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GAME-THEORETIC ANTI-SUBMARINE WARFARE MISSION PLANNER (HEURISTIC-BASED, FULLY EXCEL CAPABLE)

by

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September 2009

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This thesis introduces a Game-Theoretic Anti-Submarine Warfare Mission Planner (G-TAMP) that can quickly operate on a Navy Marine Corps Intranet (NMCI) computer without any software other than NMCI-standard Microsoft Office, Visual Basic for Applications (VBA), and a freely-available optimization package called LP-SOLVE employed as a dynamically linked library. We replace the expensive and non-NMCI approved mathematical modeling software used by Adam Thomas in his 2008 thesis with a purpose-built, fast heuristic solver implemented in VBA. This heuristic, called the Alternating Flows Heuristic, approximately solves the Thomas defender-attacker/defender (D-A/D) model, thereby deploying both visible and secret anti-submarine warfare (ASW) platforms around a high-value unit (HVU) to minimize the probability that a hostile diesel-electric submarine (SSK) penetrates these platforms undetected and reaches the HVU. We analyze five scenarios and compare our heuristic solution with the optimal ones produced by Thomas' D-A/D model.

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GAME-THEORETIC ANTI-SUBMARINE WARFARE MISSION PLANNER (HEURISTIC-BASED, FULLY EXCEL CAPABLE)

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ABSTRACT

This thesis introduces a Game-Theoretic Anti-Submarine Warfare Mission Planner (G-TAMP) that can quickly operate on a Navy Marine Corps Intranet (NMCI) computer without any software other than NMCI-standard Microsoft Office, Visual Basic for Applications (VBA), and a freely-available optimization package called LP-SOLVE employed as a dynamically linked library. We replace the expensive and non-NMCI approved mathematical modeling software used by Adam Thomas in his 2008 thesis with a purpose-built, fast heuristic solver implemented in VBA. This heuristic, called the Alternating Flows Heuristic, approximately solves the Thomas defender-attacker/defender (D-A/D) model, thereby deploying both visible and secret anti-submarine warfare (ASW) platforms around a high-value unit (HVU) to minimize the probability that a hostile diesel-electric submarine (SSK) penetrates these platforms undetected and reaches the HVU. We analyze five scenarios and compare our heuristic solution with the optimal ones produced by Thomas' D-A/D model.

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LIST OF ACRONYMS AND ABBREVIATIONS

This is the lexicon associated with the Alternating Flows Heuristic.

4W	Four-Whiskey Grid
AFH	Alternating Flows Heuristic
D-A	Defender-Attacker model
D-A/D	Defender-Attacker/Defender model
G-TAMP	Game-Theoretic ASW Mission Planner
AMS	General Algebraic Modeling Software
HVU	High-Value Unit
LP	Linear Program
MIP	Mixed Integer (Linear) Program
NM	Nautical Miles
NMCI	Navy Marine Corps Intranet
PMI	Prevention of Mutual Interference
SSK	Hunter-Killer Submarine (diesel)
SSN	Hunter-Killer Submarine (nuclear)
TDA	Tactical Decision Aid
TPZSG	Two-Person, Zero-Sum Game
VBA	Visual Basic for Application

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EXECUTIVE SUMMARY

In the summer of 2008, LT Adam Thomas presented the Game-Theoretic Anti-Submarine Warfare Mission Planner (G-TAMP), an operational planning tool to determine the optimal placement of anti-submarine warfare (ASW) screening platforms around a high-value unit (HVU). For example, the optimal placement of cruisers and destroyers around an aircraft carrier, given the characteristics of the water that the carrier traverses and sonar system performance of each screening platform. This tool is comprised of a Microsoft Excel planner interface, a mathematical programming model implemented in the algebraic modeling language GAMS, and CPLEX, a commercial mixed-integer linear program solver.

Unfortunately, G-TAMP in its current form cannot run on Navy Marine Corps Intranet (NMCI) computers because GAMS and the solver are not approved to run on NMCI systems. We therefore modify G-TAMP to run completely in Excel using heuristic algorithms implemented in Visual Basic for Applications (VBA) and employing a freely-available optimization package, LP-SOLVE, as a dynamic linked library from VBA. The goal of our project is to develop a heuristic-based algorithm that approximately solves the underlying D-A/D model acceptably well while still running at relatively high speeds and on NMCI computers.

We define "visible defender platforms" to be those ASW screening platforms easily observable by the enemy, such as surface ships, and other platforms employing active sonar and "secret defender platforms" to be those ASW screening platforms that remain hidden from the enemy, such as a friendly submarine employing passive sonar. Additionally, we define the problem of visible ASW screening platform assignments as the visible defender platform sub-problem and the problem involving the transit of a single hostile SSK towards a HVU, while avoiding detection, as the enemy SSK sub-problem.

We present the Alternating Flows Heuristic, which operates through alternation between two complimentary stages. The first stage utilizes network analysis to solve the visible defender sub-problem, identifying the best assignment of all visible defender platforms to acoustic modes and ASW missions. Once this assignment is made, our heuristic solves the enemy SSK sub-problem through minimizing detection probability from all visible defender platforms along a single path to the HVU (shortest-path problem minimizing overall detection probability). Our heuristic updates the visible defender platform sub-problem for ASW missions that intersect this path, solves the visible defender platform sub-problem again, and continues the alternation with another solve of the enemy SSK sub-problem. This alternation continues until either a visible defender platform lay-down prevents an enemy SSK from utilizing an unopposed path to the HVU, or the heuristic reaches a planner-defined iteration limit.

We supplement our heuristic's first two stages with an additional stage that models Secret defender platforms as a two-person, zero-sum game (TPZSG). Because of Excel 2007 limitations, this step can only be used if a single secret defender platform (i.e., friendly SSN) is deployed and available for mission tasking. We formulate the TPZSG as a linear program and solve it via LP SOLVE, an open-source linear programming routine. This suggests a "mixed strategy" of continuous probabilities used to produce the secret defender platform lay down and the resulting optimal attack paths of the enemy SSKs respectively. When the planner deploys secret defender platforms, our heuristic uses the "mixed strategy" of optimal enemy SSK attack paths as the solution to the enemy SSK sub-problem (in lieu of shortest-path problem).

Our heuristic algorithm implementation of G-TAMP runs on average 800% faster (8x faster) than the original G-TAMP, produces solutions that are on average within 78% of optimal, and runs completely in Excel, thereby allowing installation on NMCI computers. Heuristic G-TAMP uses no licensed software other than NMCI Microsoft Office, and the incremental cost-per-seat is zero.

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I. INTRODUCTION

A. THE G-TAMP DRAWBACK: NMCI NON-COMPATIBILITY

LT Adam Thomas (2008) presents the Game-Theoretic ASW Mission Planner (G-TAMP), an operational planning tool to determine the optimal placement of screening (hereafter defender) platforms around a high-value unit (HVU). For example, planners may use this tool to optimally deploy ASW screening platforms around an aircraft carrier, given the characteristics of the water and sonar system performance of each screening platform. G-TAMP provides defensive plans that are optimal in the sense that they maximize the probability that the defender platforms will detect any inbound enemy SSK. This tool comprises of a Microsoft Excel (2009) planner interface, a mathematical programming model implemented in the algebraic modeling language GAMS (GAMS 2009), and CPLEX (2009), a commercial mixed-integer linear program solver.

Unfortunately, the Thomas version of G-TAMP cannot run on U.S. Navy Marine Corps Intranet (NMCI) computers due to its use of GAMS and CPLEX, neither of which is approved to run on NMCI systems. We modify G-TAMP to run completely in Excel using heuristic algorithms implemented in Visual Basic for Applications (VBA) (2009) and employing a freely-available optimization package, LP-SOLVE (Berkelaar, 2009), as a dynamic linked library from VBA.

In comparison to the $8,000 per-computer licensing fees for the use of GAMS and CPLEX, the VBA heuristic algorithms we present are free. However, heuristic algorithms are not guaranteed to give optimal answers, but rather give feasible, and hopefully near-optimal, solutions.

We use the original G-TAMP optimization to tune our heuristic by testing it with the five scenarios that Thomas presents to minimize the difference between our heuristic solutions and the optimal ones. Similar to Thomas (2008), our goal is to provide defensive plans that maximize the probability of detecting enemy SSKs. Throughout, we develop the mathematics in terms of evasion probabilities and explicitly formulate our defender models to minimize enemy SSK evasion probability, and our attacker model to maximize this.

B. DEFINING THE ASW PROBLEM: THE G-TAMP LEXICON

We adapt Thomas's lexicon to describe the individual aspects of our updated G-TAMP program. The following is drawn directly from the Thomas thesis. The terms highlighted in italics describe the formal lexicon used in our work.

1. Geography

We partition a specific area of interest on the ocean into a Four-Whiskey (4W) grid. The U.S. Navy utilizes these grids to coordinate operations on, above, or below the water surface. Such grids vary in both cell length and width, and are stationary. Typical cell sizes range between 5 nm and 10 nm per side. In our models, we use index *g* to denote a grid cell. An example of a 6x6 4W grid, taken from the Thomas thesis, is shown as Figure 1.

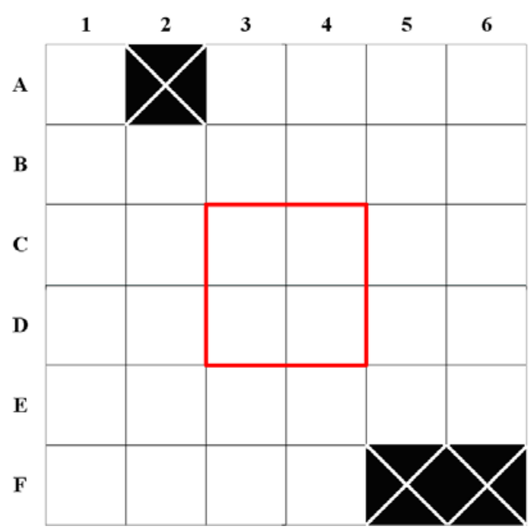

Figure 1. 4W grid (From Thomas, 2008)

A 4W grid cell is identified by its row letter and column number, e.g., C3. Each black cell with a white "x" is impassable; all other cells are traversable. The square-boxed region in the center (cells C3, C4, D3 and D4) denotes a set of protected cells.

A *traversable cell* denotes a location through which an enemy SSK may transit, or that a defender platform may patrol. In contrast, *impassable cells* prevent passage by either an enemy SSK or defender platform, representing land or shoal waters. We label each cell with an alphanumeric designator; for example, "F6" is the cell in the bottom right-hand corner of Figure 1.

A traversable cell g connects to an adjacent traversable cell, denoted as g', forming a directed *arc* (g, g') in a network model. An arc symbolizes potential feasible motion in either the horizontal, vertical, or diagonal directions for an enemy SSK. For example, in Figure 1, an SSK could move from cell F4 to cells F3, E3, E4, or E5, but not to F5.

A *protected cell* constitutes a traversable cell located such that an enemy SSK, having reached that cell, can stage an attack on the HVU. Consequently, defender platforms must be located around the protected cells in order to prevent unobserved entry by the enemy SSKs. We assume the HVU will reside and conduct operations within the region of protected cells. We also assume that defender platforms cannot patrol inside the protected cells, but rather patrol around them thus providing protection to the HVU. This assumption follows from the fact that, once the enemy has closed to within weapons range, detection is near-useless. We further assume that an attack by an enemy SSK can come from any protected cell, thus we desire to prevent unobserved entry into all protected cells. A *protected region* comprises a grouping of these cells that we illustrate by the square-boxed region in the center of Figure 1. We can accommodate more than one protected region in a 4W grid.

2. Measure of Effectiveness

Each ASW sensor employed by a defender platform has an effective detection range for an enemy SSK. This detection range varies by sensor characteristics and system performance. Twice this range defines the sensor's *sweep width* and represents the detection capability of the sensor. A second definition of sweep width, describes it as the area under the lateral range function *p(x)* of a particular sensor, a concept defined as "the probability that the target will be detected if its track relative to the searcher is a straight line, infinitely long in both directions, with closest point of approach *x*" (Washburn, 2002, p. 4–1).

In order to quantify our search effectiveness in locating enemy SSKs, we assume, following Washburn (personal communication, January, 2009), that a given defender platform requires a certain allotment of time to completely search each traversable cell. The product of the defender platform's search speed and time spent in each traversable cell constitutes the platform's search area, per unit time, in that traversable cell. From this, we define *coverage rate* as the fraction of the area searched, per unit time, for each traversable cell:

$$r = \frac{(sweep\ width)(speed)}{cell\ area}.$$

In reality, coverage rate depends on a multitude of other factors including, but not limited to: crew proficiency; environmental effects such as a cell's water temperature distribution, sea-state, water depth, bathymetry, sea-life density; shipping density; and background noise (e.g., Urick, 1983). Because coverage rates vary by defender platform-sensor-cell combinations, we use a separate, precalculated coverage rate for each such combination.

The *search pressure* in a given traversable cell results from the total amount of search effort being applied to that cell; i.e., the sum over all defender platform-sensor combinations taken for each traversable cell of the product of coverage rate and search time spent in that particular cell. Well-searched traversable cells have "high pressures" and, thus, if the enemy can sense the search effort, he will seek paths that avoid such cells. See Appendix A of Thomas (2008) for an explanation of how to convert a search pressure into a detection probability.

3. Enemy Course of Action

We assume only one enemy SSK attempts to gain access to the protected region(s). In reality, multiple enemy SSKs may make this attempt; however, protection from a single enemy SSK provides protection from any number of them.

An enemy SSK attempts to reach some cell within the protected region(s), placing itself within weapons range of the HVU. It accomplishes this by transiting from any location outside of the 4W grid, through the traversable cells, and eventually reaching a protected cell. The goal of the enemy SSK is to maneuver in order to maximize the probability that it reaches any cell within the protected region(s), while remaining undetected.

