A COMPUTATIONAL FRAMEWORK BASED ON VARIATIONAL LES METHOD FOR THE MULTI-DISCIPLINARY ANALYSIS OF MAVS WITH FLAPPING WINGS

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ABSTRACT

In this paper the three level formulation of a variational multi-scale (VMS) large eddy simulation (LES) method for compressible flow computations is extended for applications involving moving/deforming grids. A consistent method to improve the VMS-LES method by computing the small-scale Smagorinsky constant dynamically as the flow develops (dynamic VMS-LES) is also extended for dynamic grid applications. Two applications of VMS-LES for the simulation of separated flow over moving NACA-0012 extruded airfoil is then presented. The first application involves a qualitative simulation exploring the Knoller-Betz effect of the heaving airfoil at high Strouhal number. The second application is that of the pitching airfoil undergoing dynamic stall. The results predicted by the dynamic VMS-LES method are compared to those obtained with other turbulence models and to experimental data and it is found that the dynamic VMS-LES performs better than the other considered static and dynamic LES models.

1. INTRODUCTION

Since its introduction the VMS-LES method (Hughes et.al, 2000, 2001) has been successfully applied to simulate many practically relevant turbulent flow computations. In this approach the Navier-Stokes (NS) equations are treated by a variational projection instead of the classical filtering methodology. As clarified later in Collis, 2001, in VMS-LES, spatial scales are separated a priori and systems of equations, each representing the dynamics of one of the separated scales, are derived. Thus, this formalism offers the freedom to choose different closure models for equations representing different scales. The system of equations can then be recombined to obtain the final variational form of the NS equations appended with closure models. Typically in VMS-LES, the effect of the unresolved scales is modeled only in the equations representing the smallest resolved scales. Therefore, no energy is directly extracted from the large structures in the flow. These class of methods have shown great promise in simulation of different turbulent flows, such as wall bounded flows (Hughes et.al, 2001) , separated turbulent flows (Koobus and Farhat, 2004, Rajasekharan et.al. 2007, 2008) , turbulent flow control (Ramakrishnan and Collis, 2002) and simulation of bypass transition (Calo, 2004) to name a few.

Many variants of the VMS-LES method have been developed over the last few years. Initial attention focused on incompressible flows, regular grids and spectral discretizations wherein the a priori separation of scales was achieved via a frequency cutoff. For finite element approximations, a scale separation algorithm based on hierarchical bases was presented in Jansen and Martinez, 2002. For compressible turbulent flows, a more general approach for discretizing the two-level VMS-LES formulation on unstructured finite volume (FV) as well as finite element (FE) meshes was proposed in Koobus and Farhat, 2004. Later in three-level formalism in tandem to the approach in Farhat et.al., 2006 for compressible flows was derived and was shown to introduce requirement of closure terms in the continuity equation as-well.

VMS-LES involves two stages, first is a formalism stage as described in the first two paragraphs and the second is a closure stage. In most previous works on the VMS-LES method, the closure was achieved by extracting energy from the small resolved scales by the Smagorinsky eddy viscosity model (Smagorinsky, 1961) with a constant coefficient. In Farhat et.al, 2006, a dynamic VMS-LES method was constructed by computing this coefficient dynamically (Holmen et.al, 2004) using the same procedure as in the original dynamic subgrid-scale eddy viscosity model (Germano et.al., 1991). In that study, it was concluded that the dynamic multi-scale model was less sensitive to the scale partition when compared to its static counterpart. In Gravemier et.al, 2004, an iterative procedure based on the concept of residual-free bubbles (Franca et.al, 1998) was proposed for adjusting dynamically the value of the coefficient of the Smagorinsky eddy viscosity model. An alternative approach for constructing a dynamic VMS-LES method that is based on a simple variational interpretation of Germano's identity was proposed in Farhat et.al, 2006. The salient features of this alternative approach are numerous. They include: (a) consistency with the variational aspect of the VMS-LES method as opposed to the straightforward procedure adopted in for incorporating a dynamic subgrid-scale eddy viscosity model, (b) consistency of the value of the dynamically computed Smagorinsky constant with the adopted discretization method (similar to the vector-level identity presented in Oberai and Wanderer 2005 and Morinishi

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and Vasilyev 2002), and (c) incorporation of the time-history and diffusive effects by incorporation of the inertial and viscous terms in the computation of the dynamic Smagorinsky constant.

