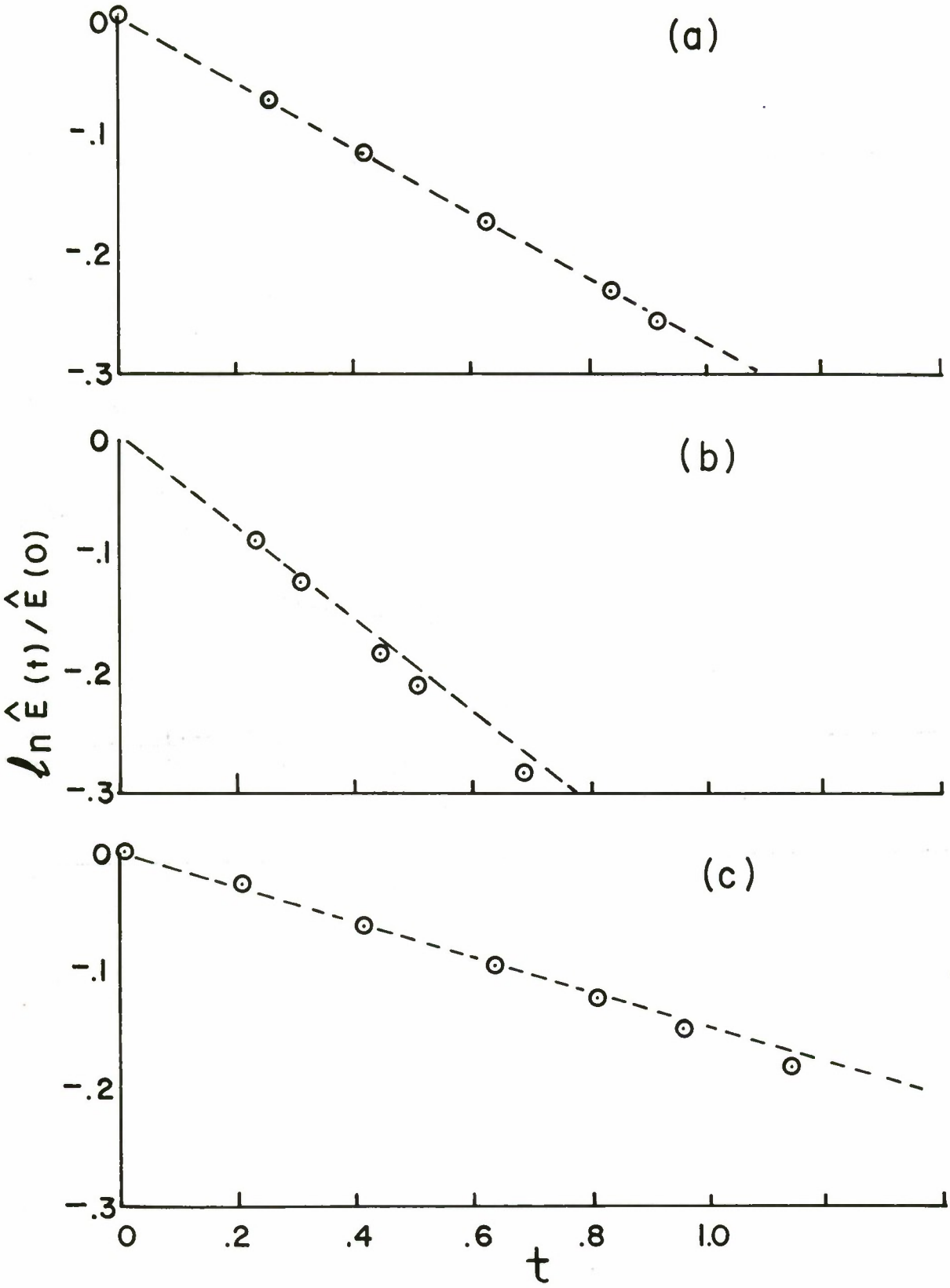


Figure 10. Computed Energy Decay for (a) Φ_{17}, (b) Φ_{28}, (c) Φ_{22}. Dashed line represents decay rate predicted from linear theory

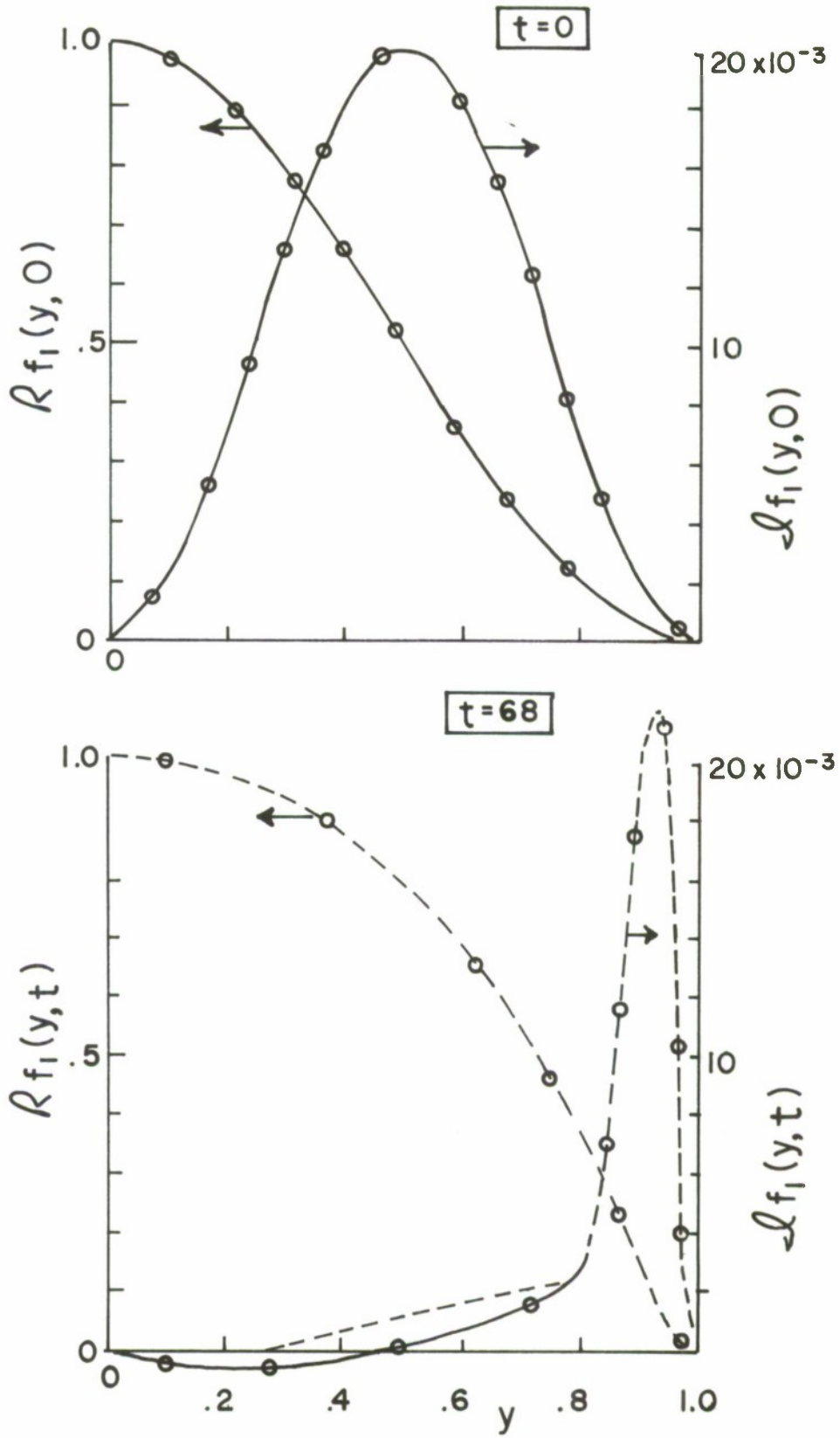


Figure 11. Primary Mode Shape, f_1, for a Low Energy (Linear) Run at Times t=0 and t=68. Dashed Line Represents Φ_{11}

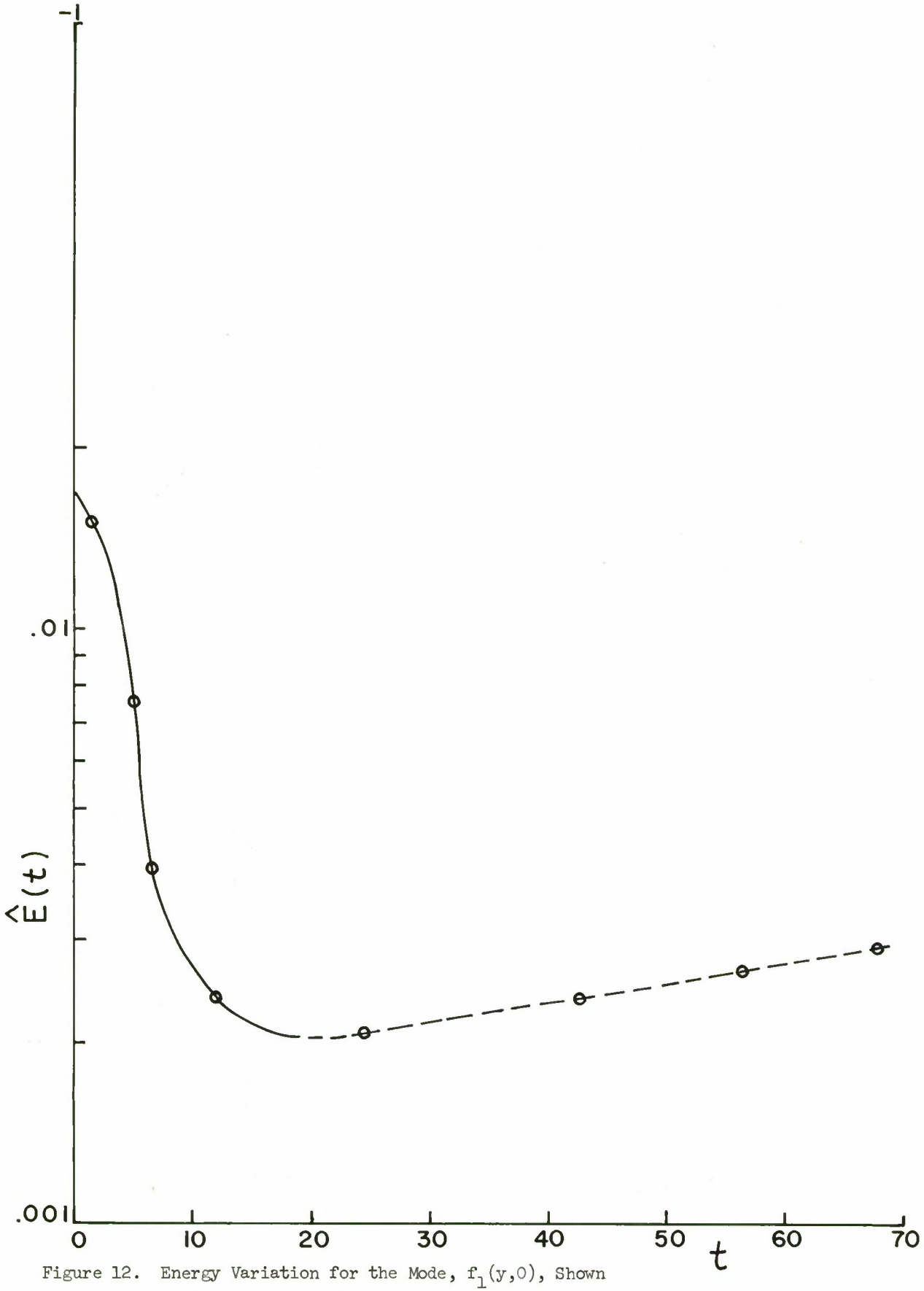


Figure 12. Energy Variation for the Mode, $f_1(y,0)$, Shown in Figure 11. Dashed Line Represents Predicted Growth Rate for ϕ_{11}

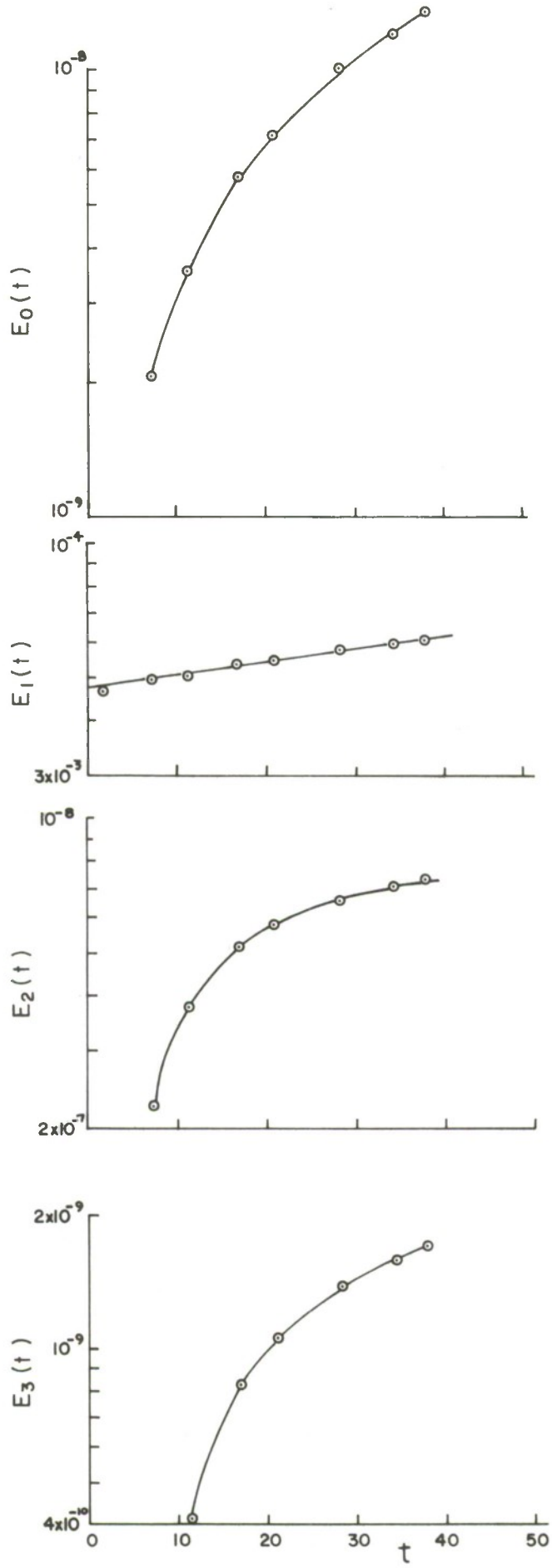


Figure 13. Energy Variation for the Modes E_0, E_1, E_2 and E_3. $R_e = 6667$, $\alpha = .875$

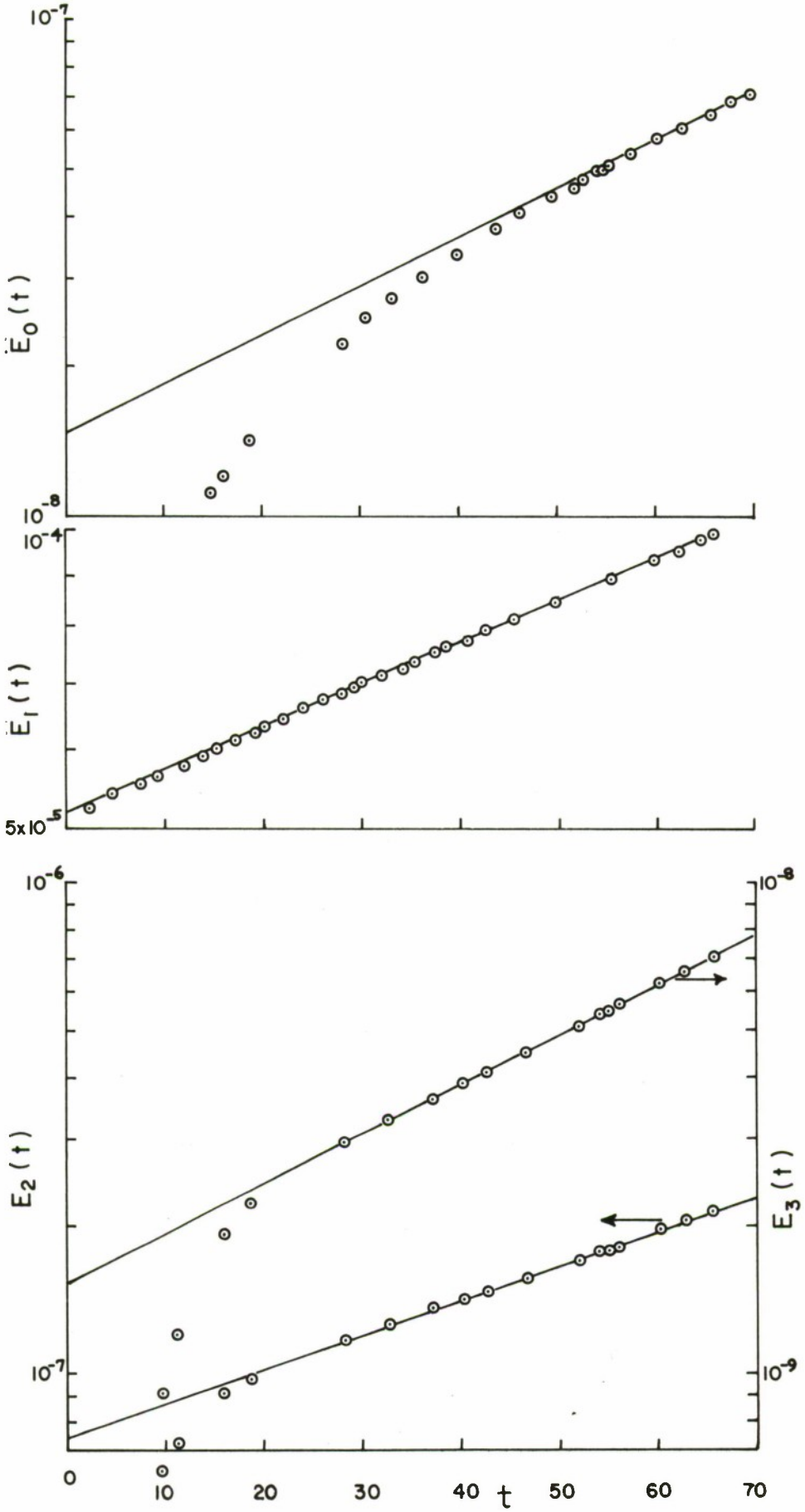


Figure 14. Energy Variation for the Modes E_0, E_1, E_2 and E_3. R_e=6667, α = 1.0

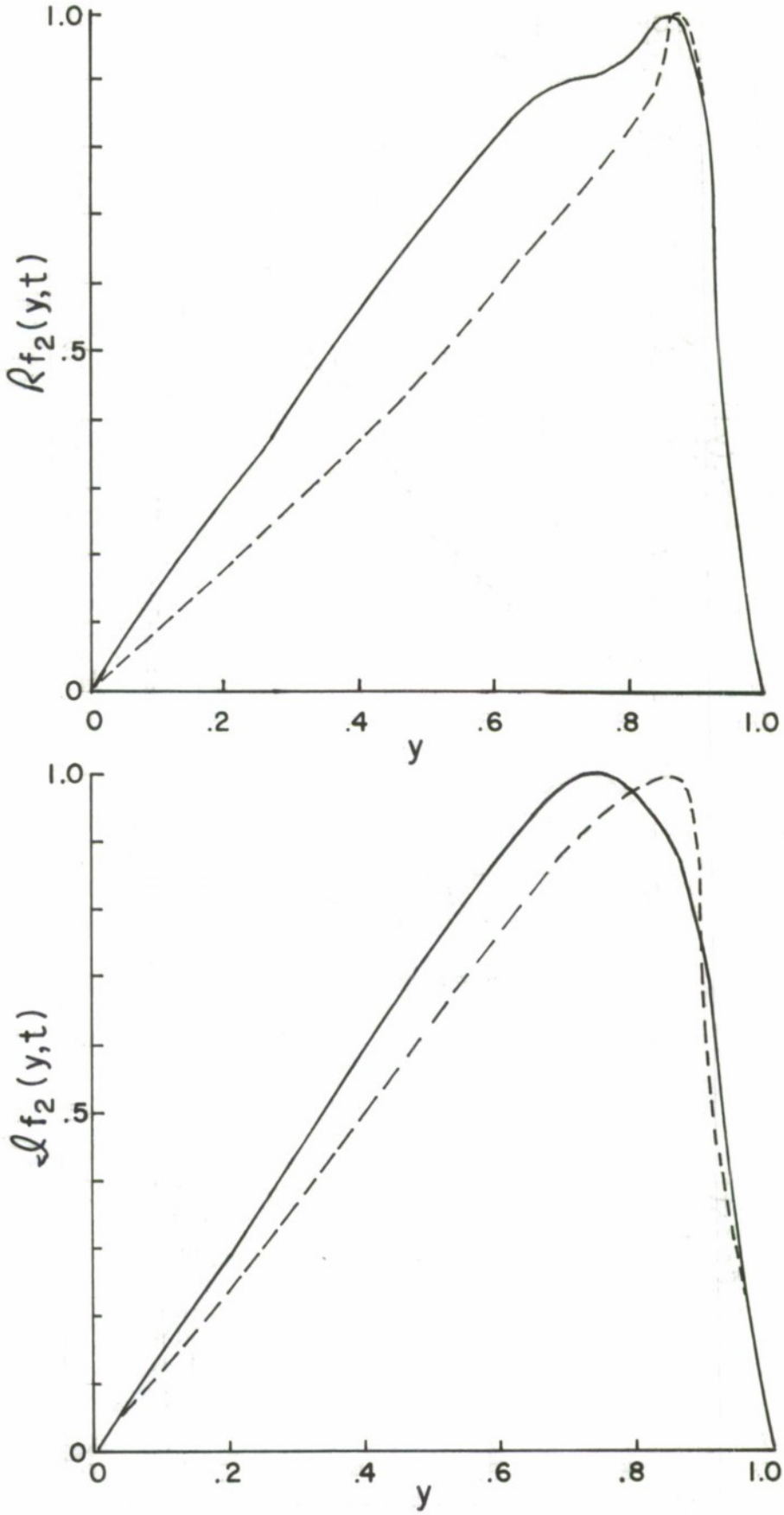


Figure 15a. Mode Shape for f_2(odd) at t = 85. α = 1.0, R_e = 6667.

Dashed Line Represents Φ_{26}(odd)

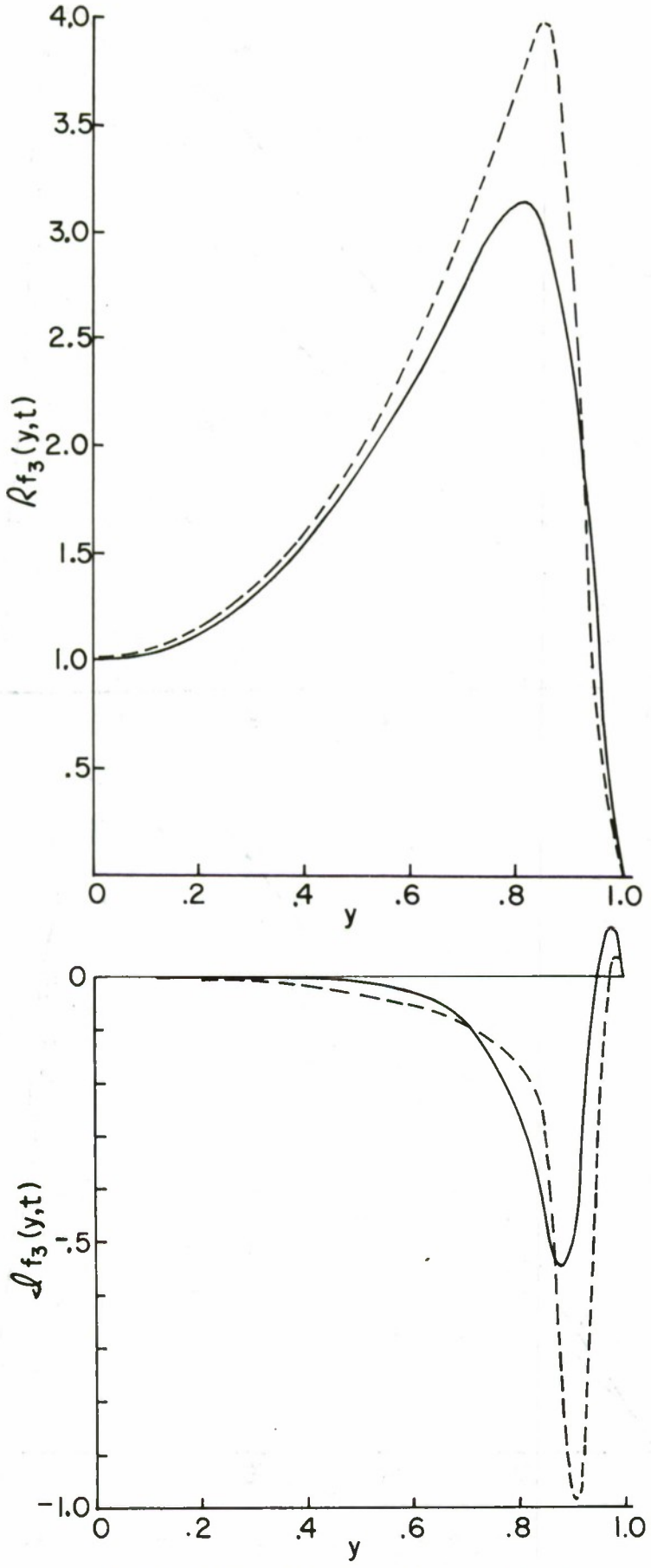


Figure 15b. Mode Shape for f_3(even) at t = 85. α = 1.0, R_e = 6667.

