ARMY RESEARCH LABORATORY

Ballistics Filtering

by Andrew A. Thompson

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There are many models of ballistics trajectories. The high-resolution 6-degree-of-freedom (6-DOF) models require many computations and a small time increment. The modified point mass models ignore the spinning of the round to reduce the computational requirements. Selecting a model to use for ballistics estimation or tracking requires tradeoffs between system accuracy and computation expense. The purpose of this report is to present reduced state models (simpler than the 6-DOF model) that can be used to model the trajectories using an extended Kalman filter. These filters can be used to enhance the performance of smart munitions.

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1. Introduction

There are many models of ballistics trajectories. The high-resolution 6-degree-of-freedom (6-DOF) models require many computations and a small time increment. The modified point mass models ignore the spinning of the round to reduce the computational requirements. Selecting a model to use for ballistics estimation or tracking requires tradeoffs between system accuracy and computation expense. For example, using a 6-DOF model for a tactical (real-time) system is not currently possible; however, for experimental work, it is possible to interpret the data using a 6-DOF model. The most basic type of polynomial filter is a straight-line predictor. These are accurate over short distances, but they will ignore any curvilinear behavior (typically needed for intercept prediction), and their performance can suffer when the prediction time is increased. In a similar fashion, higher-order polynomial predictors will diverge as prediction time increases. Reducing the fidelity of the model saves time and expense associated with computational requirements, but does so by sacrificing accuracy. The purpose of this report is to present ballistics models (simpler than the 6-DOF model) that can be used to model the dynamics in an extended Kalman filter (EKF). The EKF linearizes the dynamics at the system operating point and then proceeds as a Kalman filter (KF).

Intercept systems require the tracking of the ballistics threat and the interceptor. When there is a limited amount of control authority available, estimation must be very accurate. As control authority increases and terminal guidance sensors improve, it is possible to use models of lower fidelity to estimate the trajectory of a round (target or interceptor). Other situations also make reduced-order modeling possible. For the nonlinear effects that are small, it may be possible to remove them from the dynamic model. This includes estimation with a high sampling rate where the time between observations is small and trajectories where the drag coefficient does not change too much.

A trajectory can be adequately modeled by a differential equation. Using the initial conditions, an expected projectile path can be generated. Range, velocity, position, and direction measurements can be used to improve the estimate of the projectile's path. The differential equation and measurement process are combined to form a KF. The differential equation models the physics of the trajectory, while the measurement is used to update the parameters of the equation through least-squares estimation.

2. One-Dimensional (1-D) Case

First it is helpful to consider the 1-D case. The 1-D case is useful because it forms the basis of understanding three-dimensional (3-D) trajectories; for some direct-fire situations, it provides an excellent model for projectile analysis. In situations where time until impact is of interest for high-velocity rounds, this model can provide timing information.

2.1 Basic Equations

If a force, F, is acting on a body and the resistance to the force is proportional to the velocity squared, we have the following straightforward differential equation from Newton's law:

$$m\frac{dv}{dt} = F - bv^2 . \tag{1}$$

If $k^2 = \dfrac{F}{b}$, then the equation can be written as follows:

$$\frac{dv}{dt} = -\frac{b}{m}(v^2 - k^2) . \tag{2}$$

Solving this equation involves a separation of variables, as shown in the following:

$$\frac{1}{v^2 - k^2} dv = -\frac{b}{m} dt . \tag{3}$$

Using a partial fractions expansion,

$$\frac{1}{(v^2 - k^2)} = \frac{p}{v-k} + \frac{q}{v+k} . \tag{4}$$

Watching your p's and q's leads to the following differential equation:

$$\frac{1}{2k}(\frac{1}{v-k} - \frac{1}{v+k})dv = -\frac{b}{m} dt . \tag{5}$$

Integrating this results in the following:

$$\frac{1}{2k}(\ln(v-k) - \ln(v+k)) = -\frac{b}{m}t + c . \tag{6}$$

Taking the exponential of each side and using properties of exponents results in the following:

$$\frac{v-k}{v+k} = e^{2kc} e^{-\frac{2kbt}{m}} . \tag{7}$$

From this, the velocity can be found as a function of time, as shown in the following:

$$v(t) = k \frac{1 + ce^{-at}}{1 - ce^{-at}}. \tag{8}$$

If $a = \dfrac{2kb}{m}$ and $c = \dfrac{v_0 - k}{v_0 + k}$, where v_0 is the initial velocity, the velocity will asymptotically approach k. This equation can be used to find the terminal velocity of a dropped object by letting F be the force due to gravity. A large value of F could be used to simulate the launch of the projectile. After launch, the value of F would be 0 unless a rocket or missile is being modeled.

In the case of flat trajectory over a short time, the only force acting on the object is drag. Solving this same equation for $F = 0$ yields the following result:

$$v(t) = \frac{1}{\dfrac{1}{v_0} + \dfrac{b}{m} t}. \tag{9}$$

This expression asymptotically approaches 0. Assuming $v(t)$ has been measured, it is possible to use the previous equation to find the initial velocity using the following:

$$v_0 = \frac{1}{\dfrac{1}{v(t)} - \dfrac{b}{m} t}. \tag{10}$$

Given knowledge of two velocities, $v(t_1)$ and $v(t_2)$, it is also possible to calculate the value of drag (b). In the following, note that b includes the effects of air pressure (everything but speed) and t is the difference between the two times:

$$b = \left(\frac{1}{v(t_2)} - \frac{1}{v(t_1)} \right) \frac{m}{t}. \tag{11}$$

Position can be found by adding distance traveled to the original position. The distance traveled is found by integration of velocity and is described by the following expression:

$$\frac{m}{b} \ln \left(\frac{1}{v_0} + \frac{b}{m} t \right) - \frac{m}{b} \ln \left(\frac{1}{v_0} \right). \tag{12}$$

