Goal-Oriented Intelligence in Optimization of Distributed Parameter Systems

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Abstract

Models of complex systems can be differentiated by their ability to reproduce or generate system behavior, by their prediction power, by their robustness, or, conversely, by their sensitivity to inputs and parameters; by their uncertainty (if captured); and by their intelligence. Even the term "prediction" is not unique. First, a first-principle (physically based) distributed parameter model could be an excellent predictor if (a) it captures the main system behavior, and (b) its parameters and inputs are known accurately; otherwise, it would fail, possibly drastically. Second, predictive power depends on the data, on the goal, and on the time scale. For example, scheduling of pumping and injection in an oilfield for maximum profit over the next 5 years; or pumping from a contaminated aquifer in order to maintain certain (low) concentration at a compliance point for the next 20 years, vs. prediction of plume migration in groundwater towards a nearby river, over time: in each case, the model has a slightly different expected function, as well as different intelligence type. The paper reviews the recent developments in subsurface fluid flow management such as optimization of oil production and groundwater remediation (both sharing similar practices, though for different purposes) as a continuous struggle to increase intelligence by (a) adapting new tools such as artificial intelligence and dynamic stochastic control; (b) attempting to integrate these tools; and (c) reducing uncertainty. Although the systems discussed seem specific to the (mathematical) geosciences (specifically to oil reservoirs and contaminated aquifers), and although these systems are very different from man-made machines, similar rigid structure and reliance on differential-integral calculus, as well as the serial processing, knowledge evolution, and uncertainty propagation from one discipline to the next exist in most science and engineering fields, and so does the need for a paradigm shift. Given the need in adaptive, intelligent control/planning/optimization of such systems, the progress of these segregated efforts towards a multiresolutional decision support system is inevitable, highly desirable, and highly promising. However, we are still facing the challenge of performing and defining optimal integration between PDE models and multiresolutional representations, and since such integration depends on model quality and data quantity and quality, this is an adaptive integration process as well. One criterion for such "optimal" integration would be uncertainty reduction (resulting from the integrated MR system), which brings up yet another challenge: to define the metrics for uncertainty reduction.

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1. Introduction

Management of distributed-parameter systems, particularly where complex natural processes intersect with human industrial practice and theoretical knowledge, is extremely difficult to analyze and optimize. From a knowledge organization perspective, this area of practical knowledge is in disarray. For example, knowledge in the petroleum industry relates to many scales of representation, yet this fact is not taken into account in an organized manner.

Indeed, the oil industry is a live example of the need to integrate multiple disciplines presently not integrated, including huge volumes of raw data (particularly in geophysics) and multiple computational and conceptual models of geology, geophysics, and reservoir dynamics. Although the need in unification of the bodies of knowledge in these disciplines and the associated (tremendous) benefits of analysis enhancement capability has been widely recognized, progress in this direction has been very slow. The main reason for this is the rigid, segregated, serial process of knowledge and model building – from geophysical signals to simulation-optimization of fluid dynamics in porous media – a process that has evolved naturally during the last century, where the simulator has become the focal junction where all the knowledge and understanding of the physical processes and material properties are being filtered and concentrated in the form of partial differential equations (PDE that describe mass balance of oil, water, and gas in 3D space) whose coefficients (or distributed parameters) suppose to capture the physical-chemical properties of the medium on a particular scale, everywhere in the modeled subsurface/reservoir domain, assuming some "known" boundary conditions and initial conditions. In the following, we will use subsurface flow systems, particularly, oil reservoirs, as examples of managed, distributed parameter systems.

Oil reservoirs are complex systems on all scales. Decisions such as pumping and injection (schedule and rates), new well placement, and (directional) drilling in an active oil field, are typical of the complex relationships between reservoir characterization and oil field/reservoir management. The solutions to such problems involve a complex system of multiphase flow equations (linked PDE) in a heterogeneous domain (reservoir), as well as economical factors such as short-term and long-term oil price, worth of information, inventory/storage/delivery, cost of drilling, maintenance, production, etc. Well drilling and construction equipment are costly and cannot be afforded as frequently as necessary, while prevalent information gaps render decision-making uncertain and hence, risky.