With the increase in computational power over the years, and the development of near wall models, it is now feasible to apply LES simulations to study flow over small sized fixed and flapping wings in the low to moderate speed aerodynamics regime. This development has opened up an opportunity to successfully study flow over Micro Air Vehicles (MAV). It is thus worthwhile in exploring the efficacy of the VMS-LES method in capturing the unsteady aerodynamic flows over moving boundaries. This paper is hence an attempt to study the applicability of the VMS-LES method for flows on moving and deforming grids. In this respect, this paper is essentially a sequel to the previous works done in .

This paper is organized as follows. Section summarizes the arbitrary Lagrangian-Eulerian (ALE) governing equations and the FV/FE formulation. Section describes the VMS-LES formulation of compressible turbulent flows developed in Koobus and Farhat, 2004 and re-derived in Farhat et.al. 2006. In particular, its agglomeration-based procedure for separating a priori the scales on unstructured meshes is highlighted. In Section, an overview of a dynamic extension of the VMS-LES based on the variational interpretation of Germano's identity is presented. Next in Section, the specifics of the spatial and temporal discretizations adopted in this work are discussed. In Section, the results from the application of the dynamic VMS-LES method to the simulation a heaving and pitching extruded NACA-0012 airfoil is presented and the results are discussed. Finally, Section concludes this paper with a summary of the findings and reflections on future improvements and challenges.

2. ARBITRARY LAGRANGIAN EULERIAN (ALE) FORMULATION OF THE GOVERNING EQUATIONS AND ITS SEMI-DISCRETIZATION

Then the ALE strong conservation form of set of mass, momentum and energy equations governing a compressible fluid can be written down as

$$\begin{cases} \dfrac{\partial(J\rho)}{\partial t} + J\nabla.(\rho(\mathbf{u}-\mathbf{w})) = 0 \\ \dfrac{\partial(J\rho\mathbf{u})}{\partial t} + J\nabla.(\rho\mathbf{u}\otimes(\mathbf{u}-\mathbf{w})) = -\nabla P + \nabla.\boldsymbol{\sigma} \\ \dfrac{\partial(JE)}{\partial t} + J\nabla.[(E(\mathbf{u}-\mathbf{w})+P\mathbf{u})] = \nabla.(\boldsymbol{\sigma}\mathbf{u}) + \nabla.(\lambda\nabla T) \end{cases} \quad (1)$$

Without any loss of generality as far as issues pertaining to unstructured meshes, the following spatial discretization is assumed:

• The bounded flow domain is discretized by a tetrahedral mesh from which a dual mesh defined by cells or control volumes is derived.

• The convective fluxes are approximated by a finite volume (FV) method

• The diffusive fluxes are approximated by a piecewise linear finite element (FE) method

Now, integrating Eq. (1) on an elementary volume of the material/reference space and transforming back to the physical space, the following mixed FE/FV formulation is obtained

$$\begin{cases} A(\mathcal{X}_i, \mathbf{W}, \mathbf{w}) = 0 \\ \mathbf{B}(\mathcal{X}_i, \Phi_i, \mathbf{W}, \mathbf{w}) = \mathbf{0} \\ C(\mathcal{X}_i, \Phi_i, \mathbf{W}, \mathbf{w}) = 0 \end{cases} \quad (2)$$

