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6. AUTHOR(S) D.J. Kaup				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) DJ Kaup Inc. 903 Oak Forest Dr. Winter Springs, FL 32708-4004			8. PERFORMING ORGANIZATION REPORT NUMBER 2009-02	
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13. ABSTRACT (Maximum 200 Word) The saturated stage of a magnetron or a cross-field amplifier is when the device has powered up and is steadily delivering power. Theoretical studies of this stage had suggested that when the dc current would become sufficiently large in comparison with the rf current, then no steady state solution could be expected to exist. Attempts were made to develop a numerical code which would allow one to numerically solve for a stationary solution of the saturated stage, if it existed. However the high order of the ordinary differential equations and their singular nature had prevented any solutions from being found. It has been suggested that a hybrid method involving single particle orbits could be used. Here one would assume that a stationary solution did exist and then one would solve for the particle orbits in that stationary solution. From those orbits, one then could obtain the corresponding fluid quantities by averaging and thereby bypassing any need to solve the high order, singular differential equations. Other work completed under this contract included studies of integrable systems, particularly those integrable optical systems which govern intense laser beams and stability, and also Bose-Einstein condensates.				
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Subject: Final Report

Contract/Grant Title: Theoretical Study of the Saturated Stage of a Relativistic Magnetron

Contract/Grant #: FA9550-06-C-0030

Reporting Period: June 1, 2006 to Nov. 30, 2008

Original Objectives:

A) We shall continue our studies of the nonrelativistic, electrostatic, planar magnetron #T266 in the saturation stage. We shall (i) obtain numerical results for the operating characteristics; and (ii) compare these with the known experimental characteristics.

B) We shall extend the above studies to the saturation stage of relativistic, electromagnetic, cylindrical crossed-field devices, particularly the A6 magnetron. We shall (i) obtain results for the operating characteristics; and (ii) seek to detail and understand how each of the relativistic, and electromagnetic, and cylindrical effects affect the resulting density profiles and other operating characteristics in this stage.

C) We shall continue other studies that have potential interest to the AFOSR, such as in the areas of nonlinear optics, instabilities in higher order systems, the development of a quantitative measurement of variational results, and precursor formation in the ionosphere.

Personnel involved:

Dr. Heinz Steudel (Humboldt University, Berlin, Germany; consultant)

Dr. V. Gerjиков (Sofia, Bulgaria; consultant)

Mr. Galen T. Kaup (Research Assistant, mathematician)

Prof. Subash Antani (Edgewood College, Madison, Wisc., consultant, nonlinear interactions in the ionosphere.)

Summary of Accomplishments:

- 1) Saturated stage of magnetrons and CFAs: .The main thrust of this contract was to theoretically study the various stages of high power magnetrons (HPM) and cross-field amplifiers (CFA) by means of the cold-fluid plasma equations. The work accomplished on this project has been published in Pub01 and Pub03 listed below, which are a conference proceedings followed by publication in a standard journal. Crossed-field devices have basically two main stages: first there is the initiation stage wherein the power is turned on and the energy output of the device ramps up from zero towards its maximum output. Second is the saturation stage wherein the device has achieved the maximum power output. The growth in the initiation stage is due to the excitation of linear instabilities. The instabilities must vanish when the saturation stage is achieved, which is a stationary state. When the above theoretical results were used in the design of a Fortran code to calculate this saturated stage, it was found that regardless of the approach used (WKB expansions, direct numerical integrations, etc.), unwanted

