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EFFECT OF BANDWIDTH ON WIDEBAND-STAP PERFORMANCE (BRIEFING CHARTS)

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14. ABSTRACT
A wideband signal occupies a finite bandwidth that is significant compared to its carrier frequency. As a result, when transmitted, its returns cause bandwidth dispersion across the antenna. It is shown here that the effect of the finite bandwidth is to introduce a set of uncorrelated return signals for every physical scatter in the field. Further, each such uncorrelated return contains a set of coherent signals with different directional and Doppler components that result from a jittering effect both in angle and Doppler domain. As a result, adaptive clutter cancellation using traditional processing schemes does not work well. Although in principle it is possible to correct these decorrelating effects by 3D spacetime adaptive processing (STAP), the present day methods are quite costly and difficult to implement. In addition to the new wide band signal modeling framework mentioned above, we outline a hierarchical processing scheme which has the potential for dramatically reducing both processing and sample support burdens.

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Effect of Bandwidth on Wideband-STAP Performance

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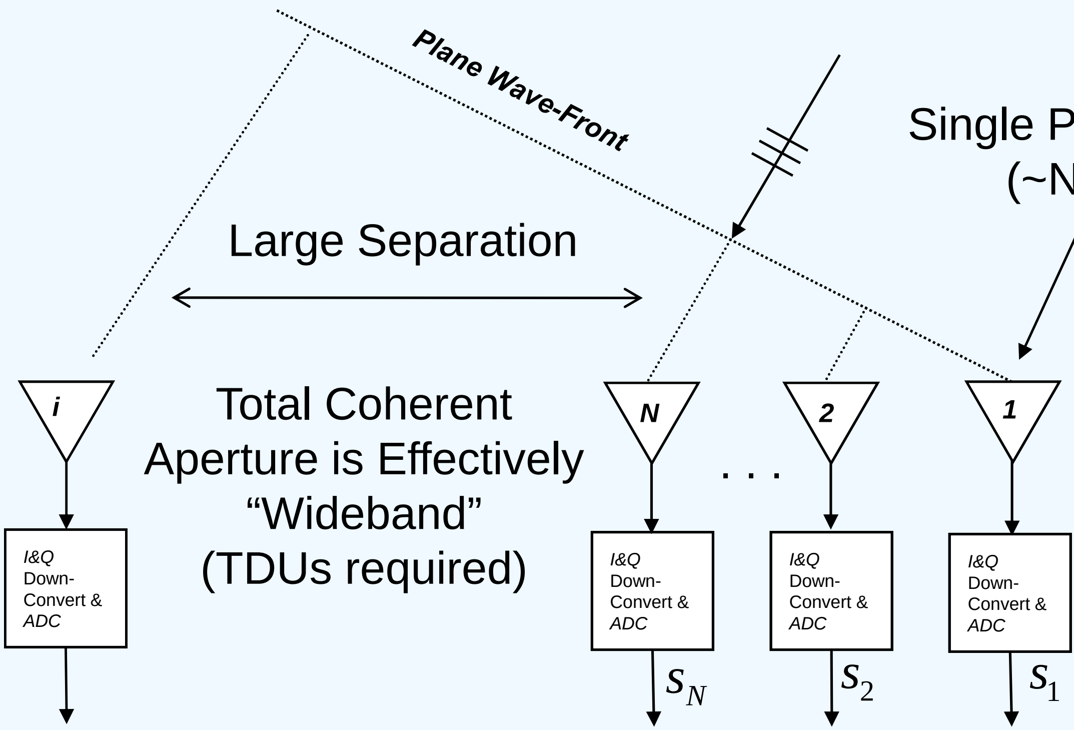
Outline

- Wideband array
- Frequency sensitive gain pattern
- Effect of wideband on STAP
- Wideband data model – A new theory
- Wideband and coherency
- Conclusions

Wideband

Large separation between multiple transmitters and/or receivers results in significant antenna dispersion even for modest bandwidths!