where

$$\begin{cases} A(\mathcal{X}_i, \mathbf{W}, \mathbf{w}) = \int_{\Omega(t)} \dfrac{\partial \rho}{\partial t}\mathcal{X}_i \, d\Omega + \int_{\partial\Omega_i(t)} \rho(\mathbf{u}-\mathbf{w}).\mathbf{n}\mathcal{X}_i \, d\Gamma \\ \mathbf{B}(\mathcal{X}_i, \Phi_i, \mathbf{W}, \mathbf{w}) = \int_{\Omega(t)} \dfrac{\partial \rho\mathbf{u}}{\partial t}\mathcal{X}_i \, d\Omega + \int_{\partial\Omega_i(t)} \rho\,\mathbf{u}\otimes(\mathbf{u}-\mathbf{w})\,\mathbf{n}\mathcal{X}_i \, d\Gamma \\ \qquad + \int_{\partial\Omega_i(t)} P\mathbf{n}\mathcal{X}_i \, d\Gamma + \int_{\Omega(t)} \boldsymbol{\sigma}\nabla\Phi_i d\Omega \\ C(\mathcal{X}_i, \Phi_i, \mathbf{W}, \mathbf{w}) = \int_{\Omega(t)} \dfrac{\partial E}{\partial t}\mathcal{X}_i \, d\Omega + \int_{\partial\Omega_i(t)} (E(\mathbf{u}-\mathbf{w})+P\mathbf{u}).\mathbf{n}\mathcal{X}_i \, d\Gamma \\ \qquad + \int_{\Omega(t)} \boldsymbol{\sigma}\mathbf{u}.\nabla\Phi_i \, d\Omega + \int_{\Omega(t)} \lambda\nabla T.\nabla\Phi_i \, d\Omega \end{cases} \quad (3)$$

and

$$\mathbf{W} = (\rho, \mathbf{u}, T)^t$$

3. VMS-LES FORMULATION OF THE ALE GOVERNING EQUATIONS

The decomposition a priori of these spaces into large resolved scales, small resolved scales, and unresolved scales (a.l.a. Collis) can be written as

$$\mathcal{V}_{FV} = \overline{\mathcal{V}}_{FV} \oplus \mathcal{V}'_{FV} \oplus \widehat{\mathcal{V}}_{FV} \quad (4)$$

$$\mathcal{V}_{FE} = \overline{\mathcal{V}}_{FE} \oplus \mathcal{V}'_{FE} \oplus \widehat{\mathcal{V}}_{FE} \quad (5)$$

where the notation "-", " ' ", and "^" designates the large resolved, small resolved, and unresolved scales,

Plugging these decompositions in Eq. (2) a coupled set of equations for large, small and unresolved scales are obtained. Neglecting the effect of unresolved scales on large scales and recombining the large and small scale equations the following set of discretized (subscript 'h') equations is achieved

$$\begin{cases} A(\mathcal{X}_{i_h}, \mathbf{W}_h, \mathrm{w}) \ + \ A'(\mathcal{X}'_i, \mathbf{W}', \widehat{\mathbf{W}}, \mathrm{w}) \ + \ R_c(\mathcal{X}'_i, \widehat{\mathbf{W}}, \mathrm{w}) \\ \quad + \ C_c(\mathcal{X}'_i, \overline{\mathbf{W}}, \widehat{\mathbf{W}}, \mathrm{w}) \qquad = 0 \\ \mathrm{B}(\mathcal{X}_{i_h}, \Phi_{i_h}, \mathbf{W}_h i, \mathrm{w}) \ + \ \mathrm{B}'(\mathcal{X}'_i, \Phi'_i, \mathbf{W}', \widehat{\mathbf{W}}, \mathrm{w}) + \mathrm{R}_m(\mathcal{X}'_i, \widehat{\mathbf{W}}, \mathrm{w}) \\ \quad + \ \mathbf{C}_m(\mathcal{X}'_i, \Phi'_i, \overline{\mathbf{W}}, \widehat{\mathbf{W}}, \mathrm{w}) \qquad = \mathbf{0} \\ C(\mathcal{X}_{i_h}, \Phi_{i_h}, \mathbf{W}_h) \ + \ C'(\mathcal{X}'_i, \Phi'_i, \mathbf{W}', \widehat{\mathbf{W}}, \mathrm{w}) + R_e(\mathcal{X}'_i, \Phi'_i, \widehat{\mathbf{W}}, \mathrm{w}) \\ \quad + \ C_e(\mathcal{X}'_i, \Phi'_i, \overline{\mathbf{W}}, \widehat{\mathbf{W}}, \mathrm{w}) \qquad = 0 \end{cases} \tag{6}$$