4. Friendly Course of Action

Each defender platform is categorized into one of several types, and will search for enemy SSKs in the cells to which it is assigned.

A *visible defender platform* can be observed by the enemy via visual sensors, acoustic sensors, or electronic surveillance measures. A visible platform can be a surface vessel, maritime patrol aircraft, submarine employing active sonar systems, or any other type of platform utilizing active search thereby making itself detectable by an enemy SSK.

In contrast, a *secret defender platform* remains hidden and therefore not observable to the enemy. These platforms include friendly submarines that employ passive sonar systems. An enemy SSK may have intelligence pertaining to the presence of secret defender platforms. For example, a carrier battlegroup is conventionally accompanied by one or two escort submarines. However, the exact location of a secret defender platform remains unknown to the enemy. In addition, the planner may choose to employ two secret defender platforms, but wish to segregate them by separate patrol areas. This concept, known as prevention of mutual interference (PMI), allows planners to utilize multiple submarines while minimizing the chance of an underwater collision between them.

A *tethered defender platform*, for example, a helicopter deployed from a surface ship, must always stay within range of its *base defender platform*. Consequently, a tethered platform only performs missions within its *tether range*.

A *mission* consists of one, two, or three contiguous cells in the 4W grid patrolled by an assigned defender platform, the time allotted for the search in each cell, and a *sensor mode* employed in the search (active or passive). Viewing the 4W grid network as an undirected graph, we require that the cells that make up a mission, along with their associated connecting arcs, constitute a connected subgraph. This ensures that individual missions contain groups of contiguous cells that are within close proximity to each other. We enumerate each subgraph containing up to three cells. A subgraph becomes a potential mission as long as it complies with the planner's requirements, which include platform speed, range, overall mission length, and pre-designated area restrictions (e.g., a submarine scheme to prevent mutual interference).

C. ASSIGNING ASW DEFENDERS TO MISSIONS: THE G-TAMP MODEL

The Thomas G-TAMP, in its original form, employs a tri-level, defender-attacker/defender optimization model (D-A/D) that minimizes the probability that any enemy submarine penetrates all defender platforms and attacks the HVU. His model allows several enemy SSKs to attempt the approach to the HVU; as previously stated, our model assumes only one. The Thomas version of this model is solved as a traditional mixed integer program (MIP), with an objective function composed of terms representing the contribution of search pressure to detect SSKs. The first two parts to the model, the defender-attacker (D-A) portion, deal with the placement of visible defender platforms (i.e., destroyers, P-3s, and/or helicopters using active acoustic search methods, etc.) around the HVU, while each enemy SSK observes, reacts to evade detection, and chooses the minimum-risk route to the HVU. The third part is an extension that allows the defender to use secret defender platforms (e.g., friendly submarines using passive acoustic search) to assist in minimizing the probability that the enemy SSKs reach the

HVU undetected. The program suggests an optimal search plan, specifying the placement of all defender platforms around the HVU, and protecting it from worst-case enemy submarine attacks.

The first two stages of Thomas's G-TAMP model are optimally solved via decomposition, a series of alternating, sequential reactions between the visible defender platforms and the enemy SSKs. In the Thomas implementation, the visible defender platforms seek to deploy themselves to maximize the search pressure they exert against the enemy SSKs. The enemy SSKs in turn, observe this deployment and attempt to maneuver around the visible defender platforms to minimize the search pressure to which they are exposed. Because of the sequential nature of these moves, this is a variant of a two-stage sequential game with perfect information represented as a *defender-attacker* (D-A) model (Brown, et al. 2006). This provides a worst-case scenario in that the attacker realizes and reacts optimally to the entirety of the defensive lay down.

In a third stage, Thomas expands on the D-A model to include secret defender platforms. At first, these platforms were modeled in a true, tri-level *defender-attacker-defender* model. In this model, the secret defender platforms would respond after the attackers (enemy SSKs) had reacted solely based on the deployments of the visible defenders. Thomas solved this model; however, the recommended optimal plans were unrealistic. Specifically, if an enemy SSK was to have only one optimal path, then one or more hidden defenders would deploy to that one path to intercept. If the attacker does not have complete knowledge of the environment, compared to the secret defender platforms, the attackers may choose sub-optimal paths, thereby avoiding the secret defender platforms by chance. This suggests that both the secret defender platforms and the attackers have at least some knowledge concerning each other's presence and the environment. In addition, both the secret defender platforms and the attackers have a common, yet opposed objective: detection probability expressed linearly as additive search pressure. The enemy SSKs still want to minimize the overall search pressure exerted against them while the secret defender platforms want to maximize their search pressure in conjunction with the visible defender platforms.

These two concepts led Thomas to employ a two-person, zero-sum game (TPZSG) with simultaneous play and perfect information. Consequently, he develops the model called *defender-attacker/defender* (D-A/D), where the hyphen denotes sequential play and the forward slash signifies simultaneous play.

II. HEURISTIC TO SOLVE G-TAMP D-A/D MODEL

A. ALTERNATING FLOWS HEURISTIC

We introduce the Alternating Flows (AF) heuristic to solve G-TAMP. AF alternates between applications of two complimentary models. The first model is solved as a sequence of network models, each representing sequential assignment of a visible defender platform to a search mission to protect against any enemy SSK attack paths, followed by restrictions for following assignments, and then another assignment, until all defenders have been given missions. The second models the optimal attack path for an enemy SSK, given a known defensive lay down.

We make the following initial assumptions. First, some defender platforms may have fixed missions, and we only re-assign those that do not. All defender platforms contribute to overall detection probability. Second, visible defender platforms fully deploy prior to the approach of any enemy SSK. Third, the visible defender platforms and the enemy SSK have the same knowledge available to them concerning the situation (e.g., ocean environmental data, sensor performance, etc.). Four, as previously stated, our model assumes only one enemy SSK attempts to gain access to the protected region(s).

1. Enemy SSK Sub-Problem: Minimum-Risk Path

We formulate and solve the sub-problem of routing an enemy SSK to the protected region(s), while avoiding visible defender platforms whose positions are given by a vector $\bar{X}$ of visible defender platform mission assignments, as a shortest path problem. The enemy SSK's network represents cell-to-cell paths in the 4W grid with nodes representing cells and arcs representing allowed cell adjacency movements. Figure 2 depicts an example of this network.

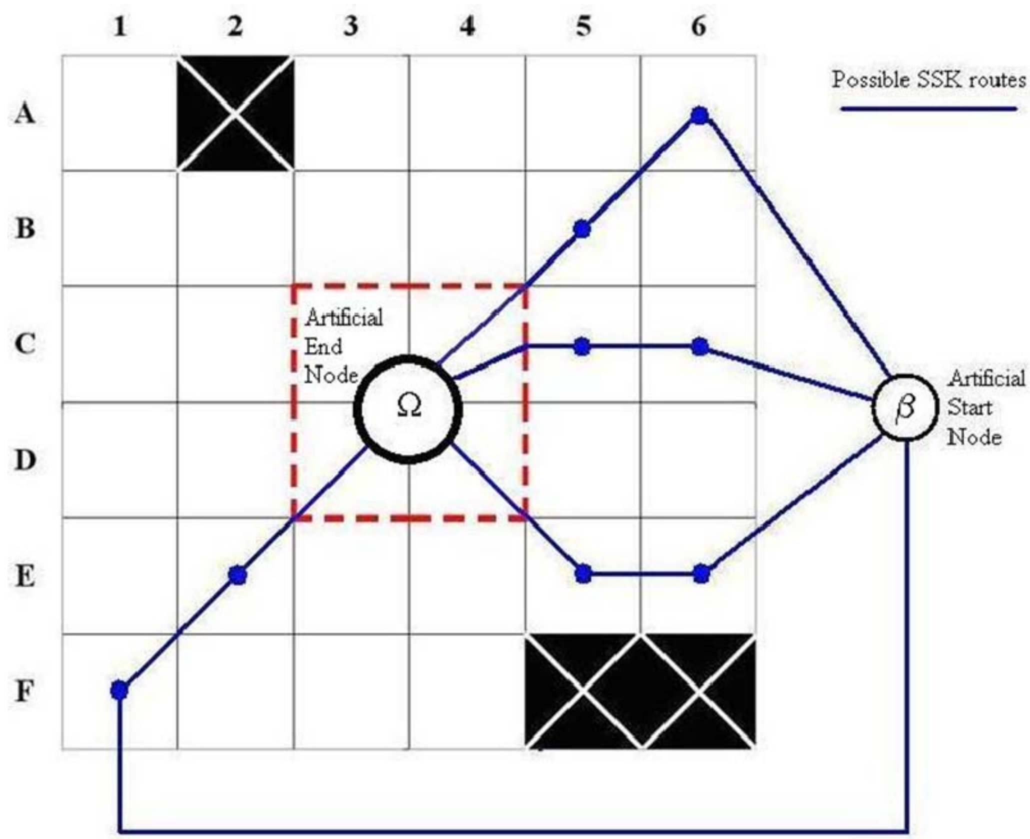

Figure 2. 4W grid depicting potential enemy SSK attack paths

Expanding on the previously defined 4W grid, we add an artificial start node, β, that is adjacent (i.e., connects to) each potential traversable entry cell along the outer edges of the 4W grid. These cells then connect to other traversable cells forming paths that lead to the cells bordering the protected region (dashed box). We add artificial arcs from each of the cells that border the protected region to an artificial end node, **Ω**. Here, we illustrate four such paths (solid lines).

Each potential SSK path can begin in an arc initiating from any designated origin cell on the border of the 4W grid, and culminate at a cell that borders the protected region. An enemy SSK cannot traverse any of the impassable cells (i.e., cells representing a geographic obstacle preventing either defender platform or enemy SSK from operating). To this network, we add an artificial supply node β and an artificial demand node **Ω**. In addition, we add an arc from β to each allowable entry cell in the 4W grid and from each protected region(s) perimeter cell to **Ω**, with arc capacity defined the same as for the network interior arcs. The search pressures, generated by the visible defensive lay down influence the arc costs on the interior arcs of this network, a concept further explained in the formulation below. We assign a capacity of one to each arc representing the traversal of a single enemy SSK.

We want to model the approach of the enemy SSK as a shortest-path problem in which a longer paths represents a higher-search-pressure traversal for the SSK. In order to avoid a situation in which there are several convoluted, but equally attractive, paths, we need to adjust the arc lengths so that, between any two paths with equal search pressures, the enemy SSK will prefer the one with the fewest arcs, and therefore the more direct route to the protected cells. We accomplish this, similarly to Thomas, by introducing a parameter called *battery* that adds a small search pressure cost (subsequently increasing detection probability) for every arc traversed by the enemy SSK.

a. Indices and Index Sets

$p \in P$	set of all defender platforms (visible or secret) [alias p', p''].
$p \in P_{Vis} \subseteq P$	set of all visible defender platforms [platform].
$m \in M$	potential enumerated missions [alias m', m'']
$s \in S$	sensor modes, with $S = \{s_1, s_2\} \equiv \{active, passive\}$ [alias s', s'']
$g \in G$	4W grid cells [alias g']
$m \in M_{ps} \subseteq M$	missions platform p can perform with sensor mode s
$g \in G_{pm} \subseteq G$	cells patrolled by visible defender platform p when executing mission m
$(g, g') \in A_{SSK}$	adjacency list specifying allowed SSK moves from grid cell g to grid cell g'
g^+_{SSK}, g^-_{SSK}	artificial start cell and terminal cell for SSKs, respectively

b. Data [Units]

r_{psg} coverage rate of platform *p* using sensor mode *s* in grid cell g [hr^{-1}]

$\bar{X}_{pmsg}$ time that visible defender platform *p* spends executing mission *m* using sensor mode *s* in grid cell *g* pre-computed based on coverage rates of cells $g \in G_{pm}$. [hr]

battery penalty incurred by an enemy SSK for traversing a single arc [pressure].

$dist_g$ distance from cell, *g*, to nearest cell within a protected region(s).

d weighting factor for distance penalty $\left[1/nm*hr\right]$

c. Variables [Units, if applicable]

$Y_{gg'}$ binary variable representing a single SSK that travels arc (g, g') [submarines]

d. Formulation SSKSP($\bar{X}$)

$$\left[\begin{array}{ll}
\min\limits_{\mathbf{Y}} & \sum\limits_{(g,g')\in A_{SSK}} \left(battery + \sum\limits_{\substack{p\in P, m\in M_{ps} \\ s\in S | g'\in G_{pm}}} \left(r_{psg} - d * dist_g\right) \bar{X}_{pmsg} \right) Y_{gg'} \\
 & \\
\text{s.t.} & \sum\limits_{g'|(g,g')\in A_{SSK}} Y_{gg'} - \sum\limits_{g'|(g',g)\in A_{SSK}} Y_{g'g} = \begin{cases} +1 & g = g^{+}_{SSK} \\ 0 & g \in G - \{g^{+}_{SSK}, g^{-}_{SSK}\} \\ -1 & g = g^{-}_{SSK} \end{cases} \\
 & Y_{ij} \in \{0,1\}
\end{array}\right]$$

In this case, the attacking SSK suffers all the search pressures exerted by the visible defending platforms, or just battery if no visible defender platform is present. The search pressures resulting from the most recent defensive lay down $\bar{X}$ determine the costs of the SSK network. If an SSK traverses from cell A1 to B2, it will suffer a cost equal to that of the pressure being exerted on B2. In the case of visible defender platforms only, the enemy SSK sub-problem network is solved to determine the path to the protected region(s) that offers the least amount of pressure (i.e., a shortest-path problem).

