



## ASSESSMENT OF A BAYESIAN MODEL AND TEST VALIDATION METHOD

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**GVSETS**

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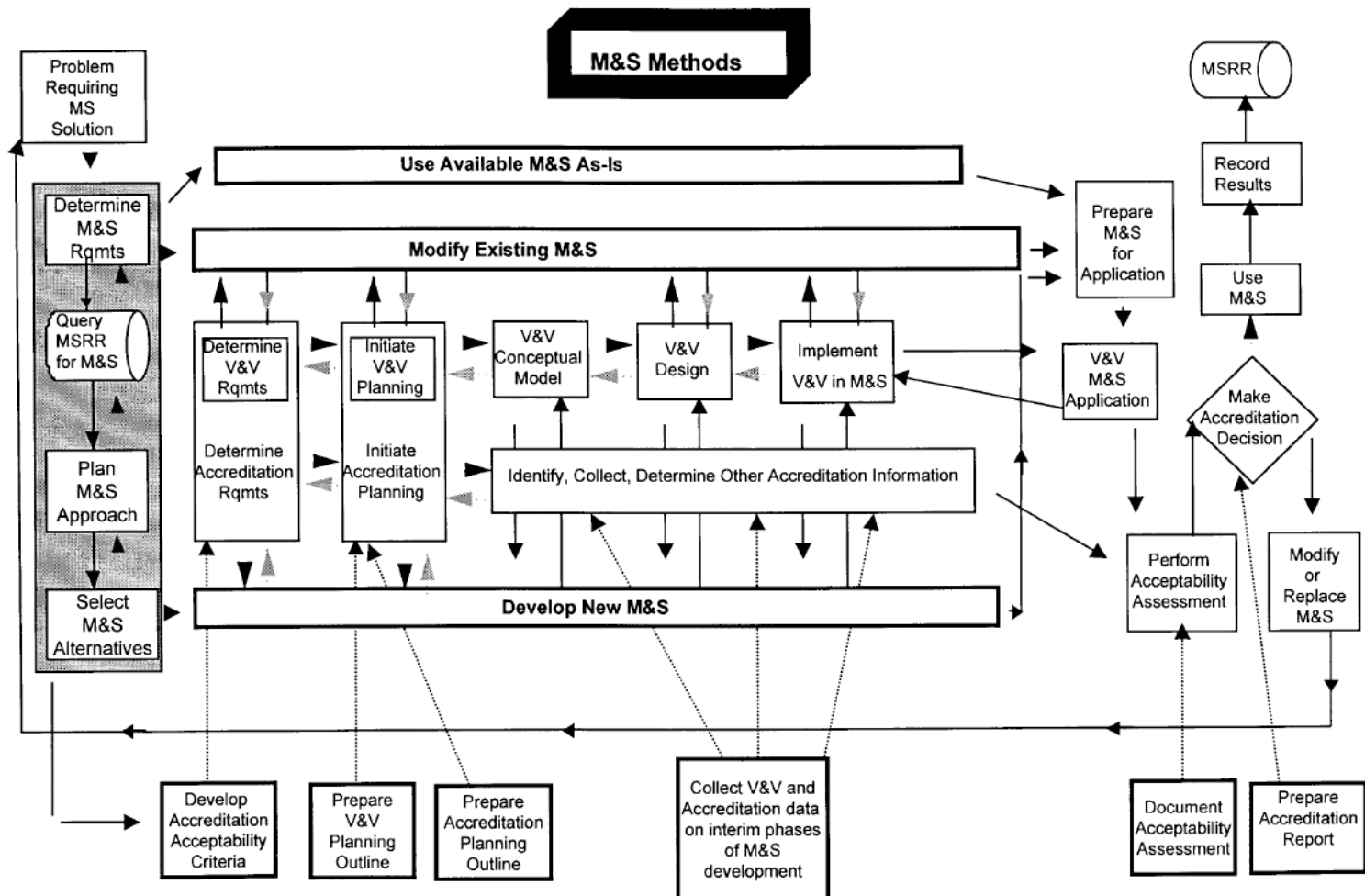
# Need for Validation Methodology

