



# Wave Chaos and Coupling to EM Structures

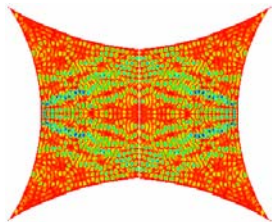


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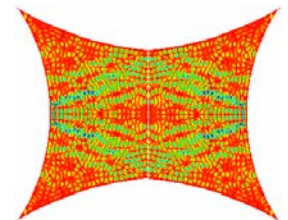
**Faculty: Steve Anlage, Tom Antonsen  
and Ed Ott**



INSTITUTE FOR RESEARCH IN  
ELECTRONICS  
& APPLIED PHYSICS



**Funded by MURI (AFOSR)**



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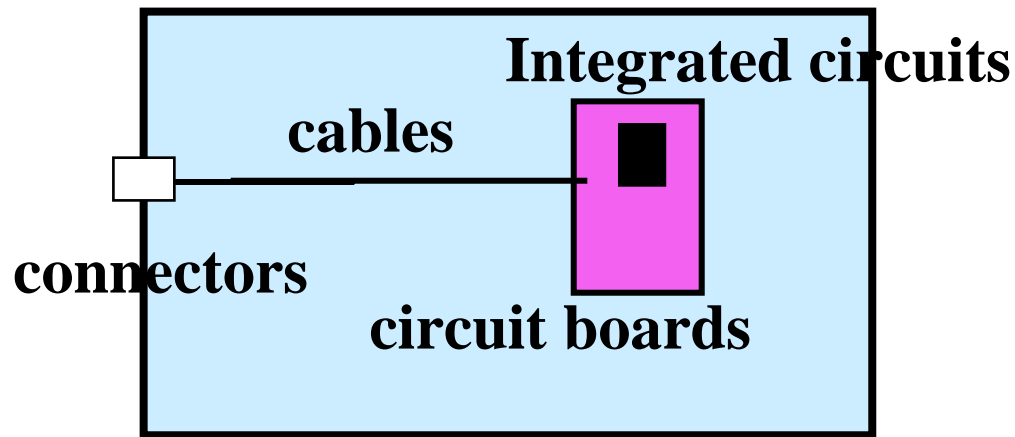
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# Electromagnetic Coupling in Computer Circuits

## Schematic



- What can be said about coupling without solving in detail the complicated EM problem ?
- Statistical description !

- Coupling of external radiation to computer circuits is a complex process:

apertures  
resonant cavities  
transmission lines  
circuit elements



# Outline

- **Part I: Frequency Domain**
  - **Extracting the universal impedance and scattering statistics**
  - **Predictions and tests**
- **Part II: Time Domain**
  - **Model**
  - **Predictions**



# Part I:

# Frequency Domain

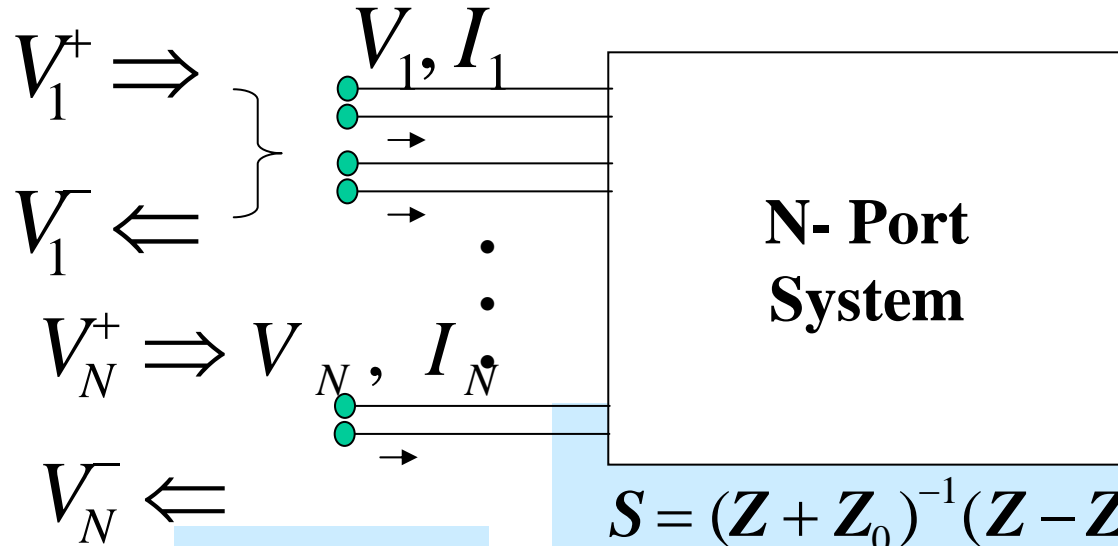


# Z and S-Matrices

## What is $S_{ij}$ ?

**N ports**

- voltages and currents,
- incoming and outgoing waves



**Z matrix**

$$\begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{pmatrix} = \mathbf{Z} \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{pmatrix}$$

**voltage current**

**S matrix**

$$\begin{pmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{pmatrix} = \mathbf{S} \begin{pmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{pmatrix}$$

**refl. inc.**

$Z(\omega) S(\omega)$

- Complicated function of frequency
- Details depend very sensitively on unknown parameters.