Yet, the advantages of reservoir simulations should not be underestimated either. Models or simulators based on PDE solutions provide physical insight into various important flow phenomena, as well as the general behavior of the fluid movements in the reservoirs, under scarcity of spatial data typical of both old and young reservoirs, particularly the latter. The ability to capture the essence of the complex physics behind the reservoir responses to pumping and injection is the strength of the simulator and the essence of its intelligence. However, this strength could promptly become its weakness where (a) natural geological heterogeneities on certain scales are being missed, or (b) physical/chemical/thermodynamic processes are being missed (e.g., leaching geochemistry; instability of the oil-water interface), or (c) uncaptured (or erroneous) boundary conditions, all of which would lead to wrong predictions. In other words, wherever an essential physical phenomenon on any scale is being missed, the simulator

looses its intelligence, i.e., its ability to predict short-term reservoir responses and long-term oil reserves and revenues.

The main question we attempt to address in this paper is how to merge the advantages and use the intelligence of existing models and interpretations in a comprehensive intelligent system that could take advantage of such physically based intelligence, while eliminating its limitations. In order to answer this question, we first need to understand the structure and limitations of current approaches to optimization/control of subsurface fluid flow and solute transport. The reader could notice that although the problems discussed here seem to be specific to optimization of oil production and groundwater remediation, similar rigid structure and reliance on differential-integral calculus exist in most engineering fields, and so is the need for a paradigm shift when planning/control/optimization become the focus. Yet, we should also keep in mind some major difference between the geosciences (or natural systems in general) and man-made machines. In hydrogeology and oil reservoirs, we deal with multiphase flow in heterogeneous formations, with transient flow and transport phenomena occurring on all scales, with nonlocal dependency on (unknown) fluid flow everywhere. However, whenever we focus on operations' scheduling, feedback and feed-forward, this complex system becomes similar to other complex operations; finding a new well location, however, requires considering the complete heterarchy of transient flow phenomena in space, often with major data gaps. Such data gaps challenge all models, and consequently, any integration of PDE models with MR knowledge representations (and/or MRDS). Such integration seems to be data-dependent, and requires optimization on its own merit, weighting the robustness, prediction/anticipation (goal-oriented) power, the uncertainty associated with different representations, as well as uncertainty reduction produced by such integration.

2. Current approaches to simulation-optimization-control of distributed-parameter, subsurface flow systems

The cutting edge subsurface fluid management such as oil reservoir optimization and groundwater remediation control under uncertainty has been moving in three major fronts: (1) operations research (including stochastic models and risk assessment); (2) stochastic-dynamic control; and (3) artificial intelligence (AI), particularly artificial neural networks (ANN), genetic algorithms (GA), and fuzzy logic (FL). However, under the current structure of serial, segregated, and isolated "disciplines" that process the information from geophysics/explorations to reservoir characterization, reservoir simulations (or flow and transport models in hydrogeology), and optimization/control, it is impossible for these three fronts to merge into a unified, integrated approach, nor could a major progress in oil field management be made. Under the current paradigm, optimization/planning/control of these complex systems have been handicapped by uncertainty on one hand, and prohibitive computer power & time requirements on the other hand, without benefiting from all available information.

The typical approach to reservoir characterization and management sketched in Figure 1 shows the different subsystems that constitute both exploration and production. The figure is highly simplified, with many subsystems not shown, such as subdivision of

exploration (remote sensing, surface- and borehole-geophysics; seismic, electrical, electromagnetic, micro-gravity, SP, etc); geologic investigations (structural geology, geochemistry, lithology, bio-stratigraphy); subdivision of production (well pattern design, injection and pumping, gas, water, and displacing fluids); and well construction, - all of which are complex, interdependent, and require real time updating and decision making. The figure illustrates current model construction and subsequent optimization.