The distance traveled is the natural logarithm of a linear function. Using this expression, an upper bound for the lateral distance traveled can be established. It is also possible to approximate the initial velocity needed to attain a specified distance. This equation provides another constraint the analyst can use in fitting data. The major problems with this formulation are the 1-D restriction and the assumption of constant drag. Typically, drag is a function of Mach number. Modeling of drag has resulted in universal drag curves. These curves give the overall shape of drag as a function of Mach number. For individual rounds, these curves are multiplied by a number, typically called a form factor, to adjust the universal curve to adequately fit the particular round. Drag changes dramatically in the region of Mach 1; thus, nonlinear

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behavior can be a concern in this region. The form factor also allows the use of drag curves of the same projectiles to be moved up or down for the round of interest. This adjustment accounts for slight perturbations of the projectile's shape and mass. The drag curves for similar shells are assumed to be similar; the form factor allows this to be incorporated into the model of motion. While the incorporation of a more complex drag model would increase the fidelity of the model, it would preclude a closed-form solution. Incorporation of more complicated drag models will require the equations to be solved numerically. The closed-form model allows a computation of time until impact and other useful quantities associated with time or distance along the trajectory. The selection and design of a 1-D model will depend on the application.

2.2 Measurements

An EKF can be designed for the 1-D case. For information on Kalman filtering, see Gelb (*1*) or Maybeck (*2*). The equations for an EKF using the notation given by Gelb follow.

System nonlinear dynamics plus plant noise q~N(0,Q):

$$\dot{\hat{x}} = f(\hat{x}(t),t) + q(t). \tag{13}$$

The observation equation v~N(0,R):

$$z_k = h_k(x(t_k)) + v_k. \tag{14}$$

Initial conditions, normal distribution:

$$x(0) \sim N(x(0), P(0)). \tag{15}$$

The covariance propagation:

$$\dot{P}_k = F(\hat{x}(t),t)P_{k-1} + P_{k-1}F(\hat{x}(t),t) + Q(t). \tag{16}$$

The gain due to an observation:

$$K_k = P_k H_k^{'}(\hat{x}_k)\left[H_k(\hat{x}_k)P_k H_k^{'}(\hat{x}_k) + R_k\right]^{-1}. \tag{17}$$

Change in the state due to observation:

$$\hat{x}_k = \hat{x}_k + K_k(z_k - h_k(\hat{x}_k)) \tag{18}$$

Updated state covariance via observation:

$$P_k = \left[I - K_k H_k(\hat{x}_k)\right]P_k. \tag{19}$$

Linearized time step:

$$F(\hat{x}(t),t) = \frac{\partial f(\hat{x}(t),t)}{\partial \hat{x}(t)}\Big|\hat{x}(t) = \hat{x}_k. \tag{20}$$

Relinearized observation:

$$H(\hat{x}_k) = \frac{\partial h_k(x(t_k))}{\partial x(t_k)} \Big| x(t) = \hat{x}_k . \tag{21}$$

The state is a set of parameters that allows the differential equation to be solved. First, define the state vector, X, as the position (X_1), the speed (X_2), and the drag coefficient (X_3). Given this set of parameters, it is possible to propagate the trajectory using the following:

$$X_1(k) = X_1(k-1) + X_2(k-1)dt + \frac{1}{2} X_2^2 (k-1) X_3 dt^2 \quad , \tag{22}$$

$$X_2(k) = X_2(k-1) + X_2^2 (k-1) X_3 dt , \tag{23}$$

and

$$X_3(k) = X_3(k-1), \tag{24}$$

where k indicates the time step. Assuming dt is small, dt^2 will be close to 0. These equations can be written as follows:

$$X(k) = X(k-1) + \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & X_2^2(k-1) \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} dt. \tag{25}$$

The 3×3 matrix is referred to as F and captures the change as a function of time. Assume $P(k–1)$ is the state covariance and using E to represent expectation, the new state covariance is as follows:

$$E(X(k)X'(k)) = (I + F)E(X(k-1)X'(k-1))(I + F)' \tag{26}$$
$$\approx P(k-1) + FP(k-1) + P(k-1)F,$$

since the expectation of $X(k-1)X'(k-1)$ is the state covariance $P(k–1)$. This formula allows a closed-form means of predicting the variation of a future state from the current conditions. By using equations 22–26, is possible to predict the striking velocity and also have statistical bounds on the variation of the striking velocity. Covariance propagation is an alternative to Monte Carlo simulation of the system. The change of the system and the covariance of the system in time are referred to as state propagation and state covariance propagation. When an observation of a state variable or combination of state variables occurs, the state can be updated in a least-squares manner.

Observations will change the perception of the state. The current example will be developed to include the effect of location or distance measurement and velocity or, in this case, speed measurement, on both the state and the state covariance. The measurement error of the location and speed are assumed to be known to a reasonable degree of accuracy.

