REPORT DOCUMENTATION PAGE

Form Approved
OMB NO. 0704-0188

1. AGENCY USE ONLY (Leave Blank)

2. REPORT DATE
21 January 2010

3. REPORT TYPE AND DATES COVERED
Final Technical, 22 Apr 2009 - 22 Oct 2009

4. TITLE AND SUBTITLE
Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials

5. FUNDING NUMBERS
55332EGII

6. AUTHOR(S)
Thomas J. Connolly, Ph.D.

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of Texas at San Antonio, 1 UTSA Circle, San Antonio, TX 78249

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U. S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

12 a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12 b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The use of energy-absorbing/dissipating materials in personnel helmets to reduce the effects of blast pressures can have unanticipated adverse effects. The presence and/or configuration of these materials can focus energy in such a way that it can cause unforeseen traumatic brain injuries. The purpose of this research is to develop a innovative modeling and simulation approach for rapidly quantifying metrics that characterize the energy absorption/dissipation capacity of candidate materials to be used in retrofitted helmets and other personnel protective equipment (PPE.) Metrics such as the saturation point of the material with respect to maximum blast loads, and the ratio of the total blast energy to the energy that is transmitted to the victim, are used to rate two candidate materials. Quantification of these metrics is accomplished using dynamic modeling and simulation technique, facilitated by impedance-based bond graphs. These models include novel elements whose constitutive laws are defined by fractional derivatives, which capture frequency-dependent viscoelastic and viscoinertial properties of energy absorbing/dissipating materials. Input forces caused by blast pressures, determined from computational fluid dynamics (CFD) analysis and simulation of common blast sources encountered in current conflicts, are used to generate the externally applied force inputs for material modeling simulations.

14. SUBJECT TERMS
material modeling, personnel protective equipment (PPE), energy absorbing materials, simulation, fractional calculus, explosive threats

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OR REPORT	18. SECURITY CLASSIFICATION ON THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev.2-89)
Prescribed by ANSI Std. 239-18
298-102

Enclosure 1

SUPPLEMENTAL REPORT DOCUMENTATION PAGE

(1) Submissions or publications under ARO sponsorship during this reporting period:

- (a) Papers published in peer-reviewed journals**: 0**
- (b) Papers published in non-peer-reviewed journals**: 0**
- (c) Presentations**: 2**
  - i. Presentations at meetings, but not published in Conference Proceedings**: 2**
    - [1] *Identifying Helmet Padding Material Dynamic Behavior in the Prevention and Mitigation of Traumatic Brain Injuries due to Improvised Explosive Devices*, Invited Presentation: The University of Texas – Pan American, Engineering Seminar Series, Spring 2009
    - [2] *Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials: Final Project Presentation*, presented to Shawn Walsh (Civ,ARL/WMRD), Aberdeen, MD, 20 Oct 2009
  - ii. Non-Peer-Reviewed Conference Proceeding publications**: 0**
  - iii. Peer-Reviewed Conference Proceeding publications**: 0**
- (d) Manuscripts**: 1**
  - Connolly, T.J., Hewitt, D., "Using Fractional Derivatives to Characterize the Response of Blast-Mitigating Protective Materials: Simulation and Experimental Validation", *Security Sensitive (withdrawn from submission for publication, per ARL request)*
- (e) Books**: 0**
- (f) Honor and Awards**: 0**
- (g) Title of Patents Disclosed during the reporting period**: 0**
- (h) Patents Awarded during the reporting period**: 0**

(2) Student/Supported Personnel Metrics for this Reporting Period:

- (a) Graduate Students**: 1**
  - Gabriel Cruz, 50% FTE Support (July-August 2009)
- (b) Post Doctorates**: 0**
- (c) Faculty**: 1**
  - Thomas Connolly, 100% FTE Support (July-August 2009)
- (d) Undergraduate Students**: 1**
  - Gabriel Cruz, 50% Full-Time Support (September-October 2009)
- (e) Graduating Undergraduate Metrics
  - i. Number who graduated during this period**: 1**
  - ii. Number who graduated during this period with a degree in science, mathematics, engineering, or technology fields**: 1**
  - iii. Number who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields**: 1**
  - iv. Number who achieved a 3.5 GPA to 4.0 (4.0 max scale) **: 1**
  - v. Number funded by a DoD funded Center of Excellence grant for Education, Research & Engineering**: 0**
  - vi. Number who intend to work for the Department of Defense**: 0**
  - vii. Number who will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields**: 1**
- (f) Masters Degrees Awarded**: 0**
- (g) Ph.D.s Awarded**: 0**
- (h) Other Research staff **: 0**

(3) Technology transfer**: N/A**

(4) Scientific Progress and Accomplishments**: CONTAINED IN ENCLOSED FINAL REPORT**

(5) Copies of technical reports: **N/A**

COPIES OF TECHNICAL MANUSCRIPT & PRESENTATION

These presentation and manuscript are contained in separate files, CONNOLLY – 55332EGII – Part2.pdf, and Part 3, respectively.

[1] *Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials: Final Project Presentation*, presented to Shawn WalshWalsh, Shawn (Civ,ARL/WMRD), Aberdeen, MD, 20 Oct 2009

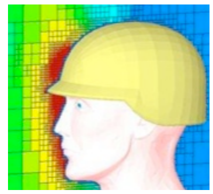
[2] Connolly, T.J., Hewitt, D., "Using Fractional Derivatives to Characterize the Response of Blast-Mitigating Protective Materials: Simulation and Experimental Validation", *Unpublished Manuscript – withdrawn from consideration for publication at the request of ARL for security concerns.*

Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials

Army Research Laboratory
STIR Project Final Presentation

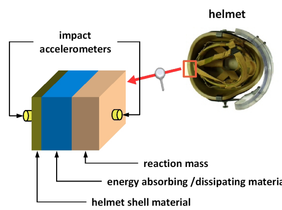
Grant # 55332EGII

20 October 2009



Thomas J. Connolly, Assistant Professor
Gabriel Cruz, Graduate Research Assistant

Department of Mechanical Engineering
The University of Texas at San Antonio



In Memory

Staff Sgt. Christopher N. Staats

Texas National Guard, 143rd Infantry Detachment out of Camp Mabry, in Austin, TX

Died October 16, 2009 of injuries suffered when his vehicle was attacked by an improvised explosive device in the Wardak province of Afghanistan.

Motivation

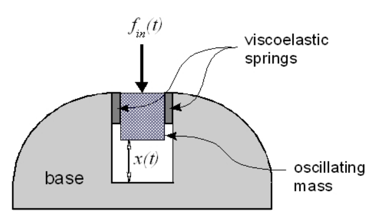
- Urgent need for effective padding materials to minimize severity of blast-related injuries due to improvised explosive devices (IEDs)
- For use in helmets, personnel protective equipment (PPE), vehicle interiors, etc.
- Difficulty in injury detection and the resulting intractability of injuries results in long-term treatment, suffering, and costs to the Army
- Need a method for rapidly evaluating efficacy of candidate materials for various applications

ARL UTSA

Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials
Short-Term Innovative Research (STIR) Grant # 55332EGII, Final Presentation, 20 October 2009

Determination of Viscoelastic Properties of Foam Materials

- GOAL: Determine material response (transfer function) in the frequency domain
- RATIONALE: Preliminary part of blast wave imparts an impulsive force on the victim
- Input provided by impact hammer; force = *F(s)*
- Output is the acceleration of the mass, *A(s)*



- Fractional-order models that feature a three parameter model in which "fractional derivatives" PROVIDE MORE ACCURACY AND FLEXIBILITY IN MODELING DYNAMIC BEHAVIOR OF FOAM.

