PASSIVE/ACTIVE HEALTH MONITORING OF FILAMENT WOUND MISSILES

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ABSTRACT

Precision long-range missile systems will be fabricated using composite materials to meet the desired weight and performance requirements. A fully featured health monitoring system is developed in this paper. Here, the term "health monitoring" is used to refer to both loads and damage identification for assessing the material state and structural performance of the missile casing pressure vessel. The initial component of the system developed in this paper is a passive monitoring system that only requires a single sensor. This passive system is able to detect, locate, and quantify an impact while also being able to use this impact to estimate the amount of damage that the impact caused in the specimen. If further evaluation of the casing is desired, an offline active nondestructive evaluation method utilizing nonlinear vibro-acoustics may be used. This active technique is advantageous because it heightens the measurement's sensitivity to damage despite widely varying boundary and environmental conditions due to the effects of propellant, temperature, humidity, etc. The passive and active sensing methods described can be combined to provide a full featured health monitoring solution.

1. INTRODUCTION

Missiles play an important role in the Army's arsenal; therefore, knowledge of the condition, or health, of the missiles and their launch systems during transport and in theater is essential. To provide situational awareness of a missile's structural integrity, a process that can determine the health and operability of each missile is desired. Newly designed missile bodies are frequently constructed using composite materials due to high strength to weight ratios of composites compared to homogeneous metallic materials. The associated decrease in weight reduces the costs and logistics burden associated with transporting and deploying these missiles and also extends their firing range. However, impacts on composite materials can induce damage (e.g., fiber breakage, delamination, matrix cracking) that is structurally compromising and difficult to detect using visual inspection. This paper focuses on the development of a health monitoring system for the detection of impacts and the resulting damage on a seven-inch diameter carbon filament wound composite missile casing that is shown in Figure 1.

To design a complete health monitoring system that is capable of both loads and damage identification on a composite missile casing, three main issues that need to be addressed are the sensing, modeling, and data analyses methods to be used. Because of the cost and weight restrictions associated with the widespread implementation of a health monitoring system on missiles, minimal sensing strategies must be used throughout the system. In addition, to identify impacts on the casing using only a minimal amount of sensors, an accurate, highly refined, and yet easy to construct model is needed. Furthermore, a robust data analysis technique is needed that is able to extract the information of interest from the acquired data.

To address these critical issues, a health monitoring method has been developed that utilizes a combination of passive and active measurements. The passive methods require only a single sensor to detect, locate, and quantify impacts as well as determine the damage done by the impacts. If further evaluation of the structure is needed, an active damage detection methodology may be utilized in offline inspections. In this paper, a nonlinear vibro-acoustic damage inspection method was utilized because it has a high sensitivity to the presence of cracks while being less sensitive than traditional vibration-based methods to variations in environmental and boundary conditions.

Figure 1. Carbon filament wound missile casing with the attached triaxial accelerometer.

2. IMPACT IDENTIFICATION

Because of the potentially damaging effects of impacts on a composite missile casing, the ability to detect and quantify these potentially damaging impacts is an important part of an integrated health monitoring system. The key to implementing such a system is the accurate and reliable estimation of impact locations despite the requirement for minimal sensor data (i.e., a single sensor). To accomplish this task, an accurate model of the missile's structural dynamics is needed. Despite the fact that models that are developed using experimental data

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accurately describe the forced response behavior of a structural component, they are often disregarded due to the time required to test a sufficient number of degrees of freedom. On the other hand, finite element models can be developed with large numbers of degrees of freedom but contain uncertainties associated with many aspects of these numerical models especially when they are comprised of complex composite materials (e.g., damping, local variability due to manufacturing).