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MODELING AND SIMULATION, TESTING AND VALIDATION

- Systematic method for validation necessary
  - Modeling and Simulation
  - Laboratory test
  - Validation of designs
- Reduce need for Subject Matter Experts
- Reduce number of field tests
- Assess cost of validation and certification
- Use existing data mines of tests, M&S, and designs

# VV&A of Army M&S

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Dept. of Army pamphlet 5-11: VV&A of Army M&S



# Bayesian Confidence Method

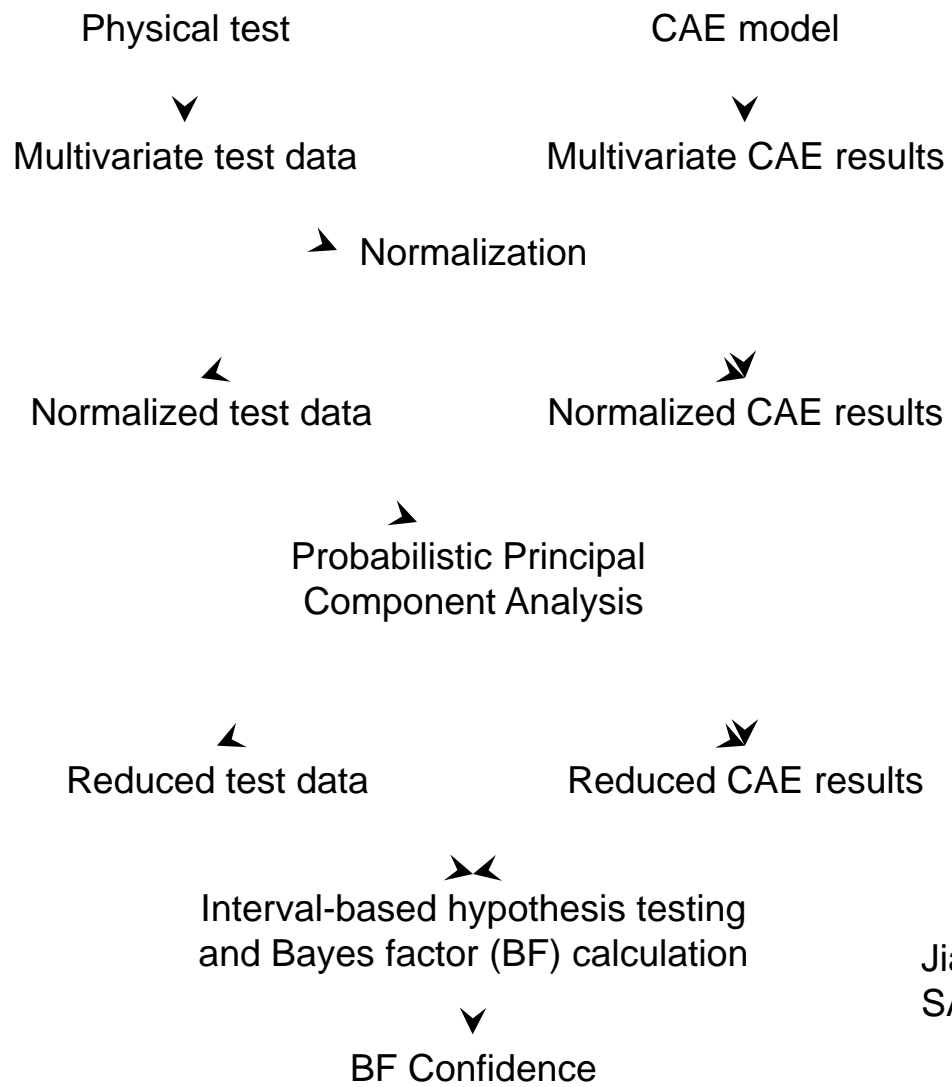
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- Model validation under uncertainty
  - Uncertainty in field data
  - Uncertainty in model data
  - Validation of designs
- Multiple, incompatible data channels can be evaluated
- Interval-based method provide more robust evaluation