ARL UTSA

Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials
Short-Term Innovative Research (STIR) Grant # 55332EGII, Final Presentation, 20 October 2009

Challenges

- Most padding materials exhibit viscoelastic behavior
- Dynamic behavior is nonlinear and frequency dependent
- Cannot be accurately modeled using simple lumped-parameter elements
  - spring, k (energy storage)
  - damper, b (energy dissipation

Determination of Viscoelastic Properties of Foam Materials

- Previously considered two approaches to modeling dynamic behavior of foam
  1. Integer-order transfer function, featuring "traditional" linear springs and dampers, i.e., Hooke's Law springs and viscous damping, respectively
  2. **Fractional-order** models that feature a three parameter model in which "fractional derivatives" are used

- SIMULATIONS INDICATE THAT **OPTION #2** PROVIDES MORE ACCURACY AND FLEXIBILITY IN MODELING DYNAMIC BEHAVIOR OF FOAM.

Fractional Derivative???

$\alpha = 0$ $\alpha = 1$

spring ⟷ damper

$F = kx$ $F = b\dot{x}$

viscoelastic material

$$F = \gamma x^{(\alpha)}$$

Fractional Derivatives to Model Foam Behavior

- Use a three-parameter impedance-based (frequency domain) model to represent viscoelastic behavior of the foam based on force input and acceleration response data
  - G_0 = shear modulus
  - G_1 = modulus that represents shear stiffness and dissipative behavior of the foam
  - α = order of the fractional derivative; used to determine G_0 and G_1

represents pure stiffness behavior of material (G_0)

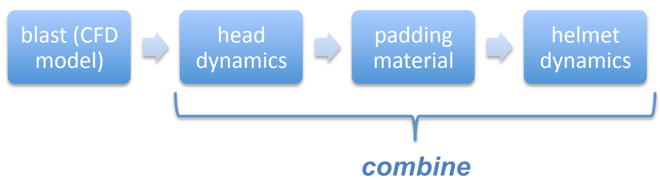
*represents a combination of stiffness **and** dissipation behavior of material* (G_1)

$$\underbrace{G_0 + s^{\alpha} G_1}_{model} = \underbrace{-\frac{\delta}{2A}\,\omega^2 \left[\left(\frac{A(s)}{F(s)} \right)^{-1} - m_t \right]}_{experimental\ data}$$

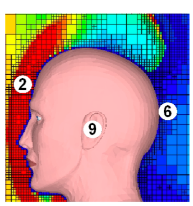
ARL UTSA
Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials
Short-Term Innovative Research (STIR) Grant # 55332EGII, Final Presentation, 20 October 2009

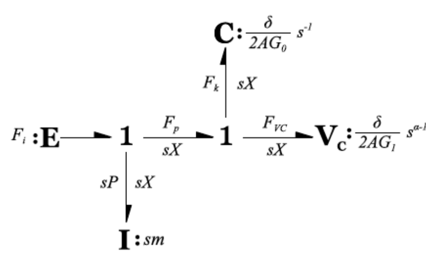
Advantages

- Ability to use lumped-parameter modeling techniques for simulations
  - simplified; increased accessibility
  - more unified modeling approach; fewer interfaces between models



Overall Modeling Approach





CFD model provides incident blast pressure time history

bond graph model with integer-order and fractional-order lumped parameter elements

Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials
Short-Term Innovative Research (STIR) Grant # 55332EGII, Final Presentation, 20 October 2009

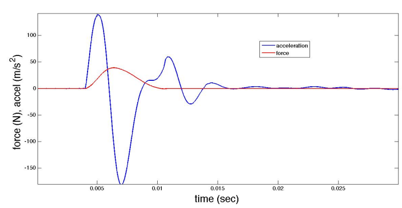
Padding Materials Used in Testing

- Used two types of commercially available foam in lieu of candidate padding materials to be provided by ARL

padding material	density (g/cm^3)	cell type	color
Ethylene Vinyl Acetate	0.032	closed	white
Acrylonitrile Butadiene Rubber / Polyvinyl Chloride	0.088	closed	black

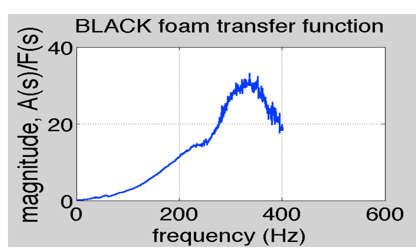
Determining Model from Experimental Data

1. Acquire input force and acceleration response data from impact testing



2. Compute averaged power spectra from five experimental runs

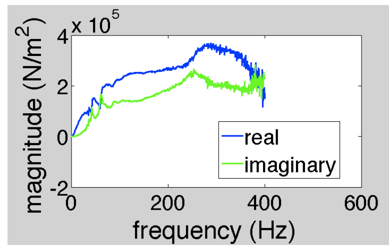
3. Apply curve-fitting algorithm to compute a plot of the transfer function between input force and acceleration of mass



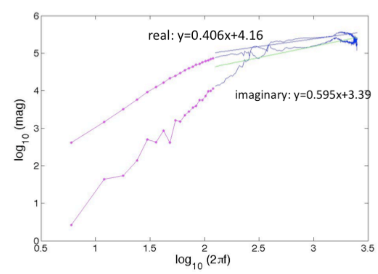
Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials
Short-Term Innovative Research (STIR) Grant # 55332EGII, Final Presentation, 20 October 2009

Determining Model from Experimental Data

4. Compute real and imaginary parts of transfer function data

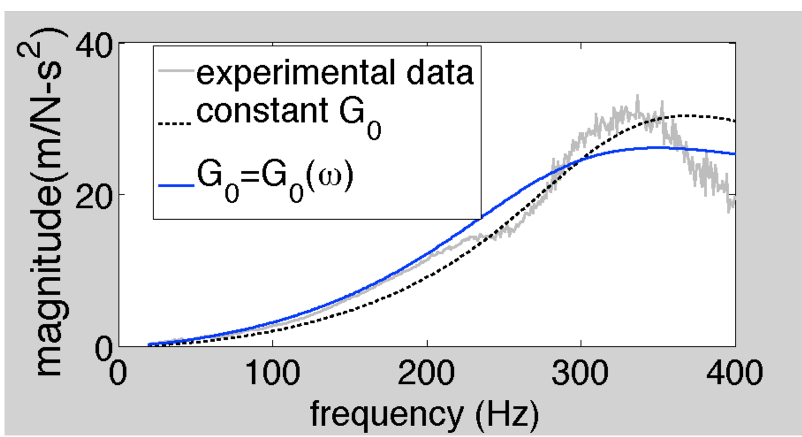


5. Perform a linear regression on each part



Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials
Short-Term Innovative Research (STIR) Grant # 55332EGII, Final Presentation, 20 October 2009