2. Visible Defender Sub-Problem

We solve the sub-problem of assigning visible defender platforms to missions as a sequence of minimum-cost network flow models, where each successive network model suggests for each unassigned platform an acoustic search mode and a particular mission. Among all suggested assignments, only the best one is adopted, and assigned. The network is then restricted and conditioned to account for this latest assignment (including accounting for consequences of this assignment on consequent ones), and we repeat with another network model. Figure 3 illustrates a generic visible defender platform network.

The rationale behind solving successive network restrictions, fixing one visible defender platform assignment per solve, is that each of these network optimizations can be solved in negligible computation time, yet offer some omniscience in that each network solve returns the local optimal synergistic assignments of the restricted problem at hand. That is, after some number of visible defender platforms have been assigned search modes and missions, the conditions facing the remaining platforms can be conditioned on this (thus accounting for constraints and computations that would otherwise require more general optimization tools) providing a restricted network successor to solve. For each restricted model in this succession, from all suggested assignments, only the best is fixed, and we continue with more restricted models until every visible defender platform is fixed. This takes advantage of the sheer speed of the network optimization, striking a balance between a purely myopic platform-by-platform assignment, and true optimization with all inter-platform relationships endogenously modeled.

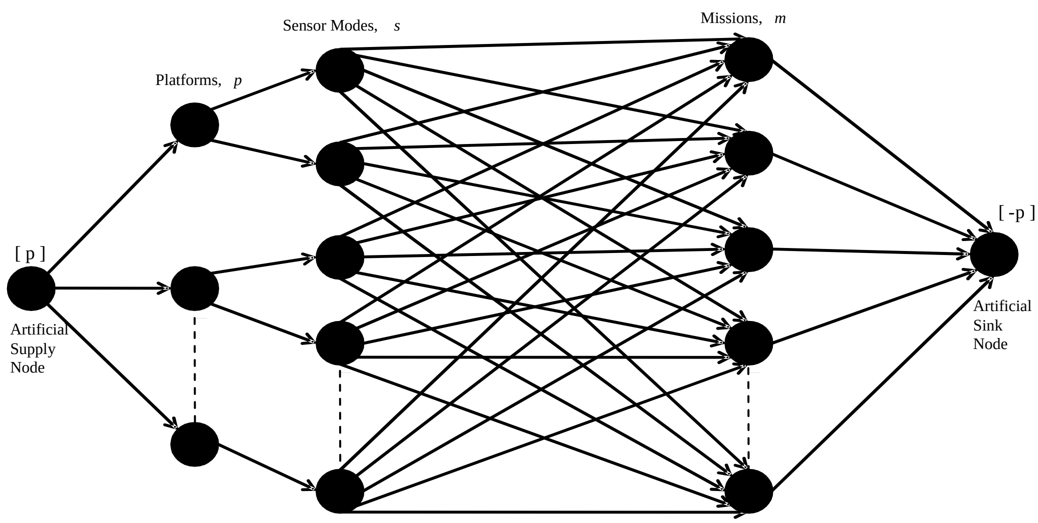

Figure 3. Generic visible defender platform network

Visible defender sub-problem network representation of Platforms, *p*, utilizing Sensor Modes, *s*, to accomplish Missions, *m*, to include artificial supply and sink nodes.

We illustrate a generic version of the visible defender platform network in Figure 3. This network contains three echelons of nodes, one for the platforms, one for the possible acoustic modes, and one for missions. Arcs denote feasible assignments from echelon to echelon and do not connect nodes within a given echelon. In addition, we specify a unit capacity on each arc to prevent the assignment of more than one platform-acoustic mode combination to any one particular mission. To this three-layer network, we add an artificial supply node with a supply of *p* visible defender platforms, and an artificial sink node with a demand for *p* visible defender platforms. We add an arc from the artificial supply node to each platform node, *p*, with unit capacity and we add an arc from each mission node, *m*, to the artificial sink node, again with unit capacity. Each arc connecting a platform-acoustic mode node, (*p*,*s*), to a mission node, *m*, has a cost, $C_{(p,s),m}$, equal to the evasion probability of an enemy SSK, determined by applying the random search model (e.g., Washburn, 2002, chap. 2, p. 1–7) to the search pressure of the defender platform, resulting from this potential assignment. All other arc costs are zero. We express this concept mathematically in the formulation below.

Initially, this probability is conditional on the passage of an enemy SSK through at least one of the mission cells and is based on the environmental characteristics of the 4W grid as given by the cells' coverage rate. When the enemy SSK sub-problem has been solved at least once, our heuristic estimates the probability that an enemy SSK passes through each cell. Our heuristic combines this with the conditional probability of detection, given an enemy SSK is present, and results in the overall probability of detection.

a. Additional Indices and Index Sets

$p \in P_{BASE} \subseteq P$	set of base defender platforms.
$p \in P_{TETH} \subseteq P$	set of tethered defender platforms where $P_{BASE} \cap P_{TETH} = \varnothing$
$(p', p'') \in PP_{TETH} \subseteq P_{BASE} \times P_{TETH}$	set defining each base and tethered defender platform pair
$i \in I$	set of all nodes within the visible defender platform network (i.e., visible defender platforms *p*, acoustic modes *s*, missions *m*, artificial start and end nodes) [alias $j \in J$]
$(i, j) \in A_{VisDef}$	adjacency list of all arcs in visible defender platform network specifying feasible combinations of visible defender platforms, acoustic modes, and missions for visible defender platforms.
$A'_{VisDef} \subseteq A_{VisDef}$	subset of visible defender platform network adjacency list specifying arcs specifically from each visible defender platform to acoustic mode.

$A''_{VisDef} \subseteq A_{VisDef}$ subset of visible defender platform network adjacency list specifying arcs specifically from acoustic modes to enumerated missions.

$A'''_{VisDef} \subseteq A_{VisDef}$ subset of defending platform network adjacency list specifying arcs specifically from enumerated missions to a super end node built into the network.

b. *Additional Data [Units]*

$time_{pm}$ time on station for any defender platform p (visible or secret) when executing mission m [hr]

n_{PLATS} number of visible defender platforms [platforms]

$\overline{n}_{PLATS}$ maximum number of visible defender platforms that can search a given cell [platforms]

$range_{m'm''}$ shortest straight-line distance between some cell in mission m' and some cell in mission m'' [nm]

$teth\_range_{p''}$ maximum range tethered defender platform p'' can travel from its base defender platform before beginning a mission [nm]

$trans_{p}$ time required for any platform p (visible or secret) to transit a grid cell [hr]

$C_{i,j}$ arc costs interior to visible defender platform network (by echelon) representing the evasion probability (via search pressure through random search model) of each feasible visible defender platform, acoustic mode, mission assignment.

$C_{i,j} = 0 \ \forall (i,j) \in A'_{VisDef}$, $C_{i,j} = 0 \ \forall (i,j) \in A'''_{VisDef}$,

$$C_{(p,s),m} = \exp\left(-\sum_{g \in G_{pm}} \left(r_{psg} - d * dist_g \right) X_{pmsg} \right) \forall \left((p,s), m \right) \in A''_{VisDef}$$

$\bar{Y}_{gg'}$ vector containing enemy SSK sub-problem optimal solutions seen thus far. This is treated as data from previous iterations.

c. *Additional Variables [Units, if applicable]*

$Z_{i,j}$ variable representing one unit of flow through visible defender platform network indicating the selection of a specific platform, acoustic mode, or mission.

d. *Formulation* DEFEND

$$\begin{bmatrix} \min\limits_{Z} & \sum\limits_{(i,j) \in A_{VisDef}} C_{i,j} Z_{i,j} & & \text{(d0)} \\ \text{s.t.} & \sum\limits_{j|(i,j) \in A_{VisDef}} Z_{i,j} - \sum\limits_{j|(j,i) \in A_{VisDef}} Z_{j,i} = \begin{Bmatrix} +n_{PLATS} & i = s \\ 0 & i \in N - \{s,t\} \\ -n_{PLATS} & i = t \end{Bmatrix} & & \text{(d1)} \\ & 0 \le Z_{i,j} \le 1 & \forall (i,j) \in A_{VisDef} & \text{(d2)} \end{bmatrix}$$

Our objective function (d0) expresses the evasion probability associated with each platform, acoustic mode, and mission combination. Each constraint (d1) is a standard network conservation of flow constraint. Each constraint (d2) sets an upper bound of one to every arc in this network. Because each visible defender platform node has exactly one inbound arc, at most one unit of flow can pass through it and, consequently, at most one acoustic mode and mission can be selected for each visible defender platform.

In addition to the (d1) and (d2) constraints, seven more exogenous constraints influence each successive visible defender network. Thomas (2008) defines variables representing the assignment of visible defender platforms to acoustic modes and to missions as follows:

e. New Variables (Thomas, 2008)

R_{pms}	1 if platform *p* executes mission *m* using sensor mode *s*, 0 otherwise
X_{pmsg}	time that platform *p* spends executing mission *m* using sensor mode *s* in grid cell *g* [hr]

He then formulates the following constraints on defender actions:

f. Constraints on Defender's Actions (Thomas, 2008)

$$\sum_{s \in S, m \in M_{ps}} R_{pms} \leq 1 \qquad \forall p \in P \qquad \text{(d3)}$$

$$\sum_{g \in G_{pm}} X_{pmsg} \leq time_{pm} R_{pms} \qquad \forall p \in P, m \in M_{ps}, s \in S \qquad \text{(d4)}$$

$$X_{pmsg} \geq trans_p R_{pms} \qquad \forall p \in P, m \in M_{ps}, s \in S, g \in G_{pm} \qquad \text{(d5)}$$

$$\sum_{\substack{p \in P, m \in M_{ps} \\ s \in S | g \in G_{pm}}} R_{pms} \leq \overline{n}_{PLATS} \qquad \forall g \in G \qquad \text{(d6)}$$

$$R_{p''m''s''} \leq \sum_{\substack{(p', p'') \in PP_{TETH} \subseteq P_{BASE} \times P_{TETH}, \\ m' \in M_{p's'}, s' \in S | \\ range_{m'm''} \leq \\ teth\_range_{p''}}} R_{p'm's'} \qquad \forall p'' \in P_{TETH}, m'' \in M_{p''s''}, s'' \in S \qquad \text{(d7)}$$

$$R_{pms} \in \{0, 1\} \qquad \forall p \in P, m \in M_{ps}, s \in S \qquad \text{(d8)}$$

$$X_{pmsg} \geq 0 \qquad \forall p \in P, m \in M_{ps}, s \in S, g \in G \qquad \text{(d9)}$$

Each constraint (d3) requires a visible defender platform to choose, at most one mission. Each constraint (d4) requires a visible defender platform to use only the available time on station for the chosen mission. Each constraint (d5) requires a visible defender platform to patrol, for at least the amount of time required to transit all cells of a chosen mission. Each constraint (d6) limits the number of visible defender platforms that can occupy a single grid cell. Each constraint (d7) requires each tethered defender platform to choose a mission within the tether range of its base defender platform; variable domains (d8) require binary decisions. Each constraint (d9) requires non-negativity of the times spent on station.

In each network solve, we refer to exogenous constraints (d3-d9) when restricting a successor network to reflect the consequences of a visible defender platform assignment. Our network flow model for the visible defender platform network does not use the variable R_{pms} explicitly. However, a single unit of flow, within this network, from start node to end node represents the assignment of a particular visible defender platform *p* to a single acoustic mode *s* and a single mission *m*. The set of all such flows in our network can therefore be interpreted directly as values for R_{pms} (namely $R_{pms} = 1$ if there is a path from the start node, to visible defender platform *p*, then to acoustic mode *s*, then to mission *m*, culminating at the end node, and is zero otherwise) that are feasible for constraints (d3-d9). We illustrate a generic example of this concept in Figure 4.

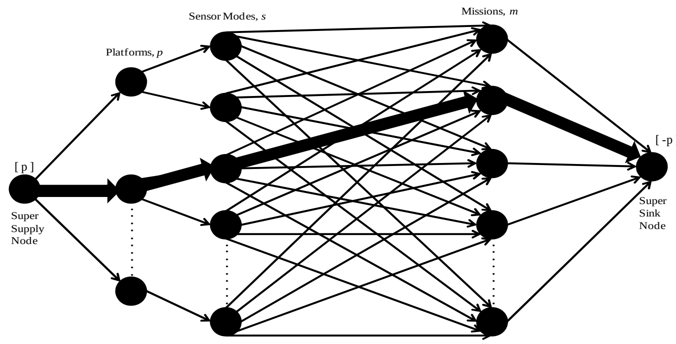

Figure 4. Generic visible defender platform network showing R_{pms}

In this figure, we graphically depict what $R_{pms} = 1$ means in our heuristic. This example demonstrates a path (thick bold arrows) from the artificial supply node, to a visible defender platform *p*, to an acoustic mode assignment *s*, to a mission assignment *m*, and ending at the artificial sink node, with one unit of flow.

Our heuristic calculates values for $C_{i,j}$ based on constraints (d4), (d5), and (d9) prior to building and solving the visible defender platform network in the following manner. Our heuristic calculates values for $\bar{X}_{pmsg}$, conditional on the choice of mission, for each platform, p, using acoustic modes, s, based on cell coverage rates via the following equation:

$$\bar{X}_{pmsg} = \left(\frac{r_{psg}}{\sum_{g \in G_{pm}} r_{psg}} * \left(time_{pm} - trans_p \right) + trans_p \right) .$$

This equation establishes a mathematical relationship between constraints (d4), (d5), and (d9), satisfying each one implicitly. Given a solution to our visible defender network flow model, we can calculate the corresponding values of R_{pms}, as defined above. These values, along with the values of $X_{pmsg} = \bar{X}_{pmsg}$ for the nonzero assignments R_{pms} (and $X_{pmsg} = 0$ for the zero values of R_{pms}), constitute a feasible solution to constraints (d3) through (d9). Therefore, our visible defender platform network flow model provides feasible solutions to the Thomas D-A/D model.