where it is noted that unlike the Favre-averaged formulation of the governing equations, the continuity equations requires modeling. Choosing to ignore this modeling, and applying Smagorinsky type closures yields

$$\begin{cases} A(\mathcal{X}_{i_h}, \mathbf{W}_h, \mathrm{w}) \qquad = 0 \\ \mathbf{B}(\mathcal{X}_{i_h}, \Phi_{i_h}, \mathbf{W}_h, \mathrm{w}) \ + \ \mathrm{M}_S\Big(\Phi'_{i_h}, \mathbf{W}'_h, (\mathcal{C}'_s \triangle'_h)^2\Big) \qquad = \mathbf{0} \\ C(\mathcal{X}_{i_h}, \Phi_{i_h}, \mathbf{W}_h, \mathrm{w}) \ + \ M_H\Big(\Phi'_{i_h}, \mathbf{W}'_h, (\mathcal{C}'_s \triangle'_h)^2, Pr_t\Big) = 0 \end{cases} \tag{7}$$

with,

$$\mathrm{M}_S\Big(\Phi'_i, \mathbf{W}', (\mathcal{C}'_s \triangle'_l)^2\Big) = \int_\Omega \overline{\rho} (\mathcal{C}'_s \triangle'_l)^2 |\mathbf{S}'| (2\mathbf{S}'_{ij} - \frac{2}{3} \mathbf{S}'_{kk} \delta_{ij}) \nabla \Phi'_i d\Omega$$

$$M_H\Big(\Phi'_i, \mathbf{W}', (\mathcal{C}'_s \triangle'_l)^2, Pr_t\Big) = \int_\Omega \frac{C_p \overline{\rho} (\mathcal{C}'_s \triangle'_l)^2 |\mathbf{S}'|}{Pr_t} \nabla T' . \nabla \Phi'_i d\Omega \tag{8}$$

3.1 *A PRIORI* SEPARATION OF SCALES ON UNSTRUCTURED ENTITIES

A priori separation of the scales according to

$$\mathcal{V}_{FE/FV_h} = \overline{\mathcal{V}}_{FE/FV_h} \oplus \mathcal{V}'_{FE/FV_h}$$

is achieved here by defining a projection

$$\overline{\mathbf{W}}_h = \mathcal{P}_h \mathbf{W}_h$$

For this, control volumes and their neighbors in the dual mesh are agglomerated to form macro-cells. Then, as proposed in Koobus and Farhat, 2004, the large-scale component is

$$\overline{\mathbf{W}}_h = \sum_k \Phi_{k_h} \widetilde{\mathbf{W}}_{k_h} = \sum_k \Phi_{k_h} \left(\frac{\sum_{j \in I_k} Vol(C_j) \mathbf{W}_{j_h}}{\sum_{j \in I_k} Vol(C_j)} \right) = \sum_k \overline{\Phi}_{k_h} \mathbf{W}_{k_h} \tag{9}$$

with,

$$\overline{\Phi}_{k_h} = \frac{Vol(C_k)}{\sum_{j \in I_k} Vol(C_j)} \sum_{j \in I_k} \Phi_{j_h} \tag{10}$$

3.2 WALL BOUNDARY TREATMENS OF LARGE AND SMALL SCALE VARIABLES

A priori separation of scales also raises the question of which boundary conditions to prescribe for the large and small resolved scales. This issue is addressed here by requiring that the large resolved scales satisfy the no-slip (Dirichlet) boundary conditions and by allowing the small scales to vanish at the wall. This is consistent with the observation that the small eddies (fluctuations) disappear at the wall. Hence, the following boundary restriction is incorporated into the definition of the large and small resolved scales

$$\begin{cases} \overline{\mathcal{V}}_{FV_h/FE_h} = \{\overline{v}_h : \overline{v}_h = \sum_k \overline{\Phi}_{k_h} v_{k_h}, \overline{\mathbf{u}}_h|_{\Gamma_{wall}} = \mathbf{u}|_{\Gamma_{wall}}\} \\ \mathcal{V}'_{FV_h/FE_h} = \{v'_h = \sum_k (\Phi_{k_h} - \overline{\Phi}_{k_h}) v_{k_h}, \mathbf{u}'_h|_{\Gamma_{wall}} = 0\} \end{cases} \tag{11}$$

The above no-slip condition is enforced naturally by choosing not to agglomerate the first layer of control volumes adjacent to the wall. This approach avoids discontinuities of the large scale velocities at the wall in comparison to strongly prescribing the no-slip conditions. In this way the Gibbs phenomenon is averted.