# Statistical Model of $\underline{\underline{Z}}$ Matrix

$$\underline{\underline{Z}}(\omega) = -\frac{j}{\pi} \sum_n \underline{\underline{R}}^{1/2}(\omega_n) \frac{\underline{w}_n \underline{w}_n^T \Delta\omega_n^2}{\omega^2 (1 - jQ^{-1}) - \omega_n^2} \underline{\underline{R}}^{1/2}(\omega_n)$$

$\underline{\underline{Z}}$  = **MxM matrix**

$\underline{\underline{R}}$  = **MxM radiation resistance matrix**

$\Delta\omega_n^2$  = **Mean spectral spacing**

$Q$  = **Quality factor**

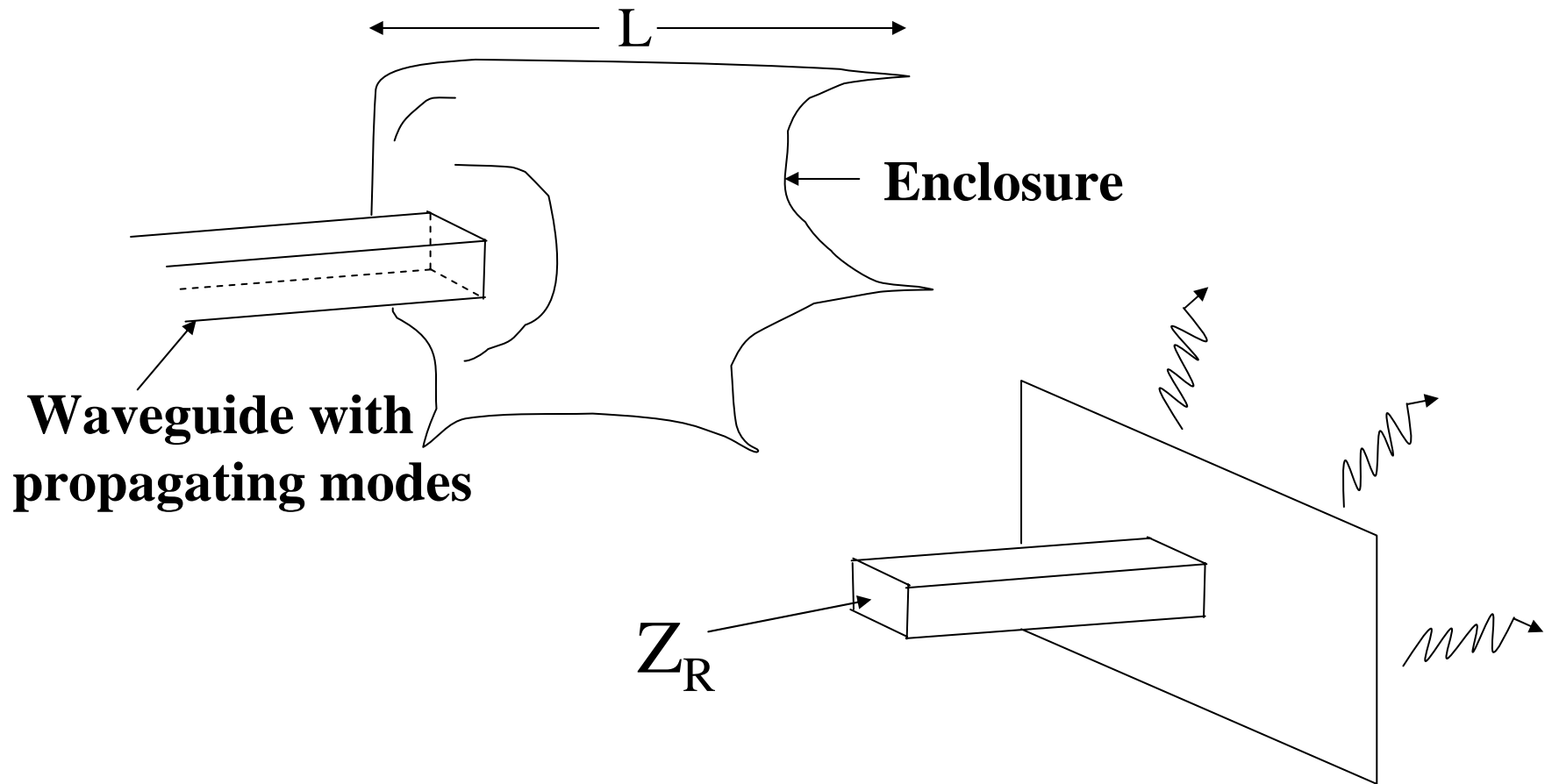
$\omega_n^2$  = **Random Spectrum from RMT**

$\underline{w}_n$  = **M vector of Gaussian random variables**

$$\left\langle \underline{w}_n \underline{w}_n^T \right\rangle = \underline{\underline{1}}$$



# Radiation Impedance



$$\underline{\underline{Z}}_R = \underline{\underline{R}}_R + j \underline{\underline{X}}_R$$





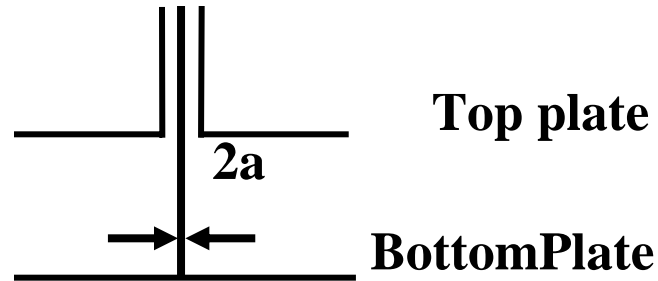
# Universal Properties of Impedance

$$\underline{\underline{\xi}} = \underline{\underline{R}}^{-1/2} (\underline{\underline{Z}} - j \underline{\underline{X}}_R) \underline{\underline{R}}^{-1/2}$$

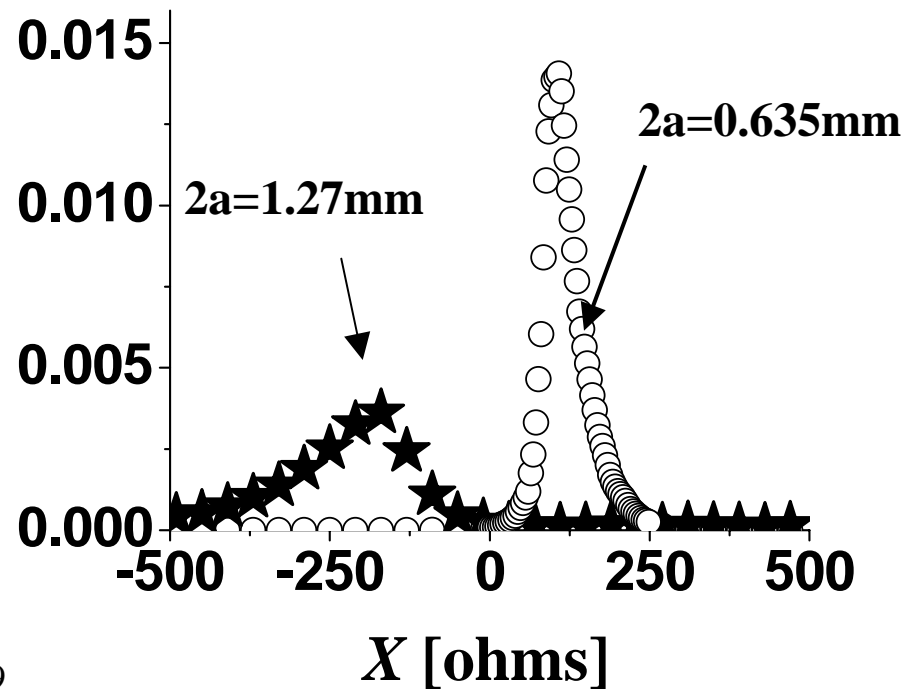
- $\xi$  is universal and obtainable from Random Matrix Theory.
- This applies for  $(\lambda/L)$  small.