Transfer Function Curve Fit Results

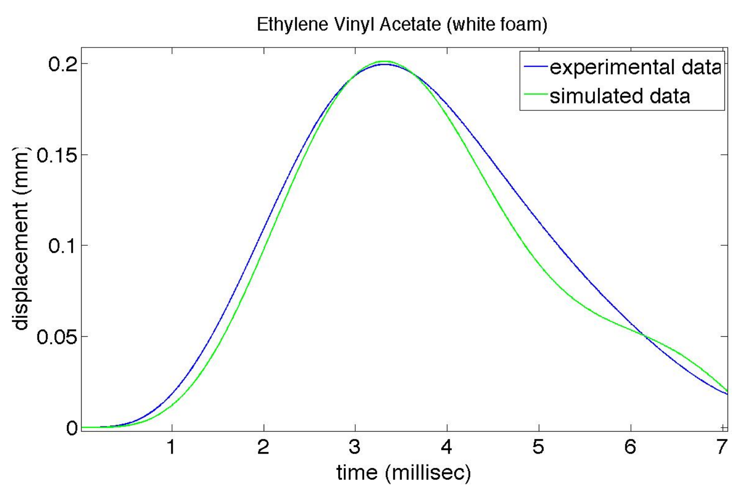


Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials
Short-Term Innovative Research (STIR) Grant # 55332EGII, Final Presentation, 20 October 2009

Results: Material Parameters

padding material	color	α	G_0 (N/m^2)	G_1 (N-s$^\alpha$/m^2)
Ethylene Vinyl Acetate	white	0.56	3.00×10^5	3.19×10^3
Acrylonitrile Butadiene Rubber / PVC	black	0.60	2.39×10^5	3.05×10^3

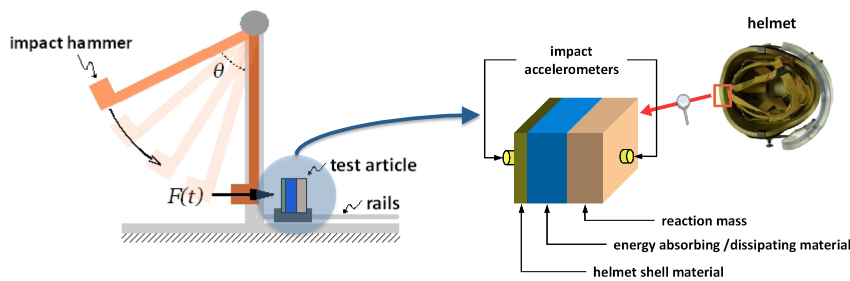
Validation of Model vs. Experimental Data



Material Model Simulations

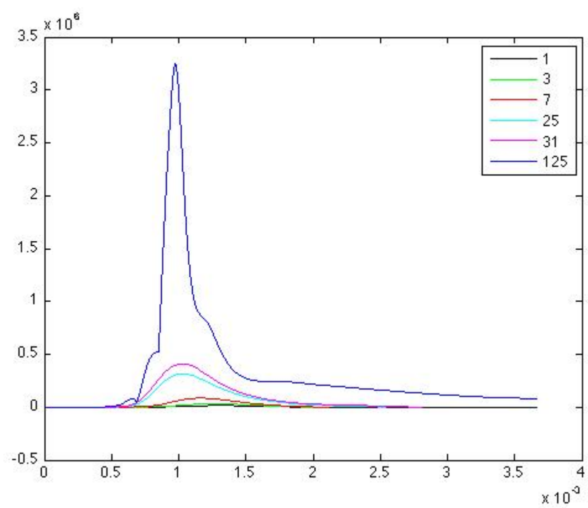
GOALS

1. Verify that three-parameter model material properties that were determined from impact tests are accurate
2. Determine onset of **saturation**: material can no longer store or dissipate energy due to blast pressures; maximally compressed during loading; acts like a mass, i.e., compliance and dissipative properties are effectively equal to zero



Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials
Short-Term Innovative Research (STIR) Grant # 55332EGII, Final Presentation, 20 October 2009

Blast Pressure Time Histories from CFD Model



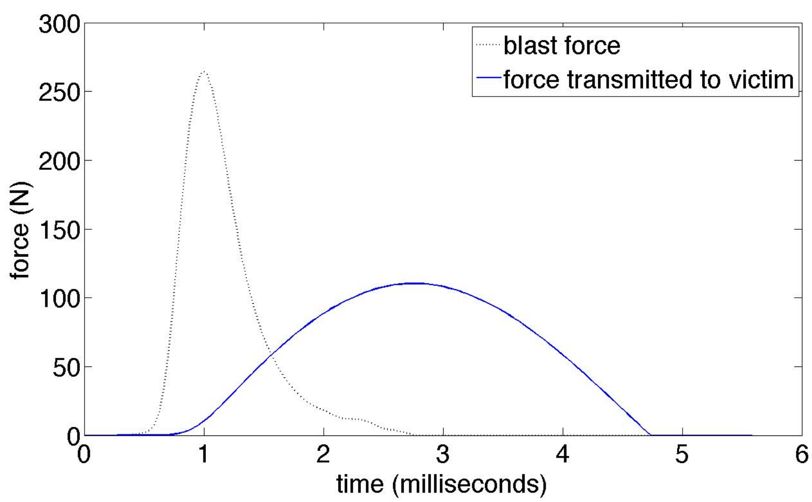
Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials
Short-Term Innovative Research (STIR) Grant # 55332EGII, Final Presentation, 20 October 2009

Model Assumptions

- Frontal area of helmet = 0.05 m^2
- Foam pad thickness = 5mm
- Saturation occurs when pad is compressed down to 1mm
- Victim is knocked over by blast, i.e., no impact with objects such as a vehicle, structure, etc.
  - results in a free-free boundary condition for helmet mass and mass of the victim's head
  - victim's resistance to motion can be easily incorporated into model

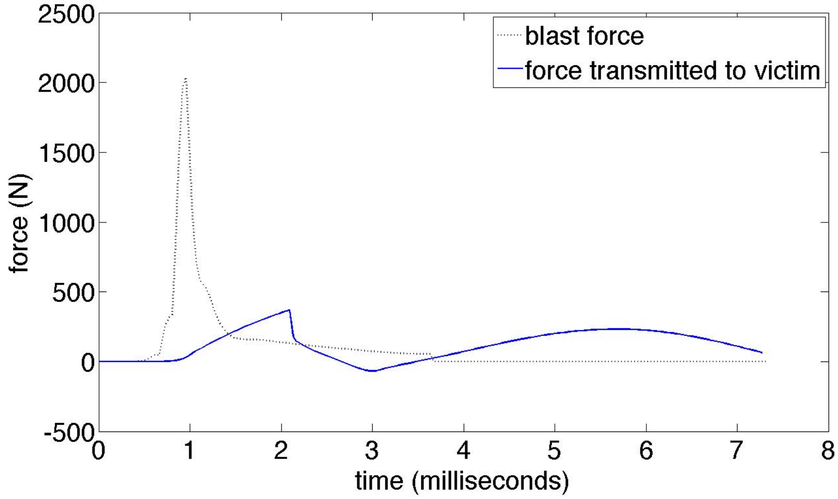
ARL UTSA *Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials*
Short-Term Innovative Research (STIR) Grant # 55332EGII, Final Presentation, 20 October 2009