Wideband Condition:

$$\frac{\text{Array Length}}{c} > \frac{1}{BW}$$

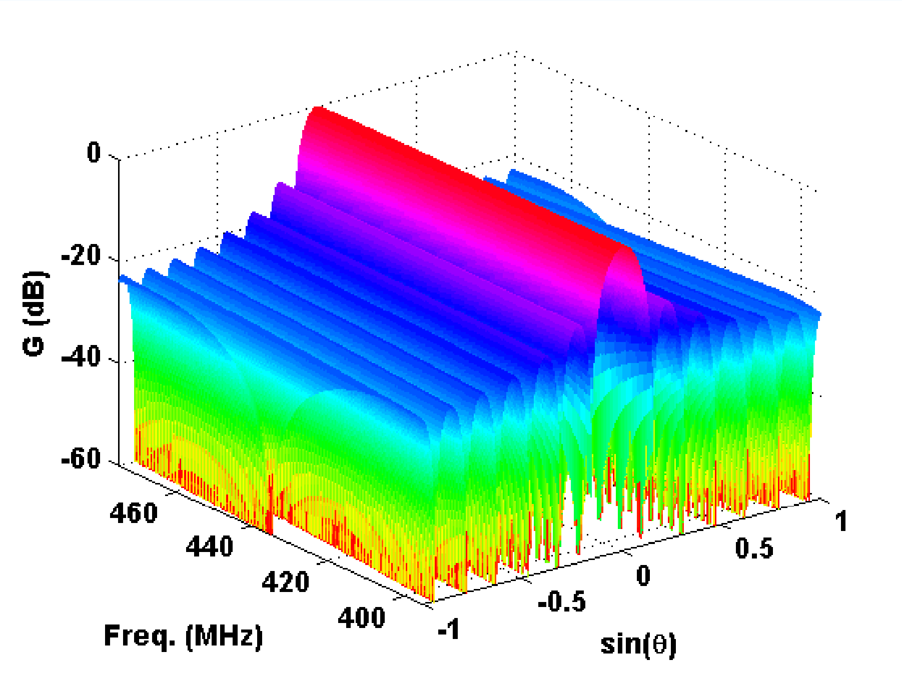
Frequency Sensitive Array Gain Pattern

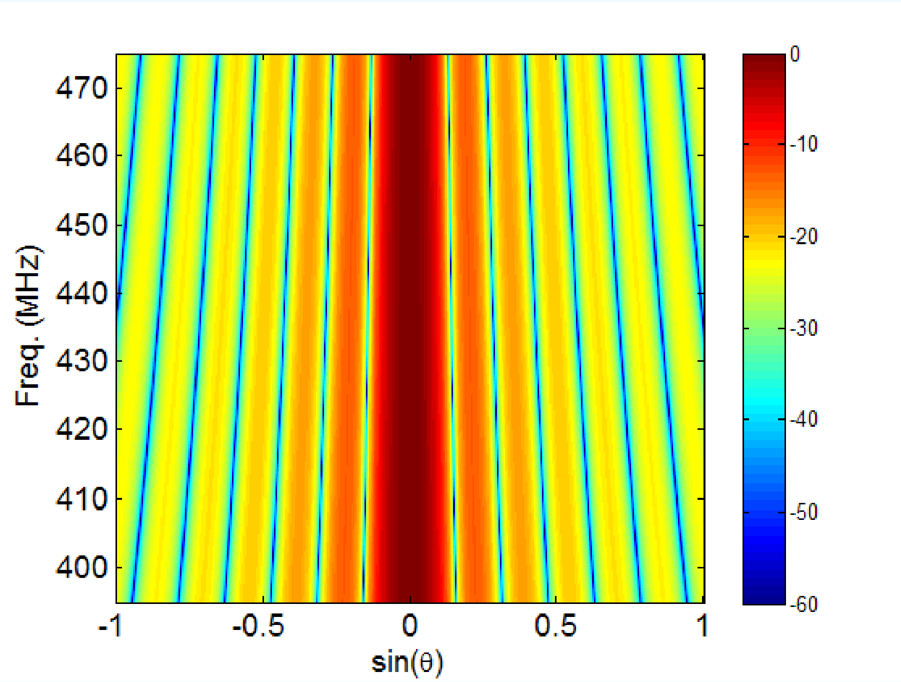
Array Amplitude Pattern

$$C(\theta, \omega_k) = \sum_{i=1}^{N} e^{-j2\pi\frac{d}{\lambda_k}(i-1)\sin\theta},$$