The modeling approach that was use for the health monitoring method developed in this paper was to derive a dynamically accurate model of the missile casing using experimental data. To reduce the time required to generate this model while retaining the ability to locate an impact with adequate spatial resolution, an interpolation algorithm was developed to expand the model using the measured FRFs (Yoder et al., 2008). The applicability of such an interpolation method to FRFs is revealed by writing the FRFs in partial fraction form as follows:

$$H_{pq}\left(j\omega\right)=\sum_{r=1}^{N}\frac{A_{pqr}}{j\omega-\lambda_r}+\frac{A_{pqr}^{*}}{j\omega-\lambda_r^{*}}\,, \qquad (1)$$

where H_{pq} is the FRF of the casing for the p^{th} output and the q^{th} input, r is the structural mode number, N is the total number of modes, λ_r is the pole (or modal frequency), and A_{pqr} is the residue. The residue can be expressed as,

$$A_{pqr}=\frac{\psi_{pr}\psi_{qr}}{2jM_r\omega_{dr}}\,, \qquad (2)$$

where M_r is the modal mass, ω_{dr} is the damped natural frequency of the mode, and ψ_{pr} and ψ_{qr} are the modal coefficients for the output and input locations, respectively. This decomposition is significant because it demonstrates that at each frequency the spatial variation of the FRFs across different excitation locations can be expressed as a function of the mode shapes of the structure for each measurement.

Accordingly, if all of the mode shapes of the structure are known, a single FRF can be used to recreate FRFs over the entire surface of the structure. However, in practice the mode shapes of the system are unknown *a priori* and are only measured at a finite number of structural degrees of freedom. These restrictions limit the accuracy with which FRFs can be recreated. For example, a structure's higher-order mode shapes may not be accurately reconstructed if only a small number of measurement locations are used. By applying the spatial version of Shannon's sampling theorem to this data, it is evident that if the minimum wavelength of a given mode shape is more than twice the spacing of the measurement points, the mode shape can be completely reconstructed using only a discrete number of measurements between which sinusoids are used for interpolation. In most structural components, the contribution of each mode to the FRF is highly localized in frequency suggesting that modes beyond the frequency range of interest have less influence. In such structures, if the mode shapes of the modes that contribute most strongly to the FRF within a given frequency range are identified, interpolated FRFs can be created with sufficient accuracy within this same frequency range.

Using a limited number of sample points, the most accurate interpolation will be achieved when the interpolating functions closely resemble the actual mode shapes of the structure. For example, at a given longitudinal location below the nose cone of the casing the dynamics of the carbon filament wound casing studied in this work (Figure 1) are likely to resemble those of a cylinder and a cylinder's mode shapes are known to be comprised of sines and cosines (Soedel, 1993). In this case it was decided to interpolate the casing's FRF model using sine and cosine functions in the circumferential direction and spline functions in the longitudinal direction (Yoder and Adams, 2008).

Once an accurate model of the casing had been created, it was used to determine the location and time history of an external impact on the casing. This inverse problem was solved by using an iterative version of the frequency domain deconvolution technique. The frequency domain deconvolution method is based on the principle that the measured response of a structure is directly related to the unknown input force acting on that structure through the frequency domain equation of motion:

$$X\left(j\omega\right)_{M\times1}=H\left(j\omega\right)_{M\times N}\,F\left(j\omega\right)_{N\times1} \qquad (3)$$

where $X(j\omega)_{M\times1}$ is a vector consisting of the Fourier transform of the M measured responses, $F(j\omega)_{N\times1}$ is a vector of the N input forces, and $H(j\omega)_{M\times N}$ is the previously determined FRF matrix relating each of the inputs to each of the outputs. If the number of possible forcing locations is larger than the number of measured responses ($N > M$), as is the case if minimal sensing methodologies are utilized, this problem is underdetermined and a unique physically meaningful solution may not be obtainable (Karlsson, 1996).

To overcome the underdetermined nature of this problem, it was assumed that only a single location was impacted during each measurement. This assumption allows each of the possible force locations to be considered individually rather than having to consider the entire distribution of possible forces at once (Martin and Doyle, 1996; Choi and Chang, 1996; Stites et al., 2007). Consequently, if more than one response is acquired the inverse equation becomes overdetermined and the force that minimizes the least squares error between the measured and reconstructed responses is given by,

$$F^R(j\omega) = H^+_{1\times M}(j\omega)\, X'_{Mx1}(j\omega) \qquad (4)$$

where $^+$ indicates the Moore-Penrose pseudoinverse, the prime indicates that the data was obtained from the impact of interest, and the ^{R} indicates that the force has been reproduced rather than measured.