Dashed Line Represents ϕ_{33}(even)

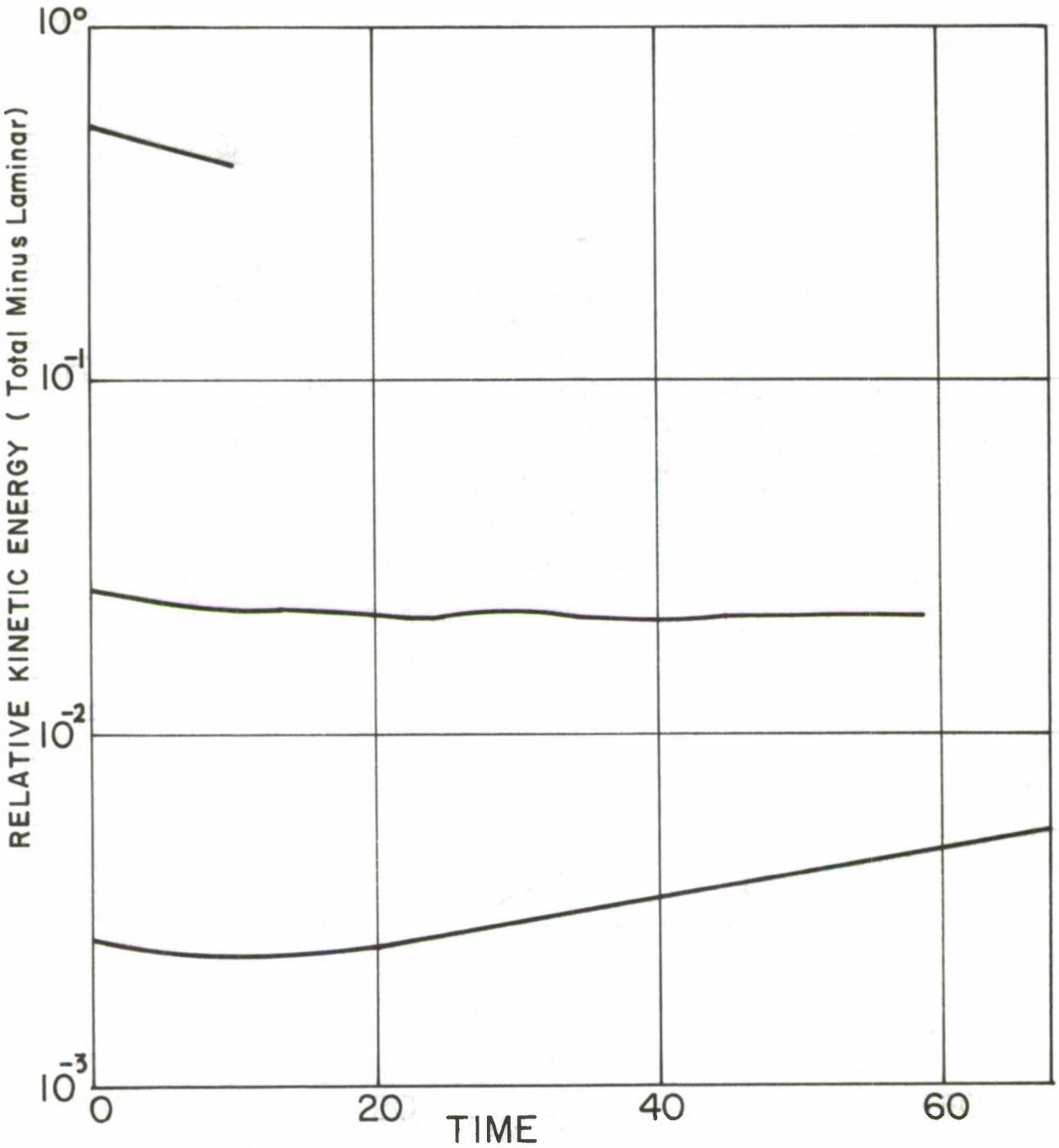


Figure 16. Total Kinetic Energy (Mean plus Turbulent) for Three Different Initial Energy Levels. The Laminar Energy (.600) has been Subtracted for Convenience.

Top Curve, $\varepsilon = .5$
Middle Curve $\varepsilon = .05\sqrt{5}$
Bottom Curve $\varepsilon = .05/\sqrt{2}$

$\alpha = 1.0$, $R_e = 6667$

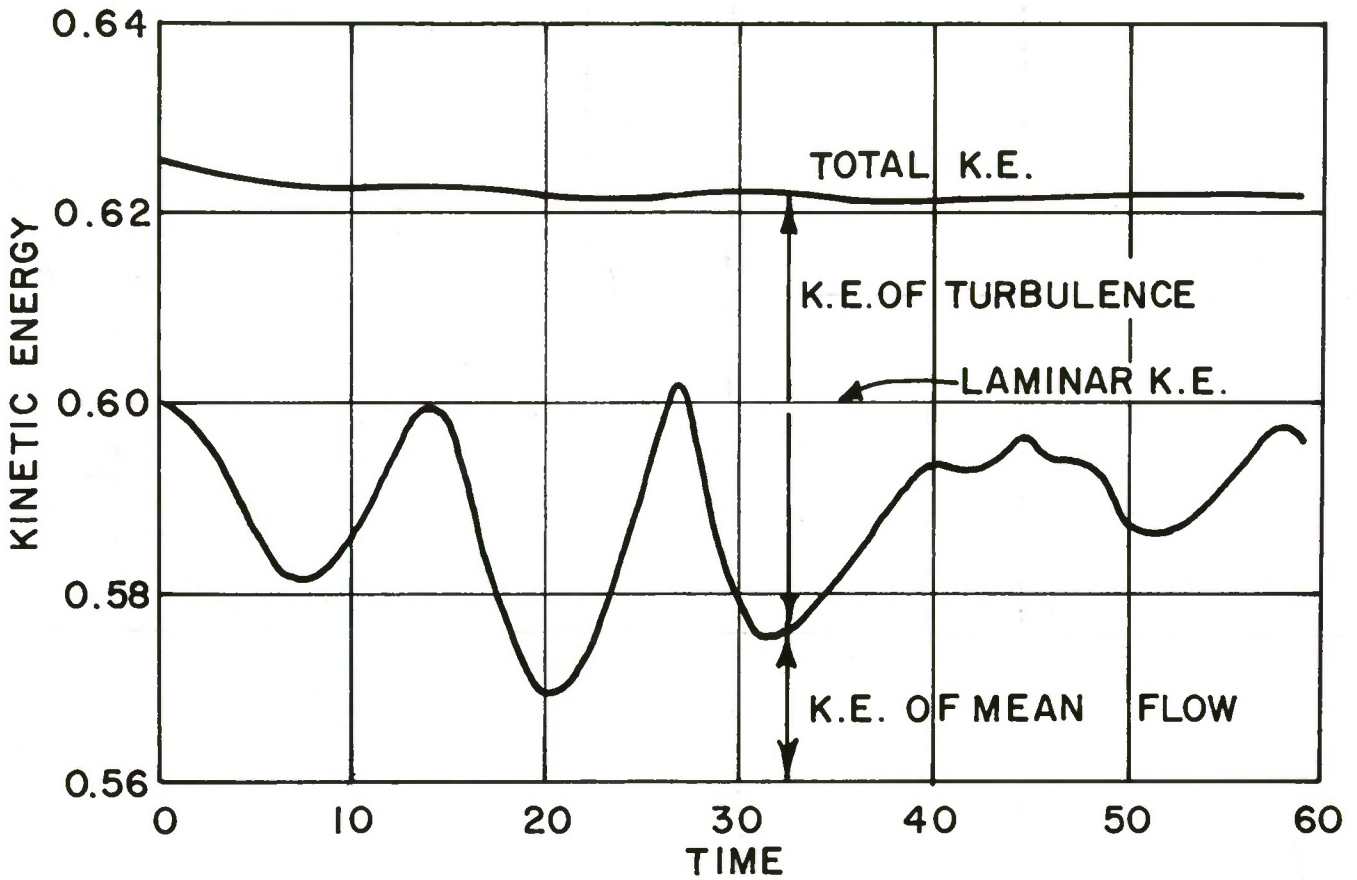
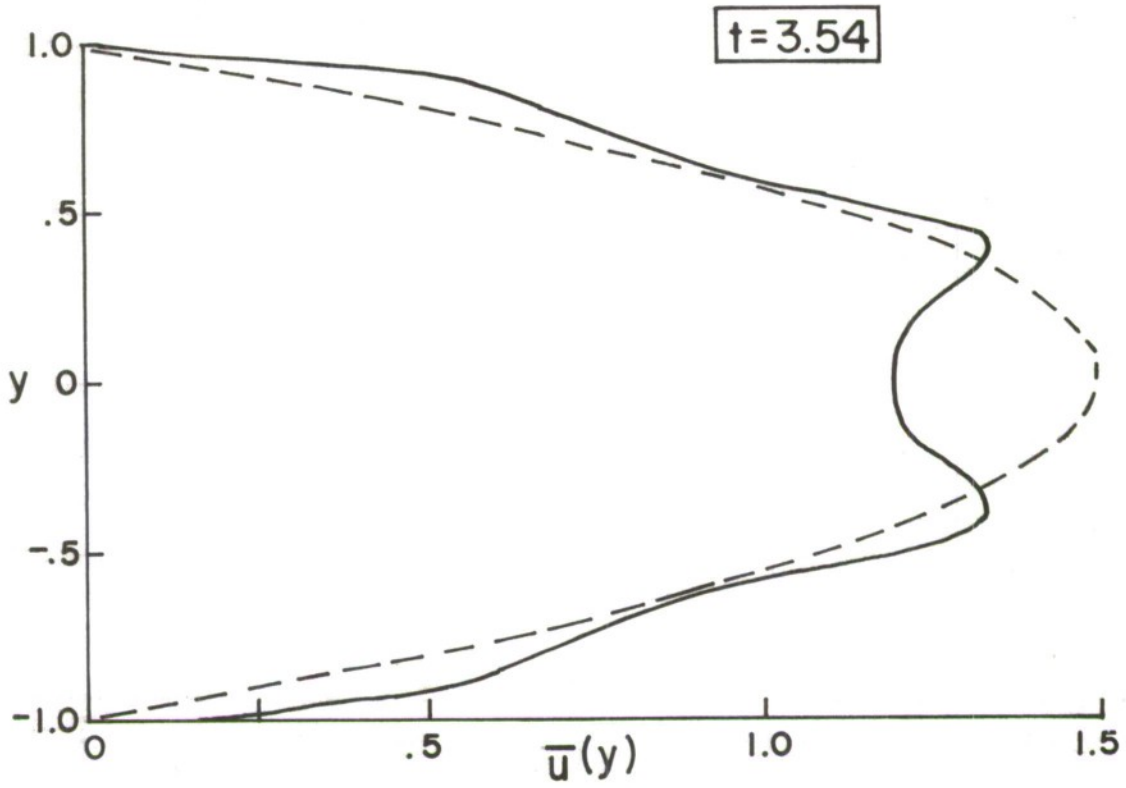


Figure 17. Energy Variation for the Run $\epsilon = .05\sqrt{5}$



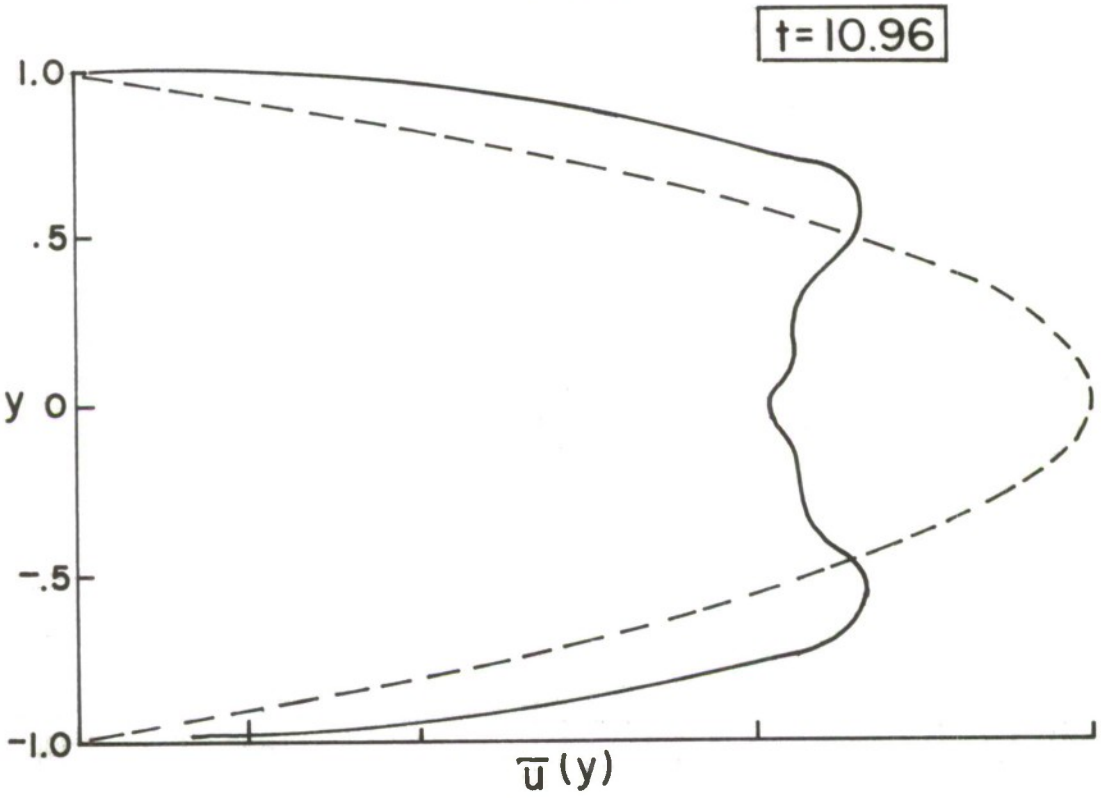


Figure 18a. Mean Velocity Profiles for the Run $\epsilon = .5$, $\alpha = 1.0$, $R_e = 6667$
Dashed Line is $U(y) = 3/2(1 - y^2)$

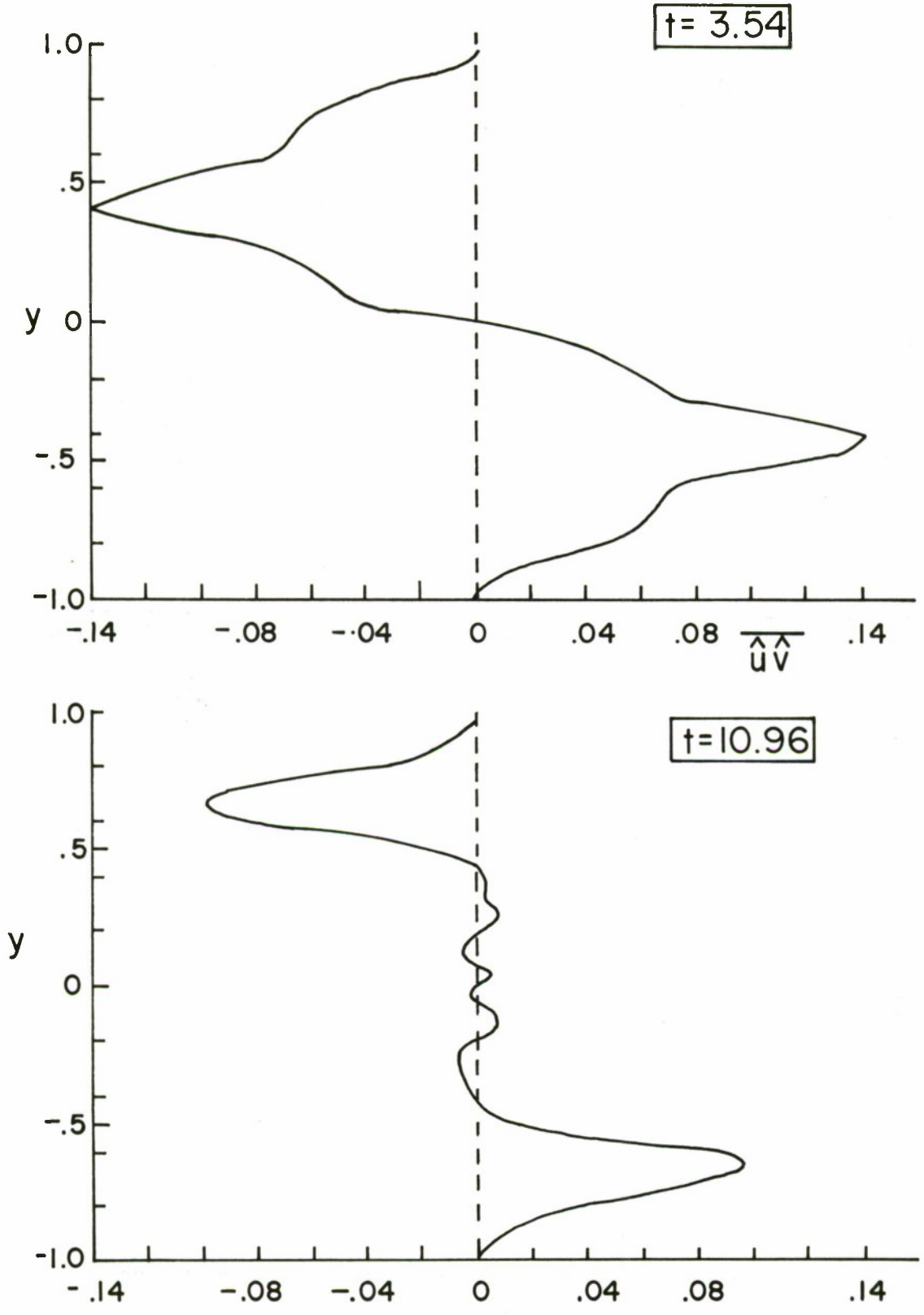


Figure 18b. Reynolds Stresses for the Run $\varepsilon = .5$, $\alpha = 1.0$
$R_e = 6667$

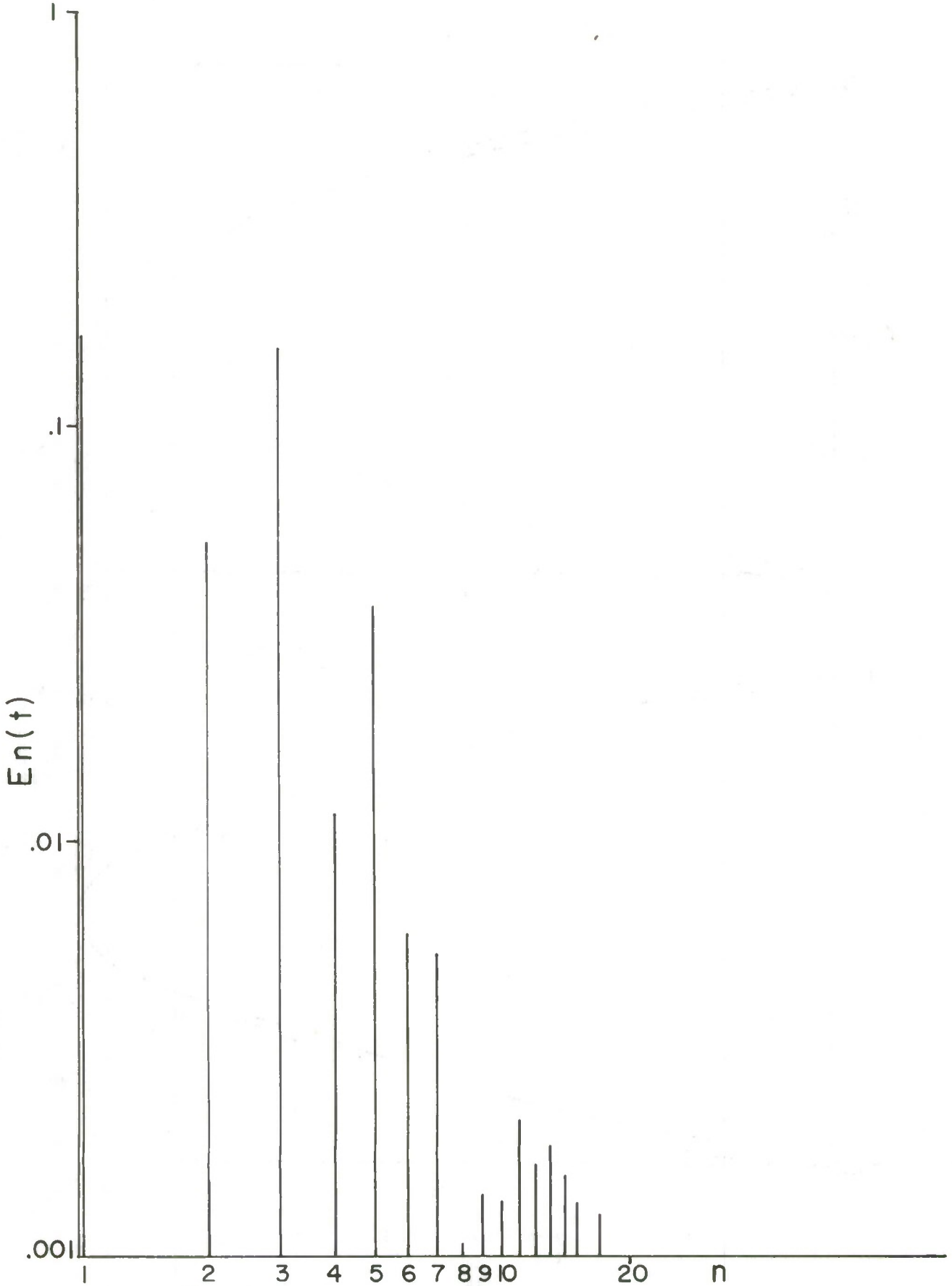


Figure 18c. Energy Spectrum for the Run $\varepsilon = .5$, $\alpha = 1.0$, $R_e = 6667$ at $t = 11$

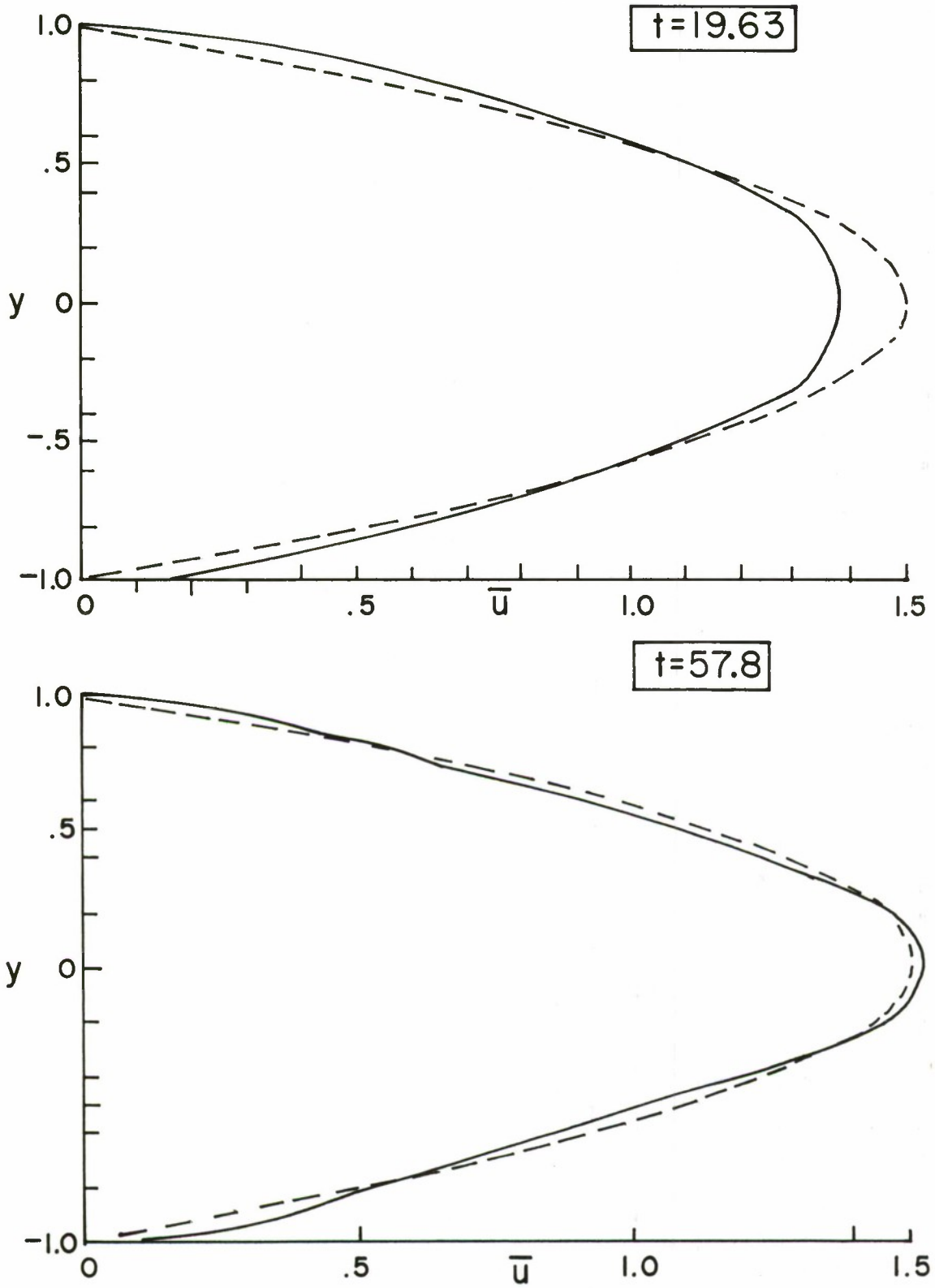


Figure 19a. Mean Velocity Profiles for the Run $\epsilon = .05\sqrt{5}$, $\alpha = 1.0$, $R_e = 6667$
Dashed Line is $U(y) = 3/2(1-y^2)$

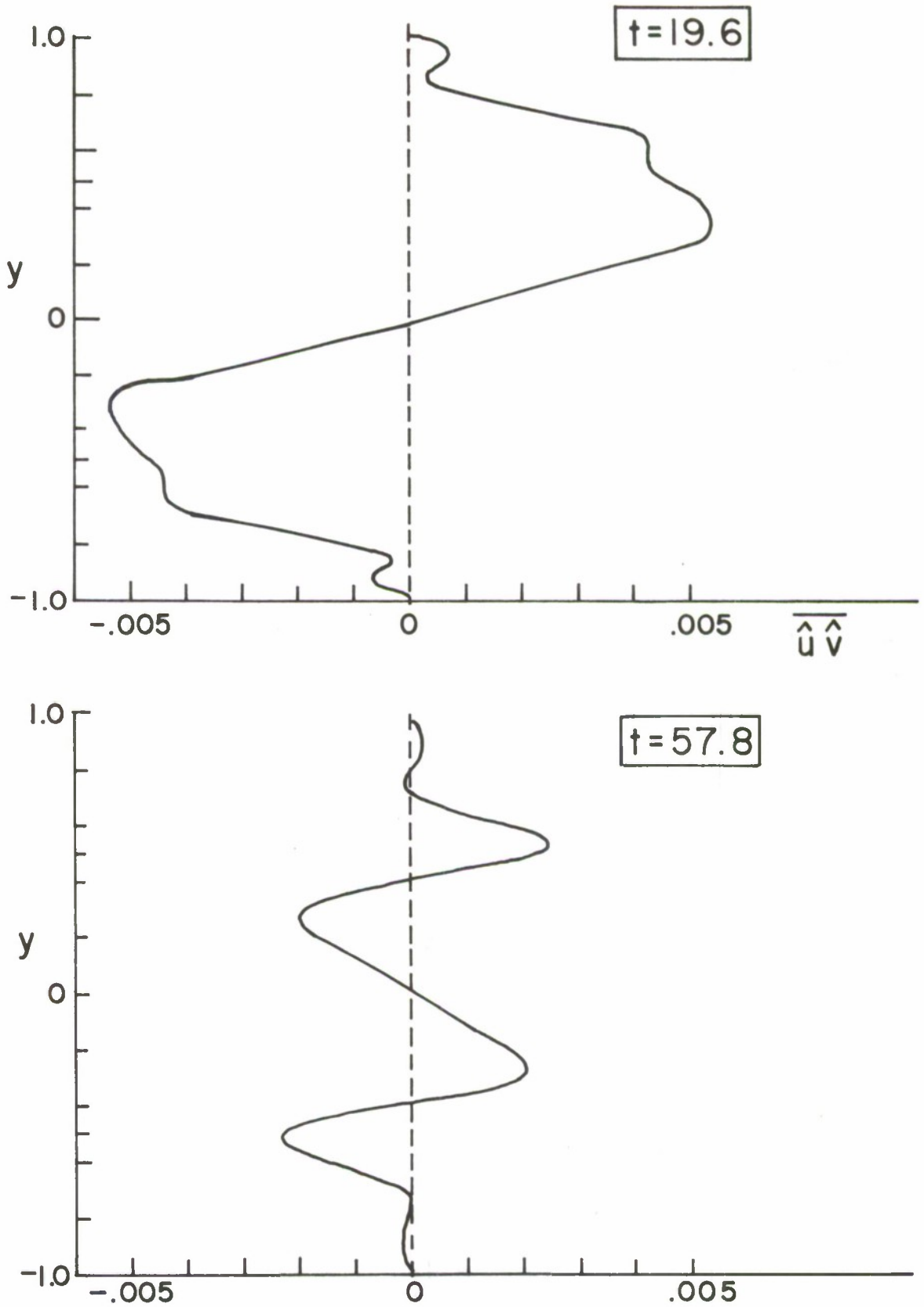


Figure 19b. Reynold's Stresses for the Run
$\varepsilon = .05\sqrt{5}$, $\alpha = 1.0$, $R_e = 6667$

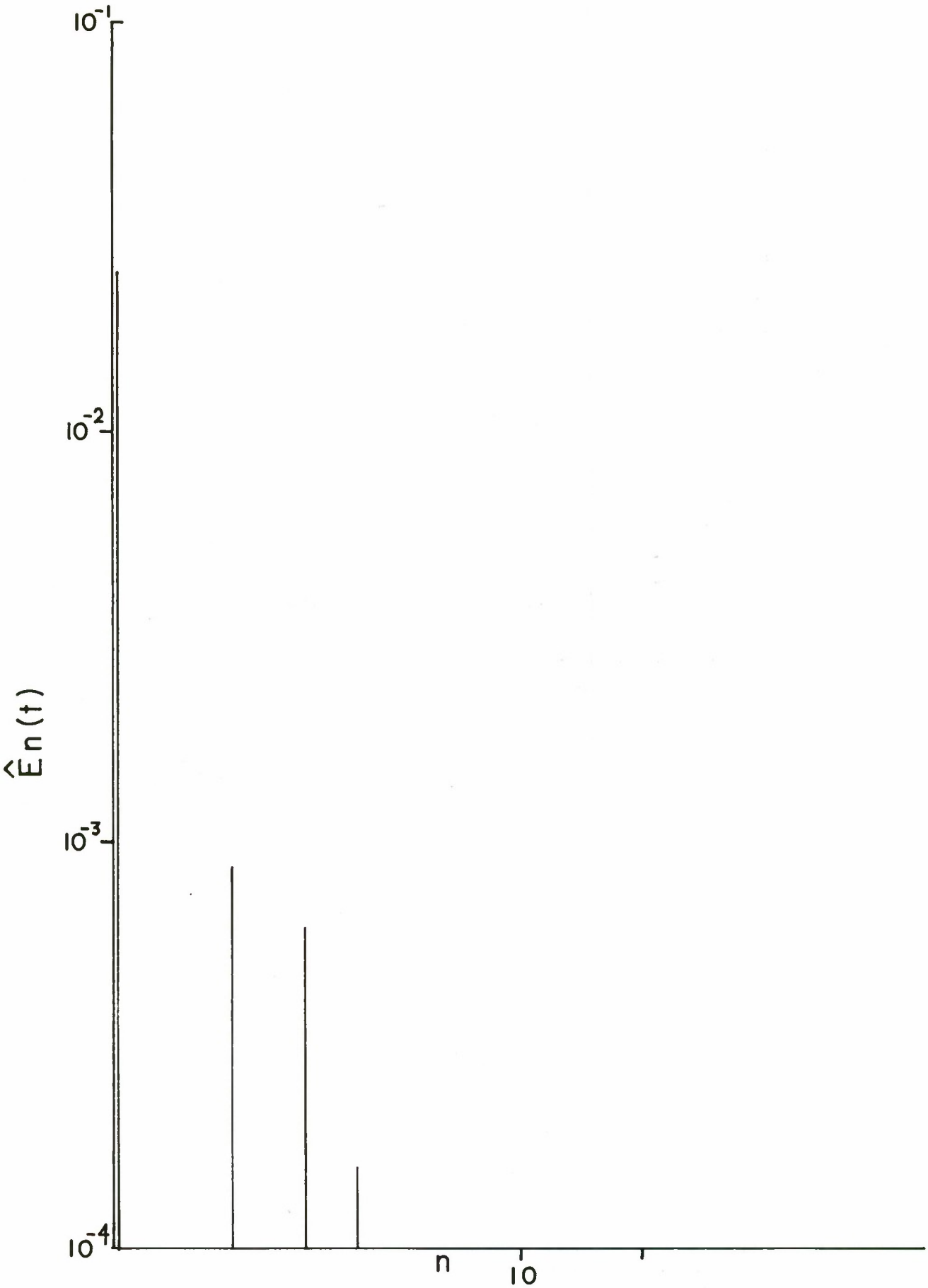


Figure 19c. Energy Spectrum for the Run $\epsilon = .05\sqrt{5}$, $\alpha = 1.0$, $R_e = 6667$ at $t = 59$

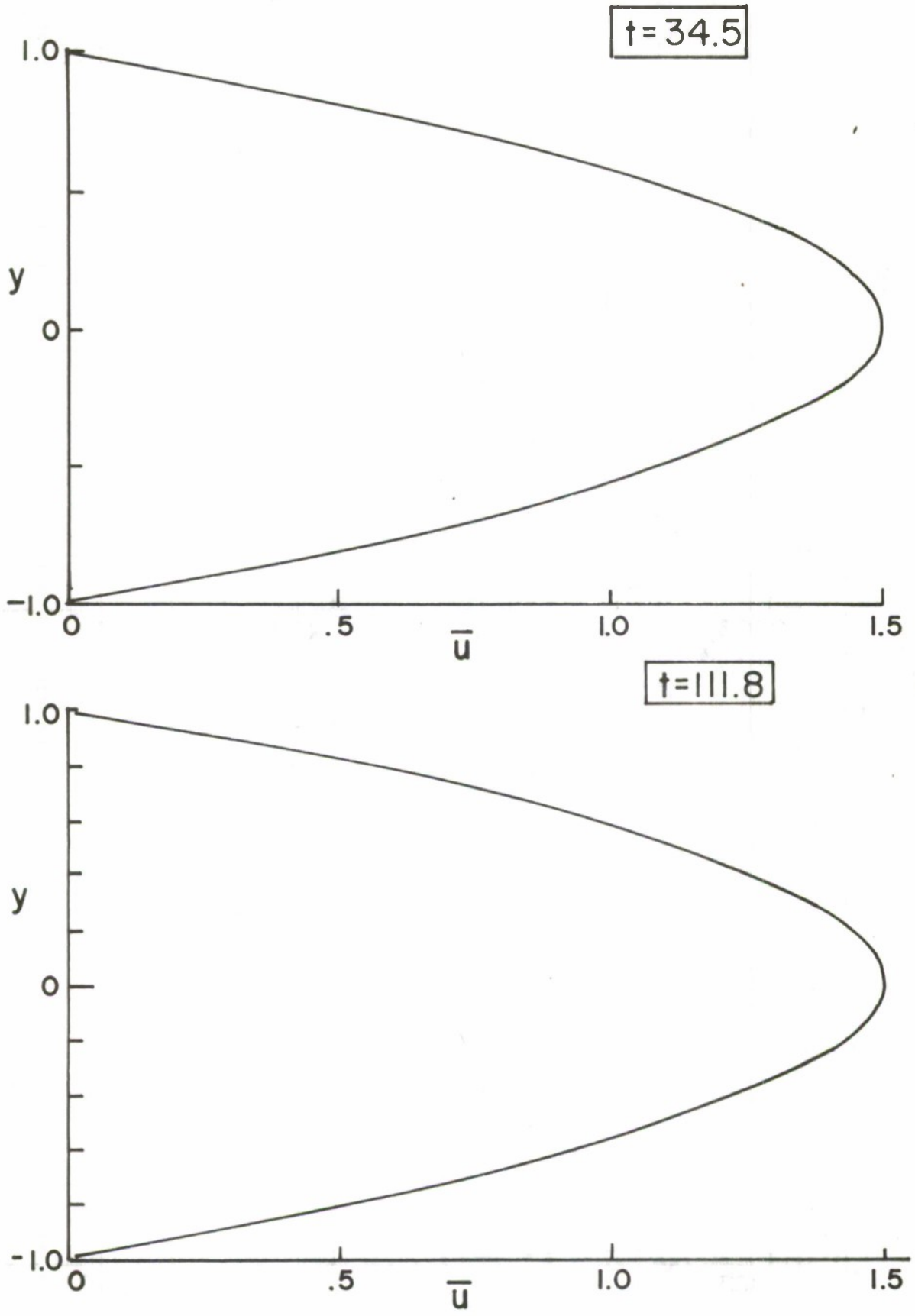


Figure 20a. Mean Velocity Profiles for the Run
$\epsilon = .05/\sqrt{2}$, $\alpha = 1.0$, $R_e = 6667$

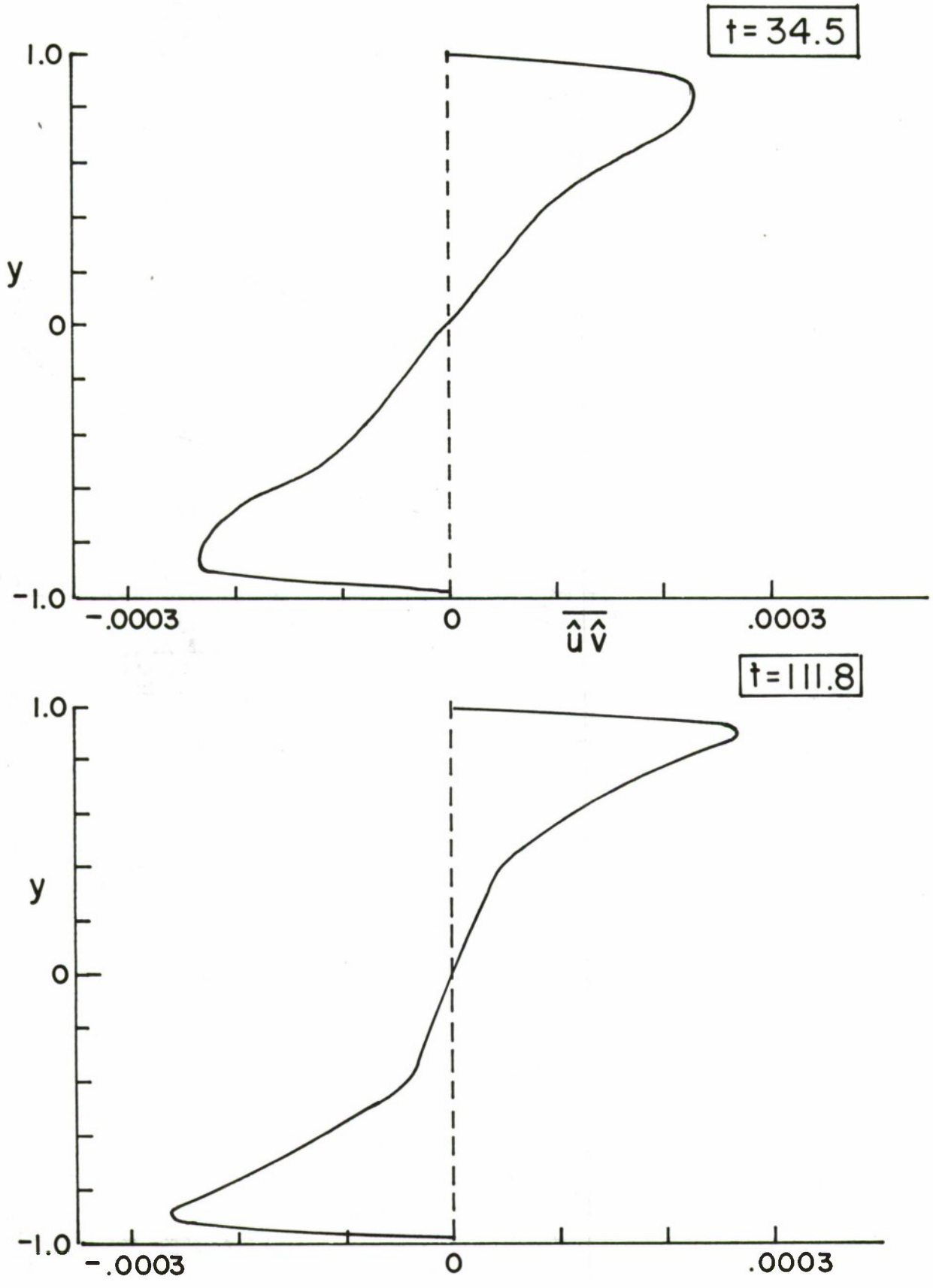


Figure 20b. Reynold's Stresses for the Run $\varepsilon = .05/\sqrt{2}$
$\alpha = 1.0$, $R_e = 6667$

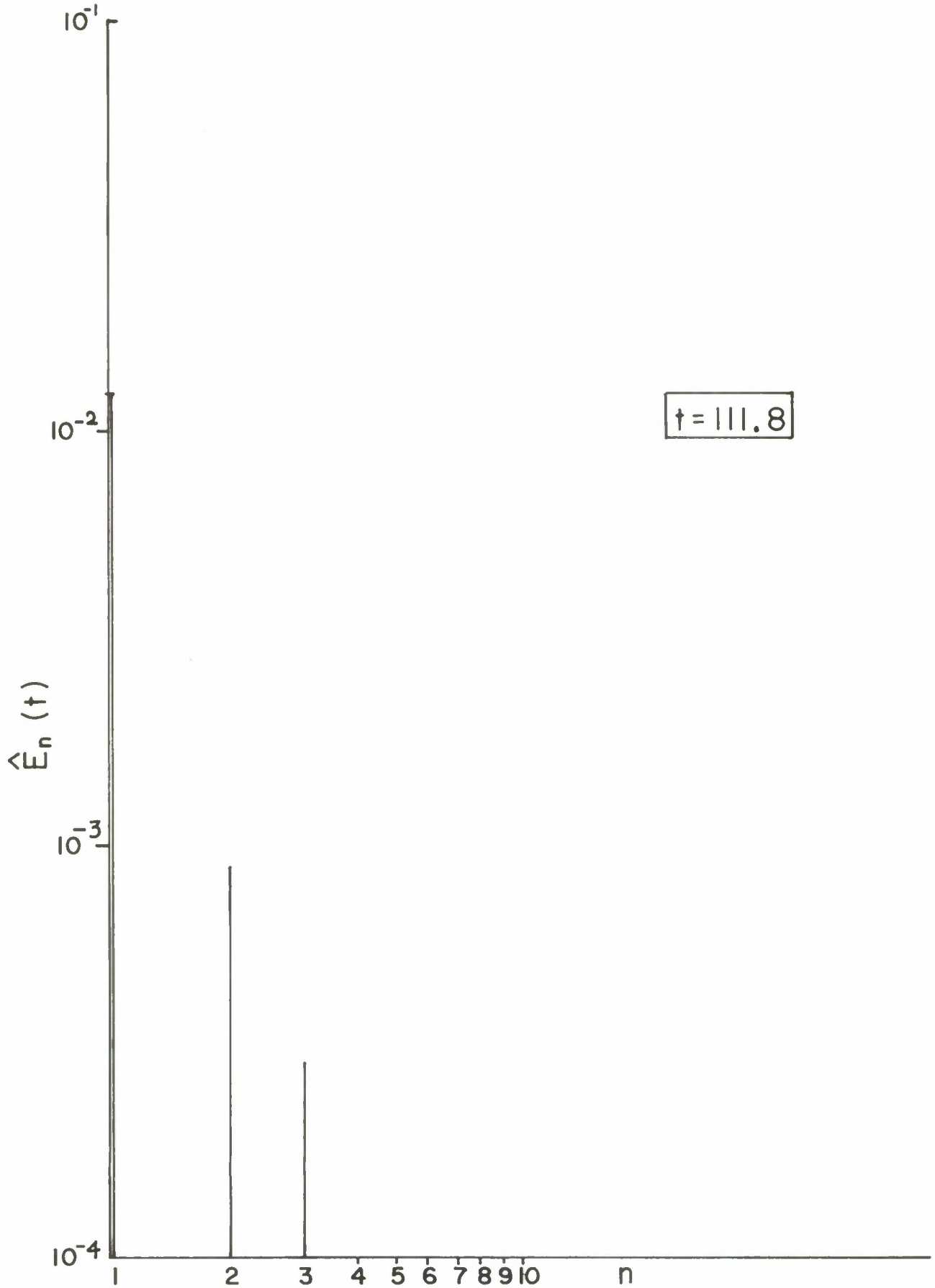


Figure 20c. Energy Spectrum for the Run $\varepsilon = .05/\sqrt{2}$, $\alpha = 1.0$, $R_e = 6667$

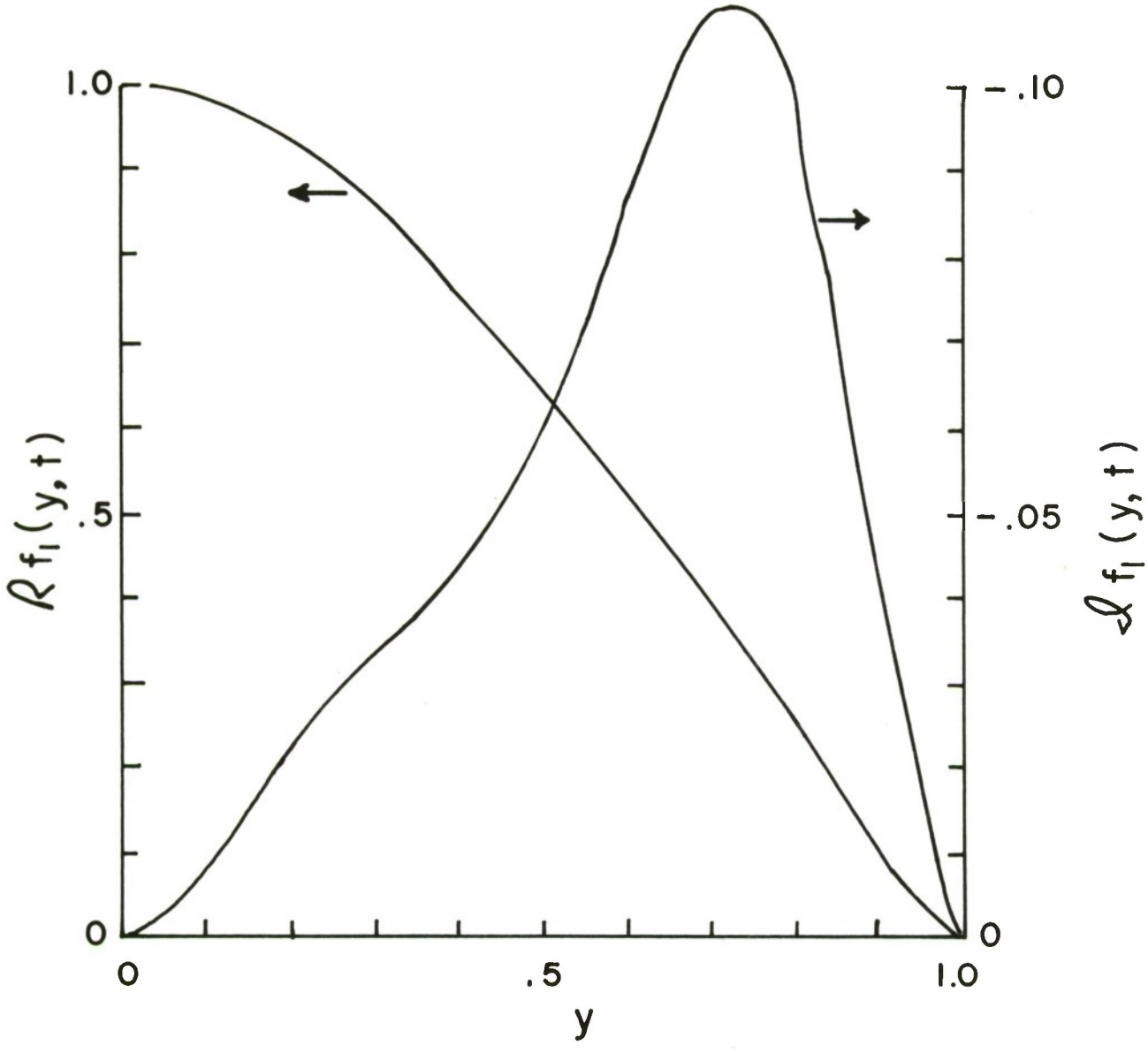


Figure 21a. Primary Mode Shape, f_1, for the Run $\varepsilon = .5$ at $t = 11$

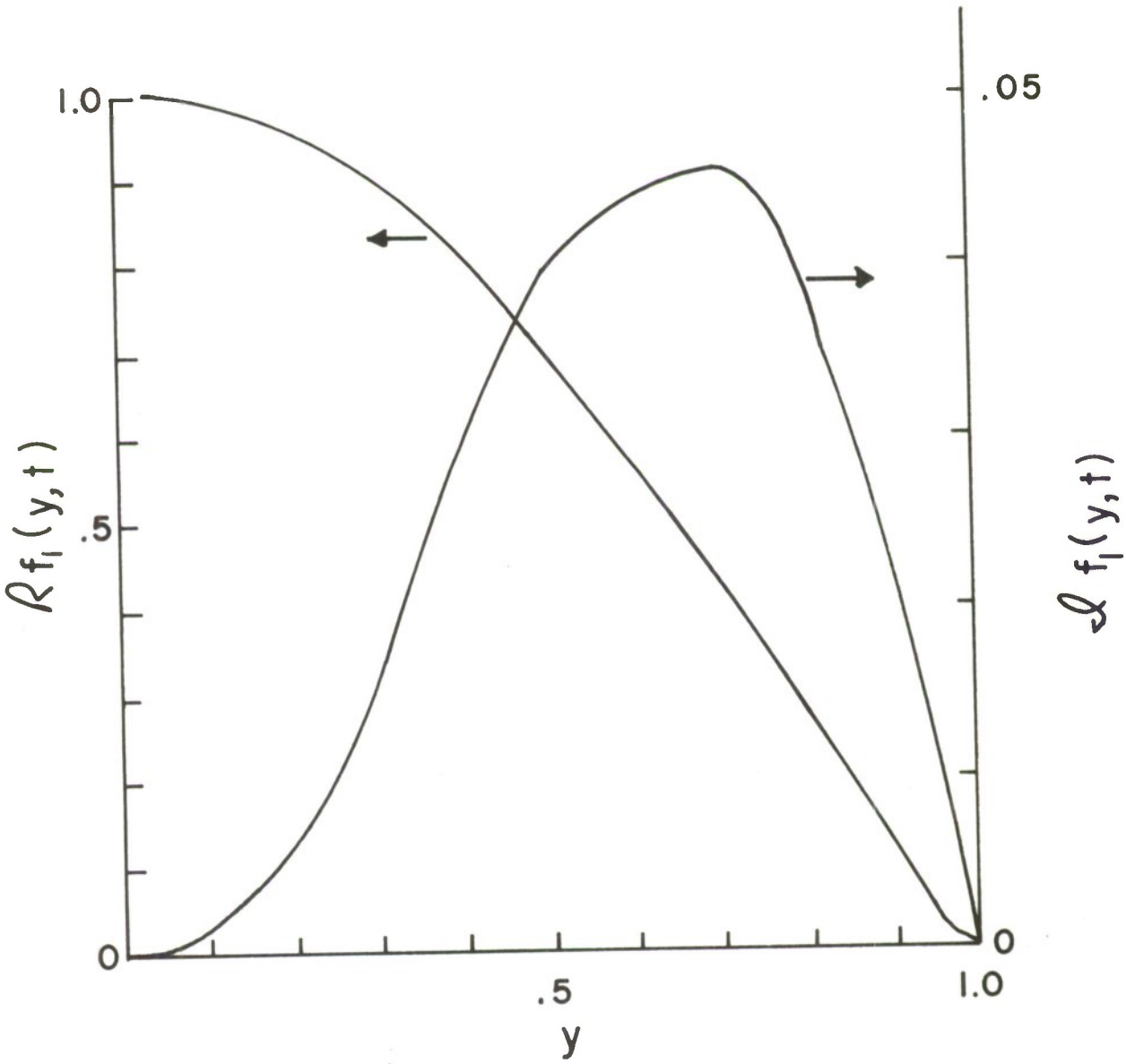


Figure 21b. Primary Mode Shape, f_1, for the Run $\varepsilon = .05\sqrt{5}$ at t = 59

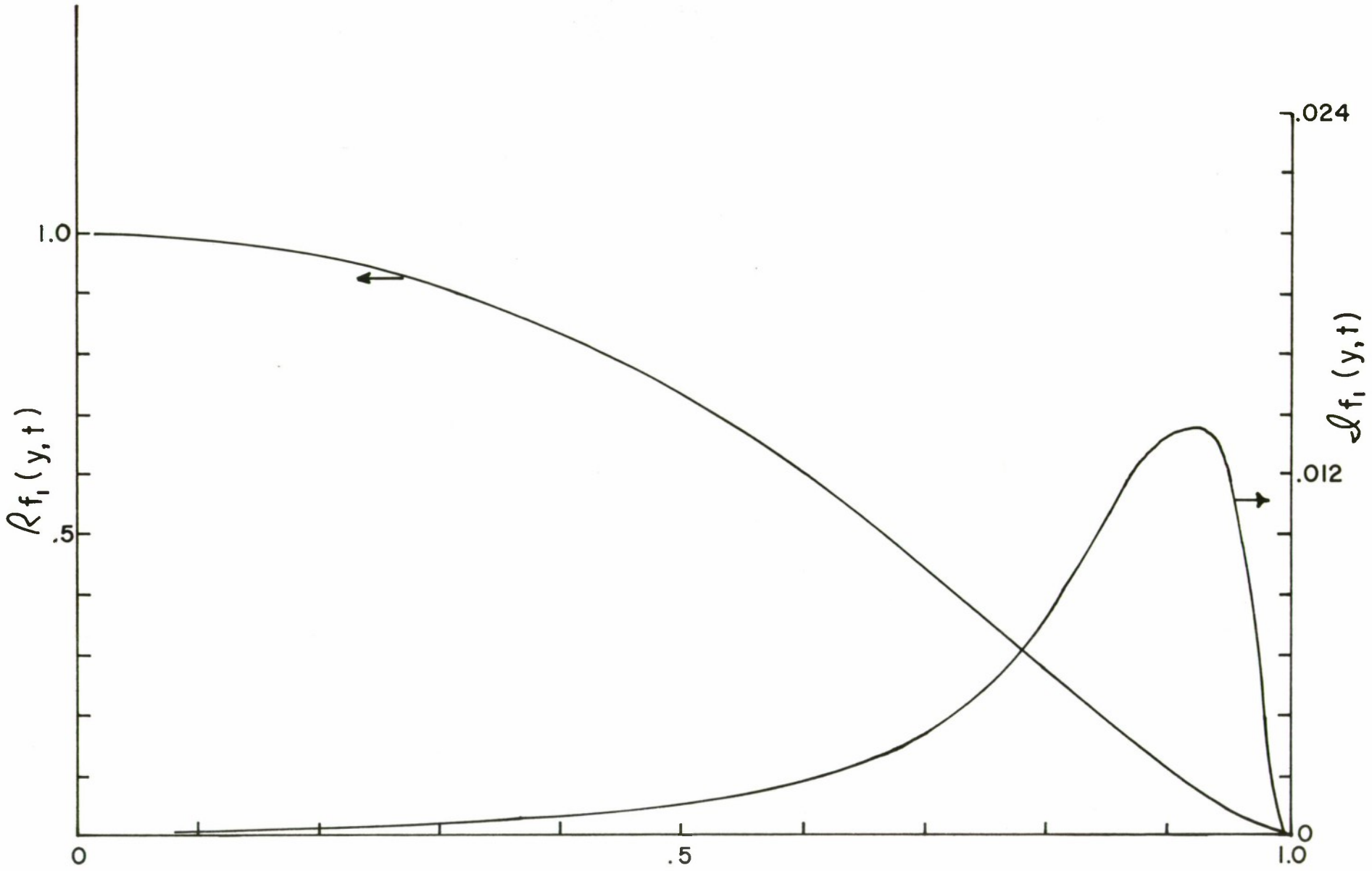


Figure 21c. Primary Mode Shape, f_1, for the Run $\varepsilon = .05/\sqrt{2}$ at t = 112

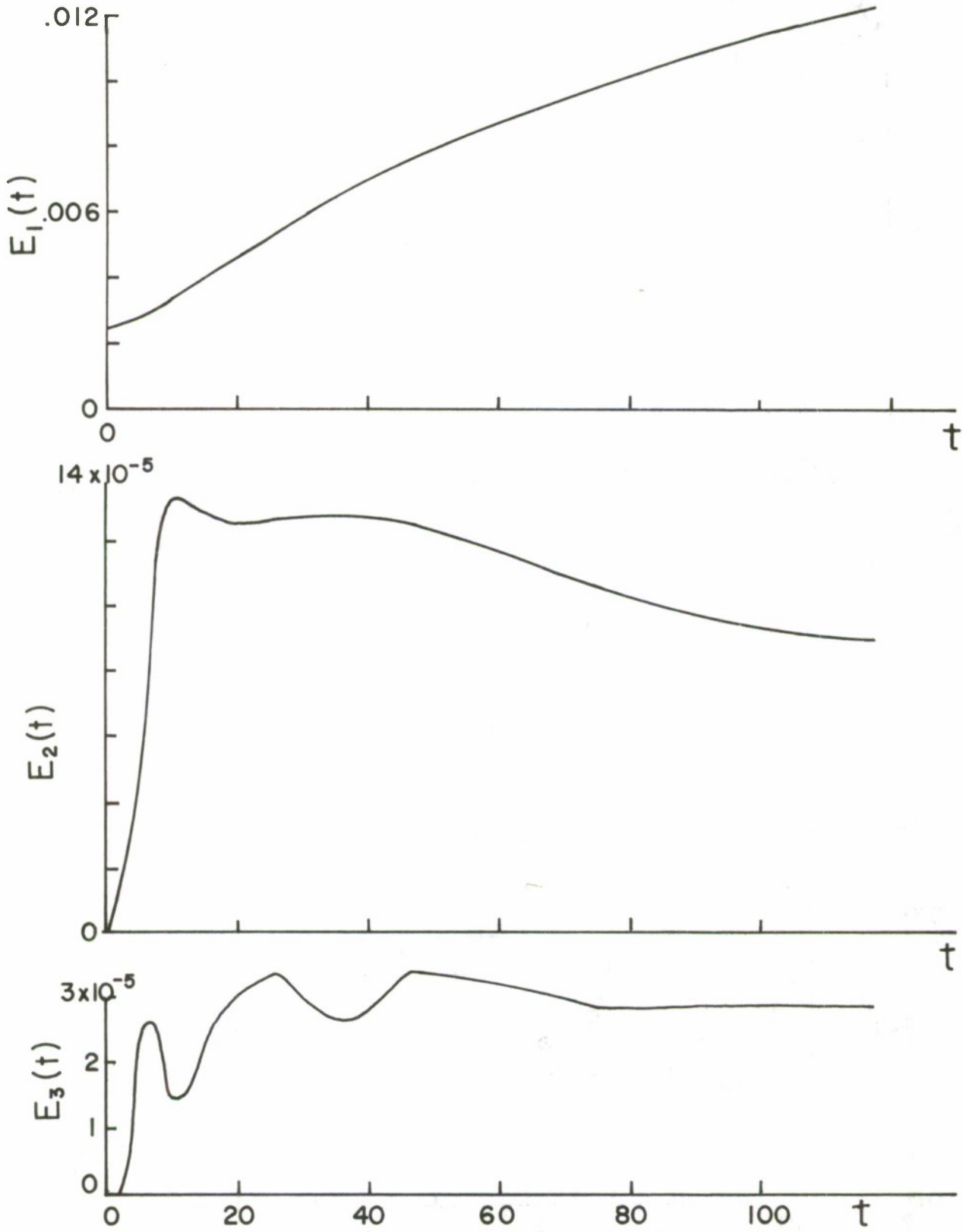


Figure 22. Energy Variation in E_1, E_2, and E_3 for the Run $\varepsilon = .05/\sqrt{2}$

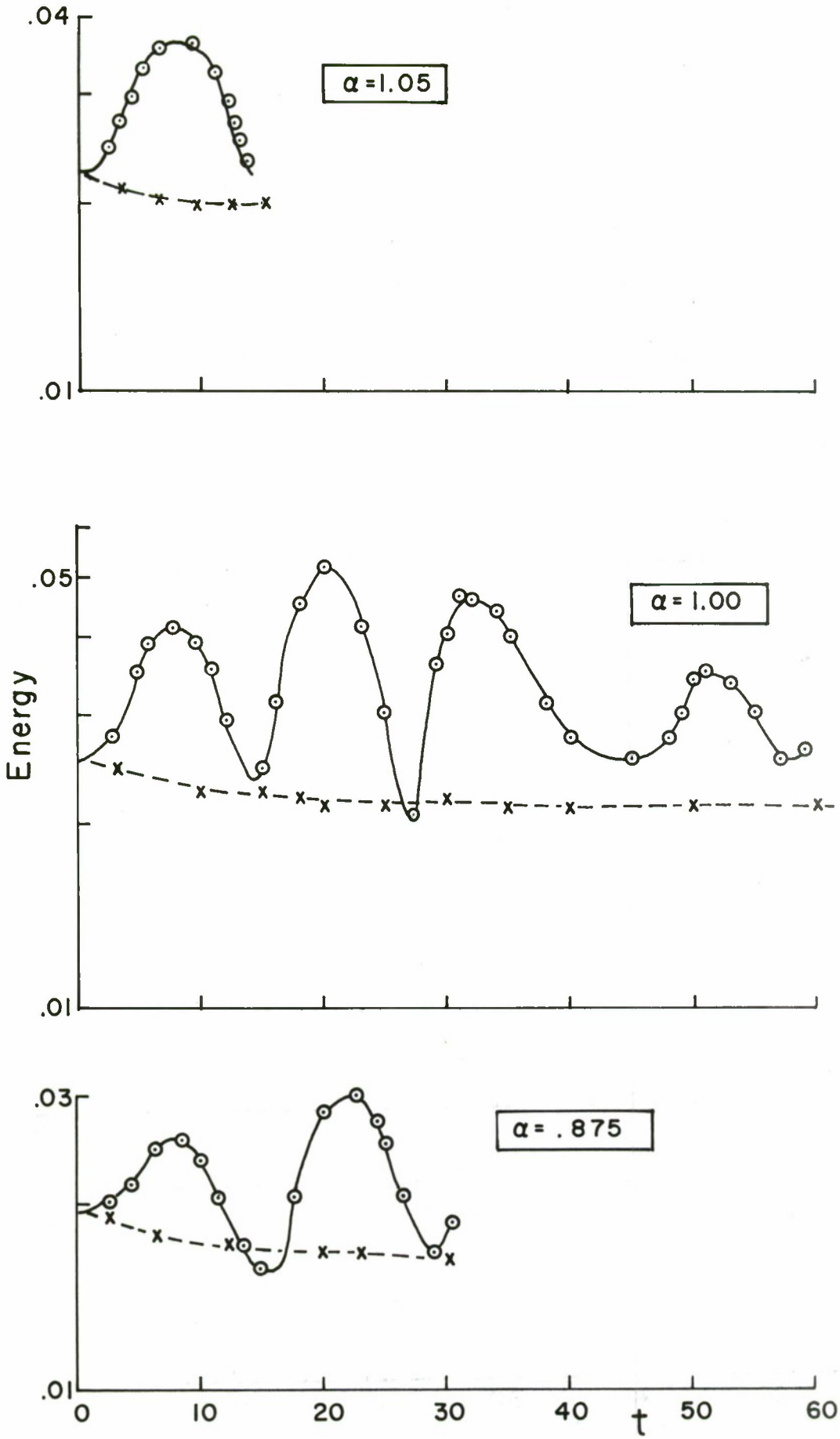


Figure 23. Energy Variation at R_e = 6667 for Three Different Wavenumbers.
-0- Turbulent Kinetic Energy
-x- Total Kinetic Energy

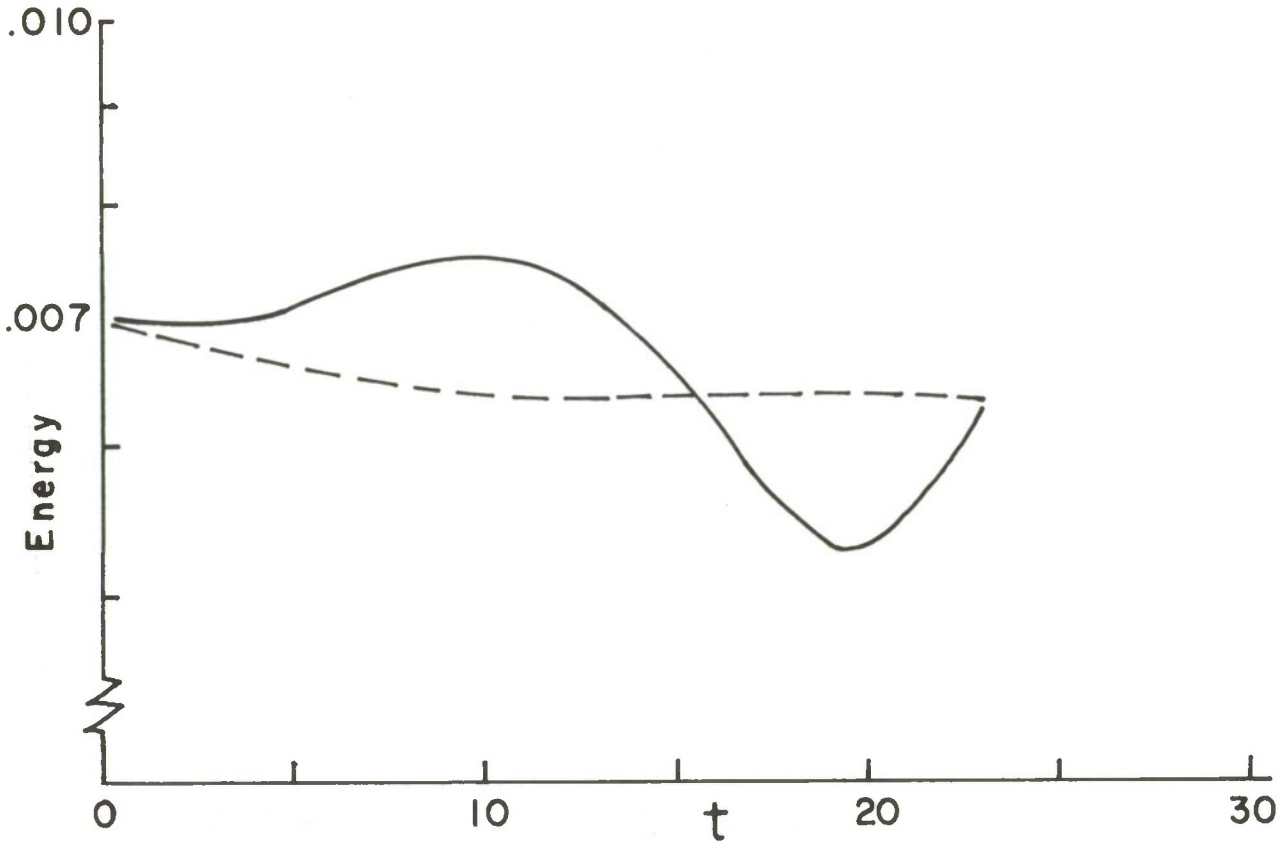


Figure 24. Energy Variation for $\alpha = .78$ $R_e = 6667$, $\epsilon = .0635$. Dashed line is the total kinetic energy while solid line is turbulent kinetic energy.

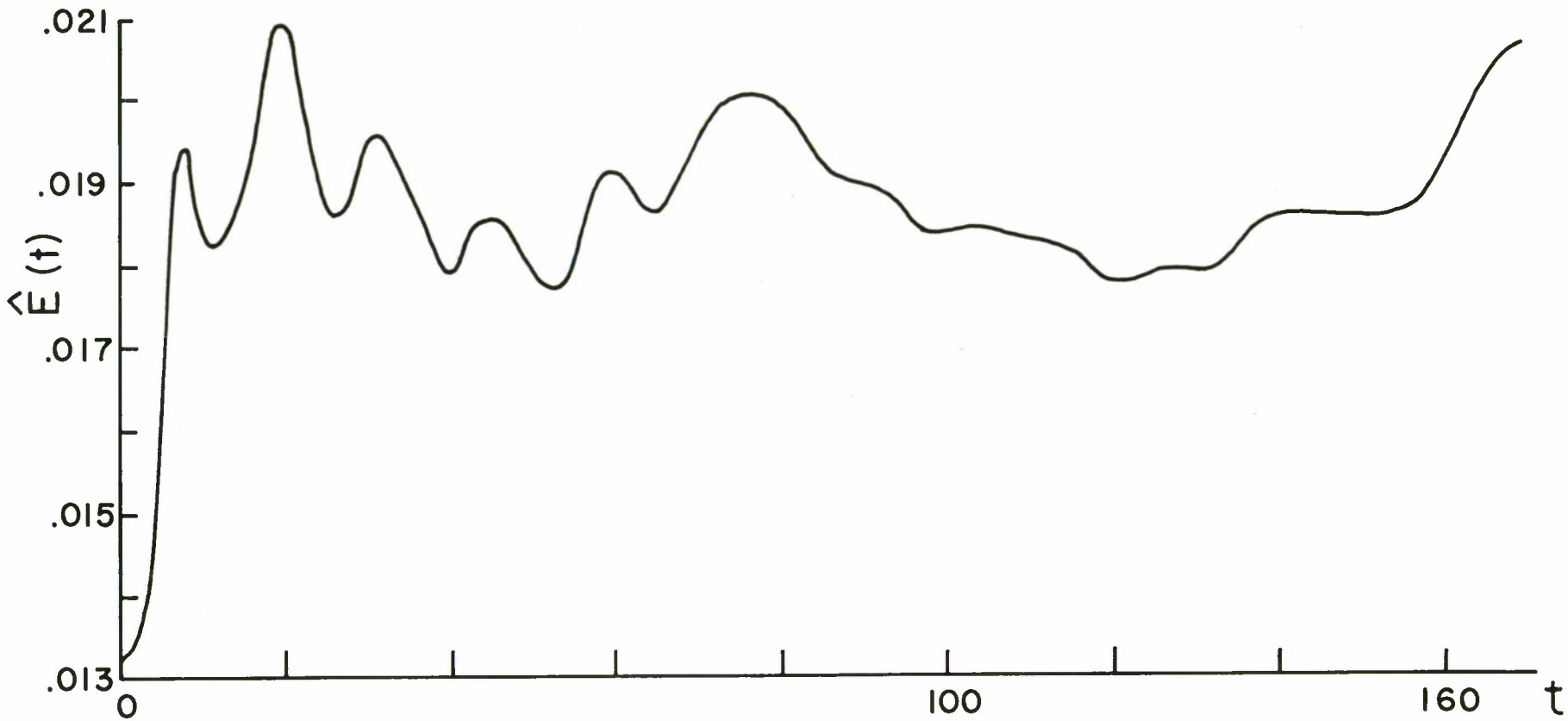


Figure 25. Turbulent Kinetic Energy for the Run with Modes f_7 and f_8 Initially Present $R_e = 6667$, $\epsilon = .059$

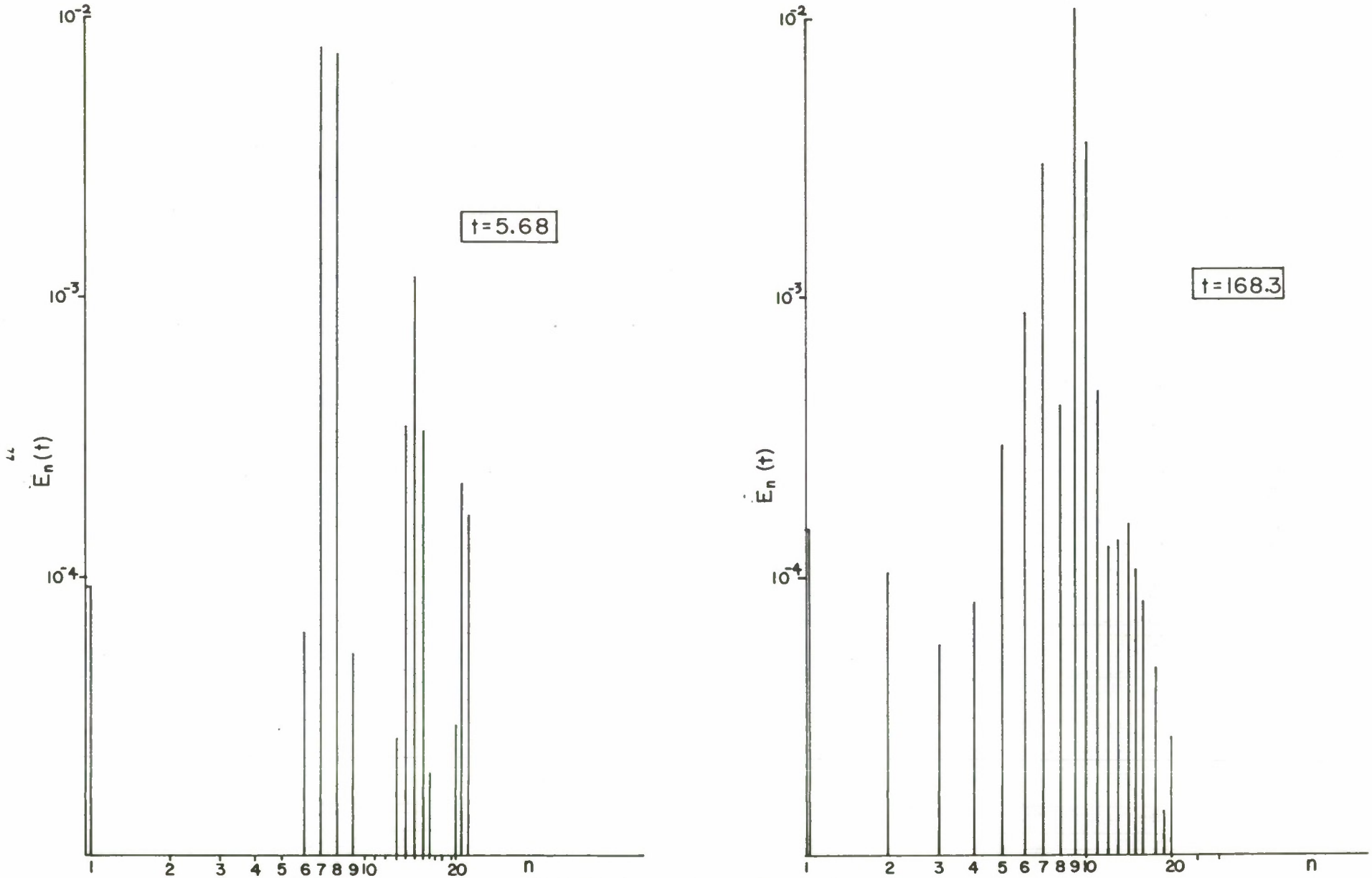


Figure 26. Energy Spectrum for the Run with Modes f_7 and f_8 Initially Present, $R_e = 6667$, $\epsilon = .059$

APPENDIX

LISTING OF THE FORTRAN CODE

```
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      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,         00000060
     1                  U,PHILAM,FF,ETA,GA,X,Y                               00000070
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                             00000080
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1,      00000090
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     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,     00000110
     3                  KINT                                                 00000120
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,      00000130
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,     00000140
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN     00000150
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6      00000160
      COMMON / SYMBOL / DOT,BLANK,XXX                                         00000170
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,      00000180
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,     00000190
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN     00000200
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6      00000210
      COMMON MRLOAD,NCYCLS,LINR
      DIMENSION     PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)        00000220
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),              00000230
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),           00000240
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),          00000250
     3                  ETA(201),GA(201)                                      00000260
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)               00000270
      REAL*4 TITL(24)/'INITIAL VALUES OF PHI'/                                00000280
C                                                                             00000290
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00000300
C                                                                             00000310
C     N      =NUMBER OF POINTS IN Y DIRECTION                                 00000320
C     M      =NUMBER OF POINTS IN X DIRECTION                                 00000330
C     NCYCLS=NUMBER OF TIME STEPS                                             00000340
C     MSCIP =NUMBER OF TIME STEPS BETWEEN CALLS OF S.R. PRINT                 00000350
C     ISCIP =NUMBER OF TIME STEPS BETWEEN CALLS OF S.R. PLOT                  00000360
C     TOL    =CONVERGENCE TOLERANCE FOR SOLUTION OF DELSQUARE PHI=GAMA        00000370
C     OMEGA =RELAXATION FACTOR                                                00000380
C     NITMAX=MAXIMUM NUMBER OF ITERATIONS IN RELAXATION                       00000390
C     REYNLD=REYNOLDS NUMBER                                                  00000400
C     FRACEL=DT*(UMAX*DY+VMAX*DX)/(DX*DY)  --- CONTROLS DT                    00000410
C     XLAMDA=4.*DT/(DX*DY*REYNLD)          --- LIMITS DT                      00000420
C     LINR       IF LINR=1 THEN NON-LINEAR TERMS ARE ZEROED OUT               
C                                                                             00000430
C                                                                             00000440
C     DATA CARDS                                                              00000450
C      1       NPROB,NCYCLS ----- 2I10                                        00000460
C              IF NPROB < 0 PROGRAM EXECUTES RESTART MODE --- NO OTHER        00000470
C              DATA REQUIRED                                                  00000480
C      2       N,M,NITMAX,ISCIP,MSCIP ----- 5I10                              00000490
C      3       TOL,OMEGA,REYNLD,FRACEL,XLAMDA ----- 5F12.0                    00000500
```

```
C                                                                       00000510
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00000520
C                                                                       00000530
C     DIMENSIONS OF ARRAYS                                              00000540
C      PHI,GAMA,FG,GAMOLD --- MXN                                       00000550
C      UMEAN,TEMP,TURBKE,U,PHILAM,T,PHMEAN,GAMEAN,Y --- N               00000560
C      XALPHA,X,IPO,IMO,IAO,IAP --- M                                   00000570
C      S,INV --- M/2                                                    00000580
C      A --- 2M                                                         00000590
C                                                                       00000600
C     THE FOLLOWING ARRAYS CONTAIN FIXED QUANTITIES AND SHOULD NEVER    00000610
C      BE USED FOR TEMPORARY STORAGE.                                   00000620
C                                                                       00000630
C      U,PHILAM,S,XALPHA,X,Y,IPO,IMO,INV,IAP,IAO                        00000640
C                                                                       00000650
C      OTHER ARRAYS MAY BE USED FOR TEMPORARY STORAGE AT CERTAIN PARTS  00000660
C      OF THE PROGRAM.ARRAYS T AND A ARE MOST SUITABLE FOR THIS         00000670
C      PURPOSE.                                                         00000680
C                                                                       00000690
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00000700
C                                                                       00000710
C                                                                       00000720
C                                                                       00000730
C     SUBROUTINE          FUNCTION                                      00000740
C                                                                       00000750
C      SETUP              ESTABLISH VALUES OF VARICUS QUANTITIES WHICH  00000760
C                         REMAIN CONSTANT THROUGHOUT THE INTEGRATION    00000770
C      START              ESTABLISH INITIAL VALUES OF STREAM FUNCTION   00000780
C                         AND VORTICITY                                 00000790
C      TITLE              WRITES OUTPUT TITLE                           00000800
C      STEP               TIME STEP BY MODIFIED EULER IMPLICIT TIME     00000810
C                         DIFFERENCING                                  00000820
C      ADVANC             TIME STEP BY DUFORT FRANKEL CENTRAL DIFFERENCING00000830
C      PRESUR             SOLVES DELSQUARE(PHI)=GAMA FOR PHI BY FINITE  00000840
C                         FOURIER TRANSFORM                             00000850
C      TRISOL             SOLUTION OF A SPECIAL TRIDIAGONAL SYSTEM OF   00000860
C                         LINEAR EQUATIONS BY GAUSS ELIMINATION.        00000870
C                         CALLED BY SUBROUTINE PRESUR                   00000880
C      TIMER              COMPUTES A NEW VALUE OF DT                    00000890
C      MEANS              COMPUTES MEAN VALUES OF DISTURBANCE STREAM    00000900
C                         FUNCTION AND VORTICITY ---PHMEAN AND GAMEAN   00000910
C      ENERGY             COMPUTES THE MEAN KINETIC ENERGY OF THE       00000920
C                         TURBULENCE AS A FUNCTION OF Y, THE INTEGRAL   00000930
C                         OVER Y OF THAT QUANTITY AND THE MEAN KINETIC  00000940
C                         ENERGY OF THE MEAN FLOW                       00000950
C      EDDYS              COMPUTES TURBULENCE STREAM FUNCTION AND       00000960
C                         VORTICITY.                                    00000970
C                         CALL EDDYS(+1) --- PUTS PHI-PHMEAN IN FG      00000980
```

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C                    CALL EDDYS(-1) --- PUTS GAMA-GAMEAN IN FG          00000990
C       STRESS       COMPUTES TURBULENT REYNOLDS STRESS                 00001000
C       PRINT        PRINTS OUTPUT LINE                                 00001010
C       PLOT         PRINTS ONLINE GRAPHS OF MEAN VELOCITY,VORTICITY    00001020
C                    TURBULENT KINETIC ENERGY AND REYNOLDS STRESS       00001030
C       SPCTRM       COMPUTES SPECTRAL DENSITY OF TURBULENT             00001040
C                    KINETIC ENERGY OR SQUARED VORTICITY AS             00001050
C                    FUNCTIONS OF Y ,AND THE MEAN OVER Y.               00001060
C                    CALLED BY PRESUR                                   00001070
C                    CALL SPCTRM(1) --- KINETIC ENERGY                  00001080
C                    CALL SPCTRM(-1) --- SQUARED VORTICITY              00001090
C       MODES        COMPUTES AND PRINTS MODAL VECTORS                  00001100
C                    CALL MODES(MOD1,MOD2,1) PRINTS VECTORS FOR         00001110
C                    FOURIER MODES MOD1 AND MOD2 NORMALIZED             00001120
C                    SO THAT AT Y=0 REAL PART = 1 ,AND IMAGINARY        00001130
C                    PART=0. CALL MODES(MOD1,MOD2,2) PRINTS             00001140
C                    MODULUS AND PHASE                                  00001150
C       PLOTSP       PRINTS ONLINE GRAPHS --- GENERAL                   00001160
C                    CALL PLOTSP(A,B,N) PRINTS GRAPHS OF                00001170
C                    POSITIVE N-DIMENSIONAL VECTORS A AND B             00001180
C       CPLOT        PRINTS CONTOUR PLOT                                00001190
C       OUTPUT       PRINTS TWO DIMENSIONAL ARRAYS.                     00001200
C                    CALL OUTPUT(1) --- PRINTS PHI                      00001210
C                    CALL OUTPUT(2) --- PRINTS GAMA                     00001220
C                    CALL OUTPUT(3) --- PRINTS FG                       00001230
C       LOAD         READS DATA FROM TAPE OR DISC                       00001240
C       RELOAD       WRITES DATA ON TAPE OR DISC                        00001250
C       DHARM        COMPUTES FINITE FOURIER TRANSFORM.                 00001260
C                    DHARM IS INCLUDED IN THE IBM SCIENTIFIC LIBRARY    00001270
C                    OF PRE COMPILED PROGRAMS AVAILABLE AT NPS.         00001280
C       PRMTRS    COMPUTES PHASES OF FIRST FIVE MODES AT SEVERAL
C                 LATERAL STATIONS
C                                                                       00001290
C       * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00001300
C                                                                       00001310
C       LOGICAL INDICES ---------------                                 00001320
C        ICOUNT          NUMBER OF COMPLETED TIME STEPS                 00001330
C        INT             CONTROLS DETAILED OUTPUT---WHEN ICOUNT=INT     00001340
C                        MAIN CALLS PRINT, PRESUR CALLS SPCTRM ETC.     00001350
C        ISCIP           INCREMENT FOR INT                              00001360
C        MCOMP           CONTROLS SINGLE LINE OUTPUT---WHEN ICOUNT=MCOMP 00001370
C                        MAIN CALLS PRINT                               00001380
C        MSCIP           INCREMENT FOR MCOMP                            00001390
C        KSTEP           CONTROLS CALL OF SUBROUTINE STEP. IF ICOUNT=   00001400
C                        KSTEP TIME STEP IS PERFORMED BY STEP           00001410
C        KINT            INCREMENT FOR KSTEP.IF KINT=1 THEN INTEGRATION 00001420
C                        IS BY MOD EULER TIME DIFFERENCING.             00001430
C                                                                       00001440
```

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C       MRLOAD     CONTROLS FREQUENCY AT WHICH DATA IS WRITTEN ON DISK
C                  THIS IS INCLUDED AS A PRECAUTION TO PREVENT LOSING
C                  A LENGTHY COMPUTATION DUE TO MACHINE FAILURE NEAR
C                  END OF COMPUTATION
C       * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00001450
C                                                                             00001460
C                                                                             00001470
        READ(5,11)NPROB,NCYCLS,LINR
        ITAPE=10                                                              00001490
        CALL ERRSET(255,1000,-1,1)                                            00001500
        IF(NPROB .LT. 0)GO TO 2                                               00001510
      1 READ(5,12) N,M,ISCIP,MSCIP                                            00001520
        READ(5,13) REYNLD,FRACEL,XLAMDA                                       00001530
        READ(5,19) DOT,BLANK,XXX                                              00001540
        INT=0                                                                 00001550
        ISTART=0                                                              00001560
        KSTEP=0                                                               00001570
        MCOMP=0                                                               00001580
        TIME=0.                                                               00001590
        NITOT=0                                                               00001600
        MRLOAD=200
        CALL SETUP                                                            00001610
        CALL START                                                            00001620
        WRITE(6,14)NPROB,NCYCLS,N,M,NITMAX,ISCIP,MSCIP                        00001630
        GO TO 3                                                               00001640
      2 CALL LOAD                                                             00001650
        MCOMP=ISTART                                                          00001660
        ISCIP=55
      3 CALL TITLE                                                            00001670
        WRITE(6,15)                                                           00001680
        WRITE(6,16)                                                           00001690
        IFIG=1                                                                00001700
        CALL PRINT                                                            00001710
        CALL PLOT                                                             00001720
C                                                                             00001730
        DO 10 ICOUNT=ISTART,NCYCLS                                            00001740
        IF(ICOUNT .NE. MCOMP) GO TO 5                                         00001750
      4 MCOMP=MCOMP+MSCIP                                                     00001760
        CALL PRINT                                                            00001770
      5 IF(ICOUNT .NE. INT) GO TO 7                                           00001780
      6 INT=INT+ISCIP                                                         00001790
        CALL PLOT                                                             00001800
      7 CALL ADVANC                                                           00001810
        IF(ICOUNT.NE.MRLOAD)GO TO 8
        MRLOAD=MRLOAD+200
        ISTART=ICOUNT+1
        CALL RELOAD
      8 CONTINUE
```

```
   10 CONTINUE                                                                  00001820
      ISTART=NCYCLS+1                                                           00001830
      CALL PRINT                                                                00001840
      WRITE(6,17)                                                               00001850
      WRITE(6,18)                                                               00001860
   11 FORMAT(3I10)
   12 FORMAT(5I10)                                                              00001880
   13 FORMAT(5G12.0)                                                            00001890
   14 FORMAT(7I10)                                                              00001900
   15 FORMAT(10X,'INITIAL VALUES OF STREAM FUNCTION'//)                         00001910
   16 FORMAT(10X,'INITIAL VALUES OF VORTICITY'//)                               00001920
   17 FORMAT(10X,'STREAM FUNCTION'//)                                           00001930
   18 FORMAT(10X,'VORTICITY'//)                                                 00001940
   19 FORMAT(3A1)                                                               00001950
      CALL RELOAD                                                               00001960
      STOP                                                                      00001970
      END                                                                       00001980


      SUBROUTINE SETUP                                                          00001990
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                                      00002000
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,           00002010
     1                  U,PHILAM,FF,ETA,GA,X,Y                                  00002020
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                               00002030
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1,         00002040
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,         00002050
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,       00002060
     3                  KINT                                                   00002070
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,         00002080
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,       00002090
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN       00002100
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6        00002110
      COMMON / SYMBOL / DOT,BLANK,XXX                                           00002120
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,         00002130
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,       00002140
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN       00002150
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6        00002160
      DIMENSION    PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)           00002170
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),                00002180
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),             00002190
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),            00002200
     3                  ETA(201),GA(201)                                        00002210
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)                 00002220
      DOUBLE PRECISION  THETA,XNN                                               00002230
C                                                                               00002240
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00002250
C                                                                               00002260
C     MISCELANEOUS PARAMETERS                                                   00002270
```

```
C                                                                       00002280
      PI=3.1415926535898                                                00002290
      H=3.1415926535898                                                 00002300
C                                                                       00002310
C                                                                       00002330
      ALPHA=1.D0
      H=H/ALPHA
      KINT=50                                                           00002350
      NITMAX=15                                                         00002360
      NP2=N+2                                                           00002370
      NP1=N+1                                                           00002380
      NM1=N-1                                                           00002390
      NM2=N-2                                                           00002400
      NM3=N-3                                                           00002410
      NM4=N-4                                                           00002420
      NH=N/2+1                                                          00002430
      MM(1)=M                                                           00002440
      MM(2)=0                                                           00002450
      MM(3)=0                                                           00002460
      M=2**M                                                            00002470
      MH=M/2                                                            00002480
      MHP=MH+1                                                          00002490
      MP1=M+1                                                           00002500
      MP2=M+2                                                           00002510
      MP3=M+3                                                           00002520
      M2M=M+M-1                                                         00002530
      MM1=M-1                                                           00002540
      XN=DFLOAT(N)                                                      00002550
      XNM1=DFLOAT(NM1)                                                  00002560
      XM=DFLOAT(M)                                                      00002570
      DX=2.D0*H/XM                                                      00002580
      DX2=2.D0*DX                                                       00002590
      DXSQR=DX*DX                                                       00002600
      DXSQR4=4.D0*DXSQR                                                 00002610
      DXSQR8=8.D0*DXSQR                                                 00002620
      DY=2.D0/XNM1                                                      00002630
      DY2=2.D0*DY                                                       00002640
      DYSQR=DY*DY                                                       00002650
      R=DX/DY                                                           00002660
      RSQR=R*R                                                          00002670
      RSQRP=RSQR+1.D0                                                   00002680
      REY=1.D0/REYNLD                                                   00002690
      TOL=.00002DC                                                      00002700
      ATERM=1.D0/(2.D0*DYSQR)                                           00002710
      BTERM=XM*DX2*DY2                                                  00002720
      FA=1.D0/(XM*XM)                                                   00002730
      FB=1.DC/(DY2**2)                                                  00002740
      FC=1.D0/(8.D0*(XM*DY)**2)                                         00002750
```

```
      THETA=2.D0*PI/XM                                                  00002760
      WRITE(6,100)R,RSQR,RSQRP                                          00002770
  100 FORMAT(8X,F12.8,3X,F12.8,3X,F12.8)                                00002780
C                                                                       00002790
CCC                                                                     00002800
CC    SET UP PERIODIC COUNTERS                                          00002810
CC    SET UP INDEXING COUNTERS, IAO &IAP.                               00002820
C                                                                       00002830
      JC=0                                                              00002840
      DO 10 I=1,M                                                       00002850
      IPO(I)=I+1                                                        00002860
      IMO(I)=I-1                                                        00002870
      IAO(I)=I+JC                                                       00002880
      IAP(I)=IAO(I)+1                                                   00002890
      JC=JC+1                                                           00002900
   10 CONTINUE                                                          00002910
      IPO(M)=1                                                          00002920
      IMO(1)=M                                                          00002930
C                                                                       00002940
CCC   SET UP COORDINATES                                                00002950
C                                                                       00002960
      DO 20 J=1,N                                                       00002970
      Y(J)=DFLOAT(J-1)*DY-1.D0                                          00002980
   20 CONTINUE                                                          00002990
      DO 30 I=1,M                                                       00003000
      X(I)=DFLOAT(I-1)*DX-H+DX/2.D0                                     00003010
   30 CONTINUE                                                          00003020
C                                                                       00003030
CCC   SET UP MODAL CONSTANTS. --- ALPHA & FF .                          00003040
C                                                                       00003050
      ALPHA=DYSQR/DXSQR                                                 00003060
      XALPHA(1)=2.D0                                                    00003070
      FF(1)=0.D0                                                        00003080
      DO 35 I=2,MH                                                      00003090
      IM=I-1                                                            00003100
      IA=IAP(IM)                                                        00003110
      IP=IA+1                                                           00003120
      XNN=DFLOAT(IM)                                                    00003130
      XALPHA(IA)=2.D0*(1.D0-ALPHA*(DCOS(THETA*XNN)-1.D0))               00003140
      XALPHA(IP)=XALPHA(IA)                                             00003150
      FF(I)=1.D0/DXSQR*(DSIN((2.D0*PI*XNN)/XM))**2                      00003160
   35 CONTINUE                                                          00003170
      XNN=DFLOAT(MH)                                                    00003180
      XALPHA(M)=2.D0*(1.D0-ALPHA*(DCOS(THETA*XNN)-1.D0))                00003190
C                                                                       00003200
CC    SET UP S AND INV TABLES FOR DHARM.                                00003210
C                                                                       00003220
      CALL DHARM(A,MM,INV,S,0,IFERR)                                    00003230
```

```
C                                                                       00003240
C     LAMINAR FLOW STREAM FUNCTION AND VELOCITY                         00003250
C                                                                       00003260
      DO 40 J=2,NM1                                                     00003270
      PHILAM(J)=.5*Y(J)*(Y(J)**2-3.)                                    00003280
      U(J)=-1.5*(Y(J)**2-1.)                                            00003290
   40 CONTINUE                                                          00003300
      PHILAM(1)=1.                                                      00003310
      PHILAM(N)=-1.                                                     00003320
      U(1)=0.                                                           00003330
      U(N)=0.                                                           00003340
C                                                                       00003350
      RETURN                                                            00003360
      END                                                               00003370

      SUBROUTINE START                                                  00003380
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                              00003390
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,   00003400
     1                  U,PHILAM,FF,ETA,GA,X,Y                          00003410
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                       00003420
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1, 00003430
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,  00003440
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,00003450
     3                  KINT                                            00003460
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR, 00003470
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,00003480
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN00003490
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6 00003500
      COMMON / SYMBOL / DOT,BLANK,XXX                                   00003510
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR, 00003520
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,00003530
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN00003540
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6 00003550
      DIMENSION    PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)   00003560
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),        00003570
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),     00003580
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),    00003590
     3                  ETA(201),GA(201)                                00003600
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)         00003610
      DOUBLE PRECISION  PI,THETA,C,B,XL                                 00003620
      DOUBLE PRECISION EPS,THETA2
C                                                                       00003640
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00003650
C                                                                       00003660
C     BOUNDARY VALUES OF STREAM FUNCTION                                00003670
C                                                                       00003680
      PI=3.1415926535898                                                00003690
```

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      DO 10 I=1,M                                                       00003700
      PHI(I,1)=0.                                                       00003710
      PHI(I,N)=0.                                                       00003720
   10 CONTINUE                                                          00003730
C                                                                       00003740
CC    ESTABLISH INITIAL VALUES OF STREAM FUNCTION                       00003750
CC    USE ARRAYS UMEAN AND TEMP FOR TEMPORARY STORAGE                   00003760
C                                                                       00003770
C       A1,B,D,D,E,F ARE DUMMY VARIALBES
      DO 60 J=1,NH                                                      00003780
      READ(5,11) PHMEAN(J),GAMEAN(J),A1,B,C,D,E,F
   11 FORMAT(8F10.0)
   60 CONTINUE                                                          00003810
      NHM=NH-1                                                          00003820
      DO 65 J=1,NHM                                                     00003830
      JC=N-J+1                                                          00003840
      PHMEAN(JC)=PHMEAN(J)                                              00003850
      GAMEAN(JC)=GAMEAN(J)                                              00003860
   65 CONTINUE                                                          00003870
   12 FORMAT(10X,3(F12.6,3X))                                           00003880
      EPS=.1D0/DSQRT(2.D0)
      EPS=EPS*1.8D0
      EPS=1.D0
      EPS=.2D0
      EPS=.01D0
      DO 71 J=1,N                                                       00003900
      C=PHMEAN(J)                                                       00003910
      B=GAMEAN(J)                                                       00003920
      DO 70 I=1,M                                                       00003930
      THETA=X(I)
      PHI(I,J)=EPS*(C*DCOS(THETA)+B*DSIN(THETA))                        00003950
   70 CONTINUE                                                          00003960
      WRITE(6,12) Y(J),C,B                                              00003970
   71 CONTINUE                                                          00003980
C                                                                       00003990
CC    COMPUTE INITIAL VALUES OF VORTICITY                               00004000
C                                                                       00004010
      DO 15 I=1,M                                                       00004020
      IP=IPO(I)                                                         00004030
      IM=IMO(I)                                                         00004040
      GAMA(I,N)=(8.D0*PHI(I,NM1)-PHI(I,NM2))*ATERM                      00004050
      GAMA(I,1)=(8.D0*PHI(I,2)-PHI(I,3))*ATERM                          00004060
      GAMA(I,N)=(1.5*PHI(I,NM2)-4.*PHI(I,NM3)/9.)/DYSQR
      GAMA(I,1)=(1.5*PHI(I,3)-4.*PHI(I,4)/9.)/DYSQR
      GAMOLD(I,1)=GAMA(I,1)                                             00004070
      GAMOLD(I,N)=GAMA(I,N)                                             00004080
      DO 15 J=2,NM1                                                     00004090
      GAMA(I,J)=(RSQR*(PHI(I,J+1)+PHI(I,J-1))+PHI(IP,J)+PHI(IM,J)       00004100
```

```
     1          -2.D0*RSQRP*PHI(I,J))/DXSQR                                    00004110
      GAMOLD(I,J)=GAMA(I,J)                                                    00004120
   15 CONTINUE                                                                 00004130
      CALL MEANS                                                               00004140
      CALL TIMER                                                               00004150
      WRITE(6,99)
      WRITE(6,100)(I,I=1,10)
      DO 131 J=1,15
      WRITE(6,101)J,Y(J),(GAMA(I,J),I=1,10)
  131 CONTINUE
   99 FORMAT(1HO,3X,'INITIAL DISTURBANCE GAMA NEAR WALL')
  100 FORMAT(1H ,5X,'I=',11X,10(I3,8X))
  101 FORMAT(1X,'J=',I3,1X,'Y=',F6.3,2X,10(E10.3,1X))
      RETURN                                                                   00004160
      END                                                                      00004170


      SUBROUTINE MEANS                                                         00012450
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                                     00012460
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,          00012470
     1                  U,PHILAM,FF,ETA,GA,X,Y                                 00012480
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                              00012490
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1,       00012500
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,       00012510
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,     00012520
     3                  KINT                                                 00012530
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,       00012540
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,     00012550
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN     00012560
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6      00012570
      COMMON / SYMBOL / DOT,BLANK,XXX                                          00012580
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,       00012590
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,     00012600
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN     00012610
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6      00012620
      DIMENSION    PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)         00012630
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),             00012640
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),          00012650
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),         00012660
     3                  ETA(201),GA(201)                                     00012670
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)              00012680
C                                                                            00012690
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00012700
C                                                                            00012710
C     THIS ROUTINE COMPUTES THE MEAN VALUES OF PHI AND GAMA                  00012720
C     ARRAY A IS USED FOR TEMPORARY STORAGE.                                 00012730
C                                                                            00012740
      DO 10 J=1,N                                                            00012750
```

```
      DO 20 I=1,M                                                       00012760
      A(I)=PHI(I,J)                                                     00012770
   20 CONTINUE                                                          00012780
      CALL ASUM(A,M,SUM)                                                00012790
      PHMEAN(J)=SUM/XM                                                  00012800
      DO 30 I=1,M                                                       00012810
      A(I)=GAMA(I,J)                                                    00012820
   30 CONTINUE                                                          00012830
      CALL ASUM(A,M,SUM)                                                00012840
      GAMEAN(J)=SUM/XM+3.D0*Y(J)                                        00012850
   10 CONTINUE                                                          00012860
      RETURN                                                            00012870
      END                                                               00012880


      SUBROUTINE TITLE                                                  00004180
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                              00004190
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,   00004200
     1                  U,PHILAM,FF,ETA,GA,X,Y                          00004210
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                       00004220
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1, 00004230
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,  00004240
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,00004250
     3                  KINT                                            00004260
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR, 00004270
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,00004280
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN00004290
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6 00004300
      COMMON / SYMBOL / DOT,BLANK,XXX                                   00004310
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR, 00004320
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,00004330
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN00004340
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6 00004350
      DIMENSION     PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)  00004360
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),        00004370
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),     00004380
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),    00004390
     3                  ETA(201),GA(201)                                00004400
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)         00004410
C                                                                       00004420
CC    * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00004430
C                                                                       00004440
      WRITE(6,11)                                                       00004450
      WRITE(6,22)                                                       00004460
      WRITE(6,12)NPROB                                                  00004470
      IF(NPROB .LT. 0) WRITE(6,23)                                      00004480
      WRITE(6,13)M,N                                                    00004490
      WRITE(6,14)REYNLD                                                 00004500
```

```
      WRITE(6,15)DX                                                          00004510
      WRITE(6,16)DY                                                          00004520
      WRITE(6,18) TOL                                                        00004530
      WRITE(6,19)XLAMDA                                                      00004540
      WRITE(6,21)FRACEL                                                      00004550
      HH=PI/H                                                                00004560
      WRITE(6,24) H,HH                                                       00004570
   24 FORMAT(1H ,10X,'H=',14X,F7.5,5X,'ALPHA=',3X,F7.5)                      00004580
   11 FORMAT(1H ,10X,'TURBCODE MARK VII   G. D. O"BRIEN'//)                  00004590
   12 FORMAT(1HO,10X,'PROBLEM NUMBER',3X,I4)                                00004600
   13 FORMAT(1HO,10X,'MESH SIZE     ',I3,' BY ',I3)                         00004610
   14 FORMAT(1H ,10X,'REYNOLDS NUMBER=',F9.1)                               00004620
   15 FORMAT(1H ,10X,'DX=',13X,F7.6)                                        00004630
   16 FORMAT(1H ,10X,'DY=',13X,F7.6)                                        00004640
   18 FORMAT(1H ,10X,'TOL=',12X,F7.6)                                       00004650
   19 FORMAT(1H ,10X,'LAMDA=',10X,F7.6)                                     00004660
   21 FORMAT(1H ,10X,'FRACEL=',9X,F7.5)                                     00004670
   22 FORMAT(1H ,10X,'DUFORT FRANKEL CENTRAL TIME DIFFERENCING')            00004680
   23 FORMAT(1H+,32X,'CONTINUED')                                           00004690
      RETURN                                                                00004700
      END                                                                   00004710


      SUBROUTINE PRINT                                                      00012020
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                                  00012030
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,       00012040
     1                  U,PHILAM,FF,ETA,GA,X,Y                              00012050
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                           00012060
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1,     00012070
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,      00012080
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,    00012090
     3                  KINT                                                00012100
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,     00012110
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,    00012120
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN    00012130
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6     00012140
      COMMON / SYMBOL / DOT,BLANK,XXX                                       00012150
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,     00012160
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,    00012170
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN    00012180
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6     00012190
      DIMENSION    PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)       00012200
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),            00012210
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),         00012220
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),        00012230
     3                  ETA(201),GA(201)                                    00012240
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)             00012250
C                                                                           00012260
```

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C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00012270
C                                                                               00012280
C     THIS ROUTINE PRINTS THE VALUES OF ICOUNT,T,DT,EKMEAN,EKTURB,UVBAR 00012290
C     NITER AND NITOT                                                           00012300
      CALL ENERGY                                                               00012310
C                                                                               00012320
      IF(IFIG .GT. 0)WRITE(6,11)                                                00012330
      EKTOT=EKMEAN+EKTURB                                                       00012340
      WRITE(6,12) ICOUNT,TIME,DT,EKMEAN,EKTURB,UFLO,EKTOT,UVBAR,UMNFLO, 00012350
     1            NITOT,ITEL                                                    00012360
   11 FORMAT(3X,'CYCLE',6X,'TIME',9X,'DT',7X,'MEAN KE',5X,'TURB KE',7X, 00012370
     1       'LOG TURB KE ',4X,'TOTAL KE',6X,'UVBAR',7X,'UMNFLO',3X,     00012380
     2       'NITOT',2X,'ITEL')                                                 00012390
   12 FORMAT(3X,I5,3X,F10.6,2(3X,F8.6),2(3X,E13.7),3X,F9.7,3X,E10.4,3X, 00012400
     1       F8.6,2X,I5,2X,I4)                                                  00012410
      IFIG=-1                                                                   00012420
      RETURN                                                                    00012430
      END                                                                       00012440

      SUBROUTINE ENERGY                                                         00013390
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                                      00013400
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,   00013410
     1                  U,PHILAM,FF,ETA,GA,X,Y                                  00013420
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                               00013430
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1, 00013440
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,  00013450
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,00013460
     3                  KINT                                                    00013470
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR, 00013480
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,00013490
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN00013500
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6 00013510
      COMMON / SYMBOL / DOT,BLANK,XXX                                           00013520
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR, 00013530
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,00013540
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN00013550
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6 00013560
      DIMENSION    PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)      00013570
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),        00013580
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),     00013590
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),    00013600
     3                  ETA(201),GA(201)                                        00013610
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)           00013620
      DOUBLE PRECISION  XM2                                                     00013630
C                                                                               00013640
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00013650
C                                                                               00013660
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C       THIS ROUTINE COMPUTES THE AVERAGE KINETIC ENERGIES OF BOTH THE     00013670
C       MEAN AND THE TURBULENT COMPONENTS OF THE FLOW                      00013680
C                                                                          00013690
C                                                                          00013700
        CALL EDDYS(10)                                                     00013710
        CALL STRESS                                                        00013720
C       MEAN FLOW. T IS USED FOR TEMPORARY STORAGE                         00013730
C                                                                          00013740
        DO 10 J=2,NM1                                                      00013750
        T(J)=UMEAN(J)**2                                                   00013760
   10 CONTINUE                                                             00013770
        T(1)=0.                                                            00013780
        CALL ASUM(T,NM1,SUM)                                               00013790
        EKMEAN=SUM/(2.D0*XNM1)                                             00013800
C                                                                          00013810
C       TURBULENT FLOW. TURBULENT COMPONENTS OF PHI ARE STORED IN FG BY    00013820
C       EDDYS. ARRAY A IS USED FOR TEMPORARY STORAGE.                      00013830
C                                                                          00013840
        XM2=2.D0*XM                                                        00013850
        TURBKE(N)=0.                                                       00013860
        TURBKE(1)=0.                                                       00013870
        DO 30 J=2,NM1                                                      00013880
        DO 20 I=1,M                                                        00013890
        IP=IPO(I)                                                          00013900
        IM=IMO(I)                                                          00013910
        A(I)=((FG(I,J+1)-FG(I,J-1))/DY2)**2+((FG(IP,J)-FG(IM,J))/DX2)**2   00013920
   20 CONTINUE                                                             00013930
        CALL ASUM(A,M,SUM)                                                 00013940
        TURBKE(J)=SUM/XM2                                                  00013950
   30 CONTINUE                                                             00013960
        CALL ASUM(TURBKE,NM1,SUM)                                          00013970
        EKTURB=SUM/XNM1                                                    00013980
        RETURN                                                             00013990
        END                                                                00014000


        SUBROUTINE EDDYS(INDEX)                                            00012890
        COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                               00012900
        COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,    00012910
       1                  U,PHILAM,FF,ETA,GA,X,Y                          00012920
        COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                        00012930
        COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1,  00012940
       1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,  00012950
       2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,00012960
       3                  KINT                                            00012970
        COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,  00012980
       1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,00012990
       2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN00013000
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     3                  ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6 00013010
      COMMON / SYMBOL / DOT,BLANK,XXX                                      00013020
      DCUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,    00013030
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,  00013040
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN  00013050
     3                  ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6  00013060
      DIMENSION    PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)      00013070
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),           00013080
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),       00013090
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),      00013100
     3                  ETA(201),GA(201)                                  00013110
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)           00013120
C                                                                          00013130
CC    * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00013140
CC                                                                         00013150
CC    ASSUMES SUBROUTINE MEANS HAS GENERATED CURRENT VALUES OF PHMEAN      00013160
CC    AND GAMEAN                                                          00013170
CC                                                                         00013180
CC                                                                         00013190
CC    IF INDEX > 0 ROUTINE PUTS TURBULENT COMPONENTS OF STREAM             00013200
CC    FUNCTION IN ARRAY FG                                                00013210
CC                                                                         00013220
CC    IF INDEX < 0 ROUTINE PUTS TURBULENT COMPONENTS OF VORTICITY          00013230
CC    IN ARRAY FG                                                         00013240
CC                                                                         00013250
C                                                                          00013260
      IF(INDEX .LT. 0) GO TO 15                                            00013270
    1 DO 10 J=1,N                                                          00013280
      DO 10 I=1,M                                                          00013290
      FG(I,J)=PHI(I,J)-PHMEAN(J)                                           00013300
   10 CONTINUE                                                             00013310
      GO TO 25                                                             00013320
   15 DO 20 J=1,N                                                          00013330
      DO 20 I=1,M                                                          00013340
      FG(I,J)=GAMA(I,J)-GAMEAN(J)                                          00013350
   20 CONTINUE                                                             00013360
   25 RETURN                                                               00013370
      END                                                                  00013380


      SUBROUTINE STRESS                                                    00014010
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                                 00014020
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,      00014030
     1                  U,PHILAM,FF,ETA,GA,X,Y                             00014040
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                          00014050
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1,    00014060
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,    00014070
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,  00014080
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     3                 KINT                                                          00014090
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,            00014100
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,           00014110
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN           00014120
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6            00014130
      COMMON / SYMBOL / DOT,BLANK,XXX                                               00014140
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,            00014150
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,           00014160
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN           00014170
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6            00014180
      DIMENSION     PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)            00014190
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),                  00014200
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),               00014210
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),              00014220
     3                  ETA(201),GA(201)                                          00014230
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)                   00014240
C                                                                                   00014250
CC    * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00014260
CC                                                                                  00014270
CC    ASSUMES THAT FLUCTUATING COMP. OF PHI IS STORED IN FG                         00014280
CC    COMPUTES REYNOLDS STRESSES AND MEAN VELOCITIES.                               00014290
C                                                                                   00014300
      TEMP(1)=0.                                                                    00014310
      TEMP(N)=0.                                                                    00014320
      DO 20 J=2,NM1                                                                 00014330
      DO 10 I=1,M                                                                   00014340
      IP=IPO(I)                                                                     00014350
      IM=IMO(I)                                                                     00014360
      A(I)=(1.DO*FG(I,J-1)-FG(I,J+1))*(1.DO*FG(IP,J)-FG(IM,J))                      00014370
   10 CONTINUE                                                                      00014380
      CALL ASUM(A,M,SUM)                                                            00014390
      TEMP(J)=SUM/BTERM                                                             00014400
   20 CONTINUE                                                                      00014410
      CALL ASUM(TEMP,NM1,SUM)                                                       00014420
      UVBAR=SUM/XNM1                                                                00014430
C                                                                                   00014440
CC    MEAN VELOCITY                                                                 00014450
C                                                                                   00014460
      DO 30 J=2,NM1                                                                 00014470
      UMEAN(J)=(PHMEAN(J-1)-PHMEAN(J+1))/DY2+U(J)                                   00014480
   30 CONTINUE                                                                      00014490
      UMEAN(1)=0.                                                                   00014500
      UMEAN(N)=0.                                                                   00014510
      CALL ASUM(UMEAN,NM1,SUM)                                                      00014520
      UMNFLO=SUM/XNM1                                                               00014530
      UFLO=(2.DO+PHMEAN(2)-PHMEAN(NM1)+PHILAM(2)-PHILAM(NM1))/4.DO                  00014540
      RETURN                                                                        00014560
      END                                                                           00014570
```

```
      SUBROUTINE PLOT                                                   CCC14720
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                              00014730
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,   00014740
     1                  U,PHILAM,FF,ETA,GA,X,Y                         00014750
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                       00014760
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1, 00014770
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART, 00014780
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,00014790
     3                  KINT                                           00014800
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR, 00014810
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,00014820
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN00014830
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6 00014840
      COMMON / SYMBOL / DOT,BLANK,XXX                                   00014850
      COMMON / PICTUR / ALINE(49),BLINE(49),APLOT(201),BPLOT(201)       00014860
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR, 00014870
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,00014880
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN00014890
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6 00014900
      DIMENSION     PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)  00014910
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),        00014920
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),     00014930
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),    00014940
     3                  ETA(201),GA(201)                                00014950
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)         00014960
C                                                                       00014970
CC    THIS ROUTINE PRINTS ONLINE PLOTS OF MEAN VELOCITY,VORTICITY,      00014980
CC    TURBULENT KINETIC ENERGY AND REYNOLDS STRESS AS FUNCTIONS OF Y.   00014990
CC                                                                      00015000
CC    * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00015010
CC                                                                      00015020
C                                                                       00015030
      IFIG=1                                                            00015040
      III=-1                                                            00015050
C                                                                       00015060
CC    PUT UMEAN IN APLOT , TEMP IN BPLOT                                00015070
C                                                                       00015080
      DO 10 J=1,N                                                       00015090
      APLOT(J)=UMEAN(J)                                                 00015100
      BPLOT(J)=TEMP(J)                                                  00015110
   10 CONTINUE                                                          00015120
C                                                                       00015130
CC    TITLE                                                             00015140
C                                                                       00015150
      WRITE(6,11)                                                       00015160
   11 FORMAT(1H0,4X,'UMEAN',25X,'UMEAN',34X,'UVBAR',28X,'UVBAR')        00015170
```

```
C                                                                       00015180
C     FIND LARGEST ELEMENTS IN APLOT AND BPLOT                          00015190
C                                                                       00015200
   15 ABIG=0.                                                           00015210
      BBIG=0.                                                           00015220
      DO 20 J=1,N                                                       00015230
      ABIG=AMAX1(ABIG,ABS(APLOT(J)))                                    00015240
      BBIG=AMAX1(BBIG,ABS(BPLOT(J)))                                    00015250
   20 CONTINUE                                                          00015260
C                                                                       00015270
C     SET UP TOP HORIZONTAL AXIS                                        00015280
C                                                                       00015290
      DO 30 I=1,49                                                      00015300
      ALINE(I)=DOT                                                      00015310
      BLINE(I)=DOT                                                      00015320
   30 CONTINUE                                                          00015330
      WRITE(6,12)ALINE,BLINE                                            00015340
   12 FORMAT(14X,49A1,20X,49A1)                                         00015350
C                                                                       00015360
C     BLANK ALINE AND BLINE AND PUT PLOTTING SYMBOL IN COMPUTED ELEMENT 00015370
C                                                                       00015380
      DO 40 I=1,49                                                      00015390
      ALINE(I)=BLANK                                                    00015400
      BLINE(I)=BLANK                                                    00015410
   40 CONTINUE                                                          00015420
      ALINE(1)=DOT                                                      00015430
      BLINE(25)=DOT                                                     00015440
      J=N                                                               00015450
      DO 50 JC=1,N,2                                                    00015460
      JA=APLOT(J)/ABIG*48.+1.                                           00015470
      JB=BPLOT(J)/BBIG*24.+25.                                          00015480
      ALINE(JA)=XXX                                                     00015490
      BLINE(JB)=XXX                                                     00015500
      WRITE(6,13)APLOT(J),ALINE,J,BPLOT(J),BLINE                        00015510
   13 FORMAT(1X,E10.4,2X,49A1,2X,I3,3X,E10.4,2X,49A1)                   00015520
      ALINE(JA)=BLANK                                                   00015530
      BLINE(JB)=BLANK                                                   00015540
      ALINE(1)=DOT                                                      00015550
      BLINE(25)=DOT                                                     00015560
      J=J-2                                                             00015570
   50 CONTINUE                                                          00015580
C                                                                       00015590
C     SET UP BOTTOM HORIZONTAL AXIS                                     00015600
C                                                                       00015610
      DO 60 I=1,49                                                      00015620
      ALINE(I)=DOT                                                      00015630
      BLINE(I)=DOT                                                      00015640
   60 CONTINUE                                                          00015650
```

```
      WRITE(6,14)ALINE,BLINE                                                    00015660
   14 FORMAT(14X,49A1,20X,49A1//)                                               00015670
      IF(III .GT. 0)GO TO 100                                                   00015680
C                                                                               00015690
CC    PUT TURBKE IN APLOT , GAMEAN IN BPLOT                                     00015700
C                                                                               00015710
   65 DO 70 J=1,N                                                               00015720
      APLOT(J)=TURBKE(J)                                                        00015730
      BPLOT(J)=GAMEAN(J)                                                        00015740
   70 CONTINUE                                                                  00015750
      III=1                                                                     00015760
C                                                                               00015770
CC    TITLE                                                                     00015780
C                                                                               00015790
      WRITE(6,16)                                                               00015800
   16 FORMAT(1HO,4X,'TURBKE',25X,'TURBKE',32X,'GAMEAN',26X,'GAMEAN')            00015810
      GO TO 15                                                                  00015820
  100 RETURN                                                                    00015830
      END                                                                       00015840


      SUBROUTINE ADVANC                                                         00006750
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                                      00006760
      CCMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,           00006770
     1                  U,PHILAM,FF,ETA,GA,X,Y                                 00006780
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                               00006790
      CCMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1,         00006800
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,         00006810
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,       00006820
     3                  KINT                                                   00006830
      CCMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,         00006840
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,       00006850
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN       00006860
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6        00006870
      CCMMON / SYMBOL / DOT,BLANK,XXX                                           00006880
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,         00006890
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,       00006900
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN       00006910
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6        00006920
      DIMENSION    PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)           00006930
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),                00006940
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),            00006950
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),           00006960
     3                  ETA(201),GA(201)                                       00006970
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)                 00006980
      DOUBLE PRECISION  GJ1,GJ2,GJ3,GJ,Q                                       00006990
      REAL*8 LAMDA,LAMDAB,LAMDAC,LAMDAD,LAMDAE,LAMDAF,LAMDAG                    00007000
      COMMON MRLOAD,NCYCLS,LINR
```

```
C                                                                       00007010
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00007020
C                                                                       00007030
C                                                                       00007040
C     THIS ROUTINE GENERATES INTERIOR VALUES OF VORTICITY AT TIME=T+DT  00007050
C     BY SECOND ORDER CENTRAL TIME DIFFERENCING.                        00007060
C     GJ1,GJ2,GJ3 ARE EACH FINITE DIFFERENCE ANALOGS FOR THE CONVECTIVE 00007070
C     TERMS . Q IS THE DISSIPATION TERM                                 00007080
C     ARRAY FG IS USED FOR TEMPORARY STORAGE OF THE NEW VALUES OF       00007090
C     ARRAY GAMOLD STORES OLD VALUES OF THE VORTICITY FOR THE CENTRAL   00007100
C     VORTICITY                                                         00007110
C     TIME DIFFERENCING.                                                00007120
C                                                                       00007130
C                                                                       00007140
      IF(ICOUNT .EQ. KSTEP)GO TO 30                                     00007150
C                                                                       00007160
    1 LAMDA=2.D0*DT*RSQRP/(REYNLD*DXSQR)                                00007170
      LAMDAB=(1.-LAMDA)/(1.+LAMDA)                                      00007180
      LAMDAC=2.*DT/(DXSQR*(1.+LAMDA))                                   00007190
      LAMDAD=DT/(DX*(1.D0+LAMDA))                                       00007200
      LAMDAE=3.D0*LAMDAD                                                00007210
      DO 10 I=1,M                                                       00007220
      IP=IPO(I)                                                         00007230
      IM=IMO(I)                                                         00007240
      IMM=IMO(IM)                                                       00007250
      IPP=IPO(IP)                                                       00007260
      DO 10 J=2,NM1                                                     00007270
      JP=J+1                                                            00007280
      JM=J-1                                                            00007290
      IF(LINR.EQ.1)GO TO 8
      GJ1= (PHI(IM,J)-PHI(IP,J))*(GAMA(I,JP)-GAMA(I,JM))                00007300
     1    +(PHI(I,JP)-PHI(I,JM))*(GAMA(IP,J)-GAMA(IM,J))                00007310
      GJ2= GAMA(IP,JP)*(PHI(I,JP)-PHI(IP,J))                            00007320
     1    +GAMA(IM,JM)*(PHI(I,JM)-PHI(IM,J))                            00007330
     2    +GAMA(IP,JM)*(PHI(IP,J)-PHI(I,JM))                            00007340
     3    +GAMA(IM,JP)*(PHI(IM,J)-PHI(I,JP))                            00007350
      GJ3= GAMA(I,JP)*(PHI(IM,JP)-PHI(IP,JP))                           00007360
     1    +GAMA(IM,J)*(PHI(IM,JM)-PHI(IM,JP))                           00007370
     2    +GAMA(I,JM)*(PHI(IP,JM)-PHI(IM,JM))                           00007380
     3    +GAMA(IP,J)*(PHI(IP,JP)-PHI(IP,JM))                           00007390
      GJ=(GJ1+GJ2+GJ3)/12.D0                                            00007400
      GJ=R*GJ                                                           00007410
      GO TO 81
    8 GJ=0.D0
   81 CONTINUE
      Q=RSQR*(GAMA(I,JP)+GAMA(I,JM))+GAMA(IP,J)+GAMA(IM,J)              00007420
      Q=Q*REY                                                           00007430
      FG(I,J)=LAMDAB*GAMOLD(I,J)+LAMDAC*(GJ+Q)                          00007440
```

```
     1         -LAMDAD*U(J)*(GAMA(IP,J)-GAMA(IM,J))                          00007450
     2         -LAMDAE*(PHI(IP,J)-PHI(IM,J))                                 00007460
    10 CONTINUE                                                              00007470
C      COMPUTE GAMA AT WALL FROM TRANSPORT EQN. FOR COMPARISON WITH
C      GAMA COMPUTED FROM THE POISON EQN.
       DO 11 I=1,M
       IM=IMO(I)
       IP=IPO(I)
       LAMDA=2.DO*DT/(REYNLD*DXSQR)
       Q=RSQR*(GAMA(I,NM2)-2.*GAMA(I,NM1)+GAMA(I,N))+GAMA(IP,N)
     1   -2.*GAMA(I,N)+GAMA(IM,N)
       FG(I,N)=GAMOLD(I,N)+LAMDA*Q
    11 CONTINUE
       DO 21 I=1,M
       GAMOLD(I,N)=GAMA(I,N)
    21 CONTINUE
       DO 20 I=1,M                                                           00007480
       DO 20 J=2,NM1                                                         00007490
       GAMOLD(I,J)=GAMA(I,J)                                                 00007500
       GAMA(I,J)=FG(I,J)                                                     00007510
    20 CONTINUE                                                              00007520
       L=ICOUNT+1
       IF(ICOUNT .EQ.KSTEP)GO TO 69
       IF(L .NE. INT)GO TO 69
C      COMPARE VALUES OF GAMA COMPUTED FROM TRANSPORT EQN WITH VALUES
C      COMPUTED FROM POISSON EQN
C
       WRITE(6,105)ICOUNT
       WRITE(6,98 )(I,I=1,10)
       DO 17 J=NM4,N
       WRITE(6, 99)J,Y(J),(  FG(I,J),I=1,5 )
    17 CONTINUE
       WRITE(6,98)(I,I=11,20)
       DO 19 J=NM4,N
       WRITE(6,99)J,Y(J),(  FG(I,J),I=6,10)
    19 CONTINUE
    69 CONTINUE
    98 FORMAT(1H ,5X,'I=',11X,10(I3,8X))
    99 FORMAT(1X,'J=',I3,1X,'Y=',F6.3,2X, 5(E14.7,1X))
   105 FORMAT(1HO,'GAMA NEAR WALL COMPUTED FROM TRANSPORT EQN CYCLE',I4)
       CALL PRESUR                                                           00007530
       TIME=TIME+DT                                                          00007540
       NITER=0                                                               00007550
       GO TO 40                                                              00007560
C                                                                            00007570
    30 CALL STEP                                                             00007580
       KSTEP=KSTEP+KINT                                                      00007590
    40 RETURN                                                                00007600
```

```
      END                                                                   00007610

      SUBROUTINE PRESUR                                                     00007620
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                                  00007630
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,       00007640
     1                  U,PHILAM,FF,ETA,GA,X,Y                             00007650
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                           00007660
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1,     00007670
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,     00007680
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,   00007690
     3                  KINT                                               00007700
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,     00007710
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,   00007720
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN   00007730
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6    00007740
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,     00007750
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,   00007760
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN   00007770
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6    00007780
      COMMON MRLOAD,NCYCLS,LINR
      DIMENSION     PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)      00007790
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),            00007800
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),         00007810
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),        00007820
     3                  ETA(201),GA(201)                                    00007830
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)             00007840
C                                                                           00007850
CC    * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00007860
CC                                                                          00007870
CC    THIS ROUTINE SOLVES A POISSON EQUATION FOR THE STREAM FUNCTION        00007880
CC    BY MEANS OF THE DISCRETE FOURIER TRANSFORM.                           00007890
CC                                                                          00007900
CC     OBTAIN THE FOURIER TRANSFORM OF GAMA. STCRE IN ARRAY PHI.            00007910
C                                                                           00007920
      CC 20 J=1,N                                                           00007930
      DO 10 I=1,M                                                           00007940
      IA=IAO(I)                                                             00007950
      IP=IAP(I)                                                             00007960
      A(IA)=GAMA(I,J)                                                       00007970
      A(IP)=0.                                                              00007980
   10 CCNTINUE                                                              00007990
C                                                                           00008000
      CALL DHARM(A,MM,INV,S,2,IFERR)                                        00008010
      PHI(1,J)=A(1)                                                         00008020
      DO 20 I=2,M                                                           00008030
      PHI(I,J)=A(I+1)                                                       00008040
   20 CCNTINUE                                                              00008050
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```
C       OBTAIN THE MEAN VALUE OF GAMA FROM THE ZERO MODE                00008060
        DO 25 J=1,N                                                     00008070
        GAMEAN(J)=PHI(1,J)/XM+3.DO*Y(J)                                 00008080
   25 CONTINUE                                                          00008090
C       OBTAIN THE SPECTRUM OF SQUARED VORTICITY                        00008100
        L=ICOUNT+1                                                      00008110
        IF(L .EQ. INT) CALL SPCTRM(-1)                                  00008120
C                                                                       00008130
C       SOLVE FOR THE FOURIER TRANSFORM OF PHI. STORE IN ARRAY PHI.     00008140
C                                                                       00008150
        DO 40 I=1,M                                                     00008160
        PHI(I,N)=0.                                                     00008170
        PHI(I,1)=0.                                                     00008180
        ALPHA=XALPHA(I)                                                 00008190
        DO 30  J=1,NM4
        T(J)= PHI(I,J+2)
   30 CONTINUE                                                          00008220
        CALL TRISOL                                                     00008230
        DO 41 J=3,NM2
        PHI(I,J)= T(J-2)
   41 CONTINUE
        PHI(I,2)=PHI(I,3)*.25
        PHI(I,NM1) =PHI(I,NM2)*.25
        PHI(I,2)=.5*PHI(I,3)-PHI(I,4)/9.
        PHI(I,NM1)=.5*PHI(I,NM2)-PHI(I,NM3)/9.
   40 CONTINUE                                                          00008260
C       OBTAIN THE MEAN VALUE OF PHI FROM THE ZERO MODE                 00008270
        DO 45 J=2,NM1                                                   00008280
        PHMEAN(J)=PHI(1,J)/XM                                           00008290
   45 CONTINUE                                                          00008300
        PHMEAN(N)=0.DO                                                  00008310
        PHMEAN(1)=0.DO                                                  00008320
C       OBTAIN THE ENERGY SPECTRUM.                                     00008330
        IF(L .EQ. INT) CALL SPCTRM(1)                                   00008340
C                                                                       00008350
C                                                                       00008360
 1000 CONTINUE                                                          00008390
        IF(L.EQ.MRLOAD)CALL MODES(1,2,1)
        IF(L.EQ.MRLOAD)CALL MODES(3,4,1)
        IF(ICOUNT.EQ.NCYCLS) CALL MODES(1,2,1)
        IF(ICOUNT.EQ.NCYCLS)CALL MODES(3,4,1)
C                                                                       00008450
 1004 CONTINUE                                                          00008460
C                                                                       00008470
C       INVERT THE FOURIER TRANSFORM OF PHI                             00008480
C                                                                       00008490
        DO 70 J=2,NM1                                                   00008500
        A(1)=PHI(1,J)                                                   00008510
```

```
      A(2)=0.                                                          00008520
      DO 50 I=3,MP1                                                    00008530
      A(I)=PHI(I-1,J)                                                  00008540
   50 CONTINUE                                                         00008550
      A(MP2)=0.                                                        00008560
      IC=5                                                             00008570
      DO 60 I=MP3,M2M,2                                                00008580
      A(I)=PHI(I-IC,J)                                                 00008590
      IP=I+1                                                           00008600
      A(IP)=-PHI(IP-IC,J)                                              00008610
      IC=IC+4                                                          00008620
   60 CONTINUE                                                         00008630
      CALL DHARM(A,MM,INV,S,-2,IFERR)                                  00008640
      DO 70 I=1,M                                                      00008650
      IA=IAO(I)                                                        00008660
      PHI(I,J)=A(IA)                                                   00008670
   70 CONTINUE                                                         00008680
   11 FORMAT(10X,'**  FFT OF GAMA  **'//)                             00008690
   12 FORMAT(10X,'**  SOLUTION FOR FFT OF PHI  **'//)                  00008700
   13 FORMAT(10X,'**  SOLUTION FOR PHI  **'//)                        00008710
   14 FORMAT(10X,'**  FFT OF EXACT SOLUTION FOR PHI  **'//)           00008720
C                                                                      00008730
CC    COMPUTE NEW VALUES OF VORTICITY AT WALLS                         00008740
C                                                                      00008750
      DO 80 I=1,M                                                      00008760
      IP=IPO(I)
      IM=IMO(I)
      GAMA(I,2)=(PHI(IP,2)-2.*PHI(I,2)+PHI(IM,2))/DXSQR +(PHI(I,3)
     1  -2.*PHI(I,2))/DYSQR
      GAMA(I,NM1)=(PHI(IP,NM1)-2.*PHI(I,NM1) +PHI(IM,NM1))/DXSQR
     1  +(PHI(I,NM2)-2.*PHI(I,NM1))/DYSQR
      GAMA(I,N)=(8.D0*PHI(I,NM1)-PHI(I,NM2))*ATERM                     00008770
      GAMA(I,1)=(8.D0*PHI(I,2)-PHI(I,3))*ATERM                         00008780
      GAMA(I,N)=(1.5*PHI(I,NM2)-4.*PHI(I,NM3)/9.)/DYSQR
      GAMA(I,1)=(1.5*PHI(I,3)-4.*PHI(I,4)/9.)/DYSQR
   80 CONTINUE                                                         00008790
      IF(ICOUNT .EQ. KSTEP)GO TO 87
      IF(L .NE. INT)GO TO 87
      WRITE(6,109)ICOUNT
      WRITE(6,98 )(I,I=1,10)
      DO 18 J=NM4,N
      WRITE(6, 99)J,Y(J),(GAMA(I,J),I=1,5)
   18 CONTINUE
      WRITE(6,98)(I,I=11,20)
      DO 21 J=NM4,N
      WRITE(6,99)J,Y(J),(GAMA(I,J),I=6,10)
   21 CONTINUE
      IFIG=1
```

```
   98 FORMAT(1H ,5X,'I=',11X,10(I3,8X))
   99 FORMAT(1X,'J=',I3,1X,'Y=',F6.3,2X, 5(E14.7,1X))
  109 FORMAT(1H ,2X,'GAMA NEAR WALL COMPUTED FROM POISSON EQN CYCLE',I4)
   87 RETURN
      END                                                                00008810

      SUBROUTINE SPCTRM(INDEX)                                           00009970
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                               00009980
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,    00009990
     1                  U,PHILAM,FF,ETA,GA,X,Y                           00010000
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                        00010010
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1,  00010020
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,   00010030
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,00010040
     3                  KINT                                             00010050
      CCMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,  00010060
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,00010070
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN00010080
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6  00010090
      COMMON / SYMBOL / DOT,BLANK,XXX                                    00010100
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,  00010110
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,00010120
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN00010130
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6  00010140
      DIMENSION     PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)   00010150
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),         00010160
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),      00010170
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),     00010180
     3                  ETA(201),GA(201)                                 00010190
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)          00010200
C                                                                        00010210
CC                                                                       00010220
CC    * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *  00010230
CC                                                                       00010240
CC    THIS ROUTINE COMPUTES THE KINETIC ENERGY OF THE DISTURBANCE        00010250
CC    MOTION AS A FUNCTION OF Y AND WAVE NUMBER IN THE X DIRECTION.      00010260
CC    WAVE OR MODE NUMBER ONE CORRESPONDS TO THE MEAN VALUE OF THE       00010270
CC    DISTURBANCE. THE SPECTRUM OF THE TURBULENT ENERGY IS CONTAINED     00010280
CC    IN MODE NUMBERS TWO THROUGH NH.                                    00010290
CC    ARRAY FG IS USED FOR STORAGE OF THE SPECTRAL VALUES                00010300
C                                                                        00010310
      IF(ICOUNT .EQ. KSTEP) RETURN                                       00010320
      IFIG=1                                                             00010330
      IF(INDEX .LT. 0) GO TO 40                                          00010340
    5 L=MH+1                                                             00010350
      DO 10 J=2,NM1                                                      00010360
      JM=J-1                                                             00010370
```

```
      JP=J+1                                                            00010380
      FG(I,J)=FC*(PHI(1,JM)-PHI(1,JP))**2                               00010390
      FG(L,J)=FC*(PHI(M,JM)-PHI(M,JP))**2                               00010400
      DO 10 I=2,MH                                                      00010410
      LL=IAO(I)                                                         00010420
      LP=LL+1                                                           00010430
      LM=LL-1                                                           00010440
                                                                        00010450
      FG(I,J)=FA*(FB*((PHI(LM,JM)**2+PHI(LL,JM)**2+PHI(LM,JP)**2        00010460
     1                +PHI(LL,JP)**2)-2.D0*(PHI(LM,JP)*PHI(LM,JM)+      00010470
     2                 PHI(LL,JP)*PHI(LL,JM)))                          00010480
     3             +FF(I)*(PHI(LM,J)**2+PHI(LL,J)**2))                  00010490
   10 CONTINUE                                                          00010500
C                                                                       00010510
C     MEAN VALUES                                                       00010520
C                                                                       00010530
      T(1)=0.D0                                                         00010540
      DO 20 I=1,MHP                                                     00010550
      DO 15 J=2,NM1                                                     00010560
      T(J)=FG(I,J)                                                      00010570
   15 CONTINUE                                                          00010580
      CALL ASUM(T,NM1,SUM)                                              00010590
      ETA(I)=SUM/XNM1                                                   00010600
      GA(I)=FG(I,NH)                                                    00010610
   20 CONTINUE                                                          00010620
C                                                                       00010630
C     TITLE                                                             00010640
C                                                                       00010650
      L=ICOUNT+1                                                        00010660
      WRITE(6,22) L                                                     00010670
   22 FORMAT(10X,'ENERGY SPECTRUM FOR CYCLE NUMBER ',I5)                00010680
      WRITE(6,21)                                                       00010690
   21 FORMAT(14X,'MEAN SPECTRUM',36X,'MODE',20X,'MIDSTREAM SPECTRUM')   00010700
      CALL PLOTSP(ETA,GA,MHP)                                           00010710
      DO 30 I=1,MHP                                                     00010720
      ETA(I)=FG(I,16)                                                   00010730
      GA(I)=FG(I,186)                                                   00010740
   30 CONTINUE                                                          00010750
      WRITE(6,23) Y(16),Y(186)                                          00010760
   23 FORMAT(10X,'SPECTRUM FOR Y=',F6.4,32X,'MODE',18X,'SPECTRUM FOR Y='00010770
     1       ,F6.4)                                                     00010780
      CALL PLOTSP(ETA,GA,MHP)                                           00010790
C                                                                       00010800
      IF(ICOUNT .GT. -1) GO TO 100
C     PRINT THE ENERGY IN SEVERAL MODES AS FUNCTIONS OF Y.              00010810
      ETA(1)=0.D0                                                       00010820
      ETA(N)=0.D0                                                       00010830
      GA(1)=0.D0                                                        00010840
```

```
      GA(N)=C.DO                                                                00010850
      I1=2                                                                      00010860
      I2=3                                                                      00010870
      DO 39 KL=1,2                                                              00010880
      DO 35 J=2,NM1                                                             00010890
      ETA(J)=FG(I1,J)                                                           00010900
      GA(J)=FG(I2,J)                                                            00010910
   35 CONTINUE                                                                  00010920
      WRITE(6,36) I1,I2                                                         00010930
   36 FORMAT(14X,'ENERGY IN MODE ',I2,35X,'J',20X,'ENERGY IN MODE ',I2)         00010940
      CALL PLOTSP(ETA,GA,N)                                                     00010950
      I1=4                                                                      00010960
      I2=5                                                                      00010970
   39 CONTINUE                                                                  00010980
      GO TO 100                                                                 00010990
C                                                                               00011000
CC    SIMILAR OPERATIONS FOR THE SPECTRUM OF SQUARED VORTICITY                  00011010
C                                                                               00011020
   40 F6=1.DO/XM**2                                                             00011030
      DO 50 J=1,N                                                               00011040
      FG(1,J)=F6*PHI(1,J)**2                                                    00011050
      FG(MHP,J)=F6*PHI(M,J)**2                                                  00011060
      DO 50 I=2,MH                                                              00011070
      LL=IAO(I)                                                                 00011080
      FG(I,J)=F6*(PHI(LL-1,J)**2+PHI(LL,J)**2)                                  00011090
   50 CONTINUE                                                                  00011100
C                                                                               00011110
      DO 60 I=1,MHP                                                             00011120
      DO 55 J=1,N                                                               00011130
      T(J)=FG(I,J)                                                              00011140
   55 CONTINUE                                                                  00011150
      CALL ASUM(T,N,SUM)                                                        00011160
      ETA(I)=SUM/XNM1                                                           00011170
      GA(I)=FG(I,NH)                                                            00011180
   60 CONTINUE                                                                  00011190
C                                                                               00011200
CC    TITLE                                                                     00011210
C                                                                               00011220
      L=ICOUNT+1                                                                00011230
      WRITE(6,61) L                                                             00011240
   61 FORMAT(10X,'VORTICITY SPECTRUM FOR CYCLE NUMBER  ',I5)                    00011250
      WRITE(6,21)                                                               00011260
      CALL PLOTSP(ETA,GA,MHP)                                                   00011270
      DO 70 I=1,MHP                                                             00011280
      ETA(I)=FG(I,16)                                                           00011290
      GA(I)=FG(I,186)                                                           00011300
   70 CONTINUE                                                                  00011310
      WRITE(6,23) Y(16),Y(186)                                                  00011320
```

```
      CALL PLOTSP(ETA,GA,MHP)                                                   00011330
  100 RETURN                                                                    00011340
      END                                                                       00011350


      SUBROUTINE TRISOL                                                         00008820
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                                      00008830
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,           00008840
     1                  U,PHILAM,FF,ETA,GA,X,Y                                 00008850
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                               00008860
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1,         00008870
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,         00008880
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,       00008890
     3                  KINT                                                   00008900
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,         00008910
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,       00008920
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN       00008930
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6        00008940
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,         00008950
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,       00008960
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN       00008970
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6        00008980
      DIMENSION     PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)          00008990
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),                00009000
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),            00009010
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),           00009020
     3                  ETA(201),GA(201)                                       00009030
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)                 00009040
C                                                                               00009050
CC    * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00009060
CC                                                                              00009070
CC    THIS SUBROUTINE SOLVES THE SYSTEM OF EQUATIONS P*X=DYSQR*Y.               00009080
CC    X AND Y ARE VECTORS,DYSQR IS A SCALAR, P IS A TRIDIAGONAL MATRIX          00009090
CC    WITH ALL MAIN DIAGONAL ELEMENTS=-ALPHA,UPPER AND LOWER DIAGONAL           00009100
CC    ELEMENTS=UNITY. ON INPUT Y IS STORED IN ARRAY T. ON OUTPUT T              00009110
CC    CONTAINS THE SOLUTION VECTOR X. ALL VECTORS HAVE DIMENSION NM2            00009120
CC                                                                              00009130
CC    COLORED CARDS DENOTE REVISED TRISOL TO INCLUDE THE CHANGES
CC    RESULTING FROM ALTERING BOUNDARY CONDITIONS TO BE CONSISTENT
CC    WITH ZERO SLOPE FOR PHI AT THE WALLS
C                                                                               00009140
      DOUBLE PRECISION  XFF
C     SOLUTION BASED ON APPROX PSI AT WALL BY PSI=A1*Y+A2*YSQ
C         +A3*Y**3
C     FOR ZERO SLOPE PHI(2)=.5*PHI(3)-1/9*PHI(4)
      NM5=N-5
      XFF=8.D0/9.D0
      GA(1)=-ALPHA+.5D0
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```
      ETA(1)=T(1)/GA(1)
      GA(2)=-ALPHA-XFF/GA(1)
      ETA(2)=(T(2)-ETA(1))/GA(2)
      DO 10 J=3,NM5
      GA(J)=-ALPHA-1.DO/GA(J-1)
      ETA(J)=(T(J)-ETA(J-1))/GA(J)
   10 CONTINUE
      GA(NM4)=GA(1)-XFF/GA(NM5)
      ETA(NM4)=(T(NM4)-XFF*ETA(NM5))/GA(NM4)
      T(NM4)=DYSQR*ETA(NM4)
      J=N-5
      DO 20 JC=2,NM4
      T(J)=DYSQR*ETA(J)-T(J+1)/GA(J)
      J=J-1
   20 CONTINUE
      RETURN
      END

      SUBROUTINE MODES(IMOD1,IMOD2,ISTYLE)                                  C0009290
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                                  C0009300
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,       00009310
     1                  U,PHILAM,FF,ETA,GA,X,Y                              00009320
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                           00009330
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1,     00009340
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,     00009350
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,   00009360
     3                  KINT                                               00009370
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,     00009380
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,   00009390
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN   00009400
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6    00009410
      COMMON / SYMBOL / DOT,BLANK,XXX                                       00009420
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,     00009430
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,   00009440
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN   00009450
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6    00009460
      DIMENSION    PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)       00009470
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),            00009480
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),         00009490
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),        00009500
     3                  ETA(201),GA(201)                                    00009510
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)             00009520
C                                                                           00009530
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *   00009540
C                                                                           00009550
C     THIS ROUTINE COMPUTES AND PRINTS NORMALIZED MODAL FUNCTIONS           00009560
C      OF THE DISTURBANCE STREAM FUNCTION.                                  00009570
```

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C                                                                       00009580
      IM1=IAP(IMOD1)                                                    00009590
      IM1P=IM1+1                                                        00009600
      IM2=IAP(IMOD2)                                                    00009610
      IM2P=IM2+1                                                        00009620
      IF(ISTYLE .EQ. 2) GO TO 20                                        00009630
    5 A1=PHI(IM1,NH)**2+PHI(IM1P,NH)**2                                 00009640
      A2=PHI(IM1,NH)/A1                                                 00009650
      A1=-PHI(IM1P,NH)/A1                                               00009660
      A3= PHI(IM2,NH)
      A4= PHI(IM2P,NH)
      WRITE(6,11) IMOD1,IMOD2                                           00009700
      DO 10 J=1,N                                                       00009710
      ETA(1)=A2*PHI(IM1,J)-A1*PHI(IM1P,J)                               00009720
      ETA(2)=A1*PHI(IM1,J)+A2*PHI(IM1P,J)                               00009730
      ETA(3)=PHI(IM2,J)-A3
      ETA(4)=PHI(IM2P,J)-A4
      WRITE(6,12) J,ETA(1),ETA(2),Y(J),ETA(3),ETA(4),J                  00009760
   10 CONTINUE                                                          00009770
   11 FORMAT(1HO,5X,'J',4X,'REAL    MODE ',I3,'  IMAG',11X,'Y',9X,      00009780
     1       'REAL    MODE ',I3,'  IMAG',6X,'J')                       00009790
   12 FORMAT(4X,I3,2(3X,F10.6),6X,F7.4,6X,2(F10.6,3X),I3)               00009800
      GO TO 40                                                          00009810
   20 WRITE(6,21) IMOD1,IMOD2                                           00009820
      DO 30 J=1,N                                                       00009830
      ETA(1)=PHI(IM1,J)**2+PHI(IM1P,J)**2                               00009840
      ETA(1)=DSQRT(ETA(1))/XM                                           00009850
      ETA(2)=ATAN2(PHI(IM1,J),PHI(IM1P,J))                              00009860
      ETA(3)=PHI(IM2,J)**2+PHI(IM2P,J)**2                               00009870
      ETA(3)=DSQRT(ETA(3))/XM                                           00009880
      ETA(4)=ATAN2(PHI(IM2,J),PHI(IM2P,J))                              00009890
      WRITE(6,12) J,ETA(1),ETA(2),Y(J),ETA(3),ETA(4),J                  00009900
   30 CONTINUE                                                          00009910
   21 FORMAT(1HO,5X,'J',4X,'MAG     MODE ',I3,'  PHAS',11X,'Y',9X,     00009920
     1       'MAG     MODE ',I3,'  PHAS',6X,'J')                       00009930
   40 IFIG=1                                                            00009940
      RETURN                                                            00009950
      END                                                               00009960