It is important to make the distinction that our $\bar{X}_{pmsg}$ values are pre-computed prior to the alternation between the visible defender network flow model and the enemy SSK network flow model. Unlike Thomas, who expresses X_{pmsg} as a decision variable thus allowing it to change, our $\bar{X}_{pmsg}$ values do not change and we use them as data for the rest of the heuristic, in that, we use them to calculate $C_{i,j}$ values in accordance with the equation shown in the additional data section (chap. II, section A, subsection 2, part a.).

To satisfy constraint (d6), our heuristic checks to see if any cells contain a number of platforms equal to this platform per cell limit; if this limit has been reached we prohibit assignment of missions containing these cells (setting associated capacities to zero). After each successive restricted network solve, our heuristic checks the resulting visible defender platform-acoustic mode-mission combination to determine if this particular visible defender platform is tethered to another visible defender platform. If so, then we restrict the visible defender's network for only this unassigned tethered defender platform (or base defender platform if applicable) to prevent the choice of missions outside the associated tether range, thereby enforcing the (d7) constraint.

We solve these network flow models with a VBA rendition of GNET (Bradley, et al. 1977). Solve times for networks with 444 nodes and 5,930 arcs are negligible.

Once our heuristic solves the SSK sub-problem more than once, resulting in the accumulation of at least one enemy SSK path; it adjusts the costs of the acoustic mode to mission arcs of the visible defender platform network to reflect the probability of an enemy SSK passing through any of the respective mission cells. We represent the total search pressure applied to a given mission, *m*, by the term $searcheffectiveness_m$. We represent the detection probability of an enemy SSK in at least one of the cells of a given mission, *m*, by $P_{\det}(m)$. We represent the conditional probability of detection, per mission, given that a hostile SSK is present in at least one cell of that mission by $P_{\det}(m | SSK \in m)$. We represent the probability of hostile SSK presence in at least one mission cell by the term $P(SSK \in m)$. We set the conditional probability, $P_{\det}(m | SSK \in m)$ equal to: $1 - \exp(-search\ effectiveness_m)$ based on a random search model. Our heuristic derives an estimate of $P(SSK \in m)$ in two parts. It generates a frequency distribution of the number of times an SSK transits a particular cell and divides this number by the total number of times an SSK transits all cells, thus creating an enemy SSK probability field (by cell). From this probability field, we calculate the detection probability of a given mission, based on the detection probabilities of its individual cells, and represent this by the term *subuses*$_m$.

We set $P_{\det}(m) = P_{\det}(m | SSK \in m) * P(SSK \in m)$. Substituting *subuses*$_m$ for $P(SSK \in m)$, $P_{\det}(m)$ becomes: $subuses_m \left(1 - \exp(-search\ effectiveness_m)\right)$. The probability of evasion is one minus this: $1 - subuses_m \left(1 - \exp(-search\ effectiveness_m)\right) =$ $1 - subuses_m + subuses_m * \exp(-search\ effectiveness_m)$. The first two additive terms are exogenous constants, while the last is discretionary. Thus, we allocate *searcheffectiveness*$_m$ and minimize the probability of evasion by minimizing $subuses_m * \exp(-search\ effectiveness_m)$. Therefore, the updated costs in the defender sub-problem network are:

$$C_{(p,s),m} = subuses_m * \exp\left(-\sum_{g \in G_{pm}} r_{psg} X_{pmsg}\right) \forall ((p,s),m) \in A''_{VisDef}.$$

Occasionally, solutions to the defender sub-problem will assign platforms to missions whose cells are more distant than desired from those of the protected region(s). Carrier battlegroup commanders typically want their screening platforms close to the HVU (i.e., carrier) for air defense operations. Consequently, we adjust the coverage rate per cell in a manner inversely proportional to the distance of that cell from the cells of the protected region(s).

Accordingly, the adjustment to the updated costs in the defender sub-problem network becomes:

$$C_{(p,s),m} = subuses_m * \exp\left(-\sum_{g \in G_{pm}} \left(r_{psg} - d * dist_g \right) X_{pmsg} \right) \forall \left(\left(p,s \right), m \right) \in A''_{VisDef}$$

where d is a discretionary constant chosen by the user representing the tradeoff between coverage rates and distances. In the results we report in chapter III, d is taken to be $0.01\left[1/nm*hr\right]$.

In addition, at each network restriction iteration, a Bayesian probability update, is performed on the evasion probabilities for missions of all other non-assigned visible defender platforms to reflect the potential combination of these platforms with already-assigned visible defender platforms. We apply this update to the non-assigned missions containing cells common to those of the most recent visible defender platform-acoustic mode-mission assignment. Once all visible defender platforms are assigned to missions utilizing an acoustic mode, our heuristic updates the 4W grid with all platform assignments and subsequent search pressures assigned to each cell. Cells where no defending platform is assigned, are given a search pressure equivalent to the battery penalty.

3. General Heuristic

Our heuristic uses the cell coverage rates to initially solve the succession of restricted visible defender networks that collectively represent the visible defender platform sub-problem. Our heuristic uses the solution of each to fix the best platform, acoustic mode, and mission, and update data for the next, restricted sub-problem. Expanding on this concept, the complete visible defender network solution (after all restricted network solves, per Alternating Flows Heuristic iteration) creates a known search pressure in each visible defender platform-occupied cell. This generates an initial feasible assignment of visible defender platforms to acoustic modes (passive or active) and to missions based solely on the cell coverage rates (supplied by planner).

To avoid this initial visible defender platform lay down, the attacker intelligently routes his SSK minimizing exposure to total search pressure along a path from some entry cell of the 4W grid to the protected region(s). Our heuristic updates the visible defender platform network using the enemy SSK path in accordance with the $C_{s,m}$ equation (chap. II, section A, subsection 2, p. 22) and continues to alternate between solving the enemy SSK (attacker) shortest path problem, updating the visible defender platform network, and solving the visible defender platform sub-problem. An overall description of our heuristic follows.

Step 1: Set iteration counter (IC) of heuristic to zero.

Step 2: Determine an initial feasible assignment of visible defender platforms, acoustic modes, and missions based on cell coverage rates (X) (described above). Set incumbent solution equal to this assignment, send X data to enemy SSK's network thus becoming $(\bar{\mathrm{X}})$, update costs on all arcs of enemy SSK's network by the search pressure values corresponding to $(\bar{\mathrm{X}})$.

Step 3: Solve SSKSP($\bar{X}$) for Y (enemy SSK paths). Send Y data to visible defender's network thus becoming $(\bar{\mathrm{Y}})$.

Step 4: Update *subuses*$_m$ for all $(\bar{\mathrm{Y}})$ seen thus far.

Step 5: Update $C_{s,m}$ values based on *subuses*$_m$.

Step 6: Solve DEFEND for X. Collect X assignments as $(\bar{\mathrm{X}})$ and current iteration number.

Step 7: Solve SSKSP($\bar{X}$) for Y (enemy SSK paths). Send Y data to visible defender's network thus becoming $(\bar{\mathrm{Y}})$.

Step 8: If stopping criterion met (described after step 10 below), Go to Step 9. If not, Go to Step 4.

Step 9: Allow planner to make choice of using current iteration $(\bar{X})$ and $(\bar{Y})$ or continuing the heuristic to find a better solution. If planner chooses to continue, Go to Step 5. If planner chooses to use current solution, Go to Step 8.

Step 10: Print $(\bar{X})$ as best defense solution found, $(\bar{Y})$ as best enemy SSK path found, and iteration number associated with best $(\bar{X})$ and $(\bar{Y})$.

The stopping criterion mentioned in Step 8 is based upon creating a "barrier" of patrolling visible defender platforms in the cells bordering the protected region(s). Once our heuristic assigns all visible defender platforms to missions, it solves the enemy SSK shortest-path problem. If the solution to this problem illustrates that the enemy SSK penetrates through to the protected region(s) without encountering a defender platform, then we fail to protect the HVU; if our heuristic has not exceeded a user-defined iteration limit (currently set to 700) then it continues to look for a better solution. However, if an enemy SSK cannot penetrate through to the protected region(s) without encountering a defender, then we have successfully achieved this criterion, and the heuristic reports this solution to the planner (Step 9).

B. POSITIONING A SINGLE SECRET DEFENDER

The heuristic described above develops a viable screen of visible defender platforms around an HVU to minimize the probability of an attack on the HVU. However, this assumes that the attacker observes all ASW defensive positioning, which may be too conservative for the defender when planners deploy secret defender platforms.

As mentioned previously, G-TAMP optimally deploys secret defender platforms, for instance friendly submarines using passive sonar, in addition to those using active sonar (visible defender platforms). Thomas describes the importance of properly modeling this situation via a two-person, zero-sum game (TPZSG) with simultaneous play.

To implement this game, we begin by enumerating a TPZSG payoff matrix where the row space consists of potential secret defender platform missions and the column space consists of available enemy SSK paths from the entry cells of the 4W grid to the protected region(s). Any row-column intersection in this matrix represents the effect of the already-placed visible defender platforms, as well as the potential effect of the secret defender platform for cells that are common to that particular secret defender platform mission and enemy SSK path. The defender platforms (visible or secret) want to maximize this quantity while the enemy SSK wants to minimize it.

Because of the limitations of internal data structures within Excel 2007, only a single platform can patrol as a secret defender platform. The TPZSG setup enumerates all feasible secret defender platform missions, based on the same mission requirements as visible defender platforms. We represent the total number of these missions by the term Φ. Secret defender platform mission enumeration, when applied to just two defender platforms, requires an Δ(Δ-1) matrix where $\Delta \equiv |\Phi|$. This very quickly causes a memory overflow problem and crashes Excel 2007.

We formulate the TPZSG as a linear program, as shown by Washburn (Washburn, 1994, p. 37), maximizing the value of the secret defender's game. We use a VBA compiled version of LP SOLVE (Berkelaar, 2009) to solve the game and discover the secret defender platform mission assignment probabilities (from the primal variables' values), and enemy SSK path utilization probabilities (from the dual variables' values) (Washburn, 1994, p. 19–20). The non-zero probability results, for either the secret defender platform or the enemy SSK, form a strategy used to form the solution to the enemy SSK sub-problem (in lieu of $\text{SSKSP}(\bar{X})$) and vice-versa for the secret defender platform. A detailed description of this algorithm follows. For the purposes of this discussion, we refer to a single secret defender platform $SSN \in P_{SECRET}$. For this platform to remain secret, recall that the only possible choice of acoustic mode, *s*, is $PASSIVE \in S$.

1. Additional Indices and Index Sets

$b \in B$ — set of potential allowable enemy SSK routes from the entry cells of the 4W grid to the cells bordering the protected region(s).

$m \in M_{SSN,PASSIVE}$ — set of potential enumerated missions executable by secret defender platform patrolling in passive acoustic mode [alias m']. Also, to prevent mutual interference between submarines: $m \in M_{SSN,PASSIVE} \cap m \in M_{SSN,ACTIVE} = \varnothing$

$g \in G_{pmb} \subseteq G$ — set of cells visible defender platform $p \in P_{Vis}$ patrols while executing mission *m* that are common to enemy SSK route *b*.

2. Data [Units]

r_{psg} — coverage rate of defender platform *p* using sensor mode *s* in grid cell *g* [hr^{-1}]

$\bar{X}_{pmsg}$ — time that visible defender platform $p \in P_{Vis}$ spends executing mission *m* using sensor mode *s* in grid cell *g* pre-computed based on coverage rates of cells $g \in G_{pm}$ (described below). [hr]

$\bar{W}_{SSN,m',PASSIVE,g}$ — time that secret defender platform $SSN \in P_{SECRET}$ spends executing mission *m* using passive acoustic mode in grid cell *g* pre-computed based on coverage rates of cells $g \in G_{pm'}$ [hr].

$f_{m,b}$ — probability that a single enemy SSK traversing path, b, is detected if defender employs mission, m [probability].

3. Additional Variables [Units]

Q_m — probability that secret defender platform $SSN \in P_{SECRET}$ executes covert mission m using passive acoustic mode [probability]

π_b — probability that an enemy SSK chooses route b [probability]

v — the value of the two person, zero sum game

4. Max-Min/Max Optimization of Detection Probability along Attacker's Path

The TPZSG linear program takes the form [dual variables]

$$\max_{Q,v} v$$

$$\text{subject to:} \quad v \le \sum_{m \in M_{SSN,PASSIVE}} Q_m f_{m,b} \quad \forall b \in B \qquad [\pi_b]$$

$$\sum_{m \in M_{SSN,PASSIVE}} Q_m = 1 \qquad [\mu]$$

$$Q_m \ge 0 \quad \forall m \in M_{SSN,PASSIVE}$$

Where

$$f_{m,b} \equiv 1 - \exp\left(-battery - \sum_{\substack{p \in P_{VIS}, \\ s \in S, \\ m' \in M_{ps}, \\ g' \in G_{pm'b}}} \left(r_{psg} - d * dist_g \right) \bar{X}_{pmsg'} - \sum_{\substack{m \in M_{SSN,PASSIVE}, \\ g' \in G_{SSN,m',b}}} \left(r_{SSN,PASSIVE,g} - d * dist_g \right) \bar{W}_{SSN,m',PASSIVE,g} \right)$$

The dual of the TPZSG linear program follows.

$$
\begin{aligned}
&\min_{\pi,\mu} \mu \\
\text{subject to: } &\mu \geq \sum_{b\in B} \pi_b \, \mathrm{f}_{m,b} && \forall m && [Q_m] \\
&\sum_{b\in B} \pi_b = 1, && && [v] \\
&Z_a \geq 0
\end{aligned}
$$

Similar to the visible defender platform sub-problem, two more exogenous constraints influence the secret defender platform assignments. Thomas (2008) defines variables representing the assignment of secret defender platforms to acoustic modes and to missions as follows:

5. New Variables (Thomas, 2008)

Q_{pms} — probability that platform $p \in P_{SECRET}$ executes covert mission *m* using sensor mode *s* [probability]

W_{pmsg} — expected amount of time platform $p \in P_{SECRET}$ spends executing covert mission *m* using sensor mode *s* in cell *g* [hr]

He then formulates the following constraints on defender actions

6. Constraints on Defender's Secret Platforms (Thomas, 2008)

$$
\begin{aligned}
&\sum_{m\in M_{ps}, s\in S} Q_{pms} \leq 1 && \forall p \in P_{SECRET} && (d10') \\
&\sum_{g\in G_{pm}} W_{pmsg} \leq time_{pm} Q_{pms} && \forall p \in P_{SECRET+}, m \in M_{ps}, s \in S && (d11') \\
&W_{pmsg} \geq trans_p Q_{pms} && \forall p \in P_{SECRET+}, m \in M_{ps}, s \in S, g \in G_{pm} && (d12') \\
&0 \leq Q_{pms} \leq 1 && \forall p \in P_{SECRET+}, m \in M_{ps}, s \in S && (d13') \\
&W_{pmsg} \geq 0 && \forall p \in P_{SECRET+}, m \in M_{ps}, s \in S, g \in G && (d14')
\end{aligned}
$$

These constraints "require the same of the secret platforms as their visible counterparts, with the exception that **Q** is now continuous" (Thomas, 2008, p. 25).