Typically, the time between observations is fixed and dt is constant; however, this need not be the case. For asynchronous measurements, the value of dt can be adjusted to meet the situation. If the interval between measurements becomes large, it is prudent to update the covariance matrix between observations. When a single measurement is made, the state is updated via recursive least squares. The measurement is expressed in terms of the state variables and will be signified with the symbol, $h(X(t))$; the observation matrix will be the partial of this with respect to the state and represented by H. In terms of the state, a position observation is $(1\ 0\ 0)\ X$; thus, the H matrix is $(1\ 0\ 0)$. For a velocity measurement, the observation corresponds to $(0\ 1\ 0)\ X$. In both these cases, the measurement is a linear function of the state; when the measurement is a nonlinear function, the partial of measurement is used to define H at each time step. First, the gain matrix, k, will be defined based on elements of the state covariance matrix, P, and the measurement error, as shown in the following:

$$k = \begin{pmatrix} p_{i1} \\ p_{i2} \\ p_{i3} \end{pmatrix} \frac{1}{p_{ii}{}^{+} + \sigma^2{}_m} . \tag{27}$$

In this situation, $i = 1$ for a position measurement, and $i = 2$ for a velocity measurement, σ represents the standard deviation of the measurement, and the subscript will indicate a position or velocity measurement. The distance variance will differ from the velocity variance and may change from measurement to measurement. The state update due to the observation is as follows:

$$x(+) = x(-) + k(z - x_i). \tag{28}$$

In this equation, z is the measurement, and the $-$ sign indicates before the observation update. Notice that the terms of the k matrix are directly proportional to the corresponding terms of the covariance matrix. The observational update always reduces the state covariance, as shown in the following:

$$P(+) = P(-) - D \frac{1}{P_{ii} + \sigma_i^2} , \tag{29}$$

and

$$D_{jk} = P_{ij} P_{ik} . \tag{30}$$

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Together, propagation and observation updates form a KF. Since a KF is recursive, it needs to be initialized. Typically, the experimenter initializes the filter; it is important to have a good initial estimate of the state and the state covariance.

The drag coefficient is more realistically represented as a function of Mach number. As the projectile changes speed, its drag coefficient changes. It is usually infeasible to generate drag curves for every projectile; a characteristic curve is found for a particular projectile. Given a similar projectile, the curve is moved up or down by incorporating a form factor into the equation. In equation 25, $\mathcal{X}_3$ would become the form factor. The drag coefficient would be included in the F matrix as a factor of the square of the speed. Note that state propagation is a numeric solution of the differential equation. In the case of an extended filter, the dynamics are linearized at each time step. The 1-D model can be used when the projectile's motion stays on a line or possibly when timing is the important information. This model can also be used to find the distance along a known trajectory. Note also that the 1-D model can be enhanced by including the effects of gravity to form a simple two-dimensional (2-D) model.

3. Two-Dimensional Case

The 2-D case is more complex because the velocity term is an interaction of both the height and range terms (cross range is ignored). In a sense, this interaction steals velocity from the range dimension as gravity causes the vertical velocity to asymptotically approach its terminal velocity. In time, the direction of motion will align itself with gravity. Only drag and gravity are considered; the equations are as follows:

$$\dot{x} = -\cos(\theta)\frac{b}{m}v^2 , \tag{31}$$

$$\dot{y} = g - \frac{b}{m}v^2\sin(\theta) , \tag{32}$$

$$v^2 = \dot{x}^2 + \dot{y}^2 , \tag{33}$$

and

$$\theta = a\tan(\dot{y}/\dot{x}) . \tag{34}$$

To solve this system, the initial conditions must be stated. The previous model (equations 31–34) has been used to model submunitions being released from a cargo round. Assume the submunitions are expelled in the range dimension so that $\dot{y} = \theta = 0$ and $\dot{x} = v_0$. Choosing the expulsion velocity allows these equations to be solved numerically. The previous equation was realized in SIMULINK and solved numerically therein. For horizontally launched submunitions, the lateral speed approaches 0 as the terminal velocity is attained. The value b is based on the

drag coefficient; a discussion of this can be found in U.S. Army Special Text 9-153 (*3*) or Sabersky et al. (*4*).

The value of drag at velocities less than Mach 1 is fairly constant. If the velocity crosses Mach 1, a more complicated model for drag should be devised. A 2-D model for prediction can be useful for endgame guidance applications where the time to hit is low and the relative position information from the sensor is good. The previous model (equations 31–34) has been used to model the dispersion of submunitions (*5*). Equations 31–34 can be used, but it would be more accurate to reduce the order of the equations discussed in the next section.

4. Three-Dimensional Case

Flight dynamics are most accurately represented by 6-DOF models. These models are nonlinear and must be solved numerically. There are no closed-form solutions, and the numerical solutions require many computations at each iteration; it is difficult to develop a real-time filter based on a 6-DOF model. Many estimators for flight dynamics use simpler models focused on the parameters of interest. For example, if position is of interest, then it is possible to ignore some of the computations associated with attitude and reduce the complexity of the model.

Point mass models and modified point mass models offer a simplification of the 6-DOF models that provide excellent position accuracy. In three dimensions, a drift term must be added to the model; this term captures a projectile's motion orthogonal to range and altitude. Drift is caused by the interaction of spin and yawing motion. Spin can also be modeled, and its decrease is in proportion to the current spin rate and the speed of the round. Obtaining precise knowledge of aerodynamic coefficients can be difficult, and even with this knowledge, there can be round-to-round variation. It is prudent to include a form factor to capture the variation of aerodynamic coefficients.