Example Simulation – No Saturation



ARL UTSA *Rapid Quantification of Energy Absorption & Dissipation Metrics for PPE Padding Materials*
Short-Term Innovative Research (STIR) Grant # 55332EGII, Final Presentation, 20 October 2009

Example Simulation – Saturation



Summary of Dynamic Modeling Results

foam	blast ID	peak blast pressure	peak blast pressure	peak blast force	peak blast force	peak transmitted force	peak transmitted force	attenuation of peak force (trans/blast)	max compression	saturation
		Pa	*lbf/in^2*	*N*	*lbf*	*N*	*lbf*	--	*mm*	--
white	1	1.57E+04	2.27E+00	9.91	2.23	4.53	1.02	46%	0.013	NO
black	1	1.57E+04	2.27E+00	9.91	2.23	4.22	0.95	43%	0.014	NO
white	3	3.28E+04	4.76E+00	21.1	4.74	9.77	2.20	46%	0.028	NO
black	3	3.28E+04	4.76E+00	21.1	4.74	9.16	2.06	43%	0.028	NO
white	7	8.60E+04	1.25E+01	55.5	12.50	24.6	5.53	44%	0.071	NO
black	7	8.60E+04	1.25E+01	55.5	12.50	23.1	5.19	42%	0.071	NO
white	25	3.15E+05	4.57E+01	203	45.70	90.6	20.37	45%	0.261	NO
black	25	3.15E+05	4.57E+01	203	45.70	85	19.11	42%	0.267	NO
white	31	4.10E+05	5.95E+01	265	59.60	117	26.30	44%	0.339	NO
black	31	4.10E+05	5.95E+01	265	59.60	110	24.73	42%	0.341	NO
white	125	3.25E+06	4.71E+02	2037	457.00	376	84.53	18%	1.000	YES
black	125	3.25E+06	4.71E+02	2037	457.00	368	82.73	18%	1.000	YES

Summary

- Fractional-order modeling of material models provides accurate results
- Modeling technique provides access to multiple metrics that can be used to quantify desired padding material properties
- Overall approach can be expanded to include wider range of blast scenarios and PPE applications

Future Work

- Improve computation performance of fractional derivative algorithm
- Use actual helmet and padding materials
- Expand blast scenarios beyond helmet applications
  - vehicle interior padding
  - other PPE
- Investigate other dynamically complex materials that are of interest to the Army
  - crushable materials
  - others???

Using Fractional Derivatives to Simulate the Dynamic Behavior of Padding Materials for Blast Mitigation

Thomas J. Connolly, Assistant Professor
Donna Hewitt, Graduate Research Assistant

Department of Mechanical Engineering
The University of Texas at San Antonio
1 UTSA Circle
San Antonio, TX 78249
United States of America

This paper concerns the development of a combined experimental and simulation approach to assist in rapidly quantifying metrics that characterize the energy absorption/dissipation capacity of candidate materials to be used in retrofitted helmets and other personnel protective equipment (PPE) in an explosion environment. The dynamics of a cross section of a helmet, padding material, and a proportional mass that represents the head of the victim, are modeled using impedance-based bond graphs that feature primitives whose constitutive behaviors are described using fractional derivatives. Force inputs to the model are derived from computational fluid dynamic (CFD) models of improvised explosive devices (IED) with a range of net explosive weights (NEW.) Metrics such as the saturation point of the material with respect to maximum blast loads, and the ratio of the total blast energy to the energy that is transmitted to the victim, are used to rate two candidate materials.

1 Introduction and Thesis

The use of energy-absorbing/dissipating materials in personnel helmets to reduce the effects of blast pressures can have unanticipated adverse effects. The presence and/or configuration of these materials can focus energy in such a way that it can cause unforeseen traumatic brain injuries (TBI). There is a wide range of candidate materials, particularly viscoelastic materials, whose abilities to both dissipate and absorb energy vary considerably. These abilities are difficult to quantify and model using linear lumped-parameter modeling techniques and bulk material properties. This challenge is further complicated, given the extreme nature of the input forcing functions encountered in a blast environment, i.e., short-duration impulses that result in high rates of energy transfer to a victim.

The authors posit that using fractional-derviative-based material models for viscoelastic padding materials, allows the use of a lumped-parameter modeling approach, which will yield more accurate results than those with linear material models. This approach eliminates costly and time-consuming computations associated with finite element modeling approaches, which are not necessary for significant portions of the model that are relatively incompressible, i.e., the helmet shell material and the victim's head, as compared to the helmet padding material.

2 Overall Approach

The overall approach of the work presented in this paper is shown in Figure 1. To assess the dynamic behavior of the helmet padding material applied as a protective material in explosive environments, it was necessary to determine input forces that are imparted to a victim in a wide range of explosive events, i.e., of varying intensity. Clutter and Connolly [2] performed an interdisciplinary study in which computational

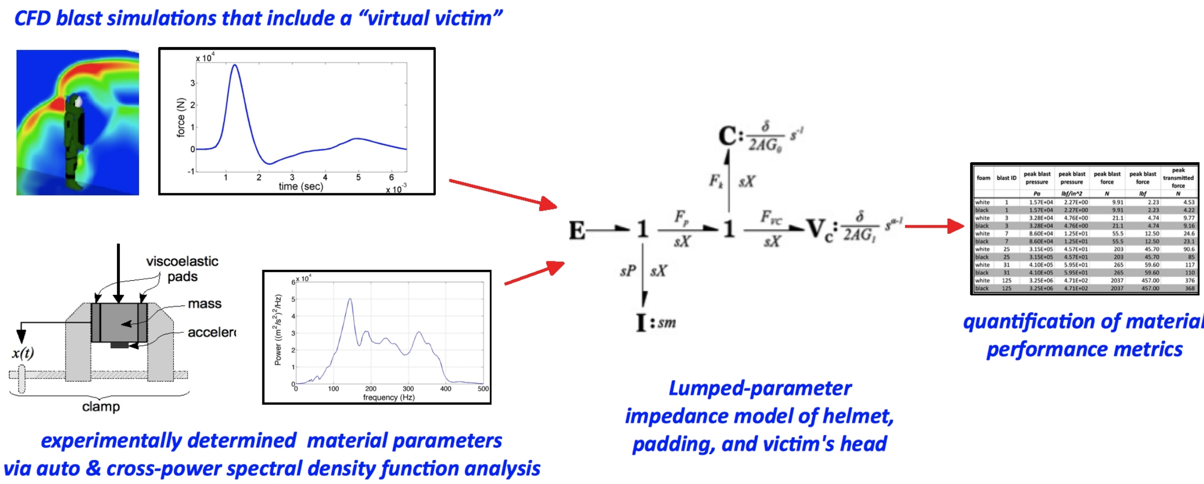


Figure 1: Overall Approach

fluid dynamic analysis (CFD) was used to generate time histories of blast pressures as experienced by a virtual victim. These time histories were used to compute Input forces, as determined by the pressures at the center of the victim's helmet, as shown in Figure 2. An example of an input force time history for a particular net explosive weight, i.e., explosion intensity, is shown in Figure 3. Material properties of the foam