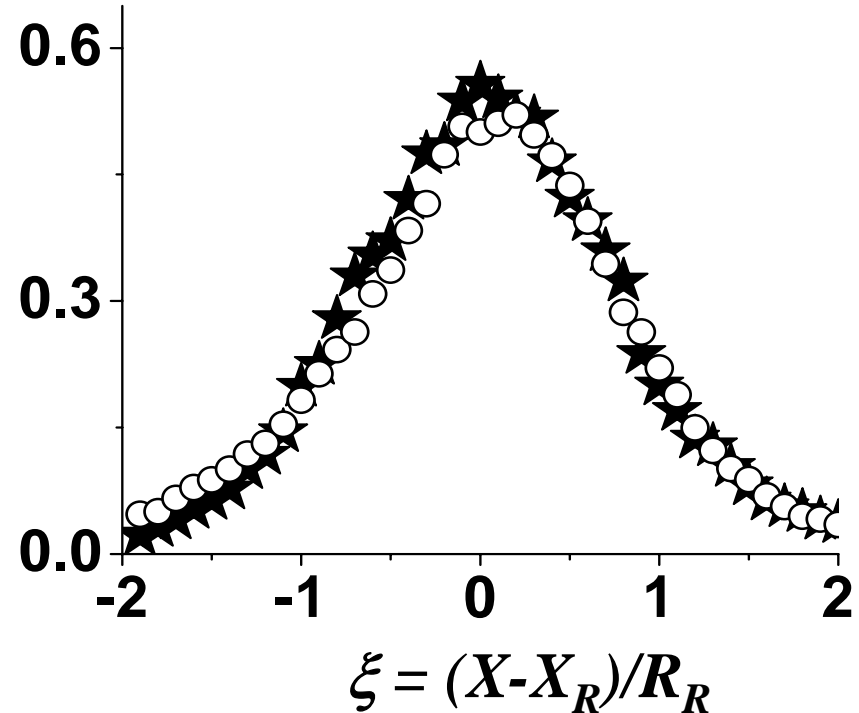
# Importance of Normalization



Raw Data



Normalized





# Past Results

- **Predictions for statistics of  $S$  and  $Z$  matrices.**
- **Tests of predictions against numerical solutions of Maxwell's equations.**
- **Tests of predictions against laboratory experiments.**



# Predictions Tested

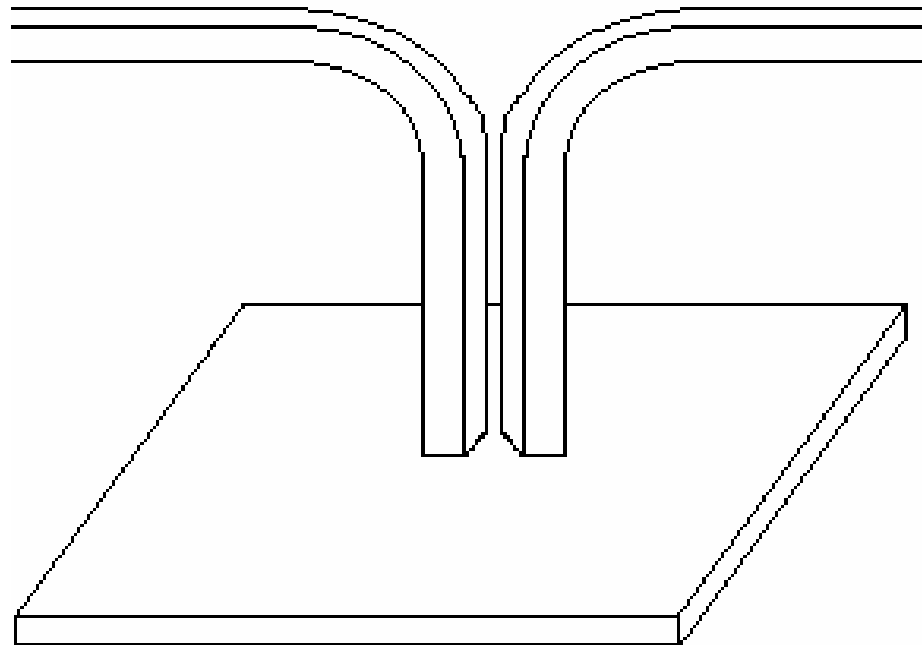
- **M=1**
  - **PDF's of normalized impedances and scattering coefficients as a funct. of loss.**
- **M=2**
  - **Statistics of normalized 2x2 impedance and scattering matrices as a funct. of loss.**
  - **Variance ratios as a funct. of loss [also Fiachetti and Michielsen, Elect. Lett. '03].**

$$VR_Z = \frac{Var[Z_{12}]}{\sqrt{Var[Z_{11}]Var[Z_{22}]}}$$



# Some Predictions Not Yet Tested

- **Situations where reciprocity does not apply**
  - **Magnetized ferrite  $\rightarrow$  different statistics**
- **Situations where off-diagonal elements of  $Z_R$  are significant.**





# Some Predictions Not Yet Tested (continued)

- **M>2: E.g.,**

**reciprocal**

$$\left\langle \left| S_{ij} \right|^2 \right\rangle = \begin{cases} \frac{2}{M+1}, & i = j \\ \frac{1}{M+1}, & i \neq j \end{cases}$$