It should also be mentioned that for computations involving wall-functions (as in moderate to high Reynolds number computations presented in this paper), the VMS-LES approach does not need any extra treatments at the displaced wall boundary. This is because, the wall-function introduces a Dirichlet-to-Neumann transformation which results in imposing a Neumann boundary condition at the displaced boundary.

4. DYNAMIC EXTENSIONS OF THE VARIATIONAL MULTISCALE METHOD

In the previous section a static VMS-LES formulation for simulation of compressible turbulent flows was presented. One of short comes of this approach is that the Smagorinsky constant is fixed. Hence, as it is, this static form of the VMS-LES model is not adaptable to the different flow features in the domain. Also this method generates turbulent viscosity even in the laminar regions of the flow domain. To overcome these shortfalls, dynamic versions of the VMS-LES method have been put forth. This section briefly describes the residual based dynamic procedure in Farhat et.al, 2006.

4.1 A RESIDUAL BASED DYNAMIC VMS-LES MODEL USING THE VARIATIONAL ANALOGUE OF GERMANO'S IDENTITY

Consider two nested meshes with element sizes h_1 and $h_2 > h_1$ respectively, and corresponding finite dimensional subspaces

$$\mathcal{V}_{FE_{h_2}} \subset \mathcal{V}_{FE_{h_1}}$$

Substituting h by h_1 in the second and third of Eqs. (6) the momentum and continuity equations for each node i is obtained. Similarly, discretizing Eqs.(6) on the coarser mesh, but using the prolongated and their associated nodal values on the coarse mesh, the corresponding momentum and continuity equations for each node i is obtained. Subtracting the first of this equations from the second and defining

$$\left\{\begin{array}{l} \mathrm{B}(\mathcal{X}_{i_{h_{2,1}}}, \Phi_{i_{h_{2,1}}}, \mathbf{W}_{h_{2,1}}, \mathrm{w}) - \mathrm{B}(\mathcal{X}_{i_{h_1}}, \Phi_{i_{h_1}}, \mathbf{W}_{h_1}, \mathrm{w}) \\ = \mathrm{M}_S\Big(\Phi'_{i_{h_1}}, \mathbf{W}'_{h_1}, \mathcal{C}'\triangle_l'^2\Big) - \mathrm{M}_S\Big(\Phi'_{i_{h_{2,1}}}, \mathbf{W}'_{h_{2,1}}, (\frac{\triangle'_{h_2}}{\triangle'_{h_1}})^2\mathcal{C}'\triangle_l'^2\Big) \\ C(\mathcal{X}_{i_{h_{2,1}}}, \Phi_{i_{h_{2,1}}}, \mathbf{W}_{h_{2,1}}, \mathrm{w}) - C(\mathcal{X}_{i_{h_1}}, \Phi_{i_{h_1}}, \mathbf{W}_{h_1}, \mathrm{w}) \\ = M_H\Big(\Phi'_{i_{h_1}}, \mathbf{W}'_{h_1}, \mathcal{C}'\triangle_l'^2, Pr_t\Big) - M_H\Big(\Phi'_{i_{h_{2,1}}}, \mathbf{W}'_{h_{2,1}}, (\frac{\triangle'_{h_2}}{\triangle'_{h_1}})^2\mathcal{C}'\triangle_l'^2, Pr_t\Big) \end{array}\right. \tag{14}$$

The above two equations relate the resolved turbulent scales to the subgrid-scales at two different levels. Hence, they can be interpreted as a variational form of Germano's identity, from which the parameters Smagorinsky constant and the turbulent Prandtl number can be dynamically computed. Choosing subspaces

$$\mathcal{V}_{FV/FE_{h_2}} = \overline{\mathcal{V}}_{FV/FE_{h_2}} \oplus \mathcal{V}'_{FV/FE_{h_2}} = \overline{\overline{\mathcal{V}}}_{FV/FE_{h_1}} \oplus \left[\overline{\mathcal{V}}\right]'_{FV/FE_{h_1}} \tag{15}$$

are constructed using a second-level agglomeration generated by applying recursive agglomerations.

5. NUMERICAL SOLUTION

The static and dynamic versions of the VMS-LES method presented in the previous sections were implemented in the AERO-F module of the nonlinear aeroelastic simulation platform AERO-F (Farhat et.al, 2003). AERO-F is a domain-decomposition-based, massively parallel, unstructured, three-dimensional, Navier-Stokes compressible flow solver with built in moving mesh capabilities. It blends an upwind scheme for the convective fluxes based on Roe's approximate Riemann solver with a P1 finite element Galerkin approximation of the diffusive fluxes and source terms. Higher-order spatial accuracy is achieved through the use of a multidimensional piecewise linear reconstruction that follows the principle of the Monotonic Upwind Scheme for Conservative Laws (MUSCL) (van Leer, 1979). Stabilization is obtained by a numerical diffusion that is based on sixth-order derivatives (Debiez and Dervieux, 1999. The associated numerical dissipation has a localized effect on high frequencies, which reduces its interaction with subgrid-scale modeling in turbulent flow simulations. Within this fluid module, RANS, LES and Hybrid turbulence models can be coupled with Reichardt's wall function (Hinze, 1959). AERO-F has the capability of performing time-integration with a three-point backward-difference implicit scheme as well as a fourth-order Runge-Kutta explicit scheme. In the low speed limit, it resorts to Roe-Turkel preconditioning (Viozat, 1997; Turkel, 1987) of dissipation terms to overcome the numerical difficulties that are usually encountered by compressible flow solvers.

6. VMS-LES OF TURBULENT FLOWS ON MOVING GRIDS

The static and dynamic versions of the VMS-LES method for moving grids presented in the previous sections are applied to heaving and pitching of a NACA-0012 extruded airfoil. These flow configurations are useful in the analysis and design of MAV wing sections. All simulations presented in this section utilize an arbitrary Lagrangian/Eulerian form of Reichardt's wall law (Tran et.al, 1998) and an implicit second-order time-stepping scheme that is provable second-order time-accurate on moving meshes and obeys its discrete geometric conservation law (DGCL) (Geuzaine et.al, 2003). The results obtained using the VMS-LES methods are also compared with the classical constant coefficient Smagorinsky LES model (Smag LES), the classical Favre averaged Germano dynamic LES model (dynamic LES, Camarri et.al. 2002) and to experimental data wherever possible.

6.1 VMS-LES SIMULATION OF THE KNOLLER-BETZ EFFECT

The Knoller-Betz effect symbolizes the ability of sinusoidally plunging airfoil to produce thrust. This is an important consideration for the design of propulsion systems of MAVs due to the excessive weight associated with traditional battery driven fixed wing propulsive systems. Various experimental and numerical investigations of this phenomenon have been carried out over the years to understand the evolution of thrust indicative wake structures . It was observed both numerically and experimentally in Jones et.al, 1998 that the wake patterns at Strouhal numbers greater than 1.0 were non-symmetric, indicating that viscous effects/flow separation may be important in understanding the wake evolution. This sub-section delves into the study qualitatively the evolution of these wake structures using the static VMS-LES method.

To this end, a qualitative simulation of a plunging extruded NACA-0012 airfoil at a Reynolds number of 61,000 based on the chord length of 1.22m is performed here. The airfoil is plunged with a reduced frequency of k=12.3 and a non-dimensional plunge amplitude of h=0.12 (resulting in a Strouhal number of 1.476). The computational domain is delimited by a circular cylinder of diameter 20 times chord length. Symmetric boundary conditions are imposed at the two wing tips. The mesh has 14,450,920 tetrahedra and 2,516,690 nodes. Preferential mesh refinement is introduced in the regions of the separated wake as shown in Fig. 1. below.