Blocks 1 and 2, and, to some extent, Block 3 (conceptual model), represent reservoir characterization, which plays a crucial role in exploration and subsequent reservoir management. Typically, the conceptual model (Block 3) of the reservoir is an undeclared part of the simulator; this is where all the geology is filtered, upscaled, and translated into the simulator's parameters, which inherently entails averaging and discarding of information (acting as a low-band filter), including small scale features that may be crucial (in which case, their large-scale influences would be modeled as different, large-scale parameters – e.g., *dispersion coefficient*). Most of the assumptions and decisions related to reservoir representation are made at the conceptual model stage, and are subject to modeler's understanding and experience.
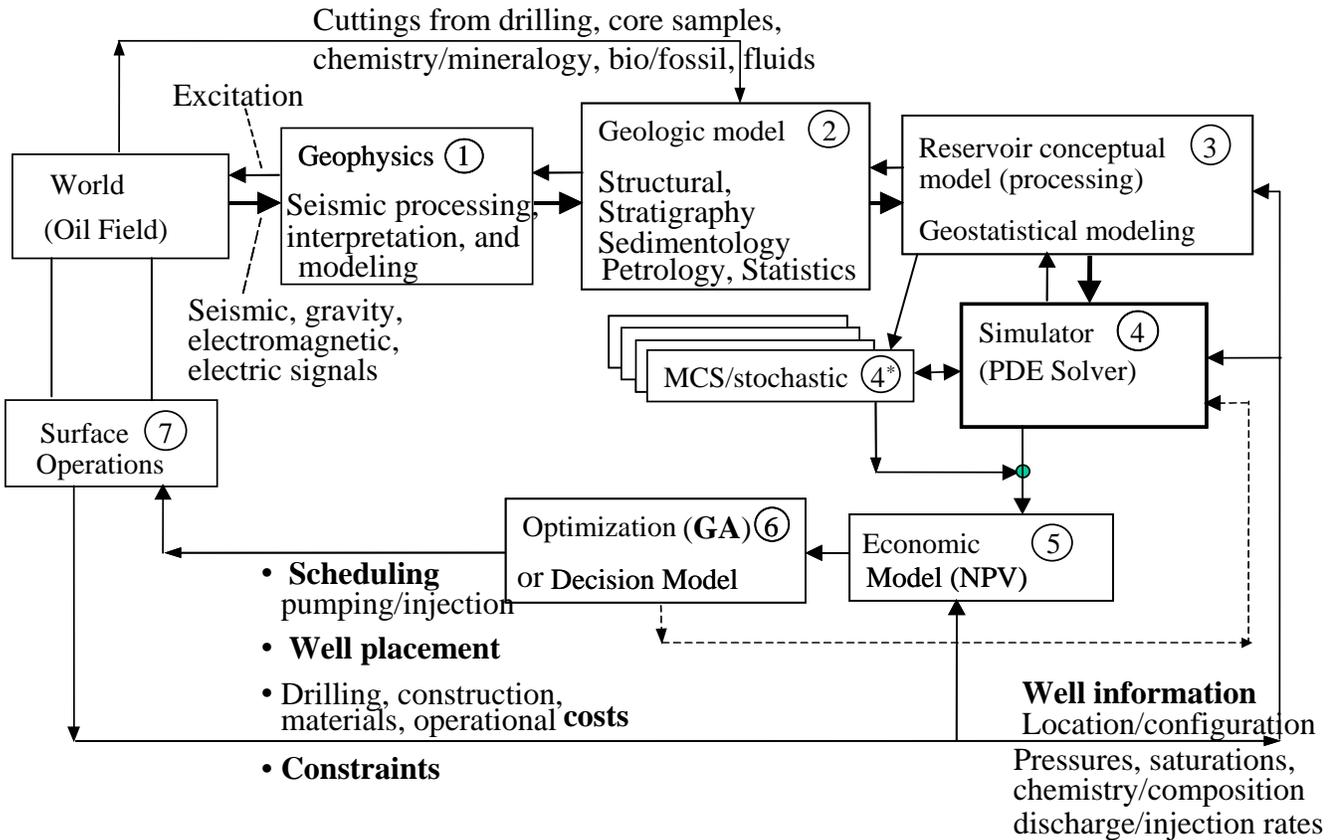
Figure 1. A schematic view of current approaches to reservoir optimization.

Block 4 is the current quantitative "brain" (or predictor), typically a distributed-parameter, complex PDE solver that may include several linked PDE with their auxiliary constitutive functions (mostly determined in the lab, on a lab-scale), or less commonly, a cell model, represented by Ordinary Differential Equations (ODE), implying a larger

scale, i.e., lower resolution). Within this computational block, additional analytical forms are utilized for computing parameters and constitutive relationships, as well as local modeling (e.g., the Buckley-Leverett model of displacement). Ideally, the simulator contains multiple analytical models functioning at different time scales, and demonstrates processes working at different levels of resolution. However, current models are far from this ideal.

Due to the embedded upscaling and loss of information, feedback from the reservoir simulator (4) to the geological model (2) is not reliable, except for special cases where certain disparity exists between the measuring window and the scale of the geologic feature, and where only a piece of the puzzle is missing (e.g., in well testing with an appropriate monitoring system in place, or in dual porosity systems where the rock properties are well characterized on all scales). Typical inverse or automatic calibration procedures determine some local reservoir-fluid interaction "properties" that fit a particular (and hence, uncertain) model. Subsequent interpretations of geologic features based on inverse modeling (or calibration of the simulator) are, therefore, speculative.

2.1 Reservoir Simulations and Groundwater Modeling Problems

Historically, modeling of fluids flow in porous media using PDE started by Muskat in the 1930's [52, 53]. Until that point, predictions of reservoir behavior over time was merely extrapolations from a (local) "production curves" or "well performance models", which describe cause-effect relationships between production, fluid content (oil, water, gas), and pressures in producing and injection wells, over time, using curve fitting/regression [66]. The introduction of PDE suddenly provided insight and extended the prediction power significantly, providing intelligence to an otherwise a black box model. This addition of intelligence has had a sweeping effect, and was extended to groundwater and multiphase flows in porous media, including geothermal reservoirs, unsaturated flow in soils, and contaminant transport in aquifers (e.g., [9]), while it has been further reinforced by a rapid development of numerical methods and ever-increasing computer power. The success of the numerical model that could explain and predict the subsidence of Venice in the early 70's [84-86] has been used as a live example of the ultimate intelligence of this new tool. By the early 80's, reservoir engineers and hydrogeologists have developed numerous numerical models (or simulators), which, with the help of new visualization tools and mainframe computers, could both predict and visualize the movement of oil, gas, water, and contaminants (in groundwater), with ultimate confidence and optimism due to the exponential growth of computer power and the prospect of optimal management. However, before long, it has become clear that once applied to geological formations on a scale where the heterogeneity cannot be neglected, the strength of the PDE-based model becomes its weakness; this sensitivity, overlooked and even welcomed initially (because sensitivity to certain inputs and parameters is consistent with the physics of the phenomenon), came to haunt the modelers later on; the exaggerated expectations have turned into disappointment and distrust.

How could this be explained? A reservoir simulator based on PDE requires accurate definitions of reservoir properties on assumed certain scales, everywhere in the reservoir, in order to reliably represent the flow, and predict reservoir responses. Such near-ideal conditions could occur in cases where the scales of heterogeneities are much smaller than the simulated domain, and given the particular question being asked, such heterogeneities

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could be lumped under unique, measurable reservoir characteristics that could be assumed uniform on a particular scale. While this could be the case in many sandy reservoirs and aquifer, on a certain scale, many more reservoirs and aquifers exhibit non-uniformity on all scales, and thereby, drastically degrade the prediction capability of PDE-based models. This has forced reservoir modelers and hydrogeologists to account for heterogeneity in any possible way. Due to lack of spatial data, heterogeneity translates to uncertainty, and uncertainty translates to randomness, and thus, the PDE become random (or stochastic) PDE [e.g., 87], severely complicating and inhibiting prediction capability. Moreover, since the uncertainty is on all scales and in all parameters, structural model errors add severe, unquantifiable uncertainty to the already uncertain solution [58, 54, 82]. Before we reach this bleak conclusion, let us further explore current treatment of heterogeneity/uncertainty in reservoir simulations.

2.2 Dealing with data gaps and uncertainty

We recognize that knowledge of all reservoir flow properties on all scales everywhere in the reservoir is impossible even without considering drilling costs. Due to the high cost of drilling, there is typically only sparse information on reservoir behavior, while geophysical (esp. seismic) data are by far more abundant. As a separate discipline, reservoir modelers have no choice but to rely on geological interpretations and limited observations of reservoir behavior, while being forced to average and upscale reservoir properties using ad-hoc estimates and geostatistical tools.

The prevalent method to account for heterogeneity (of reservoir properties) and the resulting uncertainty is to treat all the data or interpretations related to one or two dominant parameters (typically, saturated permeability) statistically, i.e., transforming uncertainty into randomness, typically under the assumptions of underlying joint probability distribution (PDF) that (assuming ergodicity) represents space and time as well. Once a certain parameter is considered random, the PDE solution becomes random (or stochastic) as well, and additional theoretical difficulties emerge [56-58, 87]. This approach has been developed and used for the last five decades in different areas of science and engineering, including stochastic optimization of oil reservoirs [1, 10-12, 20, 28, 36, 51, 59, 65-66, 80-81] and groundwater remediation [2, 3, 14-15, 19, 21-23, 25-27, 31, 39-42, 49, 68-73, 77, 83]. Despite remarkable theoretical advances in this difficult yet essential extension of the deterministic approach, developments have been limited to simple geometries and far-reaching simplifications.