**non-reciprocal**

$$\left\langle \left| S_{ij} \right|^2 \right\rangle = \frac{1}{M}$$



# Part II:

# Time Domain



# Time Domain Model

## Frequency Domain

$$Z(\omega) = -\frac{j\omega}{\pi} \sum_n \frac{R_R(\omega_n)}{\omega_n} \frac{\Delta\omega_n^2 w_n^2}{\omega^2(1-jQ^{-1}) - \omega_n^2}$$

$w_n$  - Gaussian Random variables

$\omega_n$  - random spectrum

## Time Domain

$$\left( \frac{d^2}{dt^2} + 2\nu_n \frac{d}{dt} + \omega_n^2 \right) V_n(t) = -\frac{1}{\pi} \frac{R_R(\omega_n) \Delta\omega_n^2 w_n^2}{\omega_n} \frac{d}{dt} I(t)$$

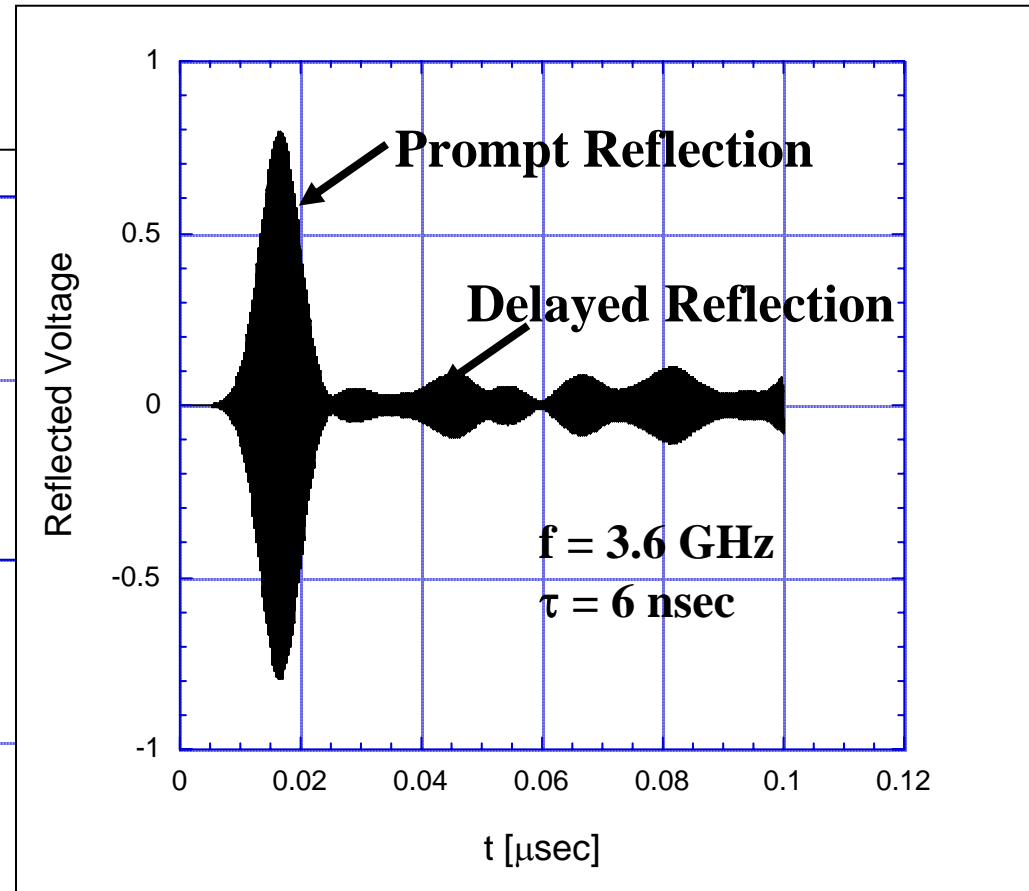
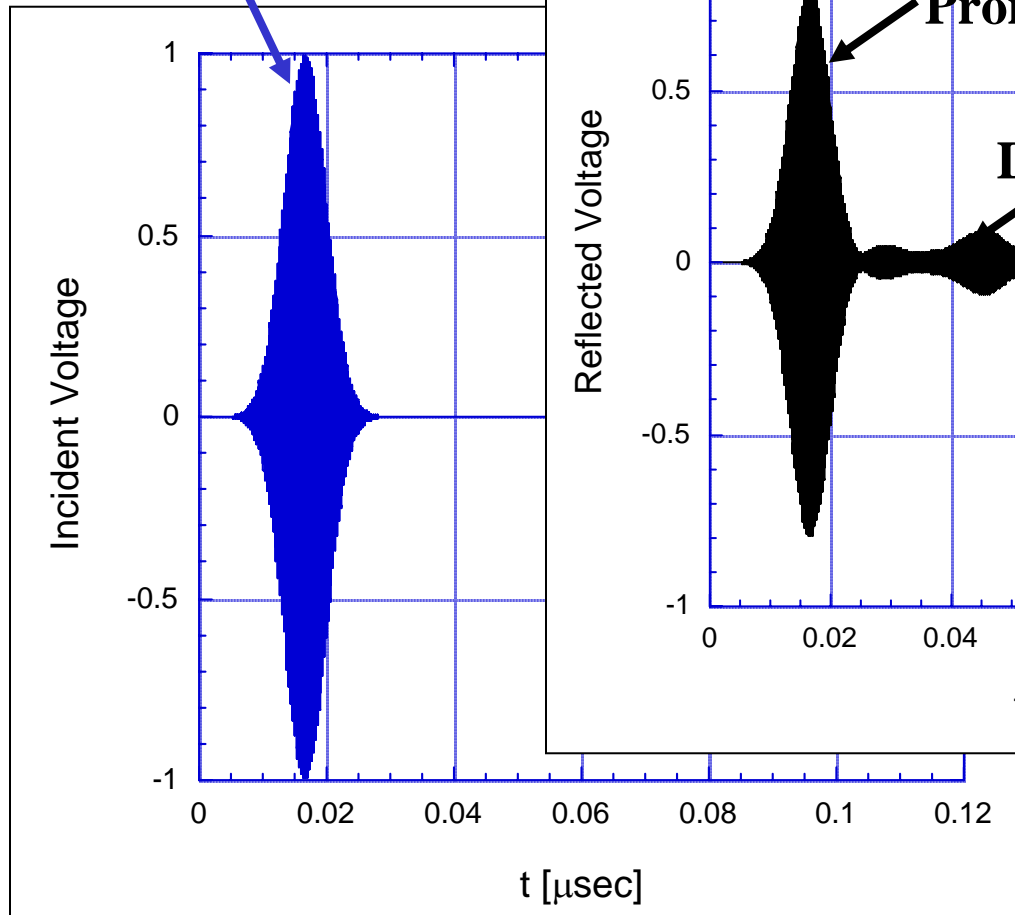
$$V(t) = \sum_n V_n(t) \quad \nu_n = \frac{\omega_n}{Q}$$