instabilities invariably occurred. First, a review of the theory in these publications revealed that the theory was defective, in that it attempted to solve the problem of the saturated stage in an unstable manner. The approach that was used was exactly the same approach which had worked successfully for solving the initiation stage: separating the problem into an rf part and a dc part, then numerically iterating back and forth between them until a solution was achieved. But what happens when one reaches the saturation stage is that the interaction between the rf wave and the dc background becomes so strong that one cannot treat these two modes separately. Rather, one must solve the coupled system as a single unit because small changes in one mode can generate larger changes in the other. The difficulty here is that the coupled system then becomes equivalent to a 15th order set of ordinary differential equations (ODEs) which are *singular ODEs*. They are singular because some of the highest derivatives in these ODEs are multiplied by small quantities. Whence any error in the evaluation of these derivatives can become magnified as one integrates forward. This set also has various boundary conditions to be set at the cathode and also at the anode. Whence this problem then becomes a two-point boundary problem. In short, this is the worst of all possible cases to attempt to solve. And all attempts by this PI to numerically integrate this 15th order system have not been successful. Let us note that in the initiation stage, one has only a fourth-order system of ODEs for the rf mode, another second-order system of ODEs for the dc mode and none of these ODEs are singular. This reduction in the order occurs because one has assumed that certain algebraic relations are satisfied between the physical quantities. These initiation ODEs can be successfully numerically integrated mainly due to being only 6th order. However the jump from 6th order to 15th order requires a much more careful formulation of the numerical approach in order to succeed. We note that at this point, we have not been able to analyze whether the instabilities involved are numerical instabilities or actual physical instabilities coming from the cold-fluid equations. If one is not to be able to numerically integrate these ODEs, then it is necessary to search for another approach. There are variational methods that could possibly work. There are methods that use single particle orbits. The same is done in PIC codes, but there one only solves for the particle orbits and averages appropriately to get the macroscopic quantities such as density and average velocities. Our approach has been to solve for that stationary state wherein the device is delivering maximum power, if such a state exists. Near the end of the contract, it was realized that there is another approach which would be somewhere between the sole use of PIC codes and fully solving the appropriate ODEs. This would be a hybrid method, whereby one would calculate the single particle orbits in an assumed stationary dc electric field and an assumed stationary rf electromagnetic field. From a knowledge of the single particle orbits in these fields, one can then obtain the macroscopic density and the average particle velocity, if such a stationary state could exist. If it could exist, then linear stability theory could be used to study its stability features.

- 2) The Camassa-Holm Equation: Certain problems can have instabilities which are related to the presence of eigensolutions of the perturbed equations whose analytical properties are not spatially uniform. This shows up in integrable systems in that the eigenvalue problem of the Lax pair will have this nature. One example of this is the Camassa-Holm equation (CH) when the initial data is sufficiently strong. Other examples are certain degenerate parametric optical

- laser problems such as degenerate two-photon propagation. A very good reason to study the CH equation first, instead of the optical problems, is that the CH equation has the advantage it lacks many of the complexities found in these optical problems. Thus its results can therefore be potentially used as a model for understanding how to treat these optical problems. In Pub02, we carry out an analysis of this feature when the initial data for the CH equation is sufficiently strong and is on an infinite spatial interval. We show that one can solve this problem if the infinite interval is broken into semi-infinite and finite regions, inside of which, each region contains its own collection of scattering data. Given the scattering data in these regions, we then demonstrate that one can, in principle, reconstruct the full solution on the infinite interval.
- 3) In Pub04, we have shown how one could solve the problem of the estimation of the accuracy of the solution of a variational problem. Variational methods have often been successful in solving complicated problems. And the method is usually more accurate than one can justify. We note that a variational formalism does exist for the HPM problem [D. J. Kaup and Gary E. Thomas, *J. Plasma Physics* **57**, 765-84 (1997)]. Consequently even if all else would fail, one would still have the option to study the HPM problem variationally. If so, then one would like to have some means for estimating the accuracy of such a solution. The results of this publication show how one can always obtain estimates of the errors in a variational solution and their associated quantities, and without having to know anything about the actual solution.
 - 4) Pub05 and Pub07 are the result of joint work between Prof. Gerdjikov and myself on eigenvalue problems. It is cast into the form of studies of integrable equations, but the problem is really one of a study of direct scattering and inverse scattering in one dimension, which are also exactly of the same nature as the problem of solving the perturbed cold-fluid equations in plasma physics. Such studies give the investigators experience in understanding these problems and experience in devising methods of solution for such problems.
 - 5) In integrable systems, one has two classes of solutions: solitons (bound states) and radiation (the continuous spectrum). Given the scattering data, in order to reconstruct the potentials, one must solve a set of linear integral equations. When the continuous spectrum is absent, then these linear integral equations can be reduced to a set of only algebraic equations. For certain integrable systems, and on finite or semi-infinite intervals, one can invariably convert any continuous spectrum into bound states, which do generate soliton-like solutions which are called "virtual solitons". Integrable systems of this nature are mainly optical parametric equations which are posed on finite intervals, such as second harmonic generation and related hyperbolic equations. In Pub06, Prof. Steudel and I have demonstrated how to do this and how it is related to earlier work in NMR (nuclear magnetic resonance). What we demonstrate here is that the scattering data for such problems can be reduced to a countable set of data. In the language of integrable equations, we show that for any reasonable initial value problem of such a system, the continuous spectrum can be converted into a countable set of bound states. Thus the inverse scattering problem can then be reduced to an algebraic problem, which is the same problem as constructing an N-soliton solution, where N is countable. In practical terms, it is further demonstrated that typically one can take N to be finite, albeit large.
 - 6) Pub08 is joint work with J. Yang and is a solution of the perturbation problem of