Array Gain Pattern

$$G(\theta, \omega_k) = \left|C(\theta, \omega_k)\right|^2$$





Bandwidth = 395 MHz - 475 MHz (80 MHz), Sensors used: 14

Space-Time Data Vector

N **sensors,** M **pulses**

Data Vector $$\mathbf{x}(t) = \sum_k \alpha_k \mathbf{s}(\theta_k, \omega_{d_k}) + \mathbf{n}(t)$$

α_k : **Random scatter return,** $\mathbf{n}(t)$: **Noise vector**

$\mathbf{s}(\theta_k, \omega_{d_k})$: $MN \times 1$ **space-time steering vector**

$$\mathbf{s}_k = \mathbf{s}(\theta_k, \omega_{d_k}) = \underline{b}(\omega_{d_k}) \otimes \underline{a}(\theta_k)$$

Spatial steering vector

$$\underline{a}(\theta_k) = \left[1 \quad e^{-j\omega_o \frac{d \sin\theta_k}{c}} \quad e^{-j\omega_o \frac{2d \sin\theta_k}{c}} \quad \cdots \quad e^{-j\omega_o \frac{(N-1)d \sin\theta_k}{c}}\right]^T$$

Temporal steering vector

$$\underline{b}(\omega_{d_k}) = \left[1 \quad e^{-j\pi\omega_{d_k}} \quad e^{-j2\pi\omega_{d_k}} \quad \cdots \quad e^{-j(M-1)\pi\omega_{d_k}}\right]^T$$

Space-Time Data Vector

Doppler frequency $\omega_{d_k} = \dfrac{2VT\sin\theta_k}{\lambda/2}$

V : **Platform velocity**

T : **Pulse repetition interval**

λ : **Operating wavelength**

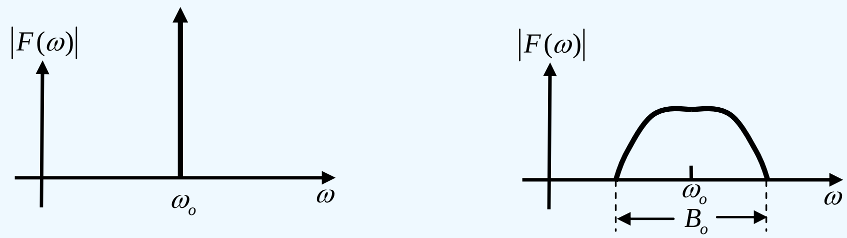
The covariance matrix in an uncorrelated clutter and noise scene

$$\mathbf{R}_x = \sum P_k \mathbf{s}_k \mathbf{s}_k^* + \sigma^2 \mathbf{I}$$

Wideband Space-Time Data Vector

Wideband transmit waveform:

$$f(t) \leftrightarrow F(\omega) = \begin{cases} F(\omega), & |\omega| \le B_o / 2 \\ 0, & |\omega| > B_o / 2 \end{cases}.$$



(a) Narrowband (b) Wideband

Let $y_1^{(1)}(t) = f(t)e^{j\omega_o t}$ **: scattered return at the reference sensor due to first pulse.**

i^{th} **sensor:** $y_1^{(i)}(t) = f\left(t-(i-1)\tau_1\right)e^{j\omega_o\left(t-(i-1)\tau_1\right)}, \quad i = 1, \; 2, \; \cdots \; N$

$\tau_1 = \dfrac{d \sin\theta_k}{c}$: **Interelement time delay for azimuth angle** θ_k

Wideband Space-Time Data Vector

The first pulse output:

$$\underline{y}_1(t) = \left[f(t)e^{j\omega_o t}, \ \cdots \ f\left(t-(N-1)\tau_1\right)e^{j\omega_o\left(t-(N-1)\tau_1\right)} \right]^T.$$

Reference sensor due to n^{th} pulse:

$$y_n^{(1)}(t) = f\left(t-(n-1)\tau_2\right)e^{-j\pi(n-1)\omega_{d\,k}} \ e^{j\omega_o t}$$

$\tau_2 = \dfrac{2VT\sin\theta_k}{c}$: **Interpulse time delay for azimuth angle** θ_k

i^{th} sensor output due to n^{th} pulse:

$$z_n^{(i)}(t) = f\left(t-(n-1)\tau_2-(i-1)\tau_1\right)e^{-j\pi(n-1)\omega_{d\,k}} \ e^{-j(i-1)\tau_1},$$

$$i = 1, 2, \cdots N, \quad n = 1, 2, \cdots M.$$

Wideband Space-Time Data Vector

Output vector for the n^{th} pulse: $\underline{z}_n(t) = \left[z_n^{(1)}(t),\ z_n^{(2)}(t),\ \cdots\ z_n^{(N)}(t) \right]^T$

Space-time vector: $$\mathbf{z}(t) = \begin{bmatrix} \underline{z}_1(t) \\ \underline{z}_2(t) \\ \vdots \\ \underline{z}_M(t) \end{bmatrix} = \mathbf{f}_k(t) \circ \mathbf{s}(\theta_k,\ \omega_{d_k})$$

Schur-Hadamard product

$\mathbf{f}_k(t)$: $MN \times 1$ **transmit signal dependent vector whose $(iN+n)^{th}$ element is given by**

$$f\left(t - n\,\tau_2 - i\,\tau_1\right),\quad i = 0,\ 1,\ 2,\ \cdots N-1,$$
$$n = 0,\ 1,\ 2,\ \cdots, M-1\ .$$

Wideband Clutter Covariance Matrix

Covariance matrix for the wideband return $\mathbf{f}_k(t)$

$$\mathbf{T}_k(\tau) = E\left\{\mathbf{f}_k(t)\mathbf{f}_k^*(t+\tau)\right\} > 0$$

Wideband array output covariance matrix:

$$\mathbf{R}_z = E\left\{\mathbf{z}(t)\mathbf{z}^*(t)\right\} = \sum_k P_k \mathbf{T}_k \circ \mathbf{s}_k \mathbf{s}_k^* + \sigma^2 \mathbf{I}$$

Low pass transmit signal gives

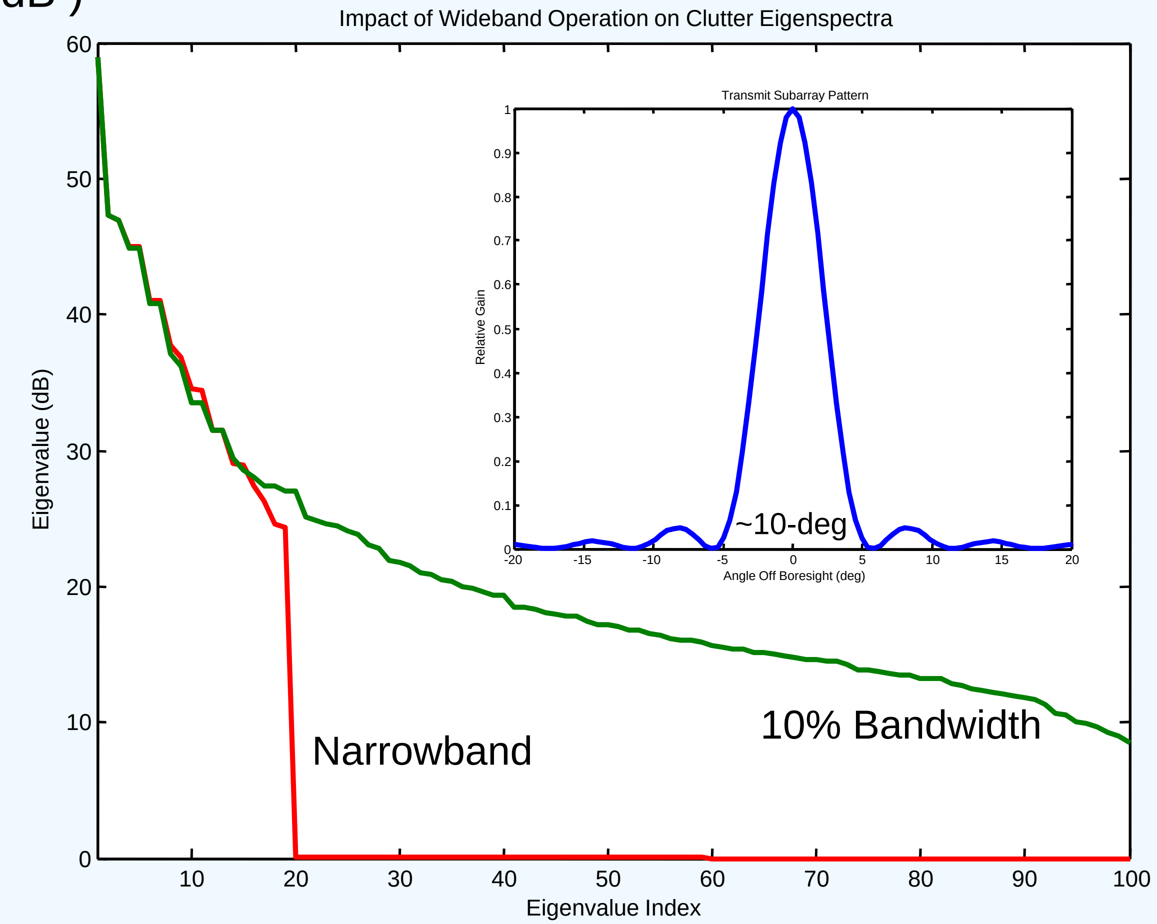
$$\mathbf{T}_k(m, n) = \operatorname{sinc} B_o\left((n_1 - n_2)\tau_2(k) + (i_1 - i_2)\tau_1(k)\right)$$