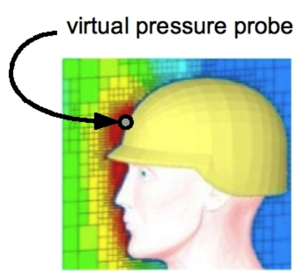


Figure 2: CFD model of blast interaction with victim

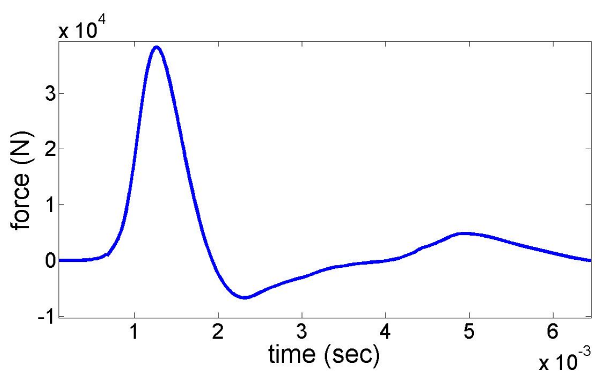


Figure 3: Time history of blast force transmitted to victim

padding materials are determined by subjecting them to a impulsive force input, which is similar to that of an explosion, albeit on a smaller scale. Acceleration and force data from these experiments are used to form experimentally determined transfer functions - that feature fractional exponents of the complex frequency, s, from which material properties are extracted. The experimental setup and protocols are detailed in Section 3 and the procedure for determination of material properties in the context of a fractional-derivative model are detailed in Section 4.

3 Experimental Setup and Protocols

The experimental test setup is shown in Figure 4. The test article consisted of two pieces of viscoelastic

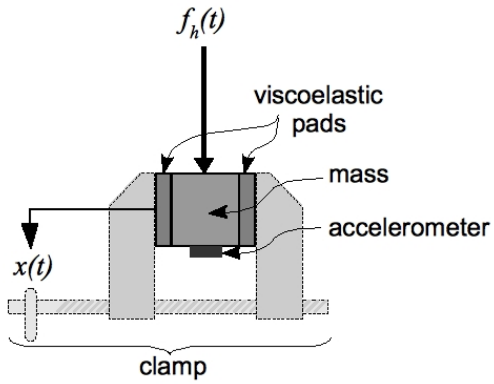


Figure 4: Experimental setup

foam, 0.25 inch thick by 1 .25 inch square, adhered to an aluminum block, 0.5 inch thick by 1.25 inch square, using a thin layer of ethyl cyanoacrylate applied with a brush. A piezoelectric accelerometer was attached in the z coordinate through a threaded hole at the bottom of the aluminum block. The test article was secured using a vice grip and held in minimal compression. An impulse hammer implemented with a piezoelectric accelerometer was used to tap the specimen. Data was collected at a rate of 1 kHz with a USB data acquisition accessory and LabView^{TM}running on a notebook computer. The experimental procedure was repeated five times for each type of padding material.

Issues that were encountered and their subsequent resolutions included:

1. A 60 Hz ground loop that was eliminated by creating one common ground among all electronics.

2. Double-hit phenomena were encountered when using harder tips because of the deformation of the aluminum. Therefore, it was necessary to use the softest tip available in the kit.

3. Aliasing issues were a problem due to the short nature of the test, which were remedied by using a higher sampling rate.

4. The natural frequency of the system was too low to obtain viable acceleration data - this required using an aluminum block instead of titanium, which had been the preferred metal, due to its lower loss coefficient. Further, this also required a thinning of the foam pads to increase the effective stiffness of the test article. Two test specimens, each with a different viscoelastic padding material were used for the tests. The physical characteristics of these materials are given in Table 1.

4 Helmet Padding Model Using Fractional Derivatives

A schematic that represents a cross section of the helmet, padding material, and a proportional mass that represents the victim's head is shown in Figure 5. The corresponding bond graph model of the dynamics of this systems is shown in Figure 6.

padding material	density (g/cm^3)	cell type	color
Ethylene Vinyl Acetate	0.032	closed	white
Acrylonitrile Butadiene Rubber / Polyvinyl Chloride	0.088	closed	black

Table 1: Padding Material Physical Characteristics

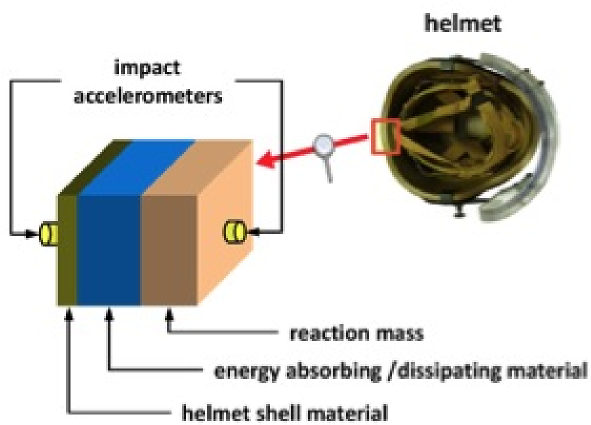


Figure 5: Helmet cross section

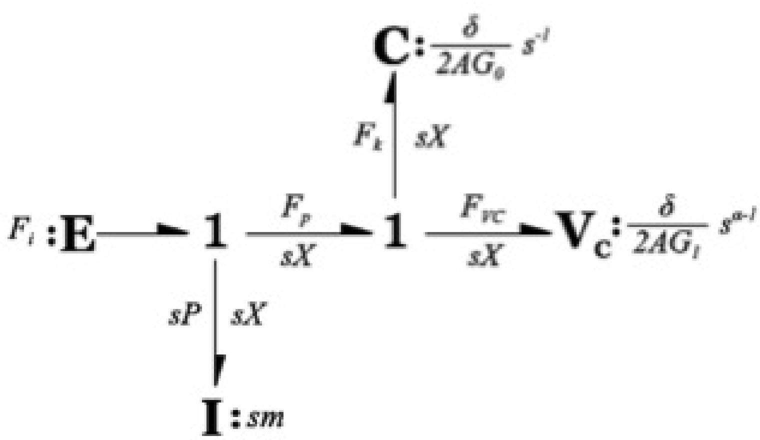


Figure 6: Impedance-based bond graph with viscoelastic primitive element

The experimentally determined transfer function is extracted using impedance-based techniques [4].

$$\frac{A(s)}{F(s)} = \frac{s^2X(s)}{F(s)} \tag{1}$$

where $A(s)$ is the acceleration of the mass and $F(s)$ is the input force to the mass. From the generalized spring-mass-damper model, we have

$$F(s) = m_t s^2 X(s) + H(s)X(s) \tag{2}$$

where the second term represents the combined damping and restoring forces, i.e.,

$$H(s)X(s) = F_{spring} + F_{damp} \tag{3}$$

For a classical mechanical oscillator, this term would simply be

$$H(s)X(s) = (k + sb)\, X(s) \tag{4}$$

If we wish to consider a general form in which the combined forces are defined by a fractional derivative, we have

$$H(s)X(s) = (\kappa + s^{\alpha}\beta)\, X(s) \tag{5}$$

where κ is a pure capacitive term, β is a combined capacitive and resistive term, and α is the order of the fractional differentiation.