- the Sasa-Satsuma equation, which is integrable and has a 3x3 eigenvalue problem. This appears to be the first time that anyone has explicitly calculated the squared eigenfunctions for anything beyond the AKNS problem. This result has a breath of complexity which goes well above that of the AKNS and the Schrodinger eigenvalue problems. It also contains certain universal features which one can see will continue into higher order problems. It has just this month appeared in the Journal of Mathematical Physics.
- 7) Pub09 is another publication in collaboration with Prof. Gerdjikov. In this case we have presented a one-dimensional, integrable system which closely models other proposed models for a one-dimensional Bose-Einstein condensate. Its basic structure is a multi-component, nonlinear Schrodinger equation, and has some freedom in assigning values to the coefficients of the nonlinear terms. This is recent work and was first presented in the Summer of 2008.
 - 8) Pub10 is work in progress and has been supported by this contract. At the present time, no variational principle has been developed for two-level atoms and the phenomenon of self-induced transparency (SIT). Although two-level SIT is an integrable system, three-level SIT is not integrable except for special level configurations and then not unless the coupling coefficients are certain special values. The advantage of having a variational principle for three-level atoms is obvious. We could execute certain studies of the three-level non-integrable cases.
 - 9) Pub11 is also work in progress by an undergraduate student. This work will complement Pub08 and will allow one to study perturbations of the integrable three-wave resonant interactions.
 - 10) Pub12 is work in progress and is concerned with ionospheric interactions when the heater wave is tuned to the second harmonic of the electron cyclotron frequency. Experiments at this frequency have recently been conducted at the HAARP transmitter. In our earlier ionospheric work of 1992, this frequency range was inadequately treated since at that time, there was no experimental data for that regime. This work is intended to fill that gap and to compare theory with the experiments for that frequency.

Archival publications (published) during the period of this contract:

Pub01) *Resonances in High Density Nonneutral Plasmas*, D. J. Kaup, "Frontiers of Nonlinear Physics, Proceedings of the Second International Conference", Nizhny Novgorod - St. Petersburg, Russia, 2004, edited by Alexander Litvak (Nizhny Novgorod, Russia, Institute of Applied Physics RAS, 2005), pp. 308-20.

Pub02) *Evolution of the Scattering Coefficients of the Camassa-Holm Equation, for General Initial Data*, D.J. Kaup, *Studies in Appl. Math.* **117**, 149-64 (2006).

Pub03) *Drift Resonance in High Density Non-neutral Plasmas*, D.J. Kaup, *Phys. Plasmas* **13**, 053113 (2006).

Pub04) *Quantitative Measurement of Variational Approximations*, D.J. Kaup and T.K. Vogel, *Phys. Lett. A* **362**, 289-97 (2007).

Pub05) *How many types of soliton solutions do we know?*, Vladimir S. Gerdjikov and D. J. Kaup, "Geometry, Integrability and Quantization", pp. 11-34, Ivailo M. Mladenov and

Manuel De Leon, Editors, (Softex, Sofia, Bulgaria, 2006).

Pub06) *Inverse Scattering for an AKNS Problem with Rational Reflection Coefficients*, H. Steudel & D.J. Kaup, *Inverse Problems* **24**, 025015 (2008).

Pub07) *On classification of soliton solutions of multicomponent nonlinear evolution equations*, V. S. Gerdjikov, D. J. Kaup, N. A. Kostov & T. I. Valchev, *J. Phys. A: Math. Theor.* **41**, 315213 (2008).

Pub08) *Derivation of Squared Eigenfunctions for the Sasa-Satsuma Equation*, Jianke Yang & D. J. Kaup, *J. Math. Phys.* **50**, 023504 (2009).

Archival publications to be completed, whose work was started during the period of this contract:

Pub09) *Bose-Einstein condensates and multi-component NLS models on symmetric spaces of **BD.I**-type. Expansions over squared solutions*, V. S. Gerdjikov, D. J. Kaup, N. A. Kostov and T. I. Valchev, [To appear in the proceedings of the Conference on Nonlinear Science and Complexity, Porto, Portugal, July 28-31 (2008)].

Pub10) *Variational Principle for Self-Induced Transparency*, D.J. Kaup and Galen T. Kaup (work in progress).

Pub11) *Perturbations and Squared Eigenfunctions for the Three-Wave Resonant Interaction*, R. A. Van Gorder and D. J. Kaup (work in progress).

Pub12) *Upper hybrid resonance phenomena at the second cyclