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Quantitative Biofractual Feedback Part I –
“Overview – Biofractals”

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ABSTRACT

Many complex networks and systems from nature can be characterized as distributed systems. We need to know more about how such schemes work? For example, we need to better understand how to characterize performance, vulnerability, and other attributes of such systems. For many networks, including those in the military, a useful measure of performance is the flow of information through such associations. Vulnerability can then be defined as the sensitivity to flow. A poorly performing network is congested and the flow is slow. A better performing system would have fast flow. Since flow is measured in units of bits/second for information systems, the flow rate times time provides the amount of bits going through the network. One can then view the performance of a distributed network system in terms of the bits actually completed in the mission in minimum time. Time is the ratio of the fixed number of bits divided by the flow rate in bits/second.

Networks are ubiquitous as noted that electric power grids, financial systems, railroad tracks, water distribution systems, food and medicine distribution systems are now highly distributed and complex in nature. Also, information systems, email systems and nature’s way of managing physiology can be characterized by networks. On the other hand, some networks can be very destructive. For example, a counter IED (improvised explosive device) network exists for a successful attack on American forces. The goal would be, in this case, to take such a network down or cause it to have a high congestion. Another example of an undesirable network would be the well-known sexual network involving the spread of disease. In this case the goal would be to make the flow through the system to be maximally congested. This represents the reduction of the disease because the flow is minimal. It is shown in this talk that the analysis of network performance can be related to the intersection of five areas: optimization theory, graph theory, information theory, fractional calculus, and bioinspired design (fractals).

The first area to be discussed in some detail involves bioinspired design or fractals. Fractals are nature’s way of characterizing certain biological processes. Fractals can be shown to satisfy the diffusion equation and there exists a fractional dimensional equation to characterize the flow of systems in nature. This provides one reason why trees disperse water from the roots to the branches and, in a similar manner, why the lung diffuses oxygen in a manner where a fractional dimension comes into play. The term “fractal” is derived from the Latin word “fractus” which means “broken” or “fractured.” It will be demonstrated that the fractal is scale free (having a self-similar property). Such entities are forever continuous and nowhere differentiable. For some example fractals, they have infinite area and finite volume. In other cases, fractals have infinite circumference but finite area. In both situations the higher dimension is finite, but the next lower dimension is not. This has advantages in nature where the goal, for the example of the lung, is that we have finite volume for the lung, but the distribution (or flow) of oxygen is proportional to the area of the lung surface, which is maximized. This provides of optimal distribution of the flow variable (oxygen), which is desired. Finally, this provides a non Euclidean Geometry method to view the world, which is vastly understudied.

In the study of fractals, originally B. Mandelbrot asked the question, “How long is the coastline of

Britain?” When measuring the coastline of Britain by a ruler that constantly decreases in size, the logarithm of the total length plotted versus the reciprocal of the ruler length shows a straight line on this semi log plot. This demonstrates the power law effect or scale free effect. The slope of the line in this semi log scale gives rise to the Hausdorff dimension, which is a non integer. Examples are worked to show how this fractional dimension occurs which makes the study of such systems non Euclidian. The discussion on Fractals will commence with the construction of several well-known fractal figures. The first fractal discussed is the Koch Snowflake which has a fractional dimension of 1.26185… Such a figure is very interesting because it has an infinite circumference and a finite area. This is principal reason why objects in nature have the fractal properties. It is to the advantage of nature to have objects with finite area and infinite circumference because they have certain optimality properties lacking from alternative designs. The discussion of fractals includes measurement of the elusive length through a box covering methodology. The second classical fractal considered is the Cantor set or “Cantor Dust” as defined by Mandelbrot. This object has a fractional dimension of 0.63092… and has the interesting property that the deleted points have a Lebesgue measure of 1.0 for an initial interval of length of unity. The final fractal set has a Lebesgue measure of 0.0.

The next important topic to relate the five topics of this talk together is the Fractional Calculus. Over 310 years ago, the concept of Fractional Calculus was conceived, originally by L. Euler. Fractional Calculus allows an analytic method to deal with functions (fractals) that are forever continuous and nowhere differentiable. Under this formulation, fractional derivatives can be shown to have the self similarity property (scale invariance) and solve familiar partial differential equations such as the heat equation. It is interesting to note that the diffusion equation (heat equation) is actually a fractional calculus representation of a dynamic process. Thus the distribution of heat by nature follows laws similar to fractals. This seems plausible since the distribution of heat must satisfy a property of optimality as most other processes in nature tend to do. A link between the fractional calculus and fractals is the well-known function called the Weierstrass function. The Weierstrass function is an example of a fractal that is always continuous and nowhere differentiable. Plots of the Weierstrass function show similarities to fractals because on increased magnifications, the same figure tends to appear. The Weirestrass function is the missing link between the Fractional Calculus and fractals. A discussion of the Fractional Calculus is then conducted using Laplace Transforms and other ways to look at this analytical method to view the dynamics of objects in nature.