$$m = i_1 N + n_1, \quad n = i_2 N + n_2 \ .$$

If $\mathbf{T}_k = \mathbf{T}$, then $\mathbf{R}_z = \mathbf{T} \circ \mathbf{R}_x$

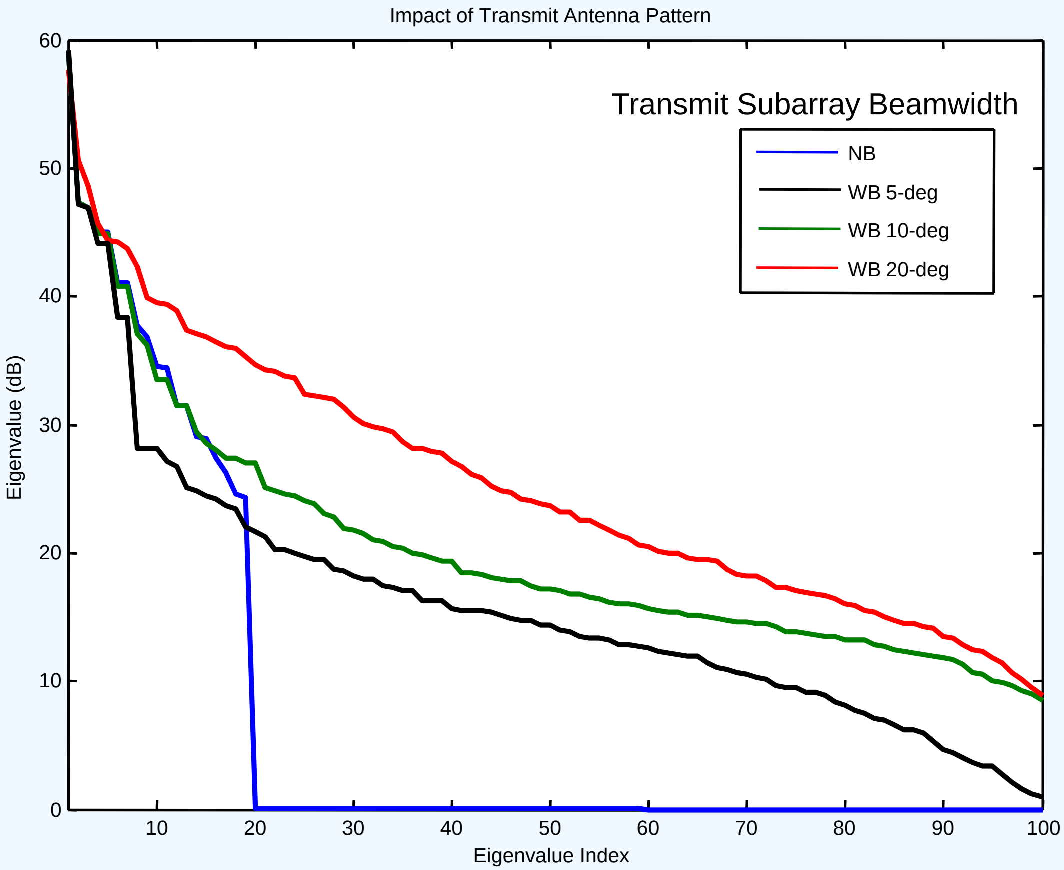
Impact of Wideband on STAP

- **X-band example**
  - (X-band case, f_0 = 10 GHz, L = 3 m, B = 10%, N = 10 subarrays, CNR = 40 dB)