For a linear model, the concept of state is used to find a Markov representation of the system. The state is a vector that incorporates the information needed to propagate forward to the next time of interest. Nonlinear dynamics are often modeled by linearization of the model followed by choosing a time step that does not result in nonlinearities of a problematic magnitude. A 14-dimensional state model will be discussed (this follows the presentation from excerpts from a portion of an unidentified report). The state is shown in appendix A. It is possible to reduce this state model to eight (position, velocity, drag, and lift) or seven (position, velocity, and drag) dimensions. Also, universal curves will be used to model aerodynamics associated with drag, lift, and spin. It is assumed that a user with more knowledge of a particular round can replace the modeled dynamics with higher fidelity models, but these will represent the default models. The 14 dimensions include three for position, three for velocity, two for wind speed, and one each for muzzle velocity, azimuth bearing, elevation bearing, drag constant, lift (or drift)

constant, and speed of sound at sea level. A seven-dimensional (7-D) model can be formed by using position, velocity, and drag. An eight-dimensional model would add lift or perhaps muzzle velocity to the previous state. The number of elements in the state can be chosen based on the purpose of the model.

The dynamics can be thought of in terms of the acceleration of the round. Additive components are grouped as drag, drift, gravity, Magnus effect, and Coriolis terms. A description of modified point mass models can be found in STANAG 4355 (6). As a frame of reference, the North-Up-East system attached to the projectile launch point on the surface of the earth will be used.

The drag term is a function of air pressure, velocity, round diameter, and aerodynamic coefficients. For the calculation of drag, velocity is squared; thus, velocity is the most important term in the drag equation. The drag coefficient is calculated from a fourth-degree polynomial of Mach number, with coefficients based on universal drag values. There are other methods that can be used to model the drag coefficient. The dynamics associated with drag expressed in terms of the state variables are as follows:

$$
\begin{bmatrix} \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = -x_{12} A C_d V \begin{bmatrix} x_4 - x_7 \\ x_5 \\ x_6 - x_8 \end{bmatrix}.
\tag{35}
$$

In this case, C_d is found using the universal drag curve. V represents the speed of the projectile relative to the ground. The symbol x_{12} is the form factor or the factor that allows the universal drag value to be moved up or down. The variable A represents the air pressure. Air pressure changes with height, so $A = f(x_2)$. The standard atmosphere model is typically used to find air pressure, although meteorological data can be used if available. Additional factors relating to the physical characteristics of the round can also be included.

The lift term in modified point mass models is most accurately described using the yaw of repose. The yaw of repose represents the projectile's average yaw. For this example, the yaw of repose will not be modeled, and the lift term will be orthogonal to the projectile velocity and the gravity vector. (See STANAG 4355 for a method to estimate yaw of repose.) The term associated with the lift will be as follows:

$$
\begin{bmatrix} \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = x_{13} C_l / V \begin{bmatrix} x_6 - x_8 \\ 0 \\ x_7 - x_4 \end{bmatrix}.
\tag{36}
$$

In this formula, C_l is the lift term. In the present situation, it will be calculated from the universal lift curve. The form factor x_{13} is used to adjust the lift curve for the current application. V is the speed of the projectile. The universal lift curve was developed using rounds with high spin rates; other methods for rounds with a low spin rate perhaps can be ignored.

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The Magnus force will be ignored in the dynamics used. This force is important for predicting impact time. With regular observations of projectile's position, this term will not have a large impact between observations. The omission of the Magnus force term will increase the model uncertainty. This term was ignored in the modified point mass model discussed by Bradley (7).

Gravity needs to be included in the model dynamics. The force of gravity is directed to the center of the earth. The magnitude of gravity changes over the earth as a function of latitude. The following formula is an approximation:

$$g = g_0(1 - g_1 \cos(2L)), \tag{37}$$

$$g_0 = 9.80665, \tag{38}$$

and

$$g_1 = .0026, \tag{39}$$

where L is the latitude at the point of launch. In terms of the state variable, the gravity vector is as follows:

$$\vec{g} = g \begin{bmatrix} x_1/R \\ 1 - (x_1^2 + x_3^2)^{.5}/(2R) \\ x_3/R \end{bmatrix} \tag{40}$$

where R is the radius of the earth.

Coriolis force is a factor when using an earth-fixed coordinate system. The rotation of the earth is $7.2921 \times 10^{-5} = \Omega$. Let $\Omega_x = \Omega \cos(Latitude)$ and $\Omega_y = \Omega \sin(Latitude)$, then the Coriolis effect can be written as follows:

$$\begin{bmatrix} \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = -2 \begin{bmatrix} \Omega_y x_6 \\ -\Omega_x x_6 \\ \Omega_x x_5 - \Omega_y x_4 \end{bmatrix}. \tag{41}$$

The dynamics discussed are summarized in appendix B. These can be used as the basis of an EKF. If a 7-D state is being used, the dynamics associated with lift can be omitted.

In situations where guidance is required, a model of the spin is necessary. STANAG 4355 proposes the following differential equation:

$$\dot{p} = \frac{\pi \rho d^4 v c_s}{8 I_x}. \tag{42}$$

where p is spin, ρ represents the air pressure, d is the diameter of the round, c_s is the spin drag coefficient, and I_x is the moment about the spin axis. C_s is a function of Mach number. This equation could be used as the dynamics of a separate EKF to estimate spin, assuming there are some measurements of spin available. In conjunction with the previous position model, the combination of the two models for position and spin could estimate much of the information desired about a projectile's flight. The combination of these two models would not capture yawing motion. This may not be an issue if the yaw is small, thus allowing the system to be modeled using a simplified or reduced state estimation.

The dynamics can be used to develop a state estimator. The theory is discussed in Gelb (*1*). The development of a KF varies based on the dynamics being modeled; thus, it is possible to have many different KF estimating the same quantities. Differences in KF are due to the state propagation equations used. For nonlinear problems, an EKF is a good first choice. An EKF is based on the same theory as a KF but uses a linearized version of the nonlinear dynamics at each time step. If the time step is made too large, the EKF may not be appropriate, and the model may need to include more terms related to the nonlinear dynamics. Another alternative is using particle filters. Particle filters do not require the propagation of the state uncertainty; this benefit is offset by the need to propagate a number of candidate states forward in time and then calculate uncertainty based on the distribution of candidate states. In addition to nonlinear dynamics associated with the state transition, there can be nonlinearities associated with the observations. An observation needs to be expressed in terms of the state variables. If position is part of the state and the distance from an object to the projectile is observed, then the observation is a nonlinear function of the state. Nonlinearities in the state dynamics and nonlinearities in a measurement expressed in terms of the state variables are mitigated through using an EKF by linearization around the current value of the state.

To develop an EKF, proceed with the following steps. First, decide on the dynamics to capture by the EKF. Next, the dynamics need to be put in the form $\dot{x} = Fx + Gu$, where x is the state of the system and F represents the change in x over a time step. The variable u represents inputs to the system, and the matrix G describes how these inputs affect the system. Another use for the Gu term is to introduce uncertainty associated with unmodeled dynamics. After these tasks have been completed, the process must be initialized. Then, as observations become available, these are incorporated into the filter through least-squares estimation. As time progresses, the state and its covariance will propagate forward in time. The state propagation equations have been discussed. Only position and velocity change. The equations for an EKF using the notation given by Gelb can be found in section 2.2.

Using the previously discussed dynamics and the EKF equations, an estimator for the trajectory of a projectile can be designed. Note that the matrix to be inverted has the size of the observation covariance; typically, this is smaller than the state covariance. The one issue not discussed is the observation equation. A GPS sensor gives position so the observation matrix

consists of a $3 \times N$ matrix of zeros, with ones in the positions corresponding to location, so that $H\hat{x}_k$ gives the position in terms of the state at the kth observation.

Many observations are nonlinear; radar typically gives the range to the target. Assuming the radar is located at (0,0,0) and the first three values of the state correspond to position, the radar observation in terms of the state is $r = (x_x^2 + x_2^2 + x_3^2)^{.5}$. If range was the only available observation, the linearized observation matrix would contain one row, the first three columns would be $H(\hat{x}) = \begin{pmatrix} \dfrac{\hat{x}_1}{r} & \dfrac{\hat{x}_2}{r} & \dfrac{\hat{x}_3}{r} \end{pmatrix}$, and the other entries would be zeros. If azimuth and elevation angles are given, they can be expressed in terms of the position, and then the proper partial derivatives can be found and included as extra rows of H.

Typically, most of the effort associated with an EKF goes into finding the matrix, $F(\hat{x}(t),t)$, the partial of the system dynamics with respect to the state. This matrix can be complex and is typically recomputed each time step or observation. If the F matrix does not change too quickly, it is possible to process several time steps before recomputing the matrix. Also, if some of the partials are numerically small, it is reasonable to drop these terms from the matrix. It is possible to have many different formulations of an EKF for the same problem. The interplay between desired accuracy and computational speed determines the final form of the EKF.

5. Example for the 3-D Case

In this section, an EKF with a 7-D state using GPS measurements will be discussed. The state variables will consist of position, velocity, and ballistics coefficient. The dynamics for lift and Magnus effect will be ignored. The state variables used from appendix A will be 1–6 (position and velocity) and 12 (ballistics coefficient). Also, wind effects will be assumed to be zero to simplify the simulation. A GPS measurement gives position; and since position is part of the state, this results in an H matrix with a 3-D identity matrix followed by zeros in positions corresponding to the fourth through seventh state variables. The f equation represents the dynamics as follows:

$$f_1 = x_4, \tag{43}$$

$$f_2 = x_5, \tag{44}$$

$$f_3 = x_6, \tag{45}$$

$$f_4 = -x_7 A k_d V x_4 - \frac{g x_1}{R_e} - 2\Omega_y x_6, \tag{46}$$

$$f_5 = -x_7 A k_d V x_5 - g(1 - \frac{(x_1^2 + x_3^2)^{.5}}{2R_e}) + 2\Omega_x x_6 , \qquad (47)$$

$$f_6 = -x_7 A k_d V x_6 - \frac{g x_3}{R_e} - 2\Omega_x x_5 + 2\Omega_y x_4 , \qquad (48)$$

and

$$f_7 = 0 . \qquad (49)$$

Using equations 43–49, the *F* matrix is found by taking the partials with respect to each state variable (see equation 20). Recall that both k_d and *V* are functions of x_4, x_5, and x_6. In equations 43–49, *V* is the speed of the projectile and the drag coefficient, k_d, is a function of Mach number, which is a function of speed. Also note that air pressure, *A*, is a function of altitude, x_2. Assume the *F* matrix starts out as a 7×7 matrix filled with zeros. The following identifies the nonzero elements:

$$F_{14} = 1 , \qquad (50)$$

$$F_{25} = 1 , \qquad (51)$$

$$F_{36} = 1 , \qquad (52)$$

$$F_{41} = \frac{-g}{R_e} , \qquad (53)$$

$$F_{42} = -x_7 k_d V x_4 \frac{\partial A}{\partial x_2} , \qquad (54)$$

$$F_{44} = -x_7 A\left(V x_4 \frac{\partial k_d}{\partial x_4} + k_d x_4 \frac{\partial V}{\partial x_4} + k_d V \right) , \qquad (55)$$

$$F_{45} = -x_7 A x_4\left(V \frac{\partial k_d}{\partial x_5} + k_d \frac{\partial V}{\partial x_5} \right) , \qquad (56)$$

$$F_{46} = -x_7 A x_4\left(V \frac{\partial k_d}{\partial x_6} + k_d \frac{\partial V}{\partial x_6} \right) - 2\Omega_y , \qquad (57)$$

$$F_{47} = -A k_d V x_4 , \qquad (58)$$

$$F_{51} = \frac{g}{2R_e} \frac{x_1}{(x_x^2 + x_3^2)^{.5}} , \qquad (59)$$

$$F_{52} = -x_7 k_d V x_5 \frac{\partial A}{\partial x_2} , \qquad (60)$$

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$$F_{53} = \frac{g}{2R_e} \frac{x_3}{(x_x^2 + x_3^2)^{.5}} , \tag{61}$$

$$F_{54} = -x_7 A x_5 \left(V \frac{\partial k_d}{\partial x_4} + k_d \frac{\partial V}{\partial x_4} \right) , \tag{62}$$

$$F_{55} = -x_7 A \left(V x_5 \frac{\partial k_d}{\partial x_5} + k_d x_5 \frac{\partial V}{\partial x_5} + k_d V \right) , \tag{63}$$

$$F_{56} = -x_7 A x_5 \left(V \frac{\partial k_d}{\partial x_6} + k_d \frac{\partial V}{\partial x_6} \right) + 2\Omega_x , \tag{64}$$

$$F_{57} = -A k_d V x_5 , \tag{65}$$

$$F_{62} = -x_7 k_d V x_6 \frac{\partial A}{\partial x_2} , \tag{66}$$

$$F_{63} = \frac{-g}{R_e} , \tag{67}$$

$$F_{64} = -x_7 A x_6 \left(V \frac{\partial k_d}{\partial x_4} + k_d \frac{\partial V}{\partial x_4} \right) + 2\Omega_y , \tag{68}$$

$$F_{65} = -x_7 A x_6 \left(V \frac{\partial k_d}{\partial x_5} + k_d \frac{\partial V}{\partial x_5} \right) - 2\Omega_x , \tag{69}$$

$$F_{66} = -x_7 A \left(V x_6 \frac{\partial k_d}{\partial x_6} + k_d x_6 \frac{\partial V}{\partial x_6} + k_d V \right) , \tag{70}$$

and

$$F_{67} = -A k_d V x_6 . \tag{71}$$

The following information is also needed to obtain numerical values for the F matrix:

$$V = (x_4^2 + x_5^2 + x_6^2)^{.5} , \tag{72}$$

$$\frac{\partial V}{\partial x_i} = \frac{x_i}{(x_4^2 + x_5^2 + x_6^2)^{.5}} \qquad i = \{4,5,6\} , \tag{73}$$

$$a_0 = 1.223 , \tag{74}$$

$$a_1 = 1.071e - 4 , \tag{75}$$

$$A = a_0 e^{-a_1 x_2} , \tag{76}$$

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$$\frac{\partial A}{\partial x_2} = -a_0 a_1 e^{-a_1 x_2} , \tag{77}$$

$$g = 9.80665(1 - .0026\cos(2Lat)) , \tag{78}$$

and

$$R_e = 6356766. \tag{79}$$

The drag coefficient is calculated using a fourth-degree polynomial of Mach number. Mach number is the speed divided by the speed of sound. Notice that in this formulation, the partial of the speed of sound with respect to height is not included in the F matrix.

$$s_0 = 340.3 \quad temperature = 59° F , \tag{80}$$

$$x_{sl} = height(launchabovesealevel) , \tag{81}$$

$$c_v = 2.26e - 5 , \tag{82}$$

$$s = s_0 (1 - c_v (x_{sl} + x_2))^{.5} , \tag{83}$$

$$k_d = \sum_{i=0}^{4} c_i m^i , \tag{84}$$

$$\frac{\partial k_d}{\partial x_i} = \left(\sum_{i=1}^{4} i c_i m^{i-1} \right) \frac{x_i}{sV} , \tag{85}$$

$$\Omega = 7.2921e - 5 , \tag{86}$$

$$\Omega_x = \Omega \cos(lat), \tag{87}$$

and

$$\Omega_y = \Omega \sin(lat). \tag{88}$$

In this example, the measurements to be used are assumed to be estimates of position from a GPS receiver, so the observation matrix H is expressed as follows:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{89}$$

When multiplied by the state vector, this matrix will select the three elements associated with position as the three observations. For these observations, there are no issues with nonlinearities. The next step is to use the preceding equations to track a projectile. The number of observations per second can be varied as a design parameter. A 6-DOF model will be used to define a trajectory.

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The initialization of the estimator will be discussed. The initial value of the state can be set using the launch position, launch speed, azimuth, and elevation of the launch. The R matrix is the covariance of the observation. For the present case, this can be found from the specifications of a GPS receiver. In other situations, this could involve building a model of the sensors and then performing a sensitivity analysis to get an accurate estimate of the R matrix. In many cases, the R matrix will change due to changes in the target sensor geometry. One R matrix will be used for this EKF. The Q matrix will be used to model the shortcomings of the dynamic model used in the EKF. Typically, a guess is made for the Q matrix. This guess is then adjusted through the information gained by repeated adjustments. The process of tuning an EKF or KF is finding a reasonable Q matrix.

The EKF can be used to approximate system performance measures based on state covariance. The state covariance matrix P is available at each time step and can be used to derive measures of effectiveness. This type of analysis can be used to determine the observation rate and sensor quality needed to meet performance specifications. This covariance analysis does not require the extensive use of replications required by Monte Carlo simulations and is closer to a closed-form solution. As an example, consider an observation rate of 10/s. Assume the observation covariance is a diagonal matrix, with 4, 9, 4 on the diagonal; also, let the Q matrix be diagonal, with ones in positions one through six and .0001 in position seven. For a mortar round shot north at 49° elevation, an aggregate measure of the state uncertainty is the trace. Figure 1 shows the trace of the state covariance matrix as a function of time in units of 0.01 s.
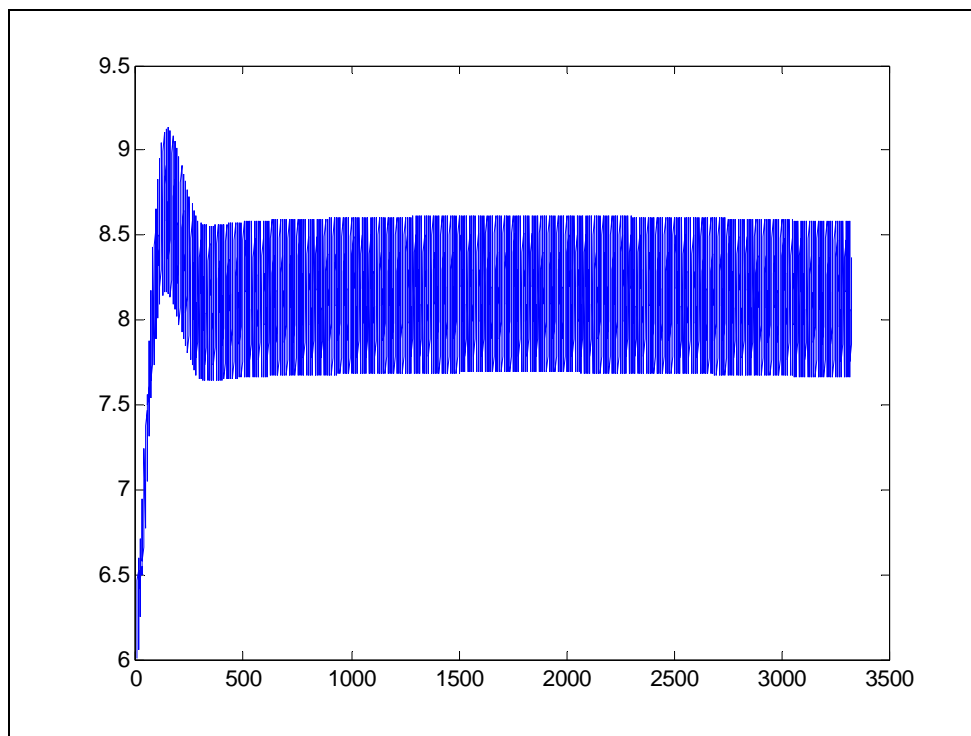

Figure 1. State covariance trace vs. time.

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From this graph, it is seen that at ~5 s the steady state behavior is achieved. The band is the result of covariance stochastically expanding due to both the Q matrix and the F matrix until an observation is made and then instantly diminishing as a result of the new information. This can be seen in the blowup of figure 1 presented as figure 2.
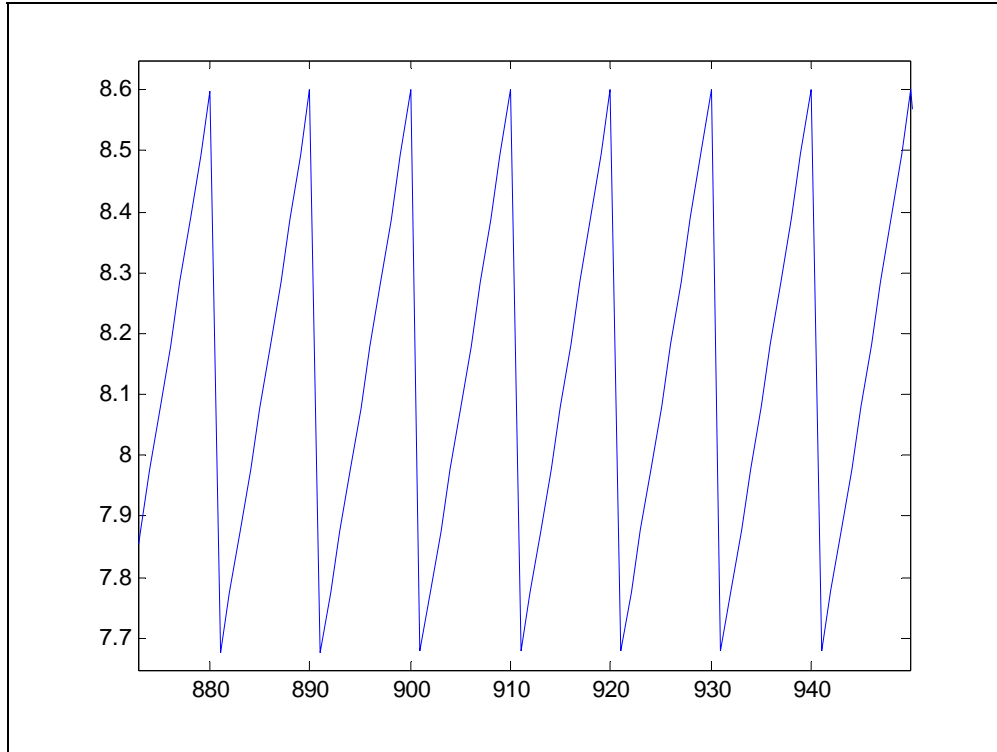

Figure 2. Expansion of figure 1.

To determine how well this filter works, data from a 6-DOF model was obtained, and the previously described 7-D EKF was used to track the data. In figure 3, the blue is the 6-DOF track, and the red is the output of the EKF. It is difficult to distinguish the two curves.