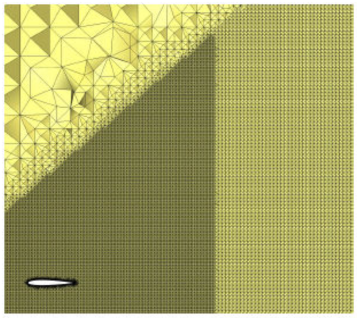

Fig 1: Refinement around the NACA-0012 airfoil

As seen in Fig. 2 the final wake pattern shows the dual mode vortex street as observed in , which is indicative of thrust and lift as seen in the plot of averaged lift and drag (Fig. 3).

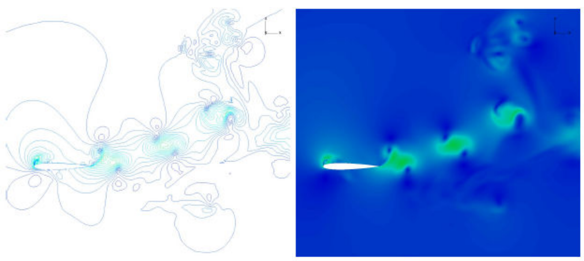

Fig 2: Contour plots and wake pattern showing the dual mode vortex street of the plunging NACA-0012 airfoil

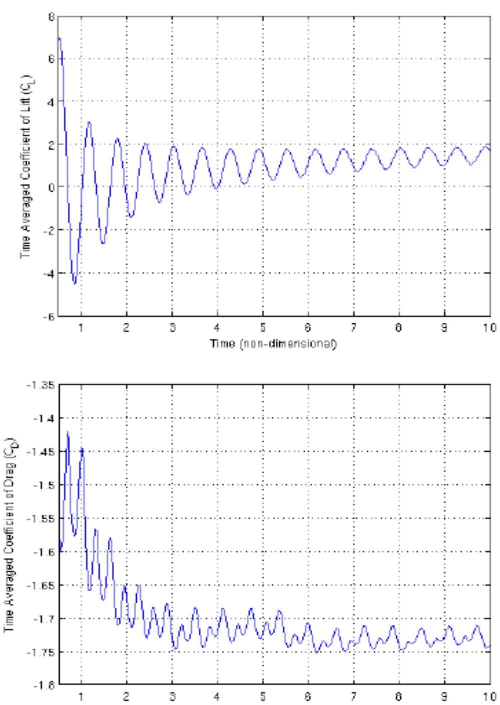

Fig 3: Time averaged lift and drag of plunging NACA-0012 airfoil using VMS-LES

6.2 VMS-LES simulation of the dynamic stall of a NACA-0012 extruded airfoil

Many low speed aerodynamics applications involve highly unsteady hovering flight regimes that call for the delay of aerodynamic stall. Dynamic stall is an unsteady phenomenon where pitching a wing beyond its static stall angle results in maximum values of lift, drag and moment coefficients that far exceed those achieved in the static case. This phenomenon is characterized by a massive flow separation and a reattachment process that results in a highly nonlinear hysteresis.

In order to test the performance of the different LES models at predicting large scale unsteady effects and to gain insight into the phenomenon of dynamic stall, the flow over a pitching NACA-0012 wing in deep stall regime is pursued here. The obtained numerical results are compared to the experimental data reported in McCroskey, 1976. The considered unswept wing section has a span of 1m and a chord length of 1.22m. It is sinusoidally pitched about the quarter chord so that the angle of attack is varied as

$$\alpha = 15^{\circ} + 10^{\circ} sin(\omega t)$$

where the reduced frequency is

$$k = \omega \times c_r / U_{\infty} = 0.15$$

The free-stream Mach number is set to 0.09 and the Reynolds number based on the chord length is set to Re=2,600,000.