In this case, Q_{pms} represents the continuous probability of a secret defender platform being assigned to a secret defender platform mission utilizing acoustic mode *s*.

In the Thomas model, recall that multiple defender platforms can choose between being visible or secret. Because of the limitations in Excel discussed earlier, our heuristic is incapable of assigning more than one secret defender platform. Therefore, our heuristic satisfies each (d10') constraint implicitly.

Our heuristic calculates values for $\overline{W}_{SSN,m',PASSIVE,g}$ in a similar manner to that of $\overline{X}_{pmsg}$. In the case of $\overline{X}_{pmsg}$, the equation is conditional on the choice of a particular mission, that is, each respective mission contains cells with varying coverage rates that influences the respective $\overline{X}_{pmsg}$ values. Despite the fact that mission choice is not a binary (Q_{pms} vice R_{pms}), the same mathematical equation used to generate $\overline{X}_{pmsg}$ applies to generating $\overline{W}_{SSN,m',PASSIVE,g}$ because the equation is still conditional on respective mission choices, that is, calculated values of $\overline{W}_{SSN,m',PASSIVE,g}$ are conditional on the choice of that particular mission, executed by the secret defender platform utilizing passive sonar, based on cell coverage rates. Therefore, we use the following formula to calculate those values:

$$\overline{W}_{SSN,m',PASSIVE,g} = \left(\frac{r_{psg}}{\sum_{g \in G_{pm}} r_{psg}} * \left(time_{pm} - trans_{p} \right) + trans_{p} \right).$$

Our definition of $f_{m,b}$ uses these $\overline{W}_{SSN,m',PASSIVE,g}$ values (along with $\overline{X}_{pmsg}$ values) to generate detection probabilities conditional on the choice of each feasible secret defender platform mission and enemy SSK route, *b.*

III. RESULTS AND ANALYSIS

This chapter evaluates our heuristic alteration of G-TAMP by comparing with the Thomas D-A/D model results solved to within a 5% optimality gap. Thomas utilizes an assortment of five examples to demonstrate the validity of his D-A/D model. We compare our results to his by solving the same five examples.

We use six of the seven fundamental scenario assumptions that Thomas used: 1) active sonar mode always offers better performance than passive in each cell; 2) each platform spends four hours on station for mission accomplishment; 3) we enumerate all possible one-, two-, or three-contiguous cell missions for each defender platform; 4) all 4W grid cells are 5 nautical mile (nm) by 5 nm in size; 5) when a helicopter is employed in a scenario, it is tethered to a specified base platform (e.g., Helo2 is tethered to Surf2); and 6) the enemy SSK transits a searched cell in 1.5 Hrs at 4 knots.

In each of the five scenarios, we compare our results to those of Thomas, based on overall detection probability, solution run time, and heuristic achievement percent of the optimal detection probability results (D-A/D model program results). For run time comparisons, we use the same 3.13 GHz processor computer to run all five scenarios in both the original Thomas G-TAMP and our heuristic G-TAMP.

In each of the examples, Thomas sets one included SSN to *flexible*, that is, a defender platform allowed to be either visible or secret determined by the D-A/D model. Our heuristic handles secret defender platforms by programming the TPZSG and solving it with LP SOLVE separately from the visible defender platform sub-problem, so we cannot automatically determine visible versus secret for defender platforms as the optimal D-A/D model does. Because of this, we repeat examples one, three, four, and five twice, once in each acoustic mode for the available submarine.

A. EXAMPLE ONE: BASIC SCENARIO

In the first scenario, we set up the 6x6 4W grid shown in Figure 5

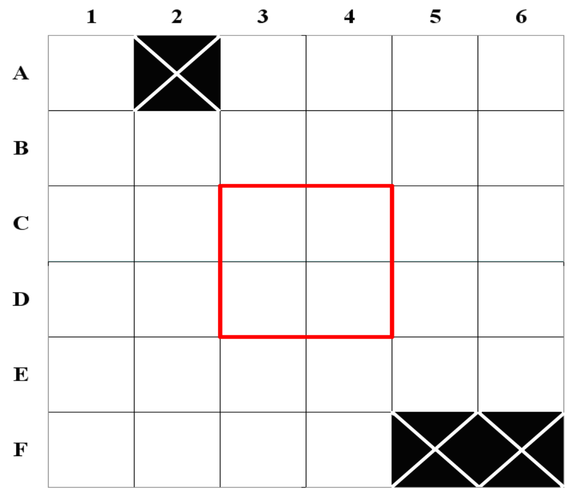

Figure 5. Basic scenario, 4W grid geography (From Thomas, 2008)

For this scenario, cells C3, C4, D3, and D4 (boxed) comprise the region of protected cells; cells A2, F5, and F6 are impassable.

We assume a non-homogenous ocean environment for this scenario; depicted graphically in Figure 6.

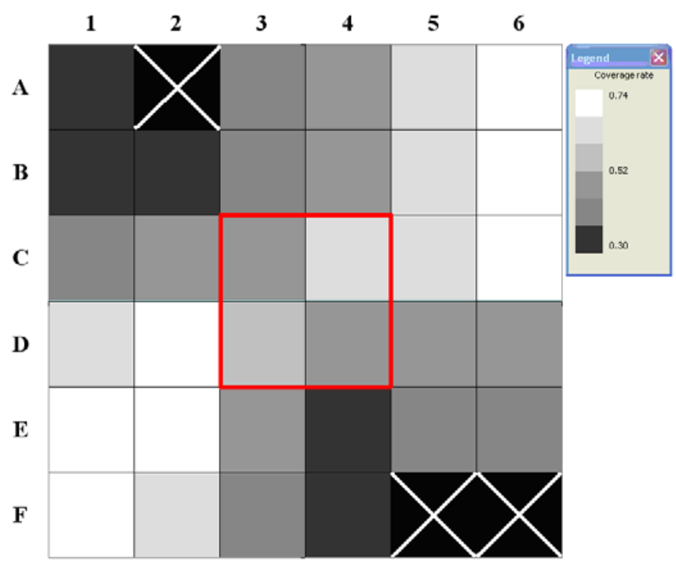

Figure 6. Basic scenario, coverage rates (From Thomas, 2008)

Friendly platforms can search light-colored cells more easily than dark-colored ones.

For this scenario, we deploy a total of six defender platforms, testing the platform settings shown in Table 1. We tether Helo1 to the base platform Surf1.

Platform	Type	Platform	Type
Surf1	Visible	Surf1	Visible
Surf2	Visible	Surf2	Visible
Surf3	Visible	Surf3	Visible
Helo1	Visible	Helo1	Visible
P31	Visible	P31	Visible
SSN1	Secret	SSN1	Visible

Table 1. Basic scenario, available defender platforms

Our heuristic deploys defender platforms around the protected region, shown in Figure 7 with corresponding mission details in Table 2. We display the heuristic results from the scenario that utilizes SSN1 in active mode, consistent with the optimal D-A/D model results.

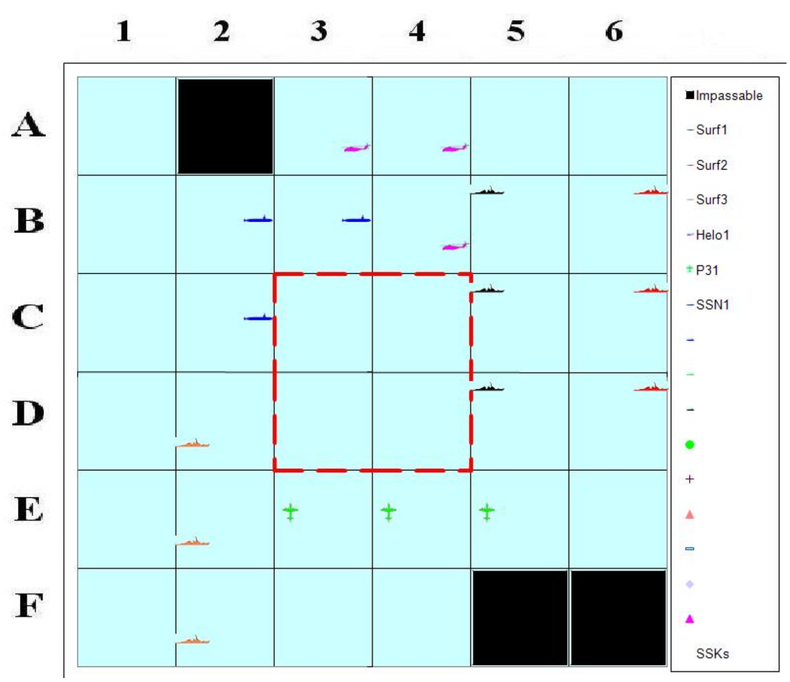

Figure 7. Basic scenario, defender platform lay down (Heuristic)

In this scenario, the defender platforms form a tight screen around the HVU and then position themselves to maximize their enemy SSK detection probabilities based on all enemy SSK paths seen thus far from all iterations.

Surf1	m76	Surf2	m77	Surf3	m83	Helo1	m138	P31	m92	SSN1	m100
	Active		Active		Active		Active		Active		Active
Cell	Time	Cell	Time	Cell	Time	Cell	Time	Cell	Time	Cell	Time
B5	1.38	B6	1.47	D2	1.38	A3	1.11	E3	1.50	B2	1.25
C5	1.42	C6	1.40	E2	1.40	A4	1.40	E4	1.17	B3	1.33
D5	1.19	D6	1.13	F2	1.21	B4	1.49	E5	1.33	C2	1.42

Table 2. Basic scenario, defender platform mission details (Heuristic)

For this scenario, each platform patrols in the active acoustic mode in each of the missions shown above. In each mission, each platform spends the shown time above (in hours) to patrol for an enemy SSK.

Our heuristic produces an overall detection probability of 0.24 and runs in 50 seconds compared to the optimal D-A/D model that produces a detection probability of 0.41 and runs in 26 seconds. Our heuristic achieves 59% of optimality.

When we run the same scenario with SSN1 as a secret defender platform, the heuristic achieves a detection probability of 0.29 in 44 seconds. This is within 71% of the optimal solution.

B. EXAMPLE TWO: SHORT-HANDED SCENARIO

In this case, we test the simple scenario of one visible defender platform and one secret defender platform. We use the same geography and ocean environment as in Example One. We fix SSN1 as a secret defender platform. We show the defensive platform set up in Table 3.

Platform	Type
Surf1	Visible
SSN1	Secret

Table 3. Short-handed scenario, available defender platforms

Figure 9 shows our heuristic solution with corresponding mission detail in Table 4, deploying a defensive platform lay down similar to the optimal D-A/D model shown in Figure 8.

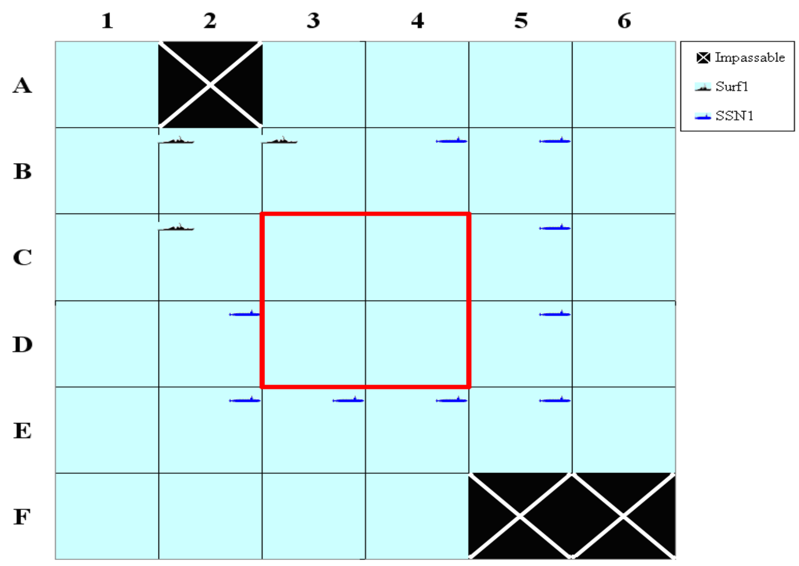

Figure 8. Short-handed scenario, D-A/D model results (From Thomas, 2008)

The D-A/D model employs a "mixed strategy" for the SSN and a three-cell mission for the single visible defender.

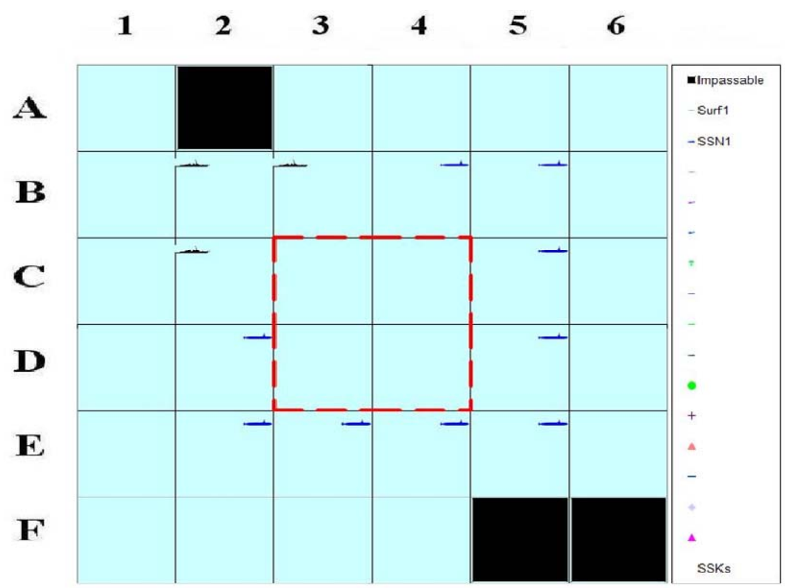

Figure 9. Short-handed scenario, defender platform lay down (Heuristic)

Two visible defender platforms, executing at most three cell missions, cannot possibly cover the entire protected region perimeter. We employ a secret defender platform (SSN1) with a mixed strategy to randomize its actions and a visible defender platform (Surf1). Each cell containing a submarine icon receives some amount of search pressure from SSN1.

Surf1	m100
	Active
Cell	Time
B2	1.22
B3	1.33
C2	1.45

SSN1					
Prob.	**0.03**	**Prob.**	**0.02**	**Prob.**	**0.01**
B4	4.00	D2	4.00	B4	1.84
				B5	2.16
Prob.	**0.04**	**Prob.**	**0.05**	**Prob.**	**0.13**
E3	2.18	E2	1.63	E3	1.42
E4	1.82	E3	1.25	E4	1.25
		E4	1.12	E5	1.33
Prob.	**0.16**	**Prob.**	**0.31**	**Prob.**	**0.25**
D2	1.42	E4	1.22	B4	1.23
E2	1.44	D5	1.47	B5	1.37
E3	1.14	E5	1.31	C5	1.40

Table 4. Short-handed scenario, defender platform mission details (Heuristic)

In this case, Surf1, a visible defender platform, selects a single mission as shown. SSN1, a secret defender platform, utilizes a mixed strategy (Probabilities in bold) which randomizes its actions.

In this scenario, two visible defender platforms cannot possibly patrol the entire protected region perimeter due to the three-cell limit of visible defender platform enumerated missions. If SSN1 patrols in active acoustic mode, then it is visible and executes enumerated missions as such. If this occurs, the enemy SSK would observe the location of both visible defender platforms and take an un-opposed path to the protected region, thereby reaching the HVU. Therefore, SSN1 remains secret, thereby randomizing its patrol pattern and remaining unpredictable to the enemy SSK. Similarly, the enemy SSK randomizes their actions consequently remaining unpredictable to the defenders.

Our heuristic produces an overall detection probability equal to the D-A/D model's optimal detection probability of 0.12 and runs in 5 seconds compared to the D-A/D model that runs in 23 seconds.

C. EXAMPLE THREE: OCEAN-INFLUENCE SCENARIO

Our third scenario focuses on ocean-influence. The cells forming the perimeter of the protected region contain water with relatively low coverage rates compared to cells outside of the perimeter. We use the same geography as examples one and two with the only difference being an update to the cell coverage rates shown in Figure 10.

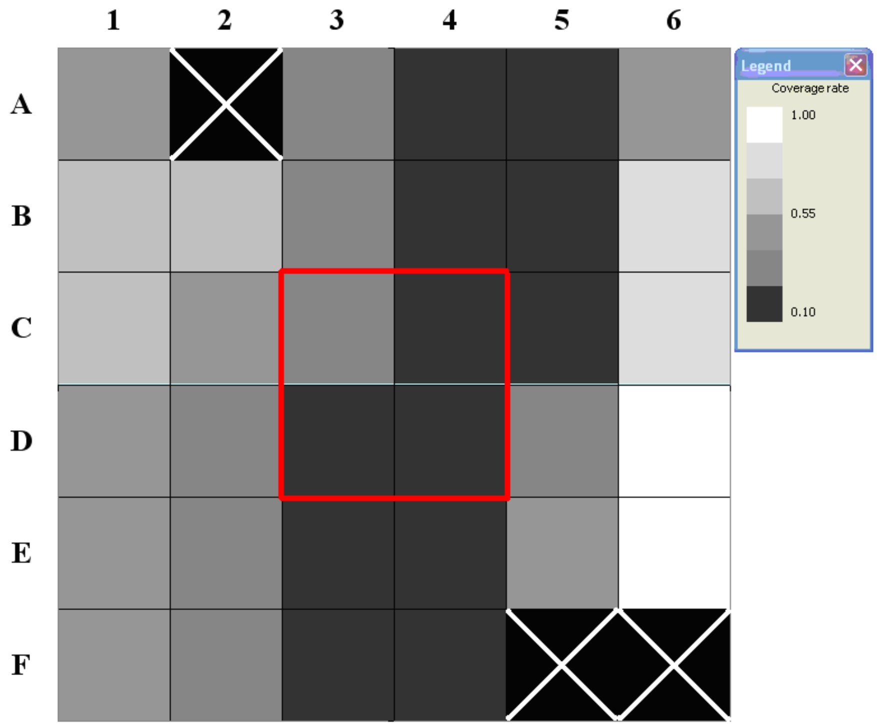

Figure 10. Ocean-influence scenario, coverage rates (From Thomas, 2008)

The protected region is surrounded by cells with low coverage rates, as would be the case if the HVU took station near an ocean current (e.g., the Gulf Stream or Kuroshio) or near an ocean front (for background on the ocean environment, see Pickard and Emery 1990).

We show the list of available defenders in Table 5. Similar to example one, we tether Helo1 to the base platform Surf1.

Platform	**Type**	**Platform**	**Type**
Surf1	Visible	Surf1	Visible
Surf2	Visible	Surf2	Visible
Surf3	Visible	Surf3	Visible
Helo1	Visible	Helo1	Visible
P31	Visible	P31	Visible
SSN1	***Secret***	***SSN1***	***Visible***

Table 5. Ocean-influence scenario, available defender platforms

With SSN1 selected as a secret defender platform, our heuristic produces an overall detection probability of 0.15, runs in twelve seconds compared to the optimal D-A/D model's run time of 3 hours, and achieves 68% of optimality based on the D-A/D model's result of 0.22. Furthermore, when we run our heuristic with SSN1 selected to be a visible defender platform, the results show a feasible visible defender platform lay down, but with a lower overall detection probability equal to 0.09 and run time of 1 minute.

We present our heuristic defensive lay down, and subsequent mission details, in Figure 12 and Table 6, respectively. We choose to illustrate the results with SSN1 as a secret defender platform to compare to the optimal D-A/D results, which we show in Figure 11.

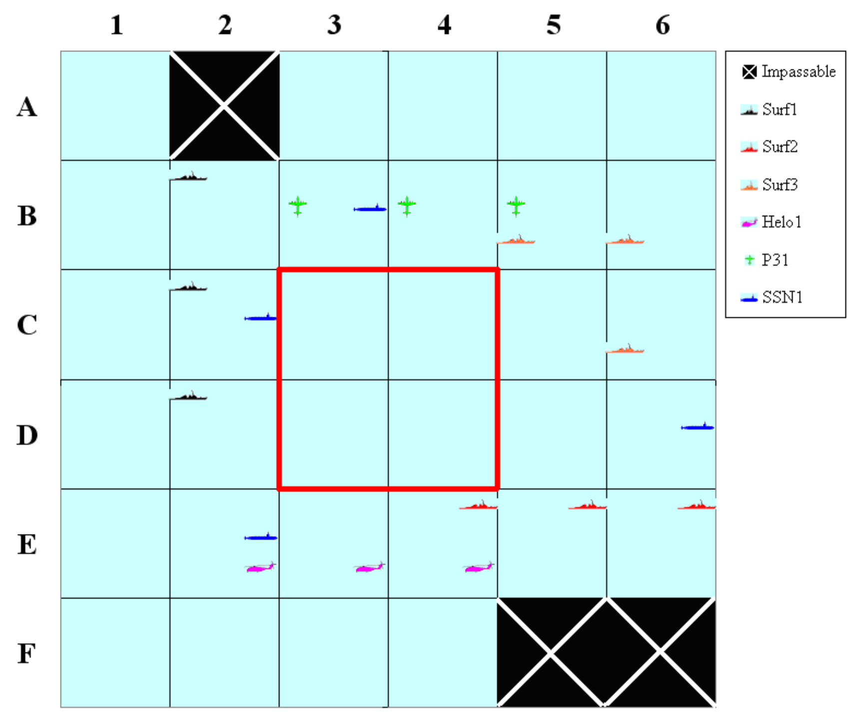

Figure 11. Ocean-influence scenario, D-A/D optimal results (From Thomas, 2008)

The D-A/D model prescribes a 'ring' of platforms around the HVU, but places search platforms outside of the perimeter cells on the east of the protected region.

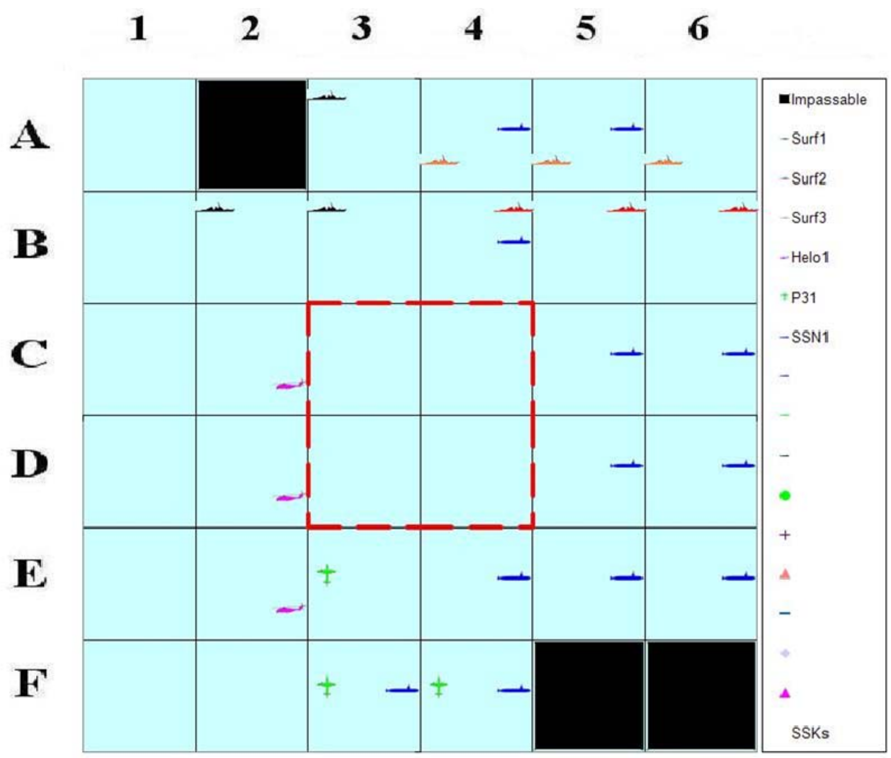

Figure 12. Ocean-influence scenario, defender platform lay down (Heuristic)

The non-uniform sound propagation in this scenario forces defensive platforms to spread out more from the protected region. An Enemy SSK takes advantage of these conditions and will preferentially choose cells with low coverage rates because defender platforms are less likely to be there.

Surf1	m124	Surf2	m89	Surf3	m85	Helo1	m79	P31	m123
	Active		Active		Active		Active		Active
Cell	Time	Cell	Time	Cell	Time	Cell	Time	Cell	Time
B2	1.78	B4	0.83	A4	0.97	C2	1.76	E3	1.48
A3	1.19	B5	0.90	A5	0.97	D2	1.16	F3	1.26
B3	1.03	B6	2.27	A6	2.06	E2	1.08	F4	1.26

SSN1											
Prob.	**0.56**	**Prob.**	**0.08**	**Prob.**	**0.15**	**Prob.**	**0.04**	**Prob.**	**0.14**	**Prob.**	**0.02**
C6	1.24	A4	1.32	C5	0.82	E4	1.11	F3	1.28	C5	0.77
D6	1.38	A5	1.32	C6	2.07	E5	1.86	E4	1.43	C6	1.50
E6	1.38	B4	1.35	D5	1.11	F4	1.03	F4	1.28	D6	1.72

Table 6. Ocean-influence scenario, defender platform mission details (Heuristic)

Here we see divergence from the optimal D-A/D model. The optimal model is omniscient, in that it simultaneously places all defender platforms, maximizing their simultaneous, synergistic probability of detection as expressed in the linear objective. By contrast, our heuristic is myopic, sequentially positioning defender platforms based on detection probability estimates achieved by alternating between successive visible defender platform network restrictions and an enemy SSK network flow model. Our heuristic attempts to approximate myopically through Bayesian probability updating what the optimal model states exactly.

Our heuristic uses the running frequency distribution of enemy SSK path cells seen thus far to apply the Bayesian probability updates to the appropriate visible defender platform network costs (cost on arcs between acoustic mode *s* and missions *m*). Our heuristic generates this frequency distribution by either the enemy SSK shortest path problem (when only visible defender platforms are present) or by the TPZSG solved from the enemy SSK's perspective (secret defender platforms and visible defender platforms present). In this scenario, this frequency distribution produces estimates of enemy SSK cell presence probability, whose magnitudes dominate those of the conditional probability of detection based on the cell's coverage rate.

Therefore, the estimate of enemy SSK cell presence probability affects the overall detection probability more so than the conditional probability of detection based on the associated cell's coverage rate. An analysis of the frequency distribution (which is one of the outputs to our model) supports this deduction, allowing us to ascertain why certain platforms seem to favor cells with comparatively low coverage rates. The analysis shows defender platforms (visible or secret) patrolling cells with relatively low coverage rates because of a remarkably higher estimate of enemy SSK cell presence. We believe this explains why our results show defender platforms patrolling cells with relatively low coverage rates or cells that are far away from the protected region.

D. EXAMPLE FOUR: MULTIPLE-HVU SCENARIO

In this scenario, we model the situation where two carrier battle groups occupy two separate protected cells and compare to the optimal D-A/D model results. We expand the 4W grid to an 8x8 grid, with two protected regions, shown in Figure 13.

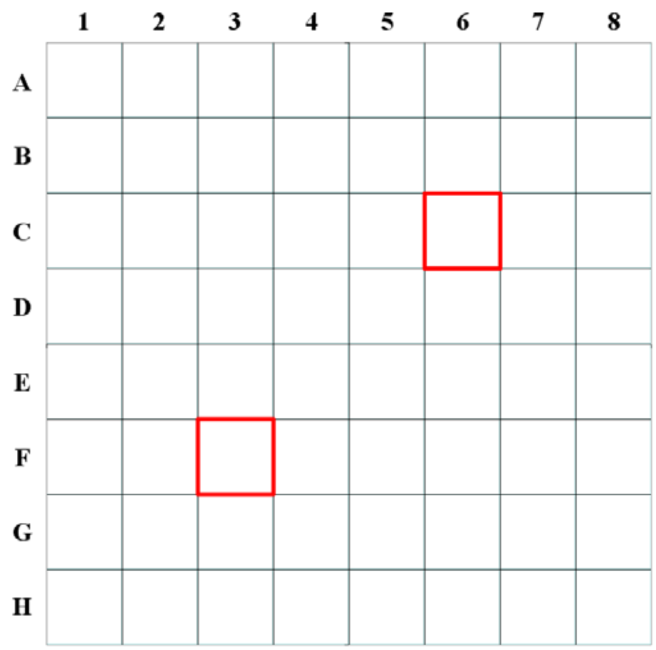

Figure 13. Multiple-HVU scenario, 4W grid geography (From Thomas, 2008)

The defender stations one HVU in each of two geographically-separated, protected cells (F3, C6), and must allocate defensive platforms among them.

For this scenario, we assume the cell coverage rates (as does Thomas) illustrated in Figure 14.

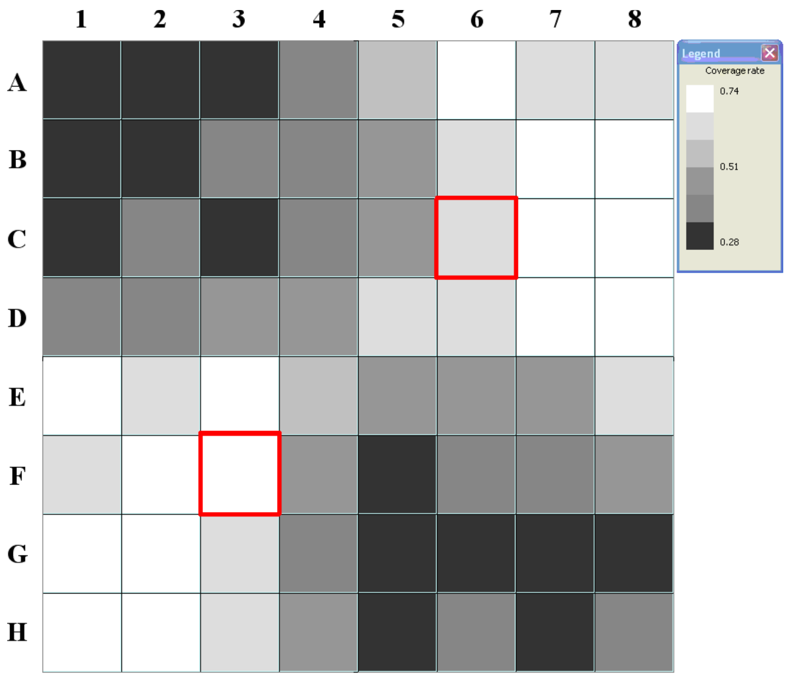

Figure 14. Multiple-HVU scenario, coverage rates (From Thomas, 2008)

As before, the defender can easily search light-colored cells, while dark cells are harder to search. The defender wisely places his carrier battlegroups in easily-searched water to facilitate detection of approaching SSKs.

We use the same available defender set-up as Thomas's example four, with one exception. Because of the previously discussed limitations of internal data structures within Excel 2007, we only allow SSN1 to patrol as a secret defender platform. We deploy the platforms listed in Table 7.

Platform	Type
Surf1	Visible
Surf2	Visible
Surf3	Visible
Helo1	Visible
Helo2	Visible
P31	Visible
SSN1	Secret
SSN2	Visible

Table 7. Multiple-HVU scenario, available defender platforms

We tether the helicopters (Helo1 and Helo2) to base platforms Surf1 and Surf2 respectively. In addition, we restrict SSN1 and SSN2 to patrol in their respective PMI schemes shown in Figure 15.

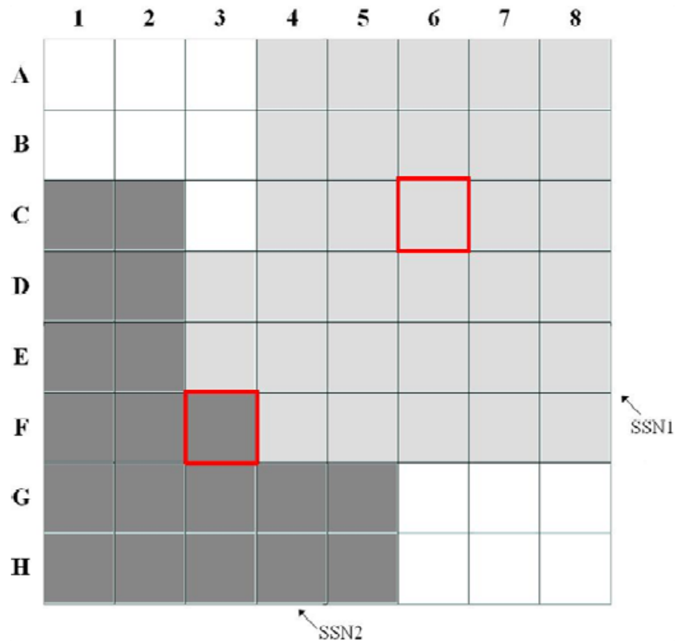

Figure 15. Multiple-HVU scenario, allowed cells for SSNs (After Thomas, 2008)

To prevent interference between the submarines, SSN2 must remain in its assigned area (dark-shaded cells on lower left), while SSN1 must stay in the light-shaded cells at upper right. The original Thomas set up reversed the placement of these submarines such that SSN1 patrols in the dark-shaded cells while SSN2 patrols in the light-shaded cells. In order to produce results consistent with Thomas, our secret defender (SSN1) must patrol the upper right protected cell.

Our heuristic algorithm achieves an overall detection probability of 0.35 and runs in 21 minutes, as compared to the optimal D-A/D model's result of 0.48 and run time of 2 minutes. Our heuristic achieves 73% of optimality. Figure 16 shows our defender platform lay down with associated mission detail shown in Table 8.

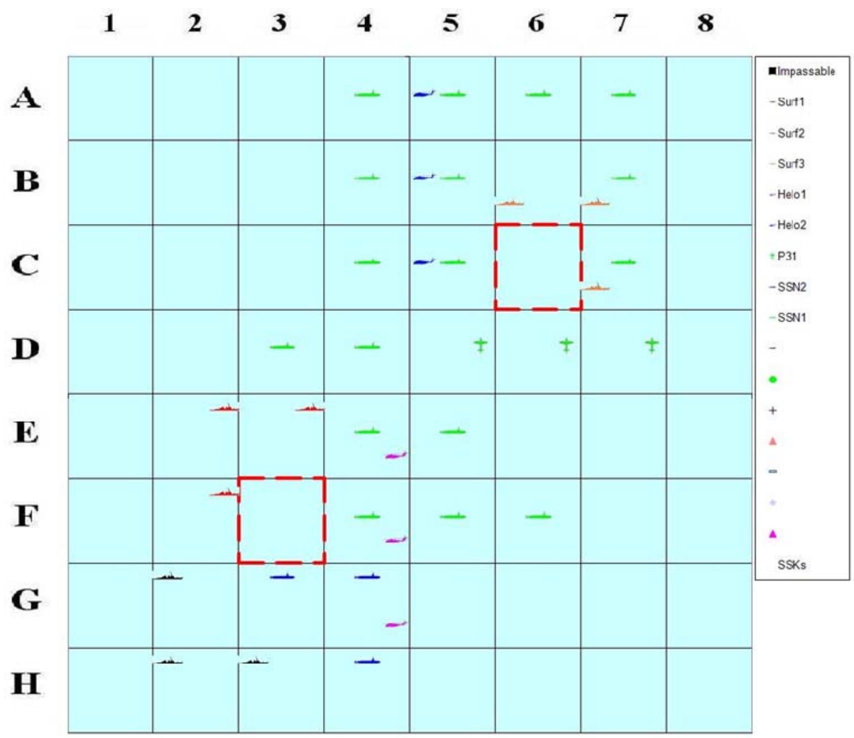

Figure 16. Multiple-HVU scenario, defender platform lay down (Heuristic)

In this case, two separate groupings of defender platforms patrol around protected regions C6 and F3. The secret defender platform, SSN1, plays a mixed strategy to protect C6, while SSN2 patrols as a visible defender platform.

Surf1	m331	Surf2	m277	Surf3	m391	Helo1	m197	Helo2	m171	P31	m228	SSN2	m418
	Active		Active		Active		Active		Active		Active		Active
Cell	Time	Cell	Time	Cell	Time	Cell	Time	Cell	Time	Cell	Time	Cell	Time
G2	1.4696	E2	1.279	B6	1.2847	E4	1.6899	A5	1.3187	D5	1.2386	G3	1.496
H2	1.3645	E3	1.3696	B7	1.3212	F4	1.3843	B5	1.3407	D6	1.3381	G4	1.285
H3	1.1659	F2	1.3514	C7	1.3942	G4	0.9259	C5	1.3407	D7	1.4233	H4	1.219

SSN1													
Prob.	0.39	Prob.	0.04	Prob.	0.39	Prob.	0.06	Prob.	0.06	Prob.	0.04	Prob.	0.03
A7	1.23	A4	1.13	F4	1.49	A4	1.23	B4	1.26	D4	1.31	D3	1.34
B7	1.35	A5	1.32	F5	1.24	B4	1.27	C4	1.26	E4	1.44	C4	1.25
C7	1.41	A6	1.54	F6	1.27	B5	1.50	C5	1.48	E5	1.25	D4	1.41

Table 8. Multiple-HVU scenario, defender platform mission details (Heuristic)

E. EXAMPLE FIVE: CHOKE-POINT SCENARIO

In this scenario, we seek to defend the HVU from a specific incoming direction (west) of an enemy SSK transiting a navigational choke-point. We show the geography of this scenario in Figure 17.

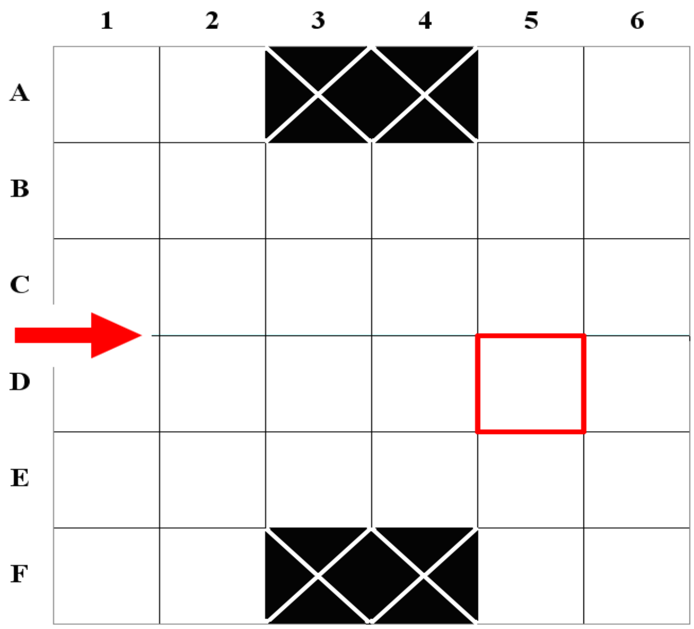

Figure 17. Choke-point scenario, 4W grid geography (From Thomas, 2008)

The attacker sends an SSK through the choke-point from left to right. The defender desires to prevent the enemy SSK from crossing through the choke-point and attacking the HVU.

We employ a similar defender platform set-up as Thomas, running our heuristic with SSN1 first set to active acoustic mode then to passive acoustic mode, shown in Table 9. We tether Helo1 to base platform Surf1.

Platform	Type	Platform	Type
Surf1	Visible	Surf1	Visible
Helo1	Visible	Helo1	Visible
SSN1	Secret	SSN1	Visible

Table 9. Choke-point scenario, available defender platforms

While utilizing SSN1 as a visible defender platform, our heuristic achieves an overall detection probability of 0.49 and runs in 15 seconds, compared to the optimal D-A/D model result of 0.54 and run time of 10 seconds. Our heuristic achieves 91% of optimality. In contrast, we ran our heuristic with SSN1 as a secret defender platform and only achieved an overall detection probability of 0.19 after running for 26 seconds. Because this detection probability is significantly lower than deploying SSN1 as a visible defender platform, we choose the former deployment rather than the latter. Figure 19 shows our defender platform lay down with associated mission detail shown in Table 10. For comparison, we also show the D-A/D model results in Figure 18.

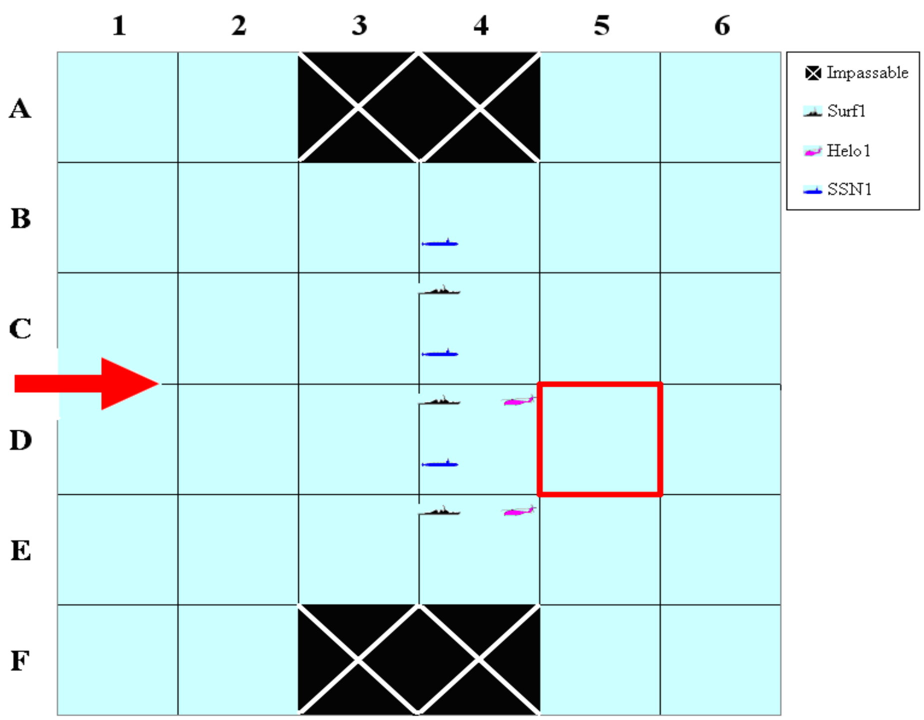

Figure 18. Choke-point scenario, D-A/D model results (From Thomas, 2008)

The defender's platforms form a classic barrier patrol in easily searched water.

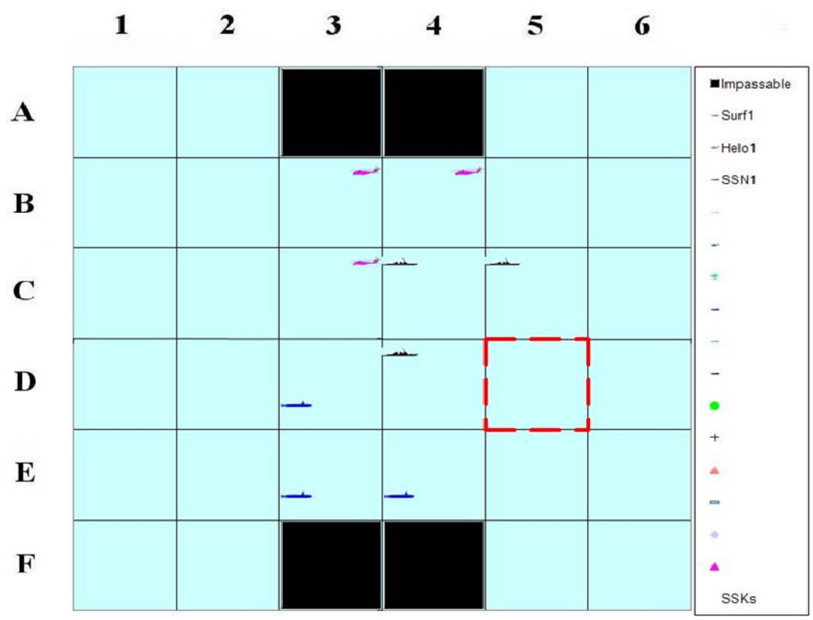

Figure 19. Choke-point scenario, defender platform lay down (Heuristic)

In this scenario, the defender platforms deploy to execute a barrier patrol between the HVU and the inbound enemy SSKs.

Surf1	m119	Helo1	m113	SSN1	m139
	Active		Active		Active
Cell	Time	Cell	Time	Cell	Time
C4	1.38	B3	1.13	D3	1.48
C5	1.47	B4	1.43	E3	1.32
D4	1.15	C3	1.43	E4	1.20

Table 10. Choke-point scenario, defender platform mission details (Heuristic)

F. HEURISTIC BINARY OUTPUT PHENOMENUM

We observe a rather interesting, yet not completely unexpected, result from our heuristic. In all five example problems, our heuristic exhibits a kind of 'binary' behavior in terms of overall detection probability. Figure 20 illustrates this behavior.

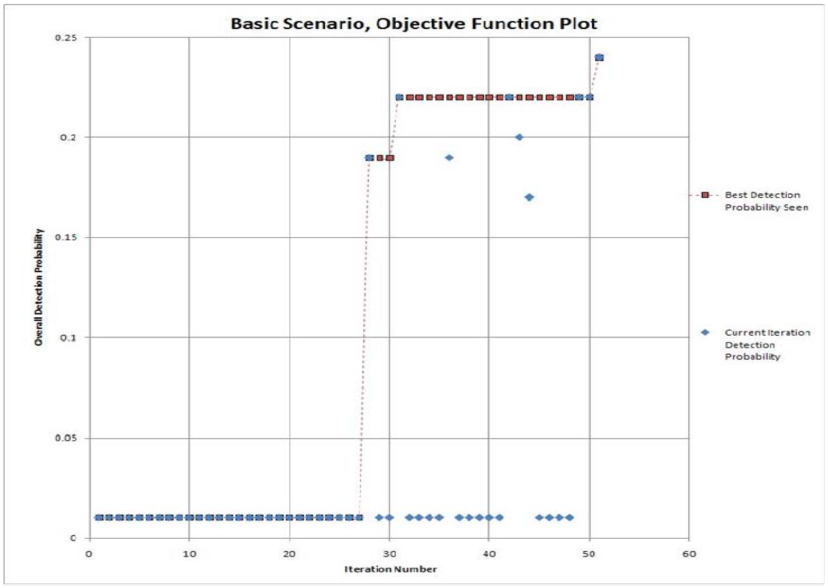

Figure 20. Basic scenario, objective function plot

This plot shows the overall detection probability, as a function of iteration number, produced over the course of Example One. The solid line connects successive objective function values as iterations proceed. The dashed- line represents the upper envelop of detection probabilities (the best incumbent solution overall detection probability seen thus far).

In example one, the objective function value takes on one of two values: 0.01, or a value proportional to the search pressure seen by the enemy SSKs. The objective function oscillates back and forth between these values. Our heuristic requires twenty-eight iterations before it produces a defensive lay down that the enemy SSKs cannot completely evade. Over the course of iterations 28 through 51, the objective function alternates between 0.01 (battery penalty previously discussed) and values indicating at least some search effort is being assigned to every enemy SSK path to the protected region.

We would prefer our heuristic objective function to exhibit monotonic (i.e., non-decreasing) behavior as it works toward optimality. Heuristics notoriously exhibit such behavior, and we are not surprised that our heuristic objective function fails to do this. Of course, our best incumbent solution overall detection probability values seen thus far do exhibit monotonic behavior, thereby remaining consistent with the behavior of many heuristics.

IV. CONCLUSIONS AND RECOMMENDATIONS

We have presented a complete Excel implementation of G-TAMP, an operational level decision support tool for positioning ASW defending platforms and assigning them missions. Our implementation solves a Defender-Attacker/Defender (D-A/D) model in two overall phases. We introduce the Alternating Flows Heuristic, which solves the visible defender platform sub-problem through alternating network solves between a network representation of potential visible ASW defender mission choices and a network representation of potential enemy SSK paths from the entry cells of a 4W grid to any cell in the protected region containing the HVU. We solve our visible defender platform network through a sequence of steps; after each step, we fix the best visible defender platform to its assigned acoustic mode and mission, restrict the visible defender platform network to reflect this assignment and then solve the resulting restriction. This process repeats until all visible defender platforms are assigned. Once our heuristic assigns all visible defender platforms, it solves the enemy SSK sub-problem as a network shortest-path problem minimizing detection probability from visible defender platforms along a single path.

Secret defender platforms are next modeled as a two-person, zero-sum game (TPZSG). Because of Excel 2007 memory allocation limitations, this step can only be used if a single secret defender platform (i.e., a friendly SSN) is deployed and available for mission tasking. We formulate the TPZSG as a linear program and solve it via LP SOLVE (Berkelaar, 2009). This suggests a "mixed strategy" of continuous probabilities used to produce the secret defender platform lay down and the resulting optimal attack paths of the enemy SSK respectively.

The goal of this entire project is to develop a heuristic to approximately solve the underlying D-A/D model while running on NMCI computers with lower program run-times than those of Thomas. We tabulate the results from all five examples and compare them in Table 11.

Comparison Analysis Of Heuristic G-TAMP To Original G-TAMP	Example #1	Example #2	Example #3	Example #4	Example #5	Average
Optimal Detection Probability:	0.41	0.12	0.22	0.48	0.54	0.354
Heuristic Detection Probability:	0.24	0.12	0.15	0.35	0.49	0.27
% Of Optimality:	59%	100%	68%	73%	91%	**0.782**
Run Time (Heuristic):	50 seconds	5 seconds	12 seconds	21 minutes	15 seconds	**4.5 minutes**
Run Time (D-A/D):	26 seconds	23 seconds	3 hours	2 minutes	10 seconds	**37.2 minutes**

Table 11. Comparison analysis of heuristic G-TAMP to original G-TAMP

From the results shown in Table 11, our heuristic algorithm implementation of G-TAMP runs on average 800% faster (8x faster) than the original G-TAMP, produces solutions that are on average within 78% of optimal, and runs completely in Excel thereby allowing installation on NMCI computers. Based on these results, we conclude that our heuristic satisfies our goal, but with less run time improvement than we originally predicted.

We previously discussed the problem regarding two-dimensional array limits in Excel preventing use of more than one secret defender platform because total enumeration of potential options for two secret defender platforms, given the 8x8 4W grid of example four, requires more than 176,000 rows (and columns) of potential plays. In an attempt to circumvent this problem, we employ row and column TPZSG dominance rules prior to solving the TPZSG linear program. For the purposes of our discussion, we define *i* to represent any row and *j* to represent any column within the TPZSG. For our TPZSG where the row player desires to maximize the value of the game and the column player desires to minimize this value, row dominance exists if the value of each element in the i^{th} row (secret defender platform potential missions) of the matrix is greater than or equal to each value in the *i*-1 row. Conversely, column dominance exists if the value of each element in the j^{th} column (enemy SSK potential attack paths) is less than or equal to each value in the *j*-1 column. Any dominated row or column can be removed from the game, as it will not participate in any optimal solution (Washburn, 1994, p. 29–30). We apply these dominance rules in a series of alternations between row dominance and column dominance, that is, our heuristic applies the row dominance rules until no more rows are dominated, then applies the column dominance rules until no more columns are dominated, and then repeats until no more rows or columns are dominated. Our heuristic maintains a listing of the dominated rows and columns and rebuilds the TPZSG with these removed, thus creating a “reduced” TPZSG matrix. Our heuristic reduces the size of the TPZSG matrix approximately 25%; however in doing so, increases run time by 160%. This relatively large increase in run time that does not significantly reduce the size of the problem led us to reject the use of TPZSG dominance rules. We therefore recommend continued research pursuant to applying TPZSG dominance rules while the respective algorithm builds the TPZSG matrix, thereby allowing VBA and LP SOLVE to solve multi-secret defender platform situations.

The ocean-influence scenario results in Chapter III (and how they show that the estimate of enemy SSK cell presence probability dominates the conditional detection probability based on cell coverage rates, thereby dominating the overall detection probability of the enemy SSK. This behavior causes visible and secret defender platforms to patrol in waters with poor sonar characteristics (i.e., lower coverage rates), which in reality lowers the overall detection probability. If these vessels patrol in waters with relatively better coverage rates while still maintaining a defensive barrier around the HVU, the resulting overall detection probability would be greater. Therefore, we recommend an objective function that places more emphasis on the conditional detection probability based on ocean environment (perhaps through the use of weighting factors in the objective function, to emphasize cell coverage rates).

Finally, we propose incorporating our program into other decision aids such as the ASW Screen Planner TDA. The ASW Screen Planner TDA requires manual and iterative manipulation by the planner in order to run (SWDG, 2004). The incorporation of our algorithm into this program could in fact circumvent the need for constant operator input. Our algorithm is not designed to replace these current decision tools (or others like them), but rather to augment and increase their capabilities.

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