The use of Monte Carlo Simulations (MCS), where "equally probable" high-resolution worlds (or realizations) are generated and simulated in order to compute ensemble statistics has been used extensively in the areas of reservoir simulations and hydrogeology (e.g., [56-58]); however, in practice, such a procedure results in enormous (if not prohibitive) computational burden for predictions alone, and becomes practically prohibitive for optimization, unless far-reaching simplifications are being made (e.g., [28]). In addition to these limitations, the traditional stochastic approach suffers from the following drawbacks and inconsistencies: (a) it cannot overcome, nor assess the major uncertainty in the model structure (which remain rigid); and (b) it leads to additional (now statistical) models with new parameters that are also uncertain; (c) using interpretations of well tests that assume homogeneity on a "near well" scale as the basis for conditional (stochastic) simulations; (d) using a single "dominant" parameter (on a

single scale) as the only random property (otherwise, computations are prohibitive even for limited cases); (e) assuming a PDF based on sparse spatial data; (f) assuming deterministic boundary conditions despite the significant uncertainty in it; (g) the inability to capture the linked physics and chemistry on all relevant scales (thereby, missing important phenomena such as front instability (between displacing and displaced fluids during enhanced oil recovery, where micro-scale variations trigger and promote fingering and bypassing due to capillary and viscosity differences [e.g., 35] and various geochemical reactions. Nevertheless, reservoir simulations and groundwater modeling are an important basis for approximations, correlations, and physical interpretations, including understanding and highlighting of the gaps and limitations of these interpretations.

Further, repeating the MCS chain of simulations and optimization as soon as new information arrives is practically impossible under the current scheme. Thus, despite the powerful theoretical framework and insight provided by the stochastic approach, this approach is yet in infancy, and does not extend beyond a certain definition of parameter uncertainty. Indeed, when optimization is attempted, e.g., for a new well placement, the computation-intensive stochastic approach becomes impossible, while a partial use of the approach (e.g., using only a few Monte Carlo simulations, as in [28, 12, 71], not accounting for uncertainty in other parameters, in the conceptual models, and in all interpretations and decisions along the path in Figure 1, leads to largely non-optimal decisions. If the reduction of intelligence can be measured by the amount of error between optimum and non-optimal operation, such a difference implies a significant reduction in intelligence gained by physically based models.

In conclusion, we recognize relationships between uncertainty, model robustness, and intelligence; an ideal PDE-based model is highly intelligent in a sense that it can predict reservoir behavior at all points in space and time; however, as soon as the model structure is inaccurate or model parameters are uncertain, it looses its intelligence to a large degree. Practically, this implies that a sensitive model would be "intelligent" as long as all necessary data exist and are accurate, but drastically looses its intelligence where data are uncertain, inaccurate, or insufficient; on the other hand, a less sensitive (more robust) PDE-based model would also be less intelligent to begin with. Thus, we seem to face an optimization problem: what model would be the optimal model for a particular problem; or better, what combination of models would be optimal in terms of data use, (maximum) intelligence, and robustness.

2.3 Current use of Artificial Intelligence

Before we answer this question, we should be aware two other sets of models: one based on artificial intelligence (AI) methods (also commonly termed *soft computing*), mostly in the context of geologic analysis and oil explorations, and one based on a statistical framework, particularly geostatistics and Bayesian statistics. As to AI, tools such as artificial neural networks, fuzzy logic, genetic algorithms, and probabilistic reasoning, have been used in reservoir characterization [75-76], subsurface flow [29, 63-64], and well field development and optimization [1, 10-11, 13, 28-30, 59, 62-64]. Consistent with the AI approach is the excessive use of geostatistics, such as the search for best next well placement described in [30], where the authors bypassed the simulator altogether, and used indicator kriging, instead, to interpolate expected production and

make corresponding decisions. Nevertheless, one of the important outcomes from these developments is the recognition of the need in integration of methodologies rather than using them in isolation [75]. In particular, the need to address the issues of (a) integrating information from various sources with varying degrees of uncertainty; (b) finding relationships between measurements and reservoir properties; (c) reducing uncertainty and risk; and (d) using all of these to optimize reservoir development and management, in real time. However, the progress in this direction has been slow, and fragmented results still dominate the field. The main reason for this is the need to translate information among the subsystems that constitute an oil reservoir, from geophysics to geology, and from geology to reservoir flow properties, and perform all of these translations on different scales of information, with different geometric and stratigraphic representations. These integration problems, and the overwhelming problem of uncertainty due to lack of data in the presence of inherent heterogeneity, have been unresolved, to date, with only scattered use of the various computational tools for limited characterization and prediction purposes; hence, leading to non-optimal management of oil reservoirs, water resources, and environmental cleanup (groundwater remediation) operations.