To determine which of the many possible reproduced forcing functions corresponds to the actual impact location, it was assumed that this impact had to occur on the outside of the casing and that the reproduced force must maintain the coupling between the measurements. To ensure that the force is able to maintain the coupling between the measurements, all of the *M* FRFs at each of the remaining possible impact locations were reproduced using the recreated forcing function,

$$H^R_{M\times 1}(j\omega) = X'_{M\times 1}(j\omega) \big/ F^R(j\omega) \qquad (5)$$

where H^R is the vector of recreated FRFs at the given impact location. The impact location is then estimated to be the one where the reproduced force is capable of most accurately reproducing the FRFs that existed in the model of the structure (*H*). If the true impact location was different than the suspected location, a different complex scalar ($F^R(j\omega)$) would be required to transform each of the acquired responses into the respective FRF due to differences in the coupling between the measured responses and the incorrect point's FRFs (Yoder et al., 2008).

To minimize the influence of frequency ranges where the response varied significantly between impacts, the ordinary coherence (which was measured during the collection of the model's FRFs) was used to identify these areas of low confidence in each response. To obtain coherence functions for interpolated locations, the measured coherences were linearly interpolated between the measurement locations at each frequency. The coherence values were normalized such that the area under the coherence functions was equal for all impact locations to ensure that each location was equally weighted. The similarity between the measured and reproduced FRFs was calculated using the equation,

$$S_q = \prod_{p=1}^{M} \left[\frac{1}{\sum_{\omega=\omega_1}^{\omega_2} \left[\left| H^R_{pq}(j\omega) - H_{pq}(j\omega) \right| \underline{\gamma}^2_{pq}(j\omega) \right]} \right] \qquad (6)$$

where *p* indicates the response, *q* indicates the potential impact location, $\underline{\gamma}_{pq}{}^2$ is the normalized ordinary coherence, and ω ranges over the frequency points calculated using the discrete Fourier transform in the range specified. The product across measurements was used rather than the sum to exaggerate the cases in which the differences between the FRFs were small in all of the measurements as was the case when the impact location was correctly identified. Once the location at which the impact was believed to have occurred was identified, the applied force time history was recreated by taking the inverse discrete Fourier transform of the reproduced force (F^R) at that point.

The developed algorithm was implemented on the carbon filament wound missile casing using a single PCB 356B21 triaxial accelerometer with a nominal sensitivity of 10 mV/g in each direction. The sensor was mounted as shown in Figure 1. To create the FRF model, impact testing was performed at 6 equispaced circumferential locations and 6 different longitudinal locations. The resulting data was used to estimate the 3 FRFs for each of the 36 distinct impact locations on the casing using the H_2 estimator. The H_2 estimator was used because it has been shown to produce lower errors in reconstructed force time histories than the H_1 FRF estimator (H. Inoue et al. 1991; Yoder and Adams, 2008). The FRFs were then interpolated to 24 locations circumferentially (one every 15 degrees) and 21 locations longitudinally for a total of 504 different possible impact locations and a spatial resolution of less than an inch.

The responses to two impacts with a PCB 086C03 impact hammer at each of the possible impact locations were measured to evaluate the effectiveness of the impact identification algorithm in terms of the identified location and the reproduced force time history. When the iterative least squares algorithm was applied to the 1008 trial impacts using the data between 250 and 900 Hz, the location with the highest similarity measure was the exact point impacted 80.6% of the time. As the model became more refined, it was more appropriate to define the impact location as an area rather than a single point. If the impact area was defined as being the 5 cm by 5 cm square with the impact point at its center, then one of the points in the impact area had the highest similarity metric 97.0% of the time. An example of the similarity metric calculated using the described algorithm for an impact at a location whose FRF was created via interpolation is shown in Figure 2.

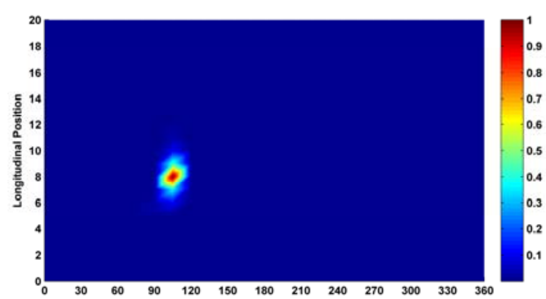

Figure 2. Normalized similarity values over the surface of the casing for an impact at a longitudinal position of 8 and a circumferential angle of 105°.