# Incident and Reflected Pulses for One Realization

Incident Pulse



Prompt reflection removed by matching  $Z_0$  to  $Z_R$



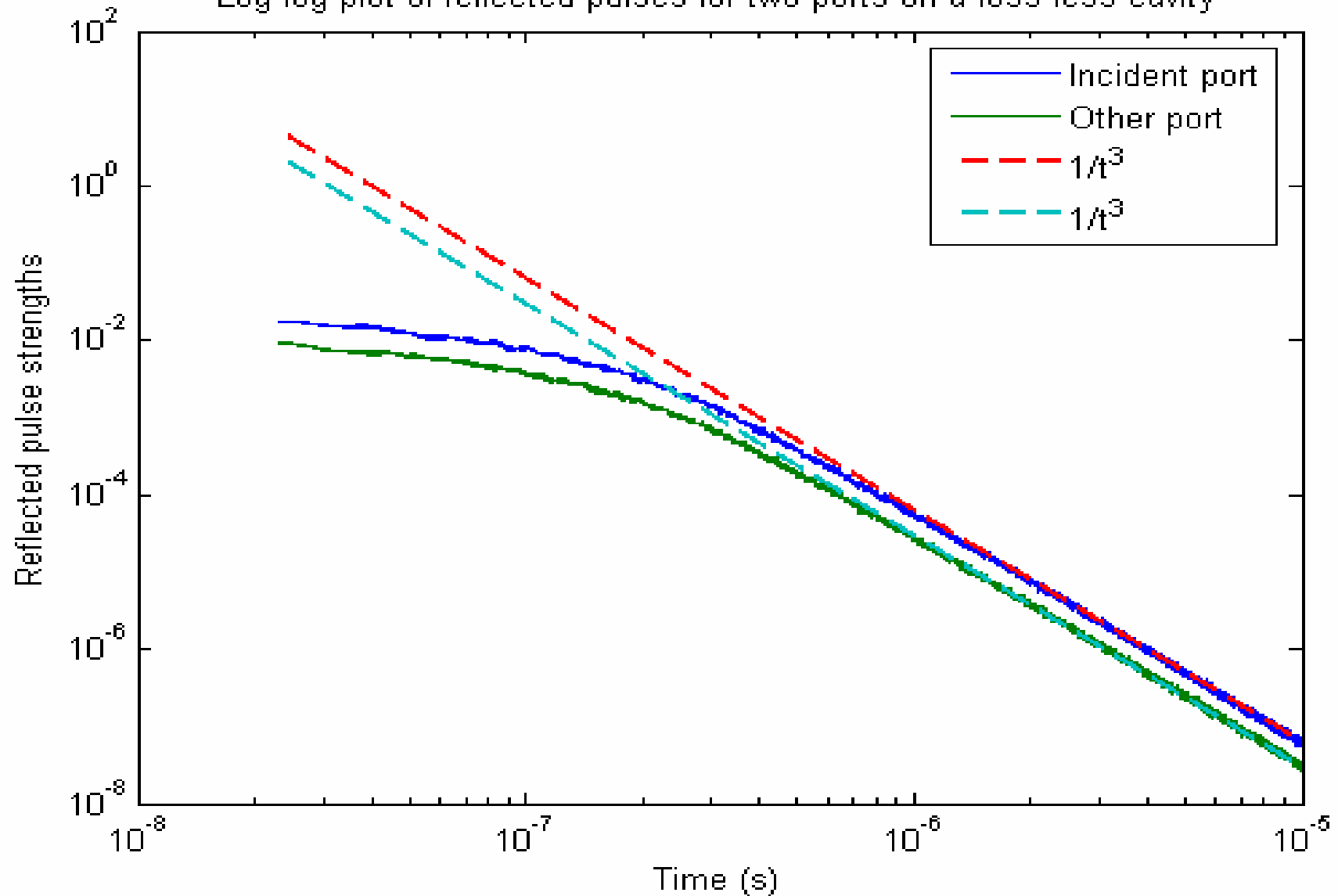
# Decay of Port Voltage - Lossless Case

- **One Port with an Incident Pulse:**  $\langle V^2(t) \rangle \approx 1/t^{5/2}$
- **Two Ports Excited Through Port 1,**
  - a) **all ports matched:**  $\langle V_1^2(t) \rangle = 2\langle V_2^2(t) \rangle \approx 1/t^3$
  - b) **Port 1 matched**  $\langle V_1^2(t) \rangle \approx 1/t^{5/2}$   
**Port 2 strongly mismatched**  $\langle V_2^2(t) \rangle \approx 1/t^{3/2}$
- **N Ports Excited Through Port 1, all ports matched:**  $\langle V_1^2(t) \rangle = 2\langle V_{i \neq 1}^2(t) \rangle \approx 1/t^{(4+N)/2}$