Impact of Wideband on STAP

- **Transmitter plays a big role in shaping clutter characteristics**



Impact of Wideband on STAP

Narrowband clutter covariance matrix (single scatter at θ_o)

$$\mathbf{R}_x = P_o\,\mathbf{s}(\theta_o,\omega_{d_o})\mathbf{s}^*(\theta_o,\omega_{d_o})$$

Wideband clutter covariance matrix:

$$\mathbf{R}_z = \mathbf{T}_o \circ \left(P_o\,\mathbf{s}(\theta_o,\omega_{d_o})\mathbf{s}^*(\theta_o,\omega_{d_o})\right).$$

$\mathbf{T}_o = \sum_{i=1}^{MN} \lambda_i\,\mathbf{e}_i\,\mathbf{e}_i^* \geq 0$ **gives**

$$\mathbf{R}_z = \sum_{k=1}^{MN} P_k\,\tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^* + \sigma^2\mathbf{I}, \qquad P_k = P_o\lambda_k > 0, \qquad \tilde{\mathbf{s}}_k = \mathbf{e}_k \circ \mathbf{s}(\theta_o,\omega_{d_o}).$$

A bunch of MN uncorrelated returns, all of them associated with the single scatter located along θ_o .

$\tilde{\mathbf{s}}_k$ **: Amplitude modulated steering vector associated with Doppler frequency ω_{d_o} and location θ_o**

Impact of Wideband on STAP

Let $\mathbf{d}_k$ represents the ordinary DFT vector associated with the eigenvector $\mathbf{e}_k$. The entries in $\mathbf{e}_k$ corresponds to a double sampling period of τ_1 followed by τ_2. Thus

$$\mathbf{e}_k = \sum_{n=1}^{MN} d_k(n) \left(\underline{b}\left(\frac{2\,j_n}{M}\right) \otimes \underline{a}\left(\frac{i_n\,c_o}{N}\right)\right)$$

Modified steering vector:

$$\tilde{\mathbf{s}}_k = \sum_{n=1}^{MN} d_k(n) \left\{ \underline{b}\left(\frac{2\,j_n}{M}\right) \otimes \underline{a}\left(\frac{i_n\,c_o}{N}\right)\right\} \circ \left(\underline{b}\left(\omega_{d_o}\right) \otimes \underline{a}\left(\theta_o\right)\right)$$

$$= \sum_{n=1}^{MN} d_k(n)\, \mathbf{s}(\theta_o + \frac{i_n\,c_o}{N}, \omega_{d_o} + \frac{2\,j_n}{M}) = \sum_{n=1}^{MN} d_k(n)\, \mathbf{s}(\theta_n, \omega_{d_n})$$

Coherent sum

$$\theta_n = \theta_o + \frac{i_n\,c_o}{N}, \quad \omega_{d_n} = \omega_{d_o} + \frac{2\,j_n}{M}.$$