Returning to Equation 2 and isolating this term yields

$$\frac{F(s)}{s^2 X(s)} = m_t + \frac{H(s)}{s^2} \tag{6}$$

$$H(s) = s^2 \left[\frac{F(s)}{s^2 X(s)} - m_t \right] = s^2 \left[\left(\frac{A(s)}{F(s)} \right)^{-1} - \ m_t \right] \tag{7}$$

Where $\left(\frac{A(s)}{F(s)}\right)^{-1}$ is the inverse of the experimentally determined transfer function, and m_t is the sum of the masses of the titanium cube, foam pads, and accelerometer, as measured using a digital scale. Thus, the experimental data that represents the bracketed term on the right hand side of Equation 7 is multiplied by s^2 to yield the data that represents $H(s)$, i.e.,

$$s \to i\omega; \quad H(s) = -\omega^2 \left[\left(\frac{A(s)}{F(s)} \right)^{-1} - \ m_t \right] \tag{8}$$

Note that $H(s)$ in Equation 8 has units of kg/s^2. Solving Equation 5 for $H(s)$ and equating with Equation 8 begins to illustrate the procedure for characterizing the material behavior

$$H(s) = (\kappa + s^{\alpha}\beta) = -\omega^2 \left[\left(\frac{A(s)}{F(s)} \right)^{-1} - \ m_t \right] \tag{9}$$

Replacing the lumped-parameter constants κ and β with the following bulk material moduli,

$$\kappa = \frac{2A}{\delta} G_0 \quad , \quad \beta = \frac{2A}{\delta} G_1 \tag{10}$$

where A and δ are the foam pad cross-sectional area and thickness, respectively, results in an equation that relates the model to experimental data

$$\underbrace{G_0 + s^{\alpha} G_1}_{model} = \underbrace{-\frac{\delta}{2A}\, \omega^2 \left[\left(\frac{A(s)}{F(s)} \right)^{-1} - \ m_t \right]}_{experimental\ data} \tag{11}$$

This demonstrates that the experimental data can be represented by a three-parameter model of the material behavior: G_0, G_1, and α , as formulated by Bagley, et al [1]. Setting $s = i\omega$ on the left side of Equation 11 and applying Euler's identity yields the following

$$G_0 + s^\alpha G_1 = G_0 + G_1\omega^\alpha \left(\cos\frac{\alpha\pi}{2} + i\sin\frac{\alpha\pi}{2}\right) \tag{12}$$

From Equation 12 , the real and imaginary parts of Equation 11 can be expressed as

$$\textbf{Re}\ (\text{data}) = G_0 + \left(G_1\cos\frac{\alpha\pi}{2}\right)\omega^\alpha \tag{13}$$

$$\textbf{Im}\ (\text{data}) = \left(G_1\sin\frac{\alpha\pi}{2}\right)\omega^\alpha \tag{14}$$

A plot of the real and imaginary parts of the experimentally determined transfer function for a particular material is shown in Figure 7 Where **Re** (data) and **Im** (data) are the real and imaginary parts of the right side

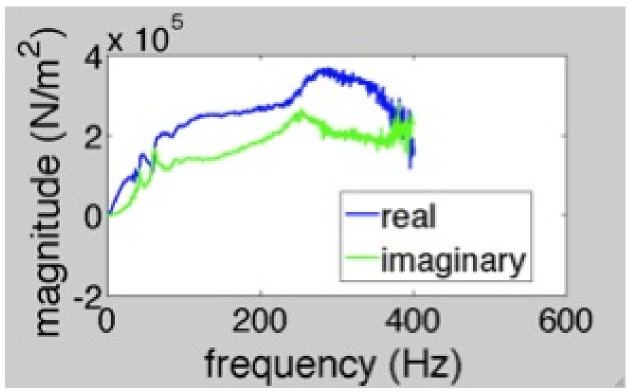


Figure 7: Real and Imaginary parts of transfer function

of Equation 11, as determined from the experimental data. Taking the logarithm of both sides of Equation 14 indicates a linear trend - where $\log\omega$ is the independent variable - from which the order of fractional differentiation, α, can be estimated

$$\log\left[\textbf{Im}\ (\text{data})\right] = \alpha\log\omega + \log\left(G_1\sin\frac{\alpha\pi}{2}\right) \tag{15}$$

Thus, α can be obtained from the following using the resulting slope and y-intercept from a linear regression, m and b, respectively

$$m = \alpha; \quad b = \log\left(G_1\sin\frac{\alpha\pi}{2}\right) \tag{16}$$

Using a base ten logarithm, the equation for G_1 becomes

$$G_1 = \frac{10^b}{\sin\frac{\alpha\pi}{2}} \tag{17}$$

Having obtained two of the three parameters of the model, G_0 remains to be determined. Substituting the expressions for α and G_1 into equation 13 and rearranging yields

$$G_0 = 10^{\textbf{Re}\ (\text{data})} - \left(G_1\cos\frac{\alpha\pi}{2}\right)\omega^\alpha \tag{18}$$

Note that, according to the above equation, G_0 is a function of the frequency, ω, and thus would *not* appear to fit into the scheme of a linear model. However, a plot of G_0 vs ω, for a typical viscoelastic foam shows that there is minimal variation, i.e., less than three percent error, over the frequency range of interest, as indicated in Figures 8 and 9 .   A plot of the curve fit for the experimentally determined transfer function

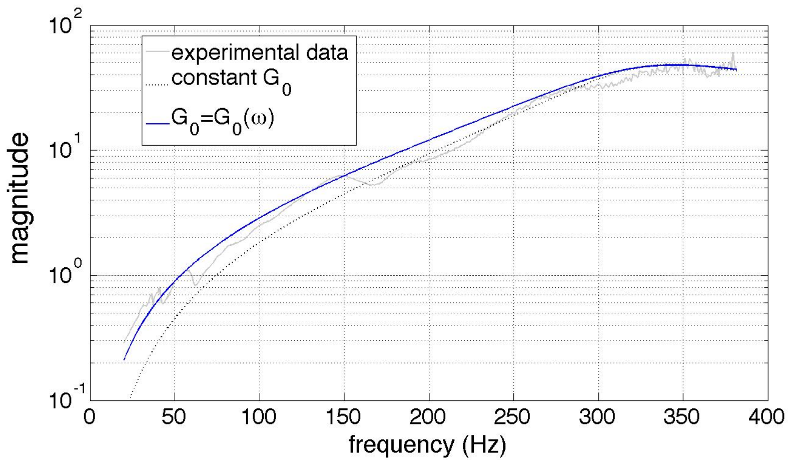


Figure 8: comparison of $G_0(\omega)$ and constant G_0

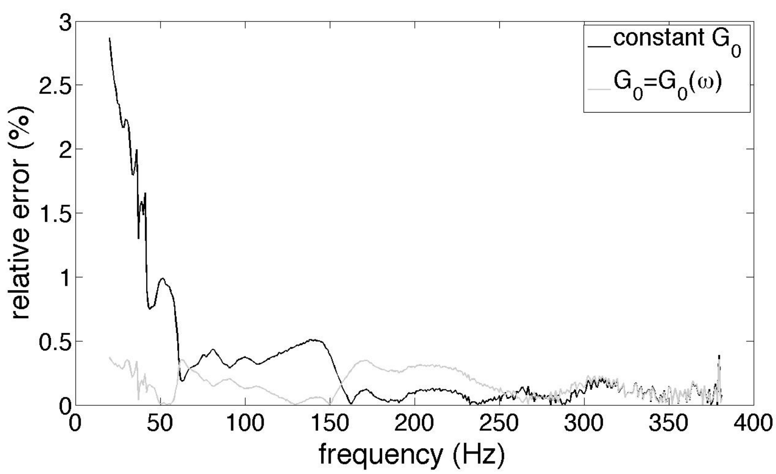


Figure 9: relative error between $G_0(\omega)$ and constant G_0

for a particular candidate material is shown in Figure 10 The curve fit parameters are the experimentally determined material properties, as given in Table 2.

padding material	**color**	α	$\mathbf{G_0}$ (N/m^2)	$\mathbf{G_1}$ (N-s$^\alpha$/m^2)
Ethylene Vinyl Acetate	white	0.56	3.00×10^5	3.19×10^3
Acrylonitrile Butadiene Rubber / PVC	black	0.60	2.39×10^5	3.05×10^3

Table 2: Experimentally Determined Properties of Padding Materials

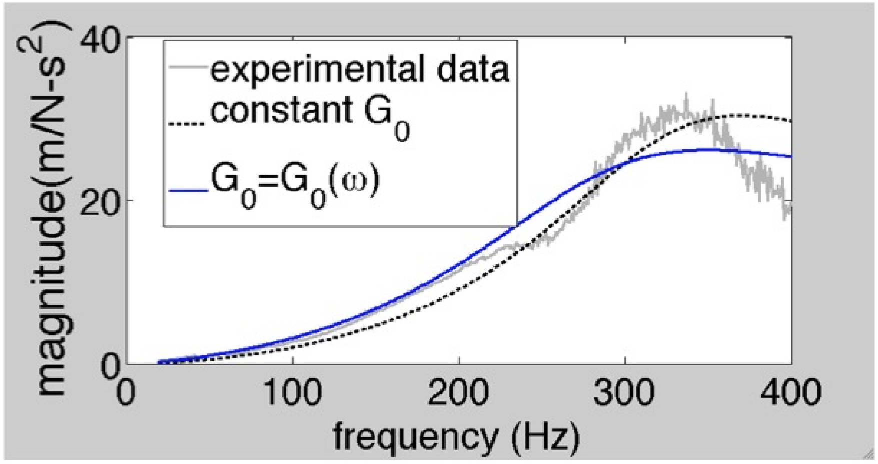


Figure 10: Curve fit for experimentally determined transfer function

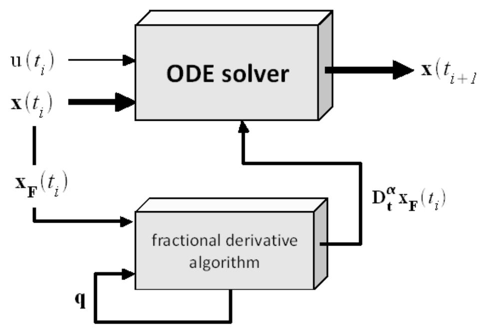


Figure 11: Computational approach for mixed state-space equations

5 Computational and Simulation Techniques

The equations of state extracted from the bond graph model constitute a *mixed state-space formulation* of the system of differential equations, whose general form is shown in Equation 19.

$$\dot{\mathbf{x}} = \mathbf{A} \begin{Bmatrix} \mathbf{x_I} \\ \cdots \\ \mathbf{x_F} \end{Bmatrix} + \mathbf{B}\, u(t) + \mathbf{\Phi}\, \mathbf{D}_t^{\alpha} \left[\begin{Bmatrix} \mathbf{0} \\ \cdots \\ \mathbf{x_F} \end{Bmatrix} \right] \tag{19}$$

The solution requires an approach where the fractional derivative of a state variable is explicitly computed [3]. Figure 11 illustrates this approach, where the differential equations are solved using an existing variable-step fourth-order Runge-Kutta algorithm which is linked to a subroutine that computes the fractional derivatives of the *necessary state variables only*, at each time step, t_i. The simulations are run using MATLAB$^{\text{TM}}$with a fixed time-step size of 10^{-5} sec.

6 Simulation Results

To assess the validity of the state equations that model the dynamics of the helmet and padding materials, simulations were run to determine whether the simulation results would match the experimental results