# Simulations of Average Decay

Log-log plot of reflected pulses for two ports on a loss-less cavity



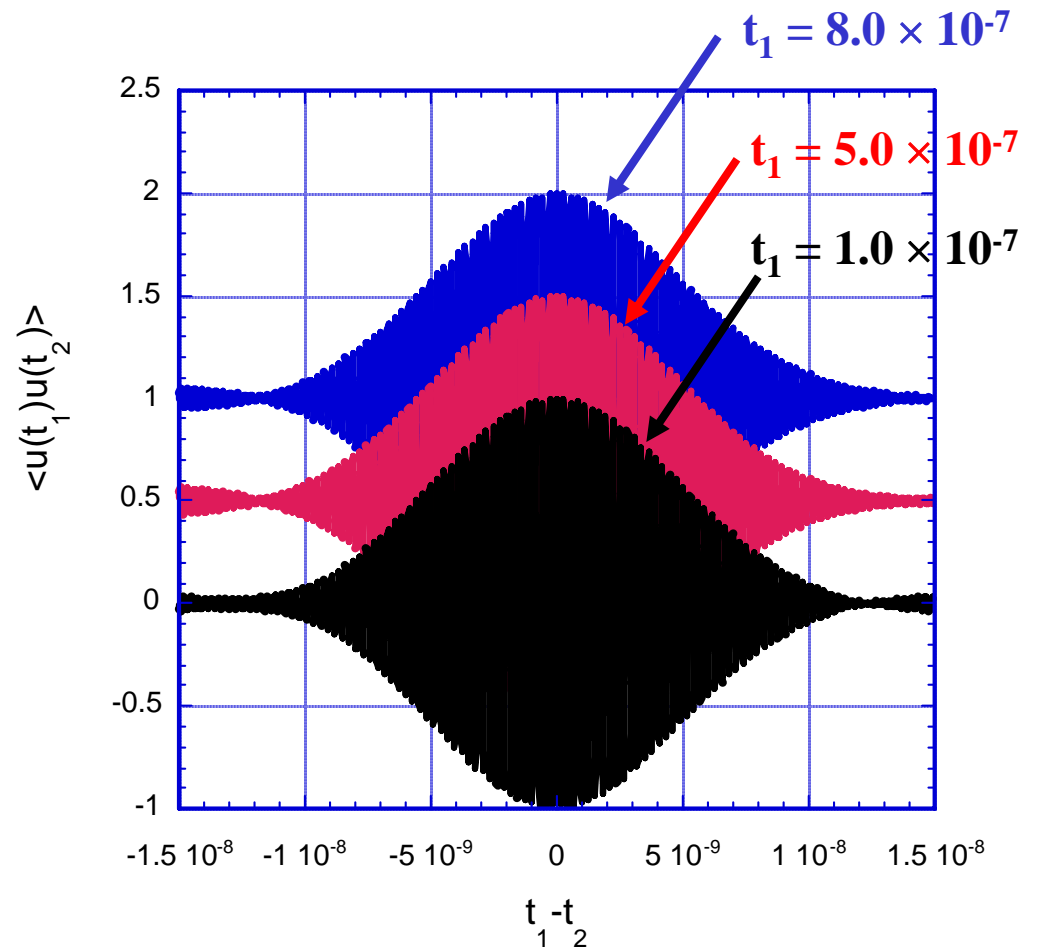
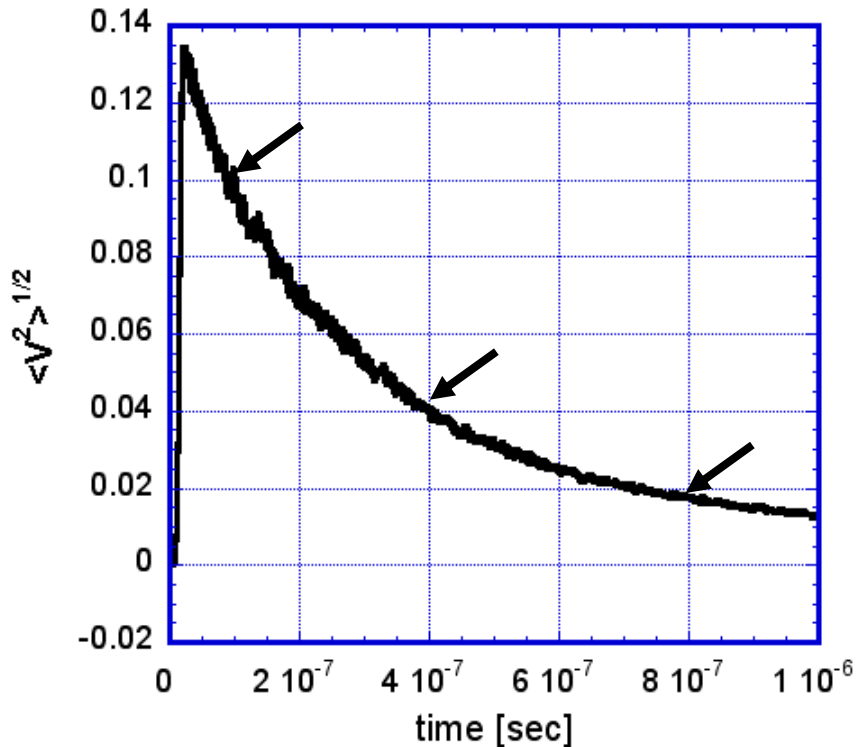