      SUBROUTINE STEP                                                   00004720
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                              00004730
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,   00004740
     1                  U,PHILAM,FF,ETA,GA,X,Y                          00004750
      COMMON / ARRAYS / IPC,IMO,INV,IAO,IAP,MM(3)                       00004760
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1, 00004770
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,  00004780
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,00004790
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     3                 KINT                                                                  00004800
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,                     00004810
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,                    00004820
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN                    00004830
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6                     00004840
      COMMON / SYMBOL / DOT,BLANK,XXX                                                       00004850
      DOUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,                     00004860
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,                    00004870
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN                    00004880
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6                     00004890
      DIMENSION    PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)                       00004900
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),                            00004910
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),                         00004920
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),                        00004930
     3                  ETA(201),GA(201)                                                    00004940
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)                             00004950
      DOUBLE PRECISION  GJ,GJ1,GJ2,GJ3,Q                                                    00004960
      DOUBLE PRECISION  CRIT,TST,PUNT,GAMAIJ                                                00004970
      COMMON MRLOAD,NCYCLS,LINR
C                                                                                           00004980
CC    * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *     00004990
C                                                                                           00005000
      REAL*8  LAMDA,LAMDAB,LAMDAC,LAMDAD,LAMDAE                                             00005010
C                                                                                           00005020
CC    THIS ROUTINE COMPUTES NEW VALUES OF VORTICITY AT TIME+DT BY                           00005030
CC    MODIFIED EULER IMPLICIT TIME DIFFERENCING.                                            00005040
CC    ARRAY FG IS USED TO STORE TERMS EVALUATED AT TIME T.                                  00005050
C                                                                                           00005060
      IF(KINT .EQ. 1) GO TO 6                                                               00005070
      IF(ICOUNT .EQ. 0) GO TO 6                                                             00005080
      DO 5 I=1,M                                                                            00005090
      DO 5 J=1,N                                                                            00005100
      GAMA(I,J)=.5*(GAMA(I,J)+GAMOLD(I,J))                                                  00005110
      GAMOLD(I,J)=GAMA(I,J)                                                                 00005120
    5 CONTINUE                                                                              00005130
      CALL PRESUR                                                                           00005140
      TIME=TIME-DT/2.D0                                                                     00005150
    6 CALL TIMER                                                                            00005160
      LAMDA=RSQRP*DT/(REYNLD*DXSQR)                                                         00005170
      LAMDAB=(1.D0-LAMDA)/(1.D0+LAMDA)                                                      00005180
      LAMDAC=DT/(2.D0*(1.D0+LAMDA)*DXSQR)                                                   00005190
      LAMDAD=DT/(4.D0*(1.D0+LAMDA)*DX)                                                      00005200
      LAMDAE=3.D0*LAMDAD                                                                    00005210
      DO 10 I=1,M                                                                           00005220
      IP=IPO(I)                                                                             00005230
      IM=IMO(I)                                                                             00005240
      DO 10 J=2,NM1                                                                         00005250
      JP=J+1                                                                                00005260
```

```
      JM=J-1                                                                    00005270
      IF(LINR.EQ.1)GO TO 8                                                      00005280
      GJ1=  (PHI(IM,J)-PHI(IP,J))*(GAMA(I,JP)-GAMA(I,JM))                       00005290
     1     +(PHI(I,JP)-PHI(I,JM))*(GAMA(IP,J)-GAMA(IM,J))                     00005300
      GJ2=  GAMA(IP,JP)*(PHI(I,JP)-PHI(IP,J))                                  00005310
     1     +GAMA(IM,JM)*(PHI(I,JM)-PHI(IM,J))                                 00005320
     2     +GAMA(IP,JM)*(PHI(IP,J)-PHI(I,JM))                                 00005330
     3     +GAMA(IM,JP)*(PHI(IM,J)-PHI(I,JP))                                 00005340
      GJ3=  GAMA(I,JP)*(PHI(IM,JP)-PHI(IP,JP))                                 00005350
     1     +GAMA(IM,J)*(PHI(IM,JM)-PHI(IM,JP))                                00005360
     2     +GAMA(I,JM)*(PHI(IP,JM)-PHI(IM,JM))                                00005370
     3     +GAMA(IP,J)*(PHI(IP,JP)-PHI(IP,JM))                                00005380
      GJ=(GJ1+GJ2+GJ3)/12.D0                                                   00005390
      GJ=R*GJ
      GO TO 81
    8 GJ=0.D0
   81 CONTINUE
      Q=RSQR*(GAMA(I,JP)+GAMA(I,JM))+GAMA(IP,J)+GAMA(IM,J)                     00005400
      Q=Q*REY                                                                  00005410
      FG(I,J)=LAMDAB*GAMA(I,J)+LAMDAC*(GJ+Q)                                   00005420
     1        -LAMDAD*U(J)*(GAMA(IP,J)-GAMA(IM,J))                             00005430
     2        -LAMDAE*(PHI(IP,J)-PHI(IM,J))                                    00005440
   10 CONTINUE                                                                 00005450
      NITSUM=0                                                                 00005460
C                                                                              00005470
C     FIRST SWEEP. START Y PASS AT LOWER WALL                                  00005480
C                                                                              00005490
      NITER=0                                                                  00005500
   15 PUNT=0.                                                                  00005510
      DO 20 I=1,M                                                              00005520
      IP=IPO(I)                                                                00005530
      IM=IMO(I)                                                                00005540
      DO 20 J=3,NM2
      JP=J+1                                                                   00005560
      JM=J-1                                                                   00005570
      IF(LINR.EQ.1)GO TO 18                                                    00005580
      GJ1=  (PHI(IM,J)-PHI(IP,J))*(GAMA(I,JP)-GAMA(I,JM))                      00005590
     1     +(PHI(I,JP)-PHI(I,JM))*(GAMA(IP,J)-GAMA(IM,J))                     00005600
      GJ2=  GAMA(IP,JP)*(PHI(I,JP)-PHI(IP,J))                                  00005610
     1     +GAMA(IM,JM)*(PHI(I,JM)-PHI(IM,J))                                 00005620
     2     +GAMA(IP,JM)*(PHI(IP,J)-PHI(I,JM))                                 00005630
     3     +GAMA(IM,JP)*(PHI(IM,J)-PHI(I,JP))                                 00005640
      GJ3=  GAMA(I,JP)*(PHI(IM,JP)-PHI(IP,JP))                                 00005650
     1     +GAMA(IM,J)*(PHI(IM,JM)-PHI(IM,JP))                                00005660
     2     +GAMA(I,JM)*(PHI(IP,JM)-PHI(IM,JM))                                00005670
     3     +GAMA(IP,J)*(PHI(IP,JP)-PHI(IP,JM))                                00005680
      GJ=(GJ1+GJ2+GJ3)/12.D0                                                   00005690
      GJ=R*GJ
```

```
      GO TO 118
   18 GJ=0.DO
  118 CONTINUE
      Q=RSQR*(GAMA(I,JP)+GAMA(I,JM))+GAMA(IP,J)+GAMA(IM,J)                00005700
      Q=Q*REY                                                             00005710
      GAMA(I,J)=LAMDAC*(GJ+Q)-LAMDAD*U(J)*(GAMA(IP,J)-GAMA(IM,J))         00005720
     1          -LAMDAE*(PHI(IP,J)-PHI(IM,J))+FG(I,J)                    00005730
   20 CONTINUE                                                            00005740
      CALL PRESUR                                                         00005750
C                                                                         00005760
C     SECOND SWEEP. START Y PASS AT UPPER WALL                            00005770
C                                                                         00005780
      DO 30 I=1,M                                                         00005790
      IP=IPO(I)                                                           00005800
      IM=IMO(I)                                                           00005810
      J=NM2
      DO 30 JC=3,NM2
      JP=J+1                                                              00005840
      JM=J-1                                                              00005850
      IF(LINR.EQ.1)GO TO 28
      GJ1= (PHI(IM,J)-PHI(IP,J))*(GAMA(I,JP)-GAMA(I,JM))                  00005860
     1     +(PHI(I,JP)-PHI(I,JM))*(GAMA(IP,J)-GAMA(IM,J))                 00005870
      GJ2= GAMA(IP,JP)*(PHI(I,JP)-PHI(IP,J))                              00005880
     1     +GAMA(IM,JM)*(PHI(I,JM)-PHI(IM,J))                             00005890
     2     +GAMA(IP,JM)*(PHI(IP,J)-PHI(I,JM))                             00005900
     3     +GAMA(IM,JP)*(PHI(IM,J)-PHI(I,JP))                             00005910
      GJ3= GAMA(I,JP)*(PHI(IM,JP)-PHI(IP,JP))                             00005920
     1     +GAMA(IM,J)*(PHI(IM,JM)-PHI(IM,JP))                            00005930
     2     +GAMA(I,JM)*(PHI(IP,JM)-PHI(IM,JM))                            00005940
     3     +GAMA(IP,J)*(PHI(IP,JP)-PHI(IP,JM))                            00005950
      GJ=(GJ1+GJ2+GJ3)/12.DO                                              00005960
      GJ=R*GJ                                                             00005970
      GO TO 128
   28 GJ=0.DO
  128 CONTINUE
      Q=RSQR*(GAMA(I,JP)+GAMA(I,JM))+GAMA(IP,J)+GAMA(IM,J)                00005980
      Q=Q*REY                                                             00005990
      TST=GAMA(I,J)                                                       00006000
      GAMAIJ=    LAMDAC*(GJ+Q)-LAMDAD*U(J)*(GAMA(IP,J)-GAMA(IM,J))        00006010
     1          -LAMDAE*(PHI(IP,J)-PHI(IM,J))+FG(I,J)                    00006020
C                                                                         00006030
C     TEST FOR CONVERGENCE                                                00006040
C                                                                         00006050
      CRIT=DABS(1.-GAMA(I,J)/GAMAIJ)                                      00006060
      PUNT=PUNT+CRIT                                                      00006070
   29 GAMA(I,J)=GAMAIJ                                                    00006080
      J=J-1                                                               00006090
   30 CONTINUE                                                            00006100
```

```
      NITER=NITER+2                                                          00006110
      TST=PUNT/(XM*NM2)                                                      00006120
      TST=PUNT/(XM*NM4)
      IF(TST .LT. TOL) GO TO 31                                              00006130
      IF(NITER .GT. NITMAX) GO TO 31                                         00006140
      CALL PRESUR                                                            00006150
      GO TO 15                                                               00006160
   31 TIME=TIME+DT                                                           00006170
      NITOT=NITOT+NITER                                                      00006180
C     IF ICOUNT EQUALS KSTEP THEN SUBROUTINE SPCTRM WILL BE BYPASSED         00006190
C     SO THAT  NO SPECTRA WILL BE CALCULATED DURING THE ITERATIONS           00006200
C     IN SUBROUTINE STEP                                                     00006210
      KSTEP=KSTEP+1                                                          00006220
      CALL PRESUR                                                            00006230
      KSTEP=KSTEP-1                                                          00006240
   40 RETURN                                                                 00006250
      END                                                                    00006260

      SUBROUTINE TIMER                                                       00006270
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                                   00006280
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,        00006290
     1                  U,PHILAM,FF,ETA,GA,X,Y                               00006300
      COMMON / ARRAYS / IPC,IMO,INV,IAO,IAP,MM(3)                            00006310
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1,      00006320
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,       00006330
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,     00006340
     3                  KINT                                                 00006350
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,      00006360
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,     00006370
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN     00006380
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6      00006390
      COMMON / SYMBOL / DOT,BLANK,XXX                                        00006400
      DCUBLE PRECISION  DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,      00006410
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,     00006420
     2                  EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN     00006430
     3                 ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6      00006440
      DIMENSION    PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)        00006450
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),             00006460
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),          00006470
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),         00006480
     3                  ETA(201),GA(201)                                     00006490
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)              00006500
C                                                                            00006510
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *    00006520
C                                                                            00006530
C     THIS ROUTINE COMPUTES A NEW VALUE OF DT.                               00006540
C                                                                            00006550
```

```
      UMAX=0.                                                           00006560
      VMAX=0.                                                           00006570
      DO 10 I=1,M,4                                                     00006580
      IP=IPO(I)                                                         00006590
      IM=IMO(I)                                                         00006600
      DO 10 J=1,NH,2                                                    00006610
      FLIP=(PHI(I,J)-PHI(I,J+2))/DY2+U(J+1)                             00006620
      FLIPR=(PHI(IP,J)-PHI(IM,J))/DX2                                   00006630
      UMAX=AMAX1(UMAX,ABS(FLIP))                                        00006640
      VMAX=AMAX1(VMAX,ABS(FLIPR))                                       00006650
   10 CONTINUE                                                          00006660
      DTT=FRACEL*DX/UMAX                                                00006670
      DT=FRACEL*DY/VMAX                                                 00006680
      DT=DMIN1(DT,DTT)                                                  00006690
      ITEL=1                                                            00006700
      IF(DT .EQ. DTT)ITEL=-1                                            00006710
      DT=FRACEL*DX*DY/(UMAX*DY+VMAX*DX)                                 00006720
      DTT=XLAMDA*DX*DY*REYNLD/4.D0                                      00006721
      DT=DMIN1(DT,DTT)                                                  00006722
      IF(DT .EQ. DTT) ITEL=0                                            00006723
      RETURN                                                            00006730
      END                                                               00006740


      SUBROUTINE PLOTSP(AX,BX,NN)                                       00011360
      COMMON / SYMBOL / DOT,BLANK,XXX                                   00011370
      COMMON / PICTUR / ALINE(49),BLINE(49),APLOT(201),BPLOT(201)       00011380
C                                                                       00011400
C     THIS ROUTINE PRINTS ONLINE PLOTS OF THE POSITIVE NN-DIMENSIONAL   00011410
C     VECTORS AX AND BX.                                                00011420
C                                                                       00011430
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 00011440
      DOUBLE PRECISION  AX(1),BX(1)                                     00011390
C     PUT AX & BX IN APLOT & BPLOT                                      00011450
      DO 10 I=1,NN                                                      00011470
      APLOT(I)=AX(I)                                                    00011480
C                                                                       00011460
      BPLOT(I)=BX(I)                                                    00011490
   10 CONTINUE                                                          00011500
C                                                                       00011510
C     FIND LARGEST ELEMENT IN APLOT & BPLOT                             00011520
C                                                                       00011530
      ABIG=0.                                                           00011540
      BBIG=0.                                                           00011550
      DO 20 I=1,NN                                                      00011560
      ABIG=AMAX1(ABIG,ABS(APLOT(I)))                                    00011570
      BBIG=AMAX1(BBIG,ABS(BPLOT(I)))                                    00011580
   20 CONTINUE                                                          00011590
```

```
C                                                                       00011600
C       SET UP TOP HORIZONTAL AXIS                                      00011610
C                                                                       00011620
        DO 30 I=1,49                                                    00011630
        BLINE(I)=DOT                                                    00011640
   30   CONTINUE                                                        00011650
        WRITE(6,31) ALINE,BLINE                                         00011660
        ALINE(I)=DOT                                                    00011670
   31   FORMAT(14X,49A1,20X,49A1)                                       00011680
C                                                                       00011690
C       BLANK ALINE & BLINE. PUT PLOTTING SYMBOL IN COMPUTED ELEMENT    00011700
C                                                                       00011710
        DO 40 I=2,49                                                    00011720
        ALINE(I)=BLANK                                                  00011730
        BLINE(I)=BLANK                                                  00011740
   40   CONTINUE                                                        00011750
        ALINE(1)=DOT                                                    00011760
        BLINE(1)=DOT                                                    00011770
C                                                                       00011780
        DO 50 I=1,NN                                                    00011790
        JA=APLOT(I)/ABIG*48.+1.                                         00011800
        JB=BPLOT(I)/BBIG*48.+1.                                         00011810
        ALINE(JA)=XXX                                                   00011820
        BLINE(JB)=XXX                                                   00011830
        WRITE(6,51) APLOT(I),ALINE,I,BPLOT(I),BLINE                     00011840
   51   FORMAT(1X,E10.4,2X,49A1,2X,I3,3X,E10.4,2X,49A1)                 00011850
        ALINE(JA)=BLANK                                                 00011860
        BLINE(JB)=BLANK                                                 00011870
        ALINE(1)=DOT                                                    00011880
        BLINE(1)=DOT                                                    00011890
   50   CONTINUE                                                        00011900
C                                                                       00011910
C       SET UP BOTTOM HORIZONTAL AXIS                                   00011920
C                                                                       00011930
        DO 60 I=1,49                                                    00011940
        ALINE(I)=DOT                                                    00011950
        BLINE(I)=DOT                                                    00011960
   60   CONTINUE                                                        00011970
        WRITE(6,61) ALINE,BLINE                                         00011980
   61   FORMAT(14X,49A1,20X,49A1//)                                     00011990
        RETURN                                                          00012000
        END                                                             00012010
```

```
      SUBROUTINE ASUM(A,NN,SUM)                                          00014580
      DOUBLE PRECISION A(1)                                              00014590
      DOUBLE PRECISION SUMM                                              00014600
C                                                                        00014610
CC    THIS ROUTINE COMPUTES THE SUM OF THE FIRST NN ELEMENTS OF A,       00014620
CC    AND STORES THE RESULT IN SUM                                       00014630
C                                                                        00014640
      SUMM=0.DC                                                          00014650
      DO 10 I=1,NN                                                       00014660
      SUMM=SUMM+A(I)                                                     00014670
   10 CONTINUE                                                           00014680
      SUM=SUMM                                                           00014690
      RETURN                                                             00014700
      END                                                                00014710


      SUBROUTINE LOAD                                                    00017630
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                               00017640
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,    00017650
     1                  U,PHILAM,FF,ETA,GA,X,Y                           00017660
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                        00017670
      COMMON / PARAMS / NUMBER(31)                                       00017680
      COMMON / FACTOR / FACTO                                            00017690
      COMMON / SYMBOL / DOT,BLANK,XXX                                    00017700
      DOUBLE PRECISION  FACTO(42)                                        00017710
      DIMENSION    PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)    00017720
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),         00017730
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),      00017740
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),     00017750
     3                  ETA(201),GA(201)                                 00017760
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)          00017770
      DIMENSION PHIO(2,101)
      COMMON/PHICOM/PHIO
      COMMON MRLOAD
      ITAPE=NUMBER(25)                                                   00017780
      REWIND ITAPE                                                       00017790
      READ (ITAPE) PHIO
      READ(ITAPE)MRLOAD
      READ(ITAPE)  PHI,GAMA,FG,GAMOLD                                    00017800
      READ(ITAPE)  T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP                 00017810
      READ(ITAPE)  TURBKE,U,PHILAM,FF,ETA,GA,X,Y                         00017820
      READ(ITAPE)  IPO,IMO,INV,IAO,IAP,MM                                00017830
      READ(ITAPE)  NUMBER                                                00017840
      READ(ITAPE)  FACTO                                                 00017850
      READ(ITAPE) DOT,BLANK,XXX                                          00017860
      REWIND ITAPE                                                       00017870
      CALL DHARM(A,MM,INV,S,0,IFER)                                      00017880
      RETURN                                                             00017890
```

```
      END                                                                   00017900

      SUBROUTINE RELOAD                                                     00017910
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD                                  00017920
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,       00017930
     1                  U,PHILAM,FF,ETA,GA,X,Y                              00017940
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)                           00017950
      COMMON / PARAMS / NUMBER(31)                                          00017960
      COMMON / FACTOR / FACTO                                               00017970
      COMMON / SYMBOL / DOT,BLANK,XXX                                       00017980
      DOUBLE PRECISION  FACTO(42)                                           00017990
      DIMENSION    PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)       00018000
      DOUBLE PRECISION  UMEAN(201),TEMP(201),TURBKE(201),U(201),            00018010
     1                  PHILAM(201),T(201),A(128),S(32),XALPHA(64),         00018020
     2                  PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),        00018030
     3                  ETA(201),GA(201)                                    00018040
      DIMENSION         IPO(64),IMO(64),INV(32),IAO(64),IAP(64)             00018050
      COMMON/PHICOM/PHIO
      COMMON MRLOAD
      DIMENSION PHIO(2,101)
      ITAPE=NUMBER(25)                                                      00018060
      REWIND ITAPE                                                          00018070
      WRITE(ITAPE)PHIO
      WRITE(ITAPE)MRLOAD
      WRITE(ITAPE) PHI,GAMA,FG,GAMOLD                                       00018080
      WRITE(ITAPE) T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP                    00018090
      WRITE(ITAPE) TURBKE,U,PHILAM,FF,ETA,GA,X,Y                            00018100
      WRITE(ITAPE) IPO,IMO,INV,IAO,IAP,MM                                   00018110
      WRITE(ITAPE) NUMBER                                                   00018120
      WRITE(ITAPE) FACTO                                                    00018130
      WRITE(ITAPE) DOT,BLANK,XXX                                            00018140
      REWIND ITAPE                                                          00018150
      RETURN                                                                00018160
      END                                                                   00018170

      SUBROUTINE PRMTRS
      COMMON / FIELDS / PHI,GAMA,FG,GAMOLD
      COMMON / VECTOR / T,A,S,XALPHA,PHMEAN,GAMEAN,UMEAN,TEMP,TURBKE,
     1                  U,PHILAM,FF,ETA,GA,X,Y
      COMMON / ARRAYS / IPO,IMO,INV,IAO,IAP,MM(3)
      COMMON / PARAMS / N,M,NP1,NP2,NM1,NM2,NM3,NM4,NH,MP1,MP2,MP3,MM1,
     1                  M2M,MH,MHP,NPROB,ICOUNT,ITEL,INT,KSTEP,ISTART,
     2                  ISCIP,MSCIP,ITAPE,NITER,NITOT,NITMAX,IFIG,MCOMP,
     3                  KINT
      COMMON / FACTOR / DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,
     1                  RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,
```

```
     2                    EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN
     3                   ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6
      DOUBLE PRECISION    DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,
     1                    RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,
     2                    EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN
     3                   ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6
      DOUBLE PRECISION    DX,DX2,DXSQR,DXSQR4,DXSQR8,DY,DY2,DYSQR,R,RSQR,
     1                    RSQRP,REYNLD,REY,FRACEL,XLAMDA,TIME,DT,DTT,PI,H,
     2                    EKMEAN,EKTURB,UVBAR,ATERM,BTERM,ALPHA,BETA,XM,XN
     3                   ,XNM1,UMNFLO,UFLO,TOL,FA,FB,FC,F1,F2,F3,F4,F5,F6
      DIMENSION      PHI(64,201),GAMA(64,201),FG(64,201),GAMOLD(64,201)
      DOUBLE PRECISION    UMEAN(201),TEMP(201),TURBKE(201),U(201),
     1                    PHILAM(201),T(201),A(128),S(32),XALPHA(64),
     2                    PHMEAN(201),GAMEAN(201),X(64),Y(201),FF(32),
     3                    ETA(201),GA(201)
      DIMENSION           IPO(64),IMO(64),INV(32),IAO(64),IAP(64)
      DIMENSION PHASE(11,6)
C
C       COMPTES PHI PHASE AT VARIOUS Y LOCATIONS
      IF(ICOUNT .EQ. KSTEP)GO TO 11
      JJ=1
      WRITE(6,10) (Y(J),J=NH,181,20)
      DO 20 J=NH,181,20
      DO 19 I=2,11
      LL=IAO(I)
      LM=LL-1
      ARGI=PHI(LL,J)
      ARGR= PHI(LM,J)
      PHASE(I,JJ)= ATAN2(ARGI,ARGR)
      IF(J .NE. 181) GO TO 19
      ARGI= PHI(LL,196)
      ARGR=PHI(LM,196)
      PHASE(I,6)=ATAN2(ARGI,ARGR)
   19 CONTINUE
      JJ=JJ+1
   20 CONTINUE
      MMX=1
      DO 30 I=2,11
      WRITE (6,40) ICOUNT, TIME,(PHASE(I,J),J=1,6),MMX
      MMX=MMX+1
   30 CONTINUE
   10 FORMAT(1HO,3X,'CYCLE',7X,'TIME',3X,'PHASE AT Y=',F6.3,4(5X,F6.3)
     1  ,10X,'.95',7X,'MODE')
   40 FORMAT(I7,3X,F10.6,3X,F8.3,10X,5(F8.3,5X),I3)
      IFIG=1
   11 RETURN
      END
```

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13. ABSTRACT

The response of a plane Poiseuille flow to disturbances of various initial wavenumbers and amplitudes is investigated by numerically integrating the equation of motion. It is shown that for very low amplitude disturbances the numerical integration scheme yields results that are consistent with those predictable from linear theory. It is also shown that because of non-linear interactions a growing unstable disturbance excites higher wavenumber modes which have the same frequency, or phase velocity, as the primary mode. For very low amplitude disturbances these spontaneously generated higher wavenumber modes have a strong resemblance to certain modes computed from the linear Orr-Sommerfeld equation.

In general it is found that the disturbance is dominated for a long time by the primary mode and that there is little alteration of the original parabolic mean velocity profile. There is evidence of the existence of an energy equilibrium state which is common to all finite-amplitude disturbances despite their initial wavenumbers. This equilibrium energy level is roughly 3-5% of the energy in the mean flow which is an order of magnitude higher than the equilibrium value predicted by existing non-linear theories.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Hydrodynamic stability						
Finite-differences						
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