How could these capable AI models be combined with all other models in an adaptive framework that will (a) account for all the information (old and new, without initial filtering), and (b) allow continual updating and improvement due to continual data accumulation. The challenge is, thus, to integrate various measurements/data and models in a comprehensive, flexible, adaptive knowledge representation that will use all the available information for optimal decision making in the most intelligent way possible. Before trying to answer this question, let us review the third approach to control/optimization of subsurface flow and transport problems. One step in that direction was made by Rogers and co-workers [63, 64] who 'trained' a ANN by using multiple deterministic flow and transport simulations of a complex aquifer under a pump & treat operation, and later [29] for oil reservoir simulation-optimization, and then used the efficient ANN as a replacement ("proxy") for the cumbersome, slow simulator. Due to the limited extrapolation power of the ANN, many model runs were needed for the training to cover the expected span of possibilities (in the search space), to enable optimization of pumping and injection schedules. Although the method used is deterministic, as pointed out by the authors, it could, in principle become stochastic by generating multiple realizations and running Monte Carlo simulations (MCS), which, however, would result in prohibitive computer power. In other words, since the unsupervised ANN used is relatively less intelligent than the PDE models, it was used only as a minor auxiliary function. We would like to reverse this ranking of intelligence in a way that will enable broader conceptualization and knowledge representation.

2.4 Applying Dynamic Control

Dynamic, stochastic control has been used and further developed mainly for groundwater remediation purposes, specifically for pump & treat operations, which are similar to oil production operations; while the goal in oil production is to maximize production profit (over a certain period), the goal of groundwater remediation is to maximize extraction of contaminated water from the aquifer over a reduced period. The dynamic-stochastic control approach in this field [6-7, 16-18, 24, 32-34, 37-38, 60-61, 74] is an extension of the more general stochastic control theories of [8, 50, 67, 78-79].

While under the dynamic control approach, inverse modeling (i.e., updating/calibrating of uncertain parameters in a simulator or in a flow and transport model) is done jointly with the optimization process (hence, dual control), while feedback control rules enable changing of pumping rates (control variables) in response to changing hydraulic heads or contaminant concentrations (state variables). The stochastic simulator varies from extended Kalman filter to PDE, using perturbation methods and dividing the cost function into deterministic and stochastic parts, with the goal of minimizing remediation or plume-containment cost while optimizing both sampling and control actions. Typically, differential dynamic programming (DDP) is used to compute the deterministic control [6] while the solution of the stochastic part of the cost function is obtained analytically using stochastic control techniques applied to the governing flow and transport equations (PDE), with challenging mathematical derivation that requires a twice differentiable cost function. The on-line parameter estimation fed into the flow equation enables updating of both state variable estimates and state covariances. In terms of formulation of the cost function, the following highlights are worth mentioning: (a) the goal is to minimize the average (estimated, probabilistic) cost function; (b) the cost function is separable in stages, and according to the dynamic programming approach, whatever the initial state and initial decision are, the remaining (future) decisions/solutions should constitute an optimal solution based on the current state; i.e., the problem is reduced to finding a current optimal control variable, given a cost function over the remaining (future) periods, and given the current information state which includes all relevant a priori knowledge of the system and its history of observations and control; probabilistically, this information state is the conditional probability density function of the state at the current period conditioned on all past information; consequently, the cost function depends on uncertainty, directly. The two hidden elements in this procedure are: (a) the Bayesian approach, and (b) learning (from past experience).

Similar works [24] emphasize the use of all available information to estimate all present and future uncertainties, solving the management problem over the designated control horizon, applying the optimal control action (pumping or injection) during the current time period, and repeating this process at the next decision time, with PDE (flow equations) treated as a dynamical state-space system using finite element and finite difference techniques, considering (both) transmissivities and boundary conditions uncertain, and hence, perturbed in a highly simplified aquifer system, with the goal of minimizing pumping (and treatment) costs while maintaining hydraulic heads that guarantee containment of the contaminant plume. The results (a) provided insight into system response under uncertainty; (b) assessed trade-offs between satisfying goals and minimizing uncertainty (based on a simplified uncertainty model); (c) emphasized the effect of management decisions at any stage on model predictions in the next step. Explicit optimization combined with sensitivity analysis appeared to be an effective management approach. Other works [37-38, 60-61] extended the methodology of optimal estimation and scheduling of aquifer remediation under uncertainty, by allowing more complexity to be introduced, while performing real time (dynamic) feedback from measurements, as well as joint (on-line) parameter estimation - optimization and stochastic optimization. Subject to constraints and a specified reliability of meeting water quality requirements for a current period, the method minimizes the expected value of the

cost in the next (remaining) periods. A comparison between (adaptive) deterministic feedback control and the stochastic control formulated by [37] showed a clear cost reduction using the stochastic control formulation, with increasing difference as the uncertainty increases. Despite the accommodation of more complexity, and more general constraints, dynamic control methods that rely on PDE models are not yet suitable for complex real world problems.

One of the important insights that emerged in this implementation of stochastic control is the "probing" and "caution" effects highlighted by Bar Shalom [8]; the effect of the stochastic/perturbation part in the dual-control example [of Lee and Kitanidis] is that of sensitivity analysis and system excitation (the "probing" effect) followed by measurements and gaining information about system parameters that resulted in a substantial improvement. A paradigm shift is embedded here: rather than focusing on general predictive power (or lack of it), the dynamic control approach anticipates how the actual (future) state will deviate from estimated state currently in hand, and steers the system to mitigate possible losses (the "caution" effect). These two effects (of probing and anticipation/caution) imply yet another effect – that of goal-oriented learning.

The advantage of dynamic control was demonstrated by [16] who used differential dynamic programming to determine the benefits of time-varying optimal groundwater pumping policies, with the goal to reduce groundwater concentrations (of a contaminant) to acceptable levels. They demonstrated that static pumping policies would cost 45-75% more than policies that allow time-varying pumping rates, where the management model can "chase" the contaminant plume. Another set of developments along this line [17-18, 74] made use of the "transition function" (TF) that models (or transforms) the system from one state to the next (in the groundwater contamination case, the TF consists of the matrices generated by the finite element model at each time step) in order to reduce the number of iterations needed for convergence and overall computational time in the differential dynamic programming.

A substantial use of the second derivatives of the transition function in a constrained differential dynamic programming (DDP in a complete form) was made [74] with respect to a general case pump & treat remediation, including pumping scheduling and finding best well location. In this work [74], these derivatives were used to generate feedback laws with the aid of the penalty function method (which converts the constrained optimal control problem to unconstrained optimization, and consequently, allows flexibility in the response of the feedback laws to violation of constraints). These feedback laws describe relationships between required corrections of the control variables and weighted deviations of observed states from the predicted states. The goal was to find the relationships between the second derivatives of the transition function and evolutionary feedback laws, where the latter relate deviations from (and hence, required corrections to) optimal pumping scheduling and deviations of heads and concentrations (state variables) from their anticipated states, through weights discovered/assigned to these state deviations. The methods requires, as a first step, to employ a (deterministic) model and initial "optimal" pumping policy, which enables to build the first transition function, and find relationships between control and state deviations. The feedback laws are obtained by adjusting the relative weight assigned to each penalty function (corresponding to each control variable).

If we disregard the evolutionary nature of the feedback laws, the simple linear relationships expressed by the feedback laws (between observed deviation and required action) resembles the inverse of action-response functions used in different works [39-40]. It is interesting to note that while the transition function is derived from the governing PDE (flow and transport) model (which could be viewed as an elaborated response function model), the feedback law represents cause-effect rules (much like the inverse of the transition function) that compensate for model errors, regardless of the source of the errors. It is also interesting to note that the evolution of the feedback laws over time has an element of memory and learning (from past cause-effect relationships). Results from a simplistic example [74] showed to be robust and efficient in terms of reducing cost (by 4-51% less than optimization without using a feedback law) as well as required computer time, for up to 25% deviations from mean parameter values (i.e., uncertainty up to CV = 0.25).

Although exercised with only small perturbations (hence, small uncertainty) and some other limitations, this particular (complete) DDP approach is the first control/optimization method that frees itself not only from the need in a rigorous, well defined statistical/uncertainty model (with assumed PDF, correlation structure, etc.) but also free from both parameter errors and model errors, yet without neglecting uncertainty, and indirectly, reducing it, which makes this work a milestone that calls for continuation. Other works have coupled optimization with network design (optimal monitoring and information extraction from new wells) [7, 31, 40-42, 68], where the former [7] coupled sequential development of the groundwater withdrawal management with sampling strategies, dynamically, which led to the solution of the withdrawal design using a closed-loop stochastic control (dual control) method that includes anticipation of future observation locations; the decomposition of the cost function into deterministic and stochastic parts, particularly, the inclusion of uncertainty in the cost function leads to trade-off between cost of new wells and uncertainty reduction. The sampling network design method sequentially selects new measurement locations based on the combined effect of the state variable (hydraulic head) uncertainty at that location, and the sensitivity of the cost function to that uncertainty. More specifically, new sampling locations are selected using the Bayesian approach (to condition new measurements on existing information) and based on the product of the sensitivity of the stochastic part of the cost function and the modeled (predicted) variance of the state-variable (hydraulic head) at that location; that is, the sensitivity of the cost function to the head uncertainty is weighted by the magnitude of the prediction error – and vice versa (the prediction error is weighted by the sensitivity of the cost function to this error). The head uncertainty is evaluated by first-order, second-moment groundwater flow model, where the head uncertainty is linked to uncertainty in hydraulic conductivity, boundary conditions, recharge, and leakage (all are inputs of the PDE).