Rather than only considering the point with the highest similarity metric, another possibility was to examine all of the points with a similarity metric above a certain threshold. This approach created a group of suspect impact locations, which could then be examined for damage after the impact had been detected. When this method was applied to the results from the casing and the threshold value was set at a quarter of the maximum similarity for the hit, the correct impact location was within the group of possible locations 99.5% of the time. The maximum number of points in this group for all 1008 trial impacts was less than 4% of the number of points on the casing. In most cases, the number of points in this group was much smaller and the median number of points in the group of possible points was only three points.

The accuracy of the force time history estimated using the impact identification algorithm was then assessed using the force time history that was measured by the impact hammer. When the maximum force that was reproduced was compared to the maximum measured force, the median percent error was 5.0% across all impacts. The root mean square (RMS) values of the reproduced and measured force time histories had only a median percent error of 3.9% across all of the trial impacts. An example of a reproduced time history compared to the measured force at an interpolated impact point is shown in Figure 3.

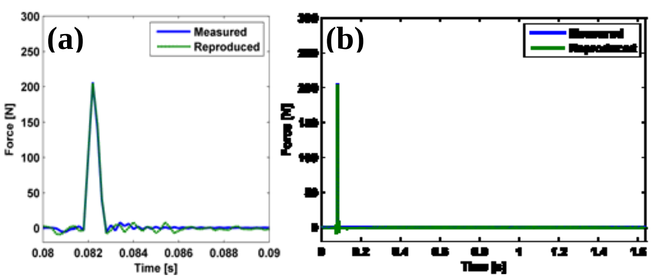

Figure 3. Experimental and reproduced time histories for a sample impact (a) during the impact event and (b) over the entire acquisition period.

3. PASSIVE DAMAGE DETECTION

Once the structure's FRFs have been recreated (Eq. (5)) and the impact location has been identified, the recreated FRF at this location can be compared to the original model to determine if the impact has damaged the casing. It has been found that the accuracy of this recreated FRF can be further increased if it is assumed the impact force is impulsive and can be modeled as a half-sine wave (Stites, 2007). If the impact does damage the casing, the overall measured response will be a combination of the healthy structural response and the damaged response. The response will be time varying because the impact changes the stiffness of the structure due to damage while the response is being measured. This change in stiffness during the measured response will result in a shift in resonant frequencies of the structure. By determining if a significant shift in resonant frequencies has occurred because of the impact, the presence of damage to the structure due that impact can be determined.

To investigate this phenomenon, the missile casing was impacted using a metal tack hammer while it was hanging to simulate a free-free boundary condition. The response of the casing to this impact was measured, and both the recreated force and the half-sine model of the impact are plotted in Figure 4. Note that because the force could not be measured when the tack hammer was applied, this estimate cannot be directly compared to a measurement. This impact created the longitudinal crack in the casing that is shown in Figure 5.

The nonstationary effects of this damage can be seen in the recreated FRF shown around the third resonance of the casing in the radial direction in Figure 6. This figure demonstrates that the reproduced FRF contains components of both the healthy and the damaged casing as hypothesized above. From the detected shift in the natural frequency of the third mode, the damage due to the impact can be estimated. To quantify the level of damage sustained during impact, the frequency shift was plotted as a function of the impulse generated by each impact as shown in Figure 7. Note that this result demonstrates that larger impulse levels do not always lead to larger damage levels due to variations in the local stress field as well as the impact location. This result demonstrates that both the force estimation and damage identification are needed to assess the true condition of the missile casing following an impulsive load.

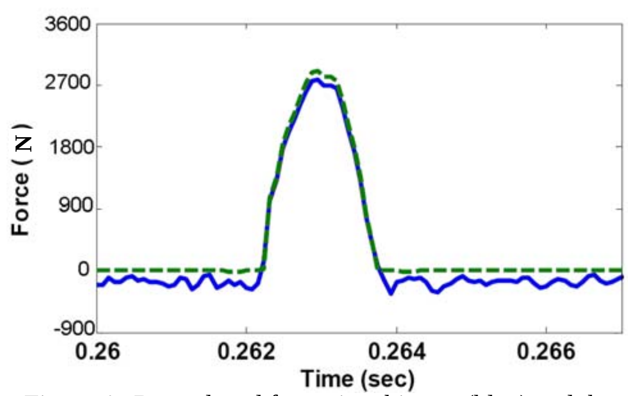

Figure 4. Reproduced force time history (blue) and the half sine curve-fit model (green) for a damaging impact with a tack hammer.