# Quasi-Stationary Process

Normalized Voltage

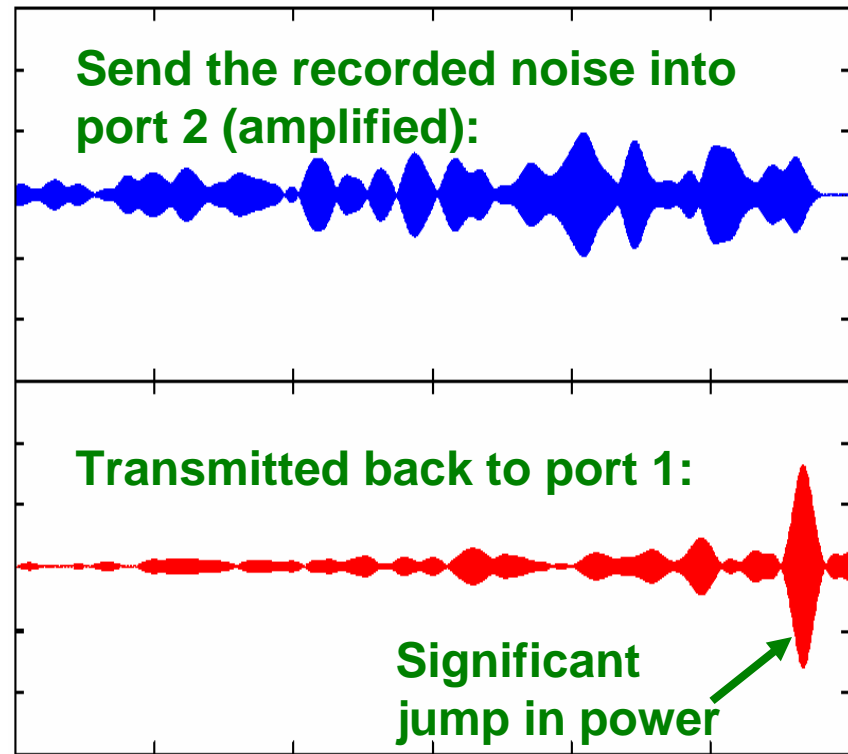
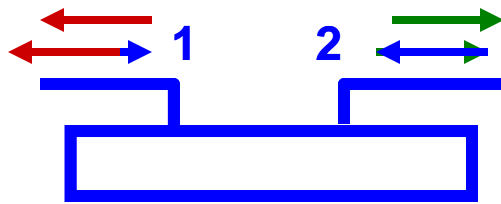
$$u(t) = V(t) / \langle V^2(t) \rangle^{1/2}$$

2-time Correlation Function  
(Matches initial pulse shape)





# Time Reversal Attack (TRA)



- Issues:**
- ‘Fidelity’ under study
  - Magnetized ferrite: breaks time reversal symmetry



# Future Theoretical Work:

- **Corrections for deviations from RMT that occur when  $(\lambda/L) \ll 1$  is not well satisfied**
- **Scars - “Anomalous” hot spots**
- **Networks formed by transmission line links**
- **Statistical aspects of coupling of pulsed signals**



# Publications

1. ***S. Hemmady, X. Zheng, E. Ott, T. Antonsen, and S. Anlage, Universal Impedance Fluctuations in Wave Chaotic Systems, Phys. Rev. Lett. 94, 014102 (2005).***
2. ***S. Hemmady, X. Zheng, T. Antonsen, E. Ott, and S. Anlage, Universal Statistics of the Scattering Coefficient of Chaotic Microwave Cavities, Phys. Rev. E 71***
3. ***X. Zheng, T. Antonsen, E. Ott, Statistics of Impedance and Scattering Matrices in Chaotic Microwave Cavities: Single Channel Case, Electromagnetics 26, 3 (2006).***
4. ***X. Zheng, T. Antonsen, E. Ott, Statistics of Impedance and Scattering Matrices of Chaotic Microwave Cavities with Multiple Ports, Electromagnetics 26, 37 (2006).***
5. ***X. Zheng, S. Hemmady, T. Antonsen, S. Anlage, and E. Ott, Characterization of Fluctuations of Impedance and Scattering Matrices in Wave Chaotic Scattering, Phys. Rev. E 73 , 046208 (2006).***
6. ***S. Hemmady, X. Zheng, T. Antonsen, E. Ott, S. Anlage, Universal Properties of 2-Port Scattering, Impedance and Admittance Matrices of Wave Chaotic Systems, Phys. Rev. E. submitted.***
7. ***S. Hemmady, X. Zheng, T. Antonsen, E. Ott and S. Anlage, Aspects of the Scattering and Impedance Properties of Chaotic Microwave Cavities, Acta Physica Polonica A 109, 65 (2006).***



Photo by Tom Antonsen



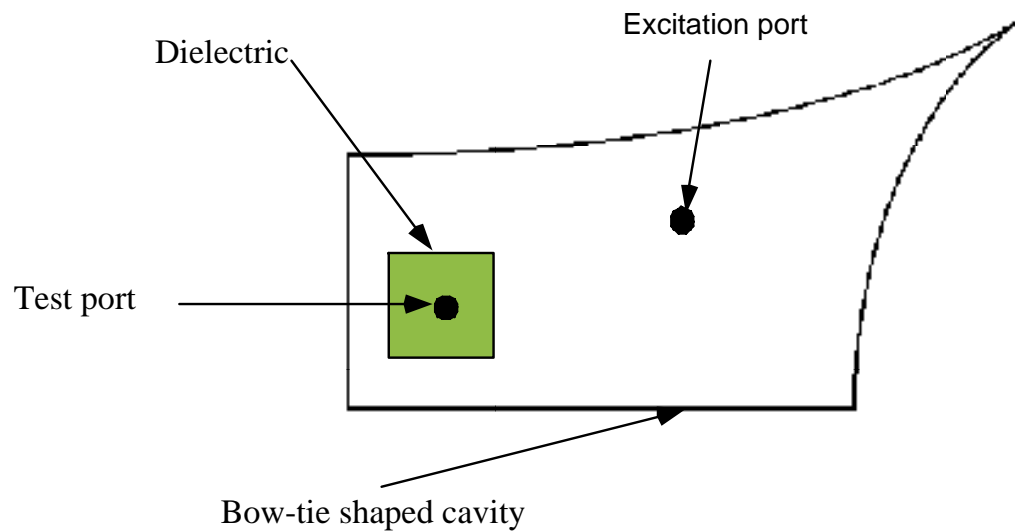


# Part III:

# Open Problems



# More Complexity in the Scatterer



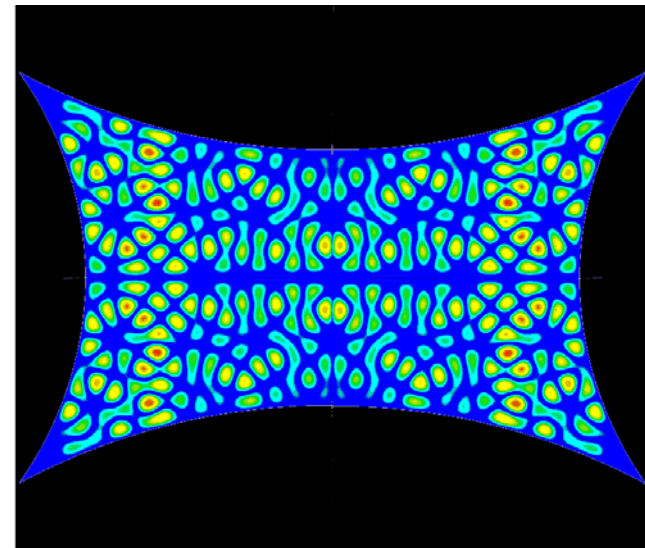
- Can be addressed
  - theoretically
  - numerically
  - experimentally