# Bayesian Confidence Method

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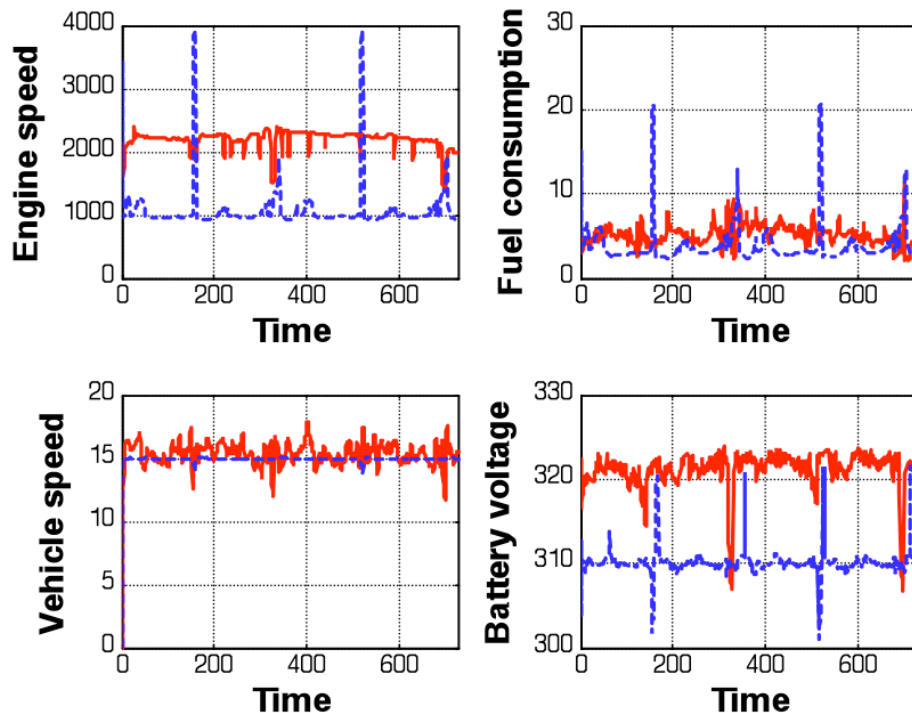
Jiang, Fu, Yang, Barbat, Li, Zhan,  
SAE 2009 World Congress

# Comparison of Model and Test

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- Model 1, Course 1



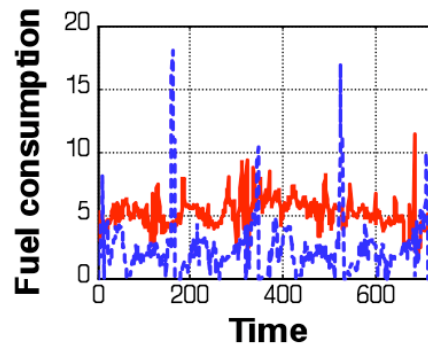
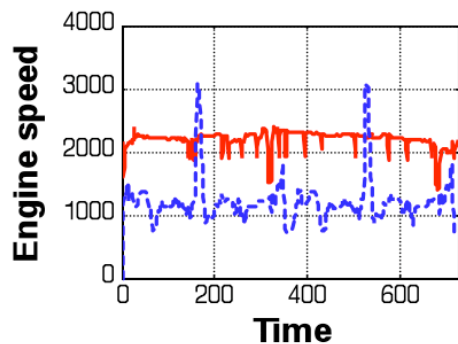
Blue = model 1  
Red = test

# Comparison of Model and Test

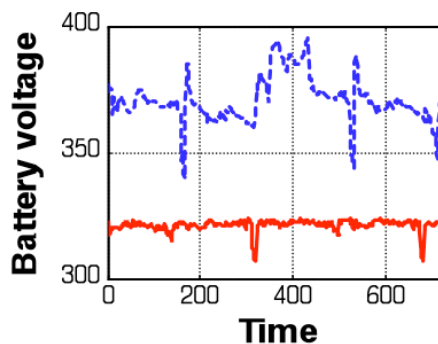
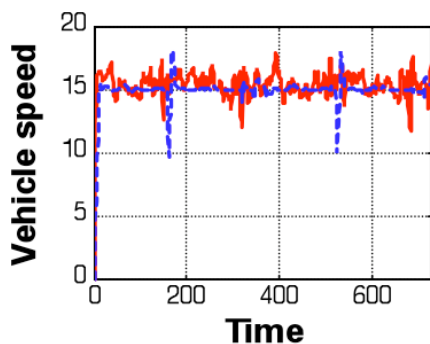
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- Model 2, Course 1



Blue = model 2  
Red = test



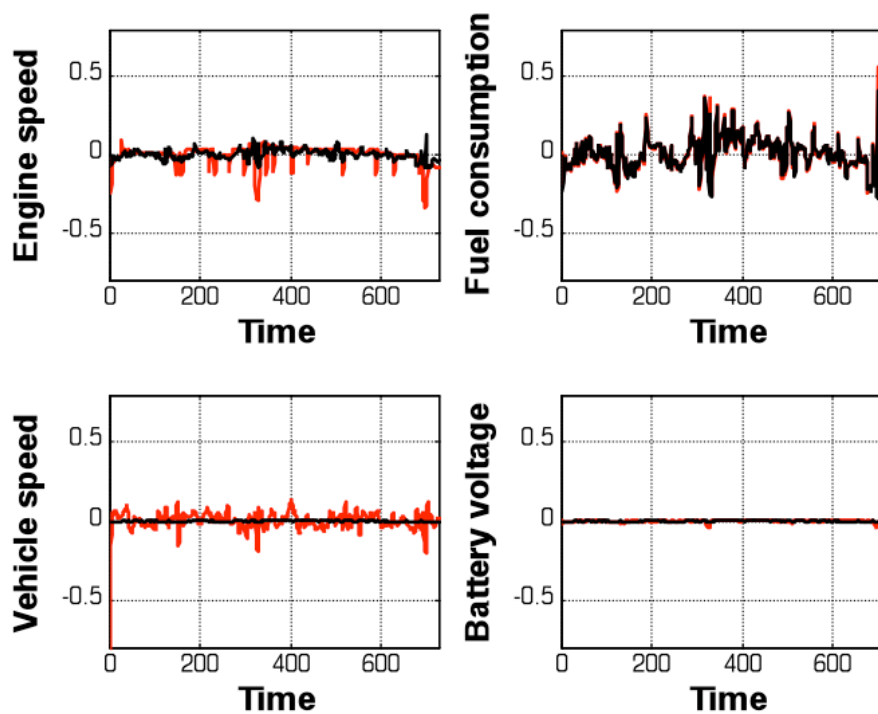


# Data Reconstruction

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- Course 1
- First principal component, 62% total variability captured



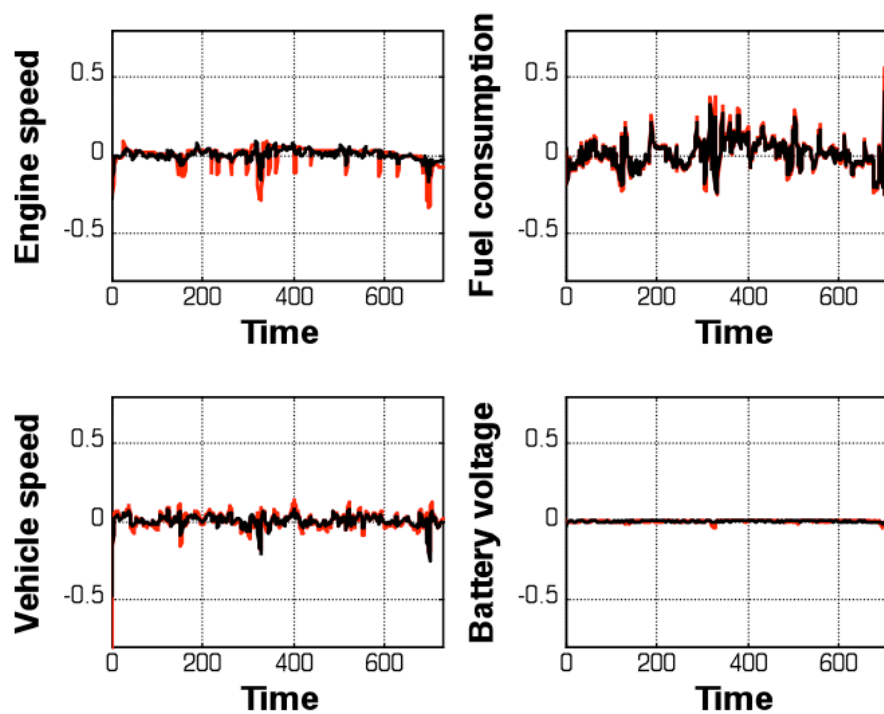
# Data Reconstruction

# MSTV

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- Course 1
- First 2 principal components, 86% total variability captured

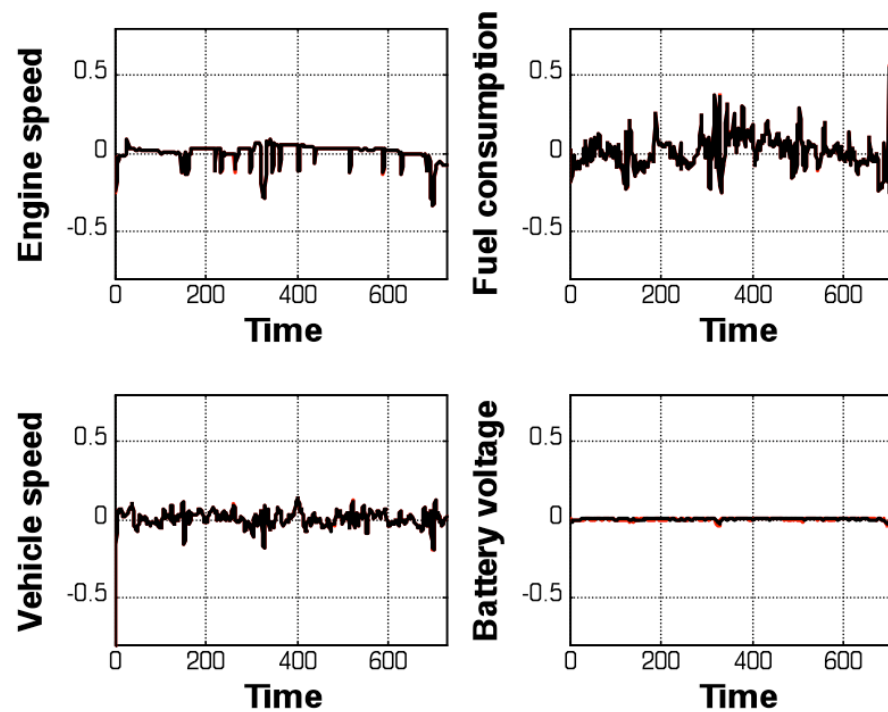


# Data Reconstruction

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- Course 1
- First 3 principal components, 99.9% total variability captured



# Bayesian Hypothesis Testing

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Reduced test data,  $\mathbf{x}_t$   
with variability  $\Sigma_t$

Reduced CAE results,  $\mathbf{x}_c$   
with variability  $\Sigma_c$

Difference  $\mathbf{d} = \mathbf{x}_c - \mathbf{x}_t$

➤ sample statistics:  $\bar{\mathbf{d}} = \text{mean}(\mathbf{d})$       <

$$\Sigma = \text{cov}(\mathbf{d}) + \Sigma_t + \Sigma_c$$

▼

Multivariate hypothesis test: Assuming prior  $\mathbf{d} \sim N(\boldsymbol{\mu}, \Sigma)$

$H_0: |\boldsymbol{\mu}| \leq \boldsymbol{\varepsilon}$  (accept)    *versus*     $H_a: |\boldsymbol{\mu}| > \boldsymbol{\varepsilon}$  (reject)