Figure 5. The damage produced by the impact reproduced in Figure 4.

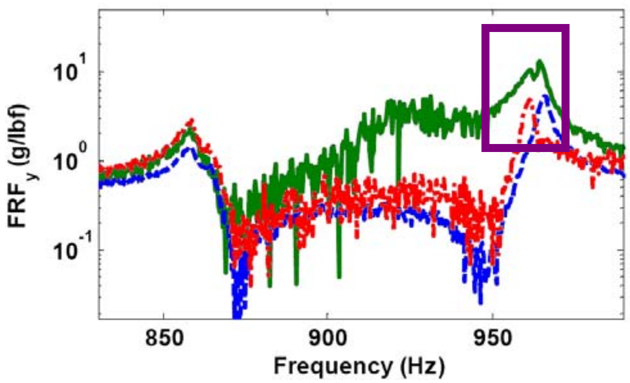

Figure 6. The FRF acquired prior to damage (blue dashed), after damage (red dashed), and reproduced using estimated curve-fit impulse (green)

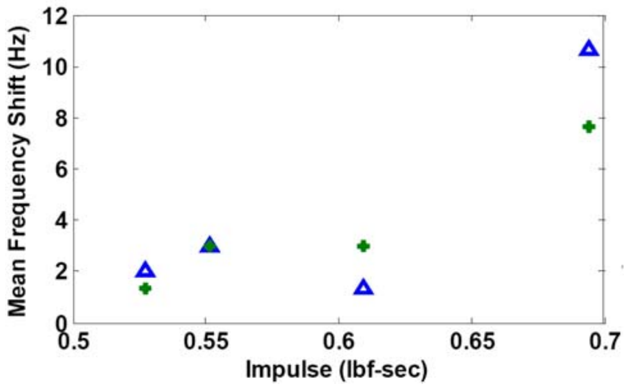

Figure 7. The mean frequency shift of the third natural frequency according to the reproduced FRFs (+) and the FRFs found via modal impact testing (Δ) plotted against the impulse of the applied force.

4. ACTIVE DAMAGE DETECTION

If further inspection of the structure is warranted, an offline method of nondestructive testing must be applied. This method should not only be highly sensitive to the presence of damage but also robust in the presence of other sources of variability. This requirement is essential because one of the major challenges to health monitoring is that changes in the structure's vibration characteristics can be caused by factors other than damage. In fact, it has been shown that if the monitoring process uses modal vibration data in the form of FRFs, most damage detection algorithms cannot detect damage in the presence of variable environmental and structural boundary conditions (Kess and Adams, 2007).

One approach to this problem is to build these environmental parameters into the model of the system (Peeters, 2001). However, this approach increases the complexity of the required measurements and the model of the system and places greater emphasis on the model's accuracy (Friswell, 2007) in multiple domains.

An alternative approach to the problem of isolating changes in the structure's dynamic response that are due to damage from changes due to the test environment is a damage detection method that relies on dynamic characteristics that are more directly related to the damage than to the environmental and boundary conditions. An example of such a method is nonlinear vibro-acoustics (also called nonlinear wave modulation spectroscopy). This method focuses on detecting an increase in the nonlinear characteristics of the structure, which are typically very small in undamaged structures but are increased significantly by damage such as cracks and delaminations (Sutin and Nazarov, 1995). This method exploits the fact that if the structure is nonlinear and is excited with two different frequencies, the low frequency signal becomes modulated around a high frequency carrier signal (Van Den Abeele et al., 2000). Although one theory posits that this modulation is due to changes in the contact area of the crack (or other defect) created by the lower frequency vibrations (Donskoy et al., 2001), recent research suggests that a more significant factor may be due to a nonlinear dissipative mechanism that effects the losses of the high frequency signal (Zaitsev and Sas, 2001). Regardless of the exact physical explanation for the cause, damage causes an increase in the nonlinearity of a structure and that this increase can be exploited to detect the presence of damage.

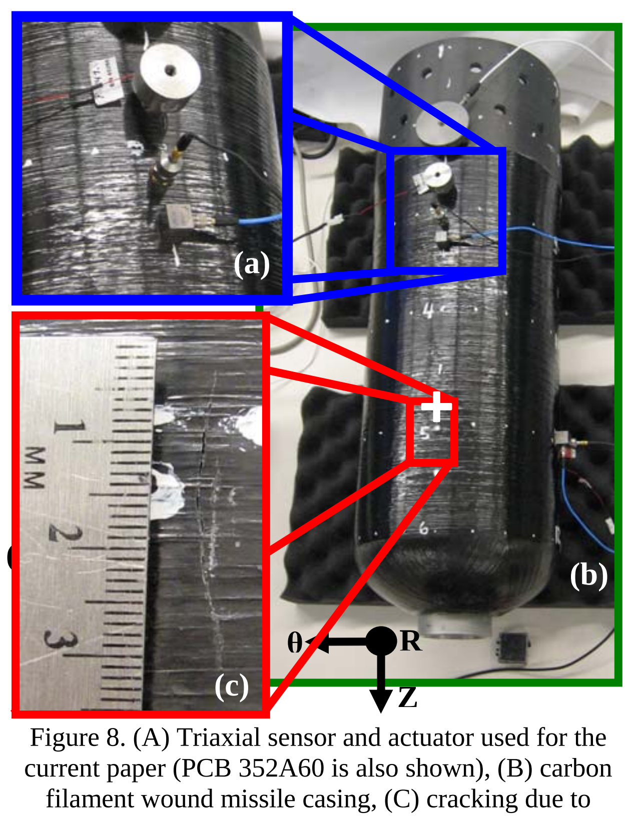

Figure 8. (A) Triaxial sensor and actuator used for the current paper (PCB 352A60 is also shown), (B) carbon filament wound missile casing, (C) cracking due to impact. Impact location is indicated by the +.

A nonlinear vibro-acoustic method was implemented on the carbon filament wound missile casing using a high frequency PI P-141.10 shear piezo actuator that was attached to the structure using LOCTITE 454 instant adhesive. A 100 gram mass was attached to the top of the actuator to provide an inertial preload. The response in all three directions was then measured using a PCB 356B21 triaxial accelerometer. The actuator and the accelerometer are shown in Figure 8(a). For all of the tests a 60 kHz carrier signal was used. To excite the low frequency vibrations that would be modulated around the high frequency carrier signal, the structure was impacted with a modal hammer. If a crack is present, the low frequency modal response of the structure to the impact will be modulated around the carrier signal, whereas if the structure is healthy, there will be no modulation. This type of nonlinear vibro-acoustics is commonly referred to as Impact-Modulation (IM).

After baseline data had been acquired, an impact was applied to the casing using a tack hammer, which produced the crack in Figure 8(c). Because of the shock loading expected due to this impact, all sensors were removed from the casing prior to impact. IM was then performed on the damaged casing when it was empty and at room temperature, when it was stuffed with foam and at room temperature, and when the empty missile casing had been cooled to a lower temperature in a humid environment (freezer). The cooling process was meant to simulate a change in environmental conditions while stuffing the casing with foam was meant to simulate a change in boundary conditions such as that due to the propellant.

The FRFs of the missile casing in each of these configurations was acquired using impact testing and are shown in Figure 9. The changes in the FRFs between each of the environmental conditions are quite significant and overwhelm the changes due to damage. For example, a closer investigation of the second resonance in the R-direction reveals that while the crack did slightly increase the magnitude and decrease the frequency of this mode the changes in the casing's modal parameters due to the other factors are far more significant. This result shows the difficulty inherent in using linear modal vibration characteristics for the detection of damage because changes in the structure's operating environment can easily mask changes in the structure due to damage.