**Features:**

**Ray splitting**

**Losses**

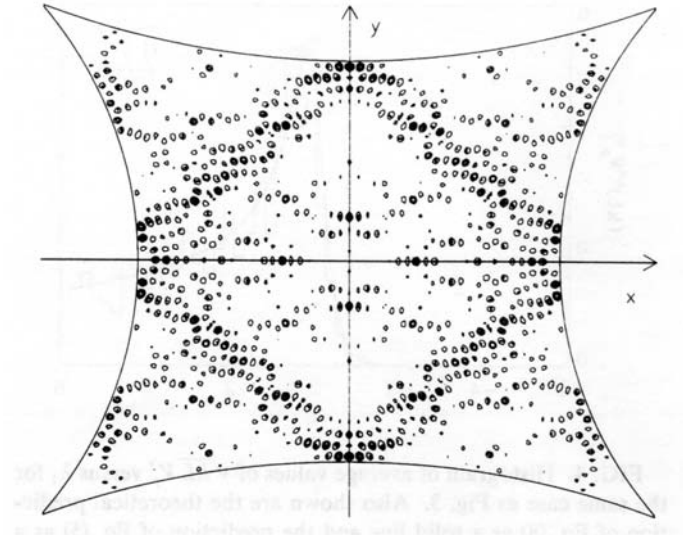
**Additional complications can be added**





## Role of Scars?

- **Eigenfunctions that do not satisfy the random plane wave assumption**
- **Scars are not treated by either random matrix or chaotic eigenfunction theory**



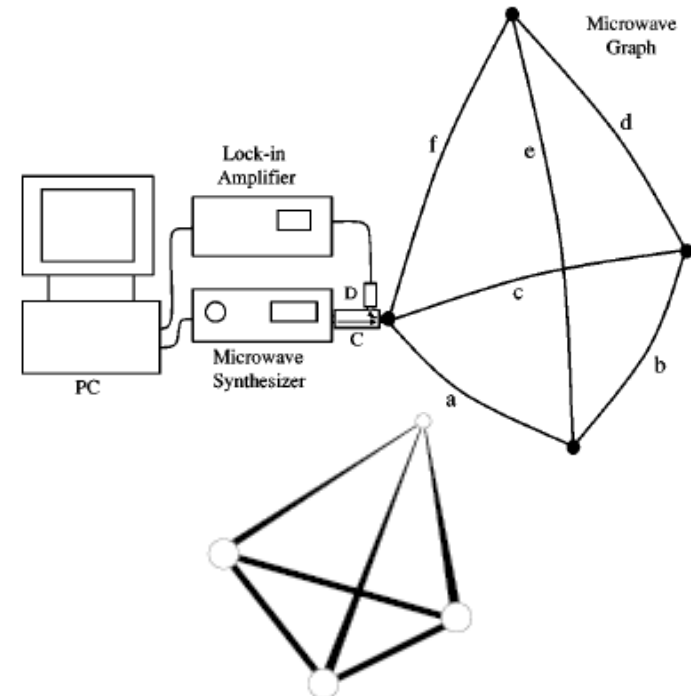
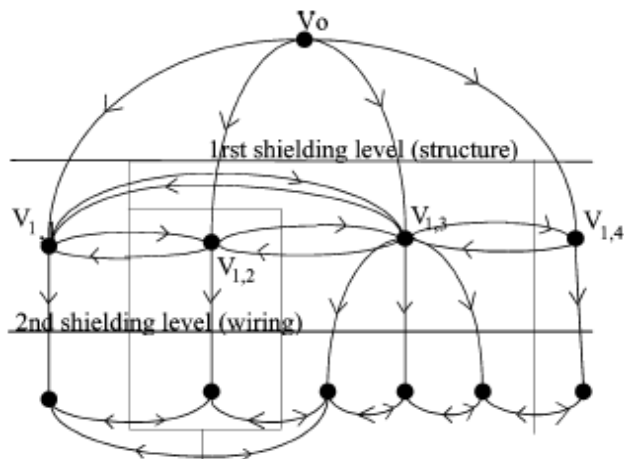
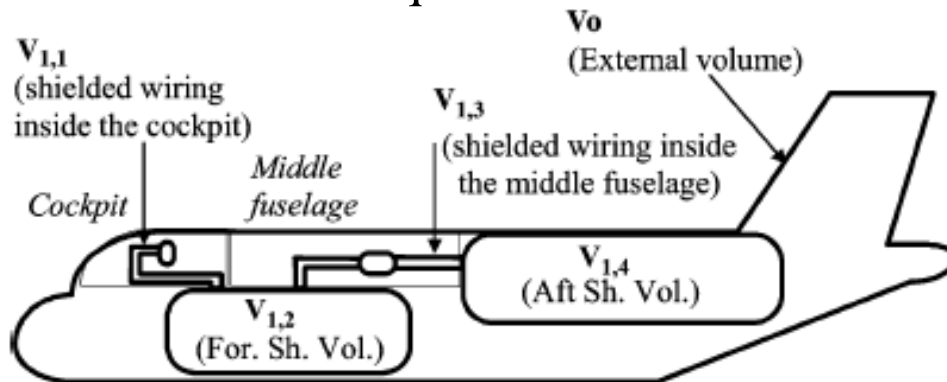
**Bow-Tie with diamond scar**

**Ref: Antonsen et al., Phys. Rev E 51, 111 (1995).**



# Electromagnetic Topology and Wave Graphs

## Electromagnetic Topology BLT Equations



J.-P. Parmentier, IEEE Trans. Electromag. Compat. 46 (3) 359-367 (2004). "Numerical coupling models for complex systems and results"

O. Hul, et al., Phys. Rev. E 69, 056205 (2004). "Experimental simulation of quantum graphs by microwave networks"