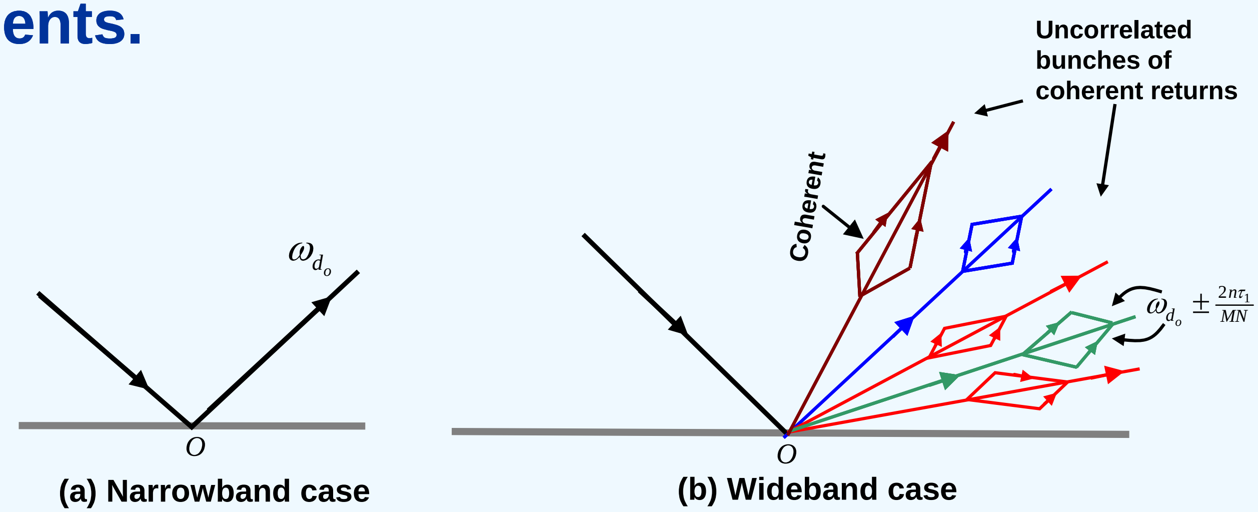
$\theta_n,\ n = 1 \rightarrow MN$

$\omega_{d_n},\ n = 1 \rightarrow MN$

Impact of Wideband on STAP

Wideband introduces jittering effect both on angle and Doppler by generating a bunch of uncorrelated returns.

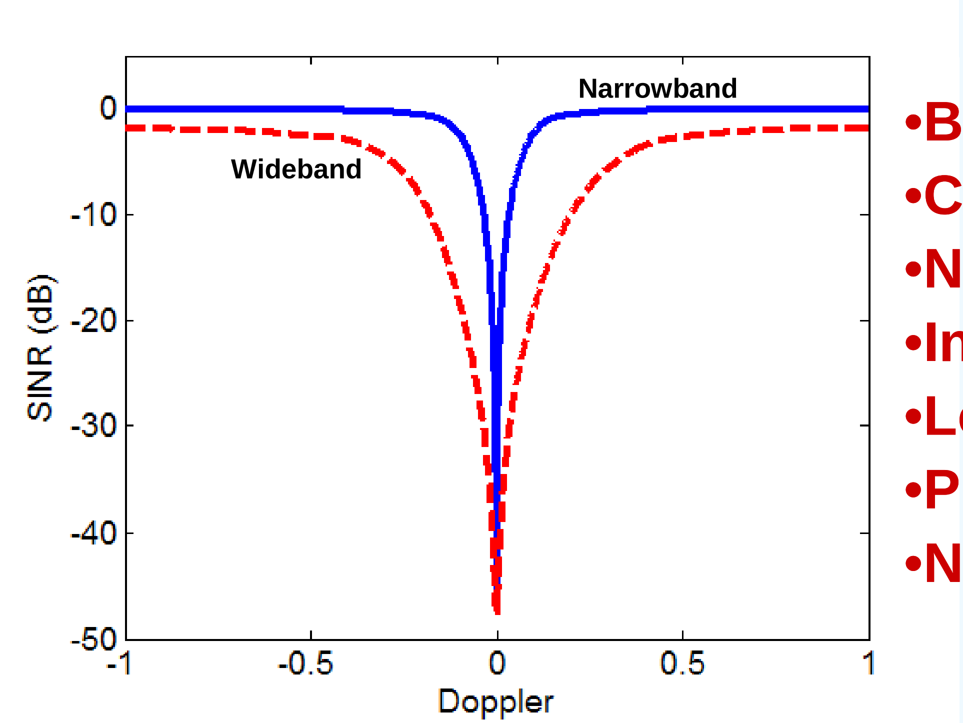
Each such uncorrelated return contains a set of coherent returns with different directional and Doppler components.



Modified steering vector $\tilde{\mathbf{s}}_k$ contains several coherent returns.

Adaptive processor will not be able to null out the clutter.

SINR Loss With and Without Bandwidth Dispersion



- **Bandwidth** $BW = 80\,MHz$
- **Center frequency** $f_c = 435\,MHz$
- **Number of sensors** $N = 14$
- **Interelement spacing** $d = 0.33\,m$
- **Look angle** $\theta_o = 0^o$
- **PRF = 625 Hz**
- **Number of pulse** $M = 16$

Conclusions

- Large aperture size contributes to wideband conditions
- Single scatter generates several uncorrelated return bunches.
- Each return bunch contains multiple coherent returns
- Adaptive processor generates wider null