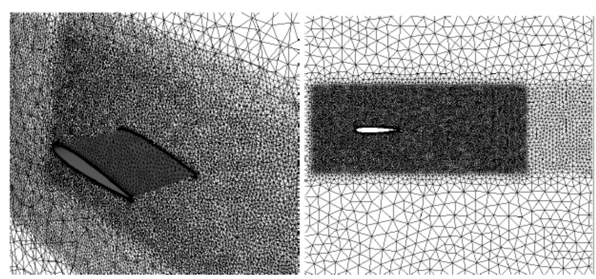

Fig 4: Computational mesh for the pitching NACA-0012 wing section

The computational domain is delimited by a circular cylinder of diameter 20 times the chord length. Symmetric boundary conditions are imposed at the two wing tips. The mesh has 8,557,236 tetrahedra and 1,504,611 nodes. Preferential mesh refinement is introduced in the regions of the separated wake as shown in Fig.4. For this flow, the estimated Taylor microscale is 2.98e-3m, the estimated Kolmogorov microscale is 1.94e-4m and the estimated Kolmogorov time scale is 2.6e-5s. The characteristic length of the smallest grid spacing near the wing and the largest grid spacing in the far field are 2e-4 m and 9.2e-2m, respectively. The time-step for the simulation was set to 2e-4s.

The number of layers is set to one in the first and second-level agglomerations and the depth of the second-level agglomeration is set to one for the dynamic VMS-LES. The lift and pitching moment hysteresis loops for the deep stall considered here is then computed using the four different LES models, namely the Smag LES, dynamic LES, VMS-LES and dynamic VMS-LES. These loops are plotted in Fig.5, along with the experimental data reported in McCroskey, 1976. It is observed that while all LES models deliver good lift hysteresis predictions, the dynamic VMS-LES model delivers the best predictions for the pitching moment coefficient. Furthermore, it is noted that the dynamic form of VMS-LES significantly improves the performance of its static counterpart in the prediction of hysterisis.

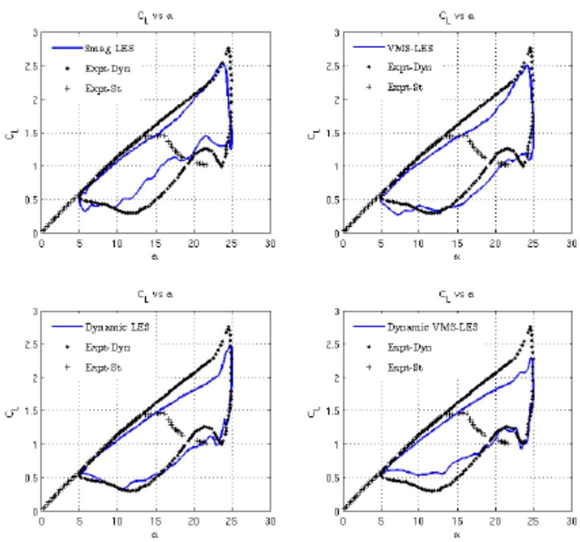

Fig 5: Refinement around the NACA-0012 airfoil

7 CONCLUSIONS

The static and dynamic VMS-LES method developed in has been applied to simulate the Knoller-Betz effect of a heaving airfoil and to simulate the dynamic stall of a pitching airfoil. The obtained numerical results suggest the superior performance of VMS-LES as a viable LES model for applications that involve turbulent flow computations over moving/deforming objects. The obtained numerical results for the dynamic stall of the pitching airfoil suggest that this dynamic VMS-LES method tends to deliver more accurate results than the static VMS-LES and the classical LES and dynamic LES methods. In particular, it is found that the dynamic VMS-LES method significantly improves the performance of its static counterpart for the prediction of lift/drag/moment characteristics.

Predicting accurate space-time correlation is an important feature that LES turbulence models need to satisfy. The dynamic version of the VMS-LES method developed here is an attempt to bring in the effect of time correlations into LES simulation, and hence it would be important to study if this model gives good space-time correlation. It would also be of interest to study turbulent flow simulations using the VMS-LES method to characterize the flow over flexible structures, there by working in the full realm of fluid-structure interaction.

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