The next topic considered is information theory. In 1948, C. Shannon developed a theory of how information can be computed between sources and received signals. A thorough discussion on how information theory variables are computed is presented. One particular variable of interest is mutual information, which is defined as the reduction in uncertainty in an input object by observing an output object. Mutual information is interesting because it provides a flow rate (bits/second) which is amenable to the study of performance in graphs and networks. One final variable brought to attention involves the information distance variable D_R. This variable is shown to have the four requisite properties of a metric, which has more powerful applicability over mutual information, which is only a measure. The advantages of using a metric to quantify flow are that a metric produces less equivalence classes as compared to a measure.

Graph theory is the next topic of discussion and the distinction is made between a random graph and a scale free graph. Most modern graphs tend to have architecture similar to scale free graphs (“the rich get richer philosophy”) and they are distinguished from the random graph architecture. Examples are given of evidence that the Internet has architecture similar to a scale free graph. It is also shown that the dynamic response of the Internet has the scale-free properties as evidenced by fractals. Other physiological evidence is provided that scale free signals exist everywhere in nature, including the variability of heart beat data over time. An example of how a lung distributes oxygen is also provided to show similarities. The distinction between random and scale free graphs are studied as an example, whereas a highway network is a random graph and an airline network has a scale-free architecture.

A number of other applications that appear as scale free networks are discussed. They include the World Wide Web where a node is the web page and the link is the hyper link between web pages. Another analogy is when nodes are computers or routers and the links are the physical wires between these routers. An interesting analogy to scale free graphs is the sexual web. In this case the nodes are people and the links are relationships between the individuals. A number of studies have been made in this area because such representations characterize how disease spreads. In this case the goal may be to reduce the flow through the network (maximize congestion) and thus prevent the spread of the disease. The next network considered involves actor connections; the nodes are the actors and the links are how the actors are cast jointly. A graph shows a power law effect through the analysis of 212,250 actors. Another complex network is the science citation index on how people reference key papers in a technical field. In a study of 1,736 papers, it is shown that a power law effect is known to occur. Similar to this effect, science co authorship is studied with the nodes as the scientists (authors) and the links are when a joint paper is written between the two scientists. Again a power law curve tends to appear in the data. Four more examples are now discussed. The email network has nodes consisting of individual email addresses and the links are the email communication. The phone-call networks has the nodes as the phone number and the links are the completed phone call. In linguistics, the nodes are words and the links are the next or one word apart from another word. Finally, networks in electronic auction (eBay) are analyzed. In this case, the nodes are agents or individuals with the links the bids for the same item. The graph theory discussion then goes to measures of resource usage and the architecture assessment (e.g. diameter of the network). Robustness of complex networks is then discussed. It is mentioned that scale-free networks are more robust under random attacks, but are very vulnerable to intentional attacks. Thus vulnerability has a special meaning to the type of attack precipitated and the architecture must be taken into account as well as the type of attack selected. The last topic mentioned with respect to networks includes their dynamic properties. This is increasingly important in networks from nature such as in metabolic systems. Again the goal is to optimize the flow or productivity of the network.

An example is worked regarding a well-known network in US Air Force applications. The problem is formulated in terms of graph theoretic means and the unknown vector of flows has to be determined. This is equivalent to using the nodes to regulate the flows and this begs the question of what class of flows optimizes the flow rate and also minimizes the flow rate? Using principals from Graph Theory such as Kirchoff’s Law, a set of constraints is determined from the cutest laws (application of Kirchoff’s Law). A sensitivity function is defined as the rate of change of cut set flow to the mutual information flow. Using genetic algorithms, the flow vectors are determined which maximize and minimize the overall mutual information flow. The sensitivity function is then easily calculated for any nodes or sets of nodes through the cut set. A discussion of the genetic algorithm code is provided with data plots of the computer simulations. The sensitivity of the different nodes is determined and examined. Finally the five areas: optimization theory, graph theory, information theory, fractional calculus, and the bioinspired methods (fractals) are summarized. The solution to the dynamic response of networks is hypothesized and discussed to an analogous problem that occurs in robotics. This concludes the Part I work with respect to biofractals.

As an introduction to Part 2, the history of the birth of QFT (quantitative feedback theory) is brought to light. Starting with the work in the early 1960’s, I. Horowitz developed a seminal paper in 1972 to introduce the QFT method. It has been now applied in the areas of flight control, power systems, unmanned air vehicles and in other numerous applications. The basics of how QFT is applied is discussed to convert the problem of sensitivity of plant variations and robust control to a method amenable to the classical control area. The three requirements of robust stability, reference tracking and disturbance rejection are quantified in control theory terms. The distinction of one and two degree of freedom systems are outlined. Finally relating the QFT methods to the networks is discussed through the common sensitivity functions. The part 2 work by Dr. R. Ewing goes further implying the reduction of parameters is common in the QRT framework and the fractal work presented in the prequel. One main strength of the QFT method is that it presents a platform to analyze robust control using classical methods much easier to understand by the design engineer.