IM was also applied to the missile casing in each of these configurations in order to examine the robustness of this technique in the face of variable environmental and boundary conditions. As can be seen from the differences in the Power Spectral Densities (PSDs) shown in Figure 10 (a) and Figure 10 (b), the casing's low frequency vibrations are modulated around the high frequency carrier signal when the crack is present especially in the θ-direction. This increase in the modulation was also seen when the crack casing was stuffed with foam (Figure 10 (c)) and when it was cooled (Figure 10 (d)).

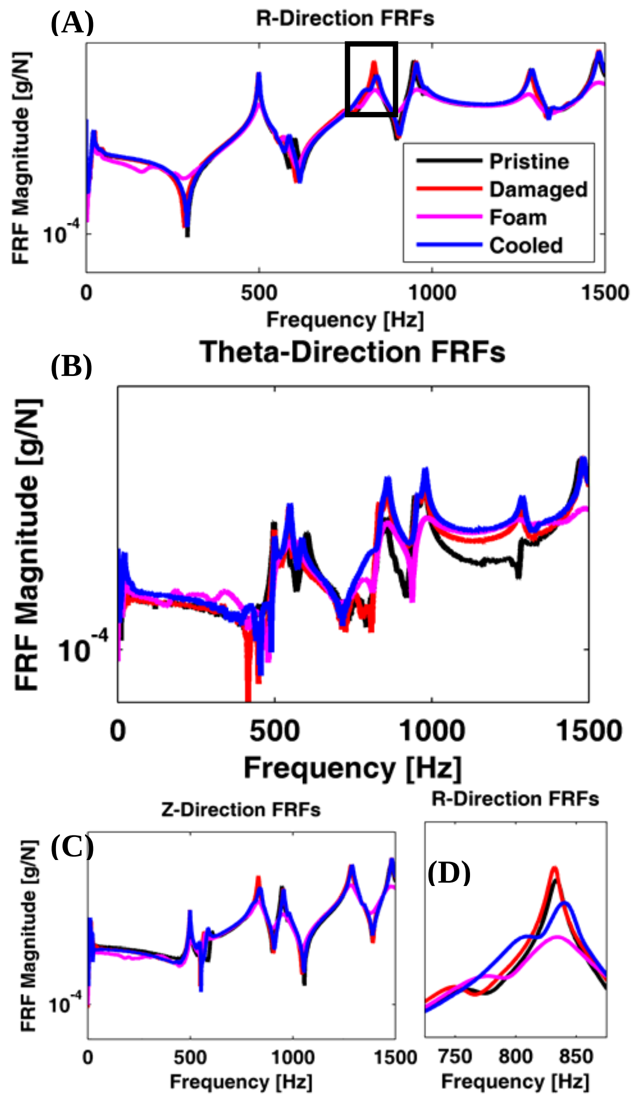

Figure 9. Experimental frequency response functions acquired using modal impact testing in (a,b,c) multiple directions with close up in (d) of peak in (a).

To quantify the amount of modulation that was present in each of these cases, a simple damage index was created based on the phenomena observed in the acquired θ-direction PSDs. The result highlights the need for multidimensional sensing and is likely due to the fact that waves traveling in this direction are perpendicular to the crack face and consequently have the largest interaction with the area in which the nonlinear interactions occur (Yoder and Adams, 2008). To remove the noise floor of the data from the analysis, the mean of the signal's energy between 58 and 58.5 kHz was calculated and subtracted from the PSDs. The damage index was then computed by summing the energy in the modulated region and dividing by the value of the PSD at the driving frequency. The resulting modulation index increased by a factor of 17 for the cracked canister with respect to the modulation index for the pristine canister as shown in Figure 11. When the damaged missile was stuffed with foam, the casing still had a modulation index 13 times as large as the pristine canister. While the canister that had been cooled showed a smaller level of modulation than the other damage cases investigated, it still had a modulation almost twice as large as that of the pristine casing. This decrease in modulation may be due to the fact that during the cooling process condensation may have filled the crack, which has been seen to decrease the magnitude of modulated peaks in other cases (Didenkulov et al., 2000). Taken as a whole, these results demonstrate that nonlinear vibro-acoustic methods have the ability to detect damage despite the changes in the environmental and boundary conditions of the structure.

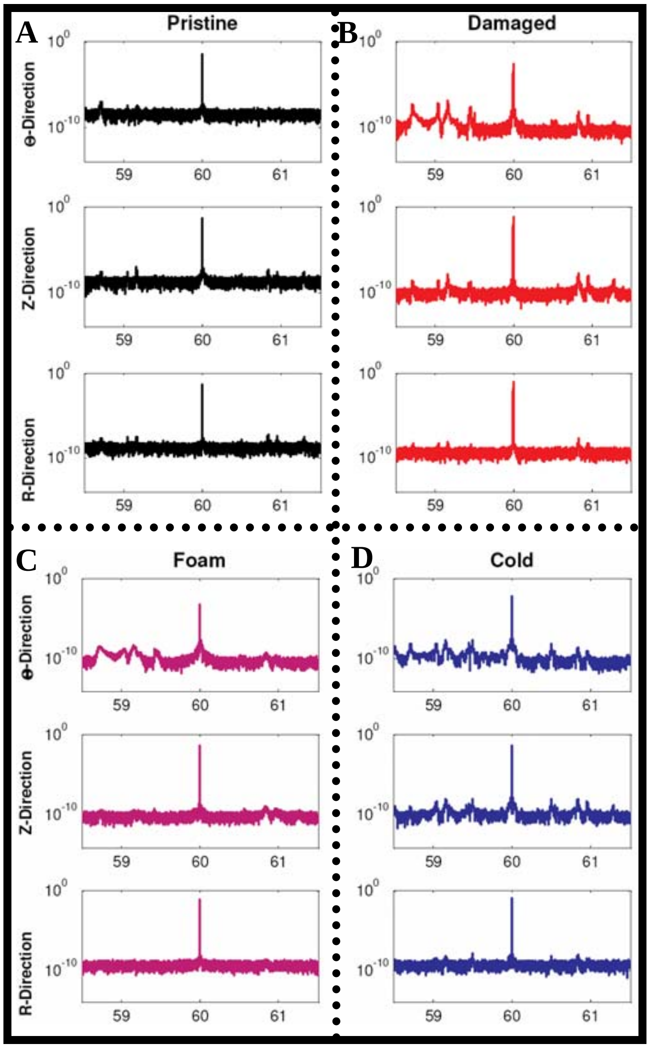

Figure 10. Power spectral densities acquired from: (a) the pristine casing, (b) the cracked casing, (c) the cracked casing stuffed with foam and (d) the cracked canister after it had been cooled. All X-axes are frequency in kHz.

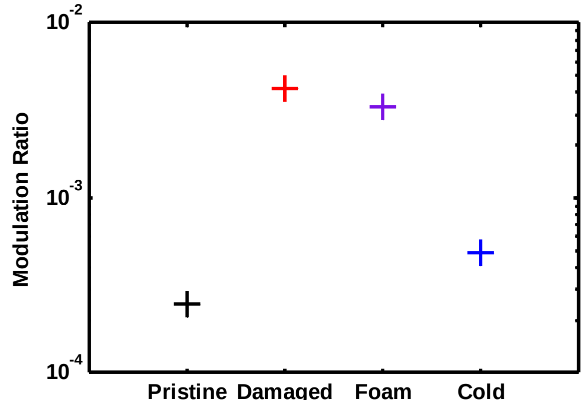

Figure 11. The modulation based damage index for all operating conditions tested.

7. CONCLUSIONS

Because of the critical role that missiles play in the Army's arsenal, the ability to monitor the health of the missiles through their life is essential. A solution to a logistical problem of this magnitude will require a variety of different approaches as part of a fully integrated system. This paper has presented a method that utilizes a combination of passive methods that utilize minimal sensing methodologies to guide the implementation of an offline active system for further evaluation of the casings. The passive system utilizes an impact identification method that allows external impacts to be detected, located, and quantified with high accuracy. This impact can then be used to reproduce the system FRFs. These FRFs allow for the identification of the time varying response of the system as it is damaged. By examining the shift in resonance frequencies that has taken place, the presence of damage can be detected. If both of these methods indicate that further inspection is necessary, an offline nondestructive test may be conducted using nonlinear vibro-acoustic modulation. This methodology was used because of its sensitivity to the presence of cracks in addition to the robustness of the technique in the face of varying environmental and boundary conditions.

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