▼

Bayesian factor calculation

$$B_M = P(\mathbf{d}|H_0) / P(\mathbf{d}|H_a) \text{ (likelihood ratio)}$$

▼

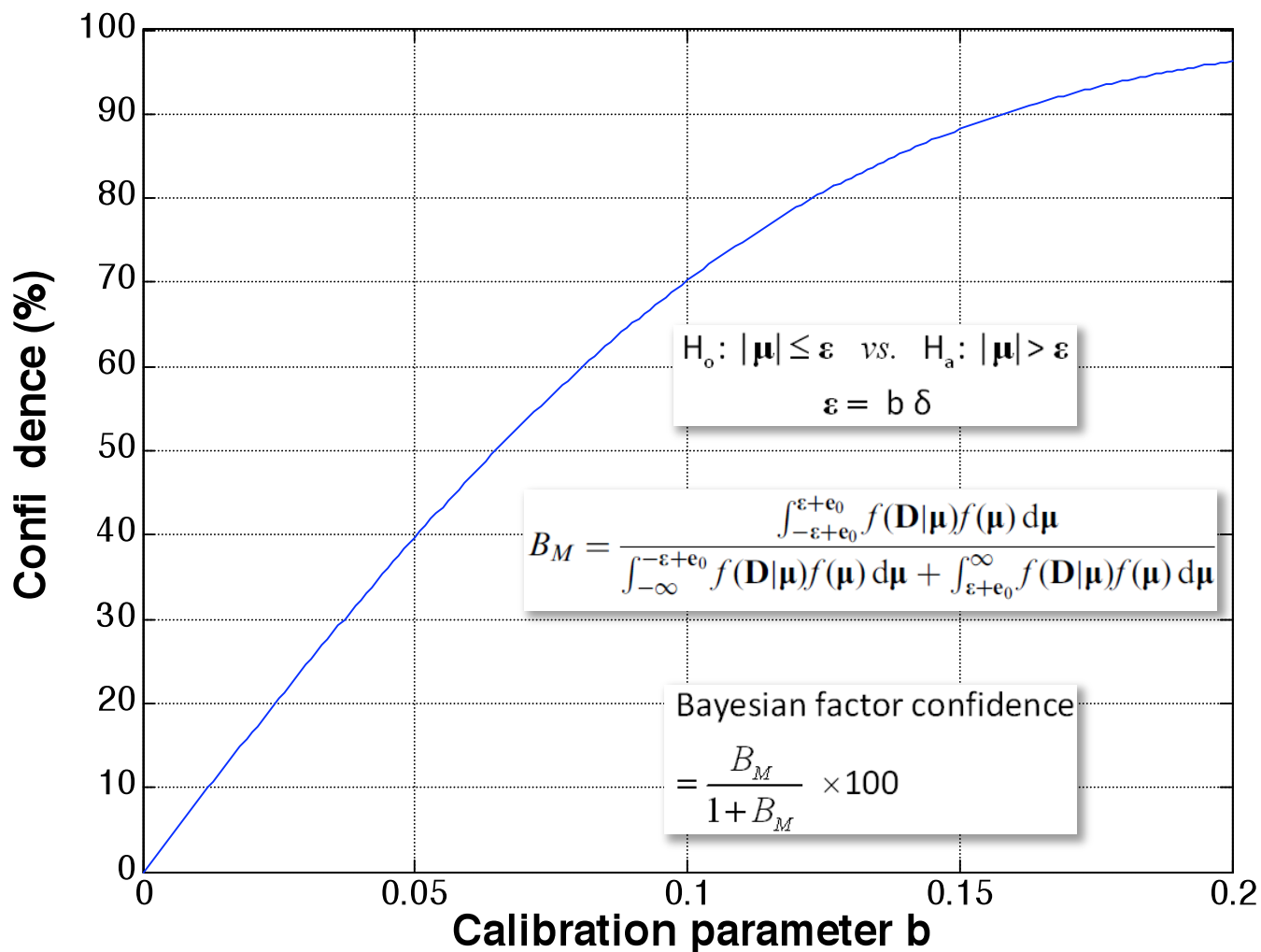
BF confidence quantification

$$\kappa = B_M / (1 + B_M) \times 100$$

# Calibration Parameter Selection

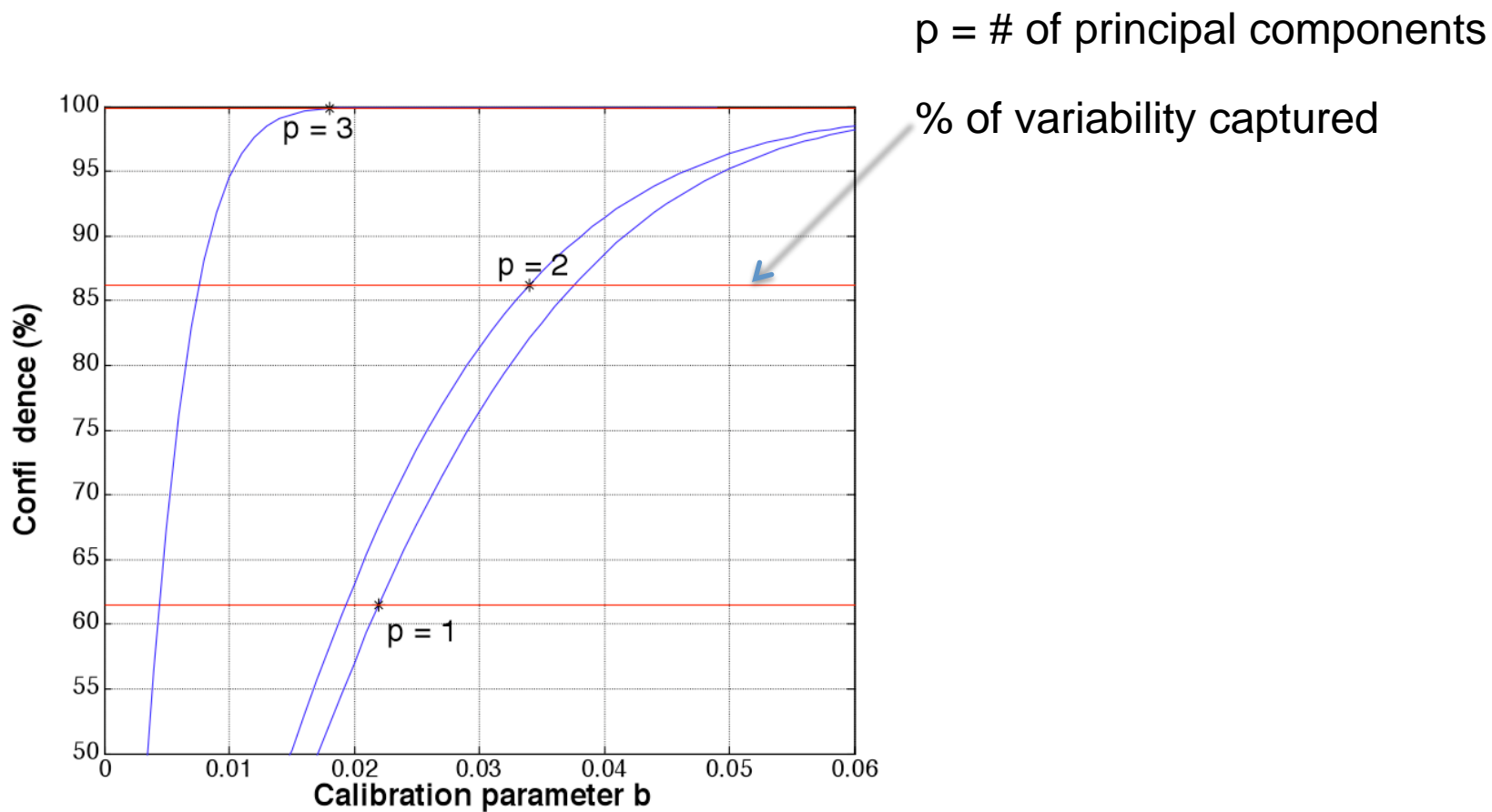
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# Calibration Parameter Selection

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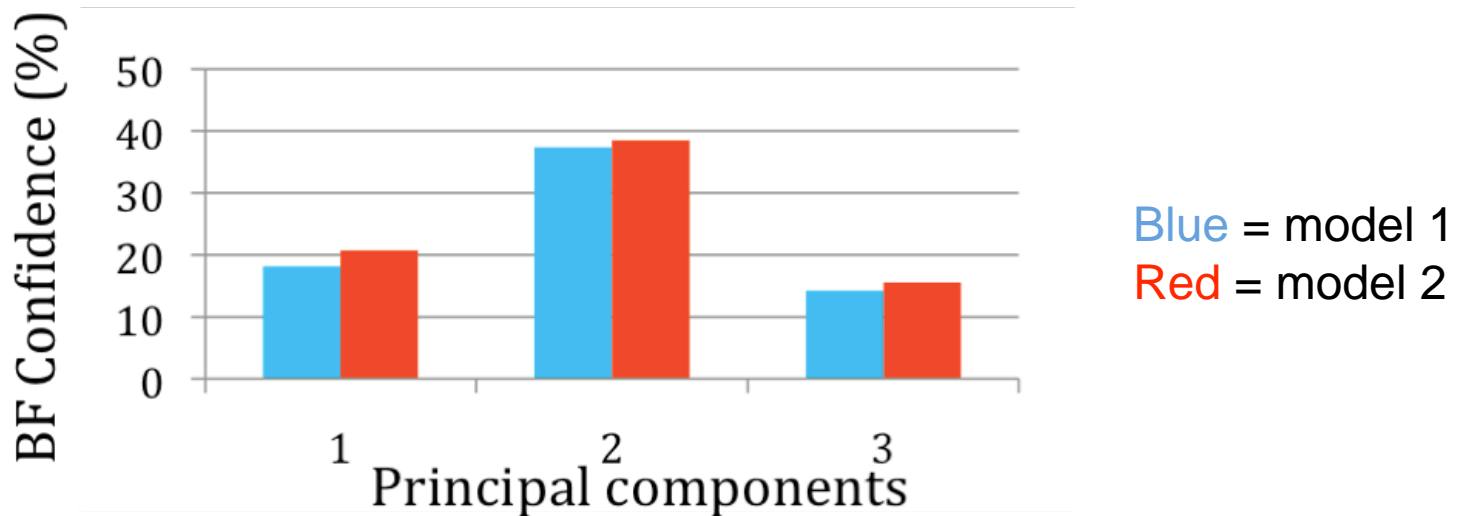


# Effect of Principal Components

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- Course 1

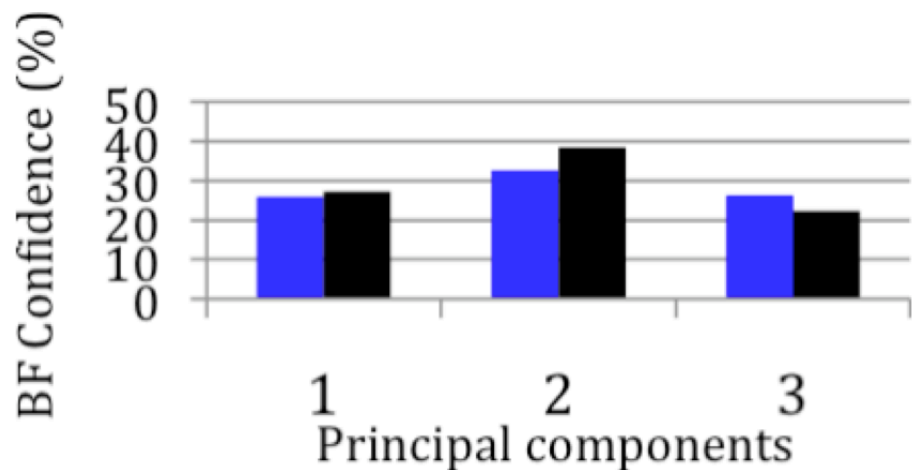


# Effect of Principal Components

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- Course 2



Blue = model 1  
Black = model 2





## Closing Remarks

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- Bayesian framework promising for validation
  - Incorporates statistics of field data
  - Incorporates statistics of M&S
  - Enables systematic evaluation of data variability
- Systematic method for accepting M&S
- Systematic method for comparing M&S
- Further refinement needed for calibration and sensitivity
- Further research required for accreditation use