3. Interim conclusions

The theoretical developments and adaptations of methods from the different disciplines of operations research, stochastic control theories, and artificial-intelligence/soft-computing for management of oil reservoirs and groundwater remediation have provided insight into

a) the effect of uncertainty (even if just in one parameter) on optimal management and cost;

b) the inseparability of the various components of optimal reservoir management, such as optimal scheduling and best new well location for either pumping/injection or new monitoring wells;

c) the inseparability between optimal management and characterization;

d) the relationships between parameter uncertainty, reliability, and risk;

e) the relationships between parameter uncertainty and cost;

f) the effect of probing the system, system anticipation, and the "caution" that follows;

g) the similarity between the components of sensitivity analyses, random perturbations, and response functions and their "inverse" - weighted feedback laws;

h) the ability to compensate for unknown model errors by determining appropriate weighted feedback policies, particularly under dynamic feedback control;

i) the hidden forms of memory and learning that exist in some statistical models (particularly Bayesian statistics), particularly where recursive/evolutionary information processing takes place, as is the case in some dynamic control systems, and particularly where such processing results in corresponding feedback;

j) the strength of Bayesian approaches in both estimation and uncertainty reduction.

The advantage of the control approach is in shifting the emphasis from one type of intelligence – that of predictions of first principle (physically-based) models to goal-oriented system anticipation (the anticipation of the effect that a control action would have on the goal, i.e., on the cost function), as well as shifting sensitivity analysis (of general model predictions) to sensitivity of the cost function (to parameter uncertainty and particularly, to state uncertainty), which changes the experimental design and overall planning. This goal-oriented intelligence is less "ambitious" than the "know-it-all" first-principle model. Our goal is to increase the intelligence of the goal-oriented anticipating model by combining/integrating knowledge and models from all disciplines in a multiresolutional decision support system (MR-DSS or MRDS), e.g., [4-5, 43-48].

4. Increasing intelligence with intelligent control

Fortunately, the area of intelligent control, particularly, the MRDS has been developing rapidly during the last two decades, combining the advantages and eliminating the limitations of control theories, operations research, and artificial intelligence. For example, an intelligent control agent such as MRDS could free the dynamic control from its ultimate dependence on the rigid PDE, and can increase its learning, accumulated memory, and speed of convergence to optimal solutions by orders of magnitude. Moreover, one of the appealing outcomes of the stochastic approach (including stochastic PDE) – the effect of conditioning on uncertainty reduction via correlations among variables – could be amplified significantly by extending the associations among variables on all relevant scales (through advanced MR clustering methods) far beyond linear the statistical correlation used in the traditional stochastic approach; the Multi-Resolutional (MR) knowledge representation in MRDS maximizes the information hidden in interdependencies among these variables on all levels of resolution, independent of any particular single-scale model. By maximizing extraction of information, the MR approach effectively reduces uncertainty and overcomes the problem of lacking and

corrupted information. Most importantly, by using intelligent control, specifically, a goal-oriented MR knowledge representation, we could eliminate the dependency on PDE models, and use them just as interpretations and general gap-filler in the process of MR rule building (the latter being based on experiences and cause-effect relationships). An adaptive MR knowledge representation is the only way to integrate all the methods from all disciplines – to benefit from the advantages of the different models and eliminate their limitations (particularly their rigid structure); to break through the rigid serial, model-building process (currently done in segregation and isolation) and (hence) to enable more powerful use of data and knowledge from all disciplines; to provide the highest uncertainty reduction possible, and efficient global stochastic optimal control of complex natural resources systems such as oil reservoirs and groundwater, with the highest intelligence and autonomy possible for particular goals.

However, we are still facing the challenge of performing and defining optimal integration between PDE models and multiresolutional representations, and since such integration depends on model quality and data quantity and quality, this is an adaptive integration process as well. One criterion for such "optimal" integration would be uncertainty reduction (resulting from the integrated MR system), which brings up yet another challenge: to define the metrics for uncertainty reduction.

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