The reduced dynamic model, coupled with the observations, tracks the 6-DOF data with limited error.

6. Conclusions

It is realistic to simplify the model when the nonlinearities being ignored do not adversely affect the estimation or result in errors that are within tolerances. Also, when observations are available, these observations, due to their accuracy, may correct the estimator to the extent that including some of the dynamics is not worth the computational expense; that is, with a high data rate, it is often possible to use a simple model and achieve acceptable results. In linear systems
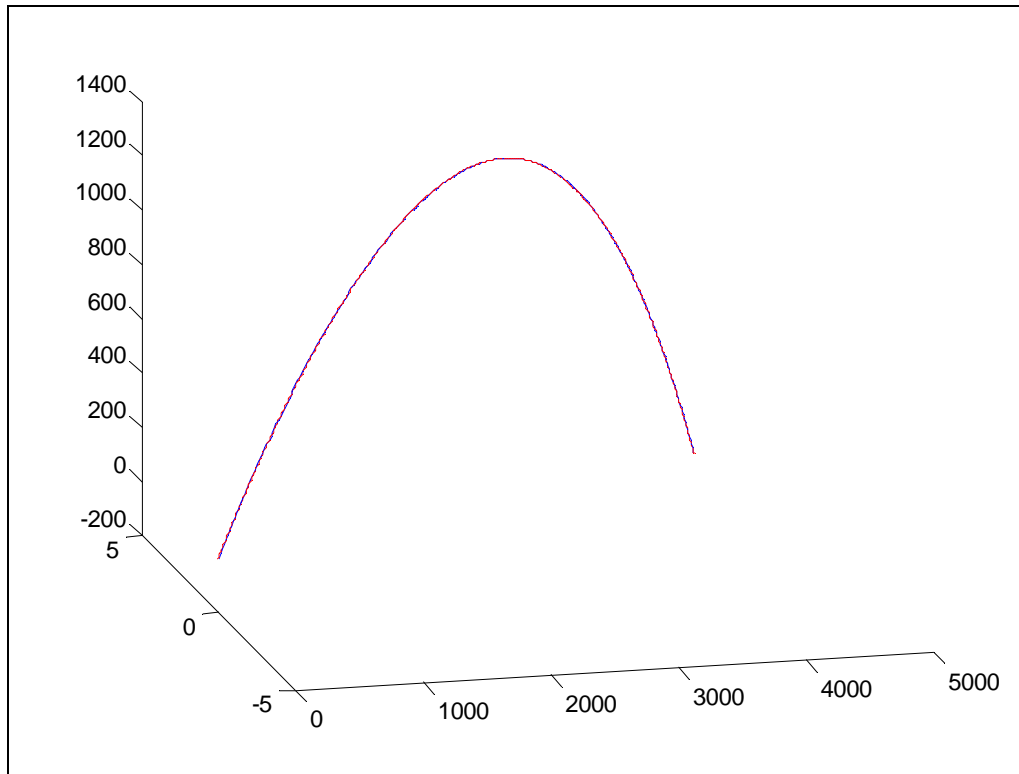
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Figure 3. True vs. estimated trajectory.

theory, the effects of unmodeled dynamics are referred to as model mismatch error. There are uncertainty terms that can account for the unmodeled dynamics; this leads to an increase in the covariance associated with the state variables. It is not possible to present all possible reduced state models; the intent was to show how it can be done and discuss some of the issues associated with designing an EKF.

It is possible to break the problem into a position, velocity model, and spin model. In many cases, using these two separate models will provide an acceptable alternative to using 6-DOF models for target tracking.

The accuracy of a ballistics estimator needs to be considered in system design. There is a tradeoff between accuracy and computation requirements. For a kinetic energy round, a 1-D model that incorporates gravity may suffice. The observations available to the system will influence the choice of dynamics. If there are many high-quality observations, a simple linear predictor may suffice. In this situation, the nonlinear effects may be small between observations. The matrix F representing the partials of the state dynamics with respect to the state variables is computationally the most difficult term. Terms in the F matrix with relatively small magnitudes are to be considered as candidates for omission.   For target interceptor systems, the overall system performance is determined by the accuracy of each estimator (target and interceptor) in conjunction with the control authority.

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Appendix A. The State for a Three-Dimensional System

Location

x_1 Position north

x_2 Position up

x_3 Position east

Velocity

x_4 Speed north

x_5 Speed up

x_6 Speed east

Wind Velocity

x_7 Speed north

x_8 Speed east

x_9 Muzzle speed

x_{10} Azimuth angle to target

x_{11} Elevation angle to target

x_{12} Drag constant

x_{13} Lift constant

x_{14} Speed of sound at sea level

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Appendix B. Change in the State Variables

$f_1 = x_4.$

$f_2 = x_5.$

$f_3 = x_6.$

$$f_4 = -x_{12} A k_d V (x_4 - x_7) - \frac{x_{13} k_l}{V} (x_6 - x_8) - \frac{g x_1}{R_e} - 2\Omega_y x_6 \ .$$

$$f_5 = -x_{12} A k_d V x_5 - \frac{g(1-(x_1^2 + x_3^2)^{.5})}{2R_e} + 2\Omega_x x_6 \ .$$

$$f_6 = -x_{12} A k_d V (x_6 - x_8) - \frac{x_{13} k_l}{V} (x_4 - x_7) - \frac{g x_3}{R_e} - 2\Omega_x x_5 + 2\Omega_y x_4 \ .$$

$f_7 \ldots f_{14} = 0 \ .$

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