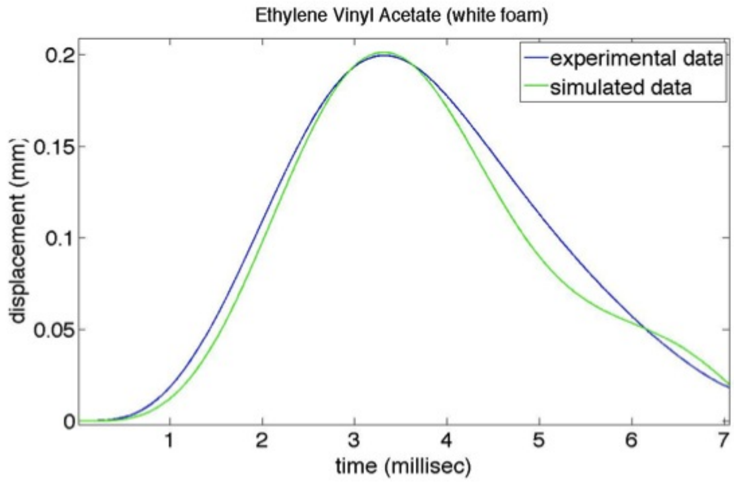


Figure 12: Validation of state equations via comparison with experimental data

foam	blast ID	peak blast pressure	peak blast pressure	peak blast force	peak blast force	peak transmitted force	peak transmitted force	attenuation of peak force (trans/blast)	max compression	saturation
		Pa	*lbf/in^2*	*N*	*lbf*	*N*	*lbf*	--	*mm*	--
white	1	1.57E+04	2.27E+00	9.91	2.23	4.53	1.02	46%	0.013	NO
black	1	1.57E+04	2.27E+00	9.91	2.23	4.22	0.95	43%	0.014	NO
white	3	3.28E+04	4.76E+00	21.1	4.74	9.77	2.20	46%	0.028	NO
black	3	3.28E+04	4.76E+00	21.1	4.74	9.16	2.06	43%	0.028	NO
white	7	8.60E+04	1.25E+01	55.5	12.50	24.6	5.53	44%	0.071	NO
black	7	8.60E+04	1.25E+01	55.5	12.50	23.1	5.19	42%	0.071	NO
white	25	3.15E+05	4.57E+01	203	45.70	90.6	20.37	45%	0.261	NO
black	25	3.15E+05	4.57E+01	203	45.70	85	19.11	42%	0.267	NO
white	31	4.10E+05	5.95E+01	265	59.60	117	26.30	44%	0.339	NO
black	31	4.10E+05	5.95E+01	265	59.60	110	24.73	42%	0.341	NO
white	125	3.25E+06	4.71E+02	2037	457.00	376	84.53	18%	1.000	YES
black	125	3.25E+06	4.71E+02	2037	457.00	368	82.73	18%	1.000	YES

Figure 13: Summary of all simulation runs and dynamic behavior metrics for various blast intensities and padding materials

obtained as described in Section 3, i.e., the input force was set to the that of the impact hammer for the material property experiments, instead of the forces obtained from the CFD models of the explosions. It was demonstrated that there was good agreement between the simulated and experimental results, as shown by the overlay plot in Figure **??**. Simulations were run for both materials subjected to a range of blast intensities, as gather from the CFD analysis study cited in Section 2. The metrics used to identify material behavior were:

1. The peak force transmitted to the victim by the padding material, expressed in newtons and lb-force
2. The attenuation factor, i.e., the ratio of the peak transmitted force and the peak blast force, expressed as a percentage
3. The maximum compression of the padding material, expressed in mm
4. Whether or not the material became saturated, i.e., the material reached its maximum possible compression

Figure 12 summarizes the simulation results for various blast intensities and padding materials. Example simulation results for cases of material non-saturation and saturation are shown in Figures 13 and 14,

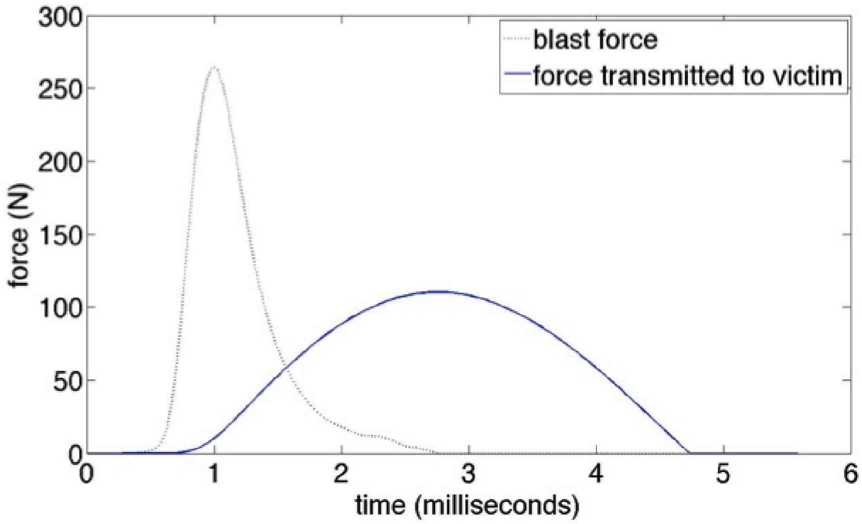


Figure 14: Case in which padding material does not reach saturation

respectively. These plots and the summary of results demonstrate how the combined experimental and simulation-based approach for assessing the efficacy of materials in blast applications for actual padding materials that are to be considered for helmet and other PPE applications.

7 Acknowledgements

The work presented in this paper was performed under the auspices of a grant from the United States Department of Defense, Army Research Office: Mechanical Sciences Division, through a Short Term Innovative Research Grant (STIR), Grant # 55332EGII. Special thanks to Dr. Ronald Bagley, Professor of Mechanical Engineering at The University of Texas at San Antonio, for his guidance in the application of fractional calculus theory used in this study.

References

[1] R.L. Bagley and P.J. Torvik. On the appearance of the fractional derivative in the behavior of real materials. *J. Applied Mechanics*, 51:294–298, 1984.

[2] T.J. Connolly; J.K. Clutter. Modeling head motion during explosive events to assess brain injury severity in a battlefield environment. In *IMECE 2007: International Mechanical Engineering Congress and Exposition*, Seattle, WA, 2007. American Society of Mechanical Engineers.

[3] T.J. Connolly; J.A. Contreras. New bond graph primitive elements for modeling systems that exhibit behavior that can be represented by fractional derivatives. *J. Fractional Calculus and Applied Analysis*, 12(4), 2009.

[4] D. Karnopp, D. Margolis, and R. Rosenberg. *System Dynamics: Modeling and Control of Mechatronic Systems*. John Wiley and Sons, Hoboken, NJ, 3d edition, 2006.

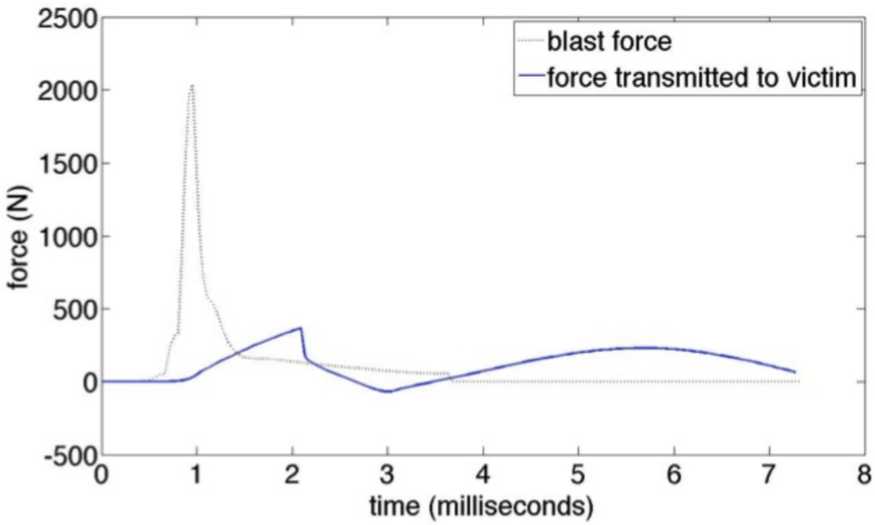


Figure 15: Case in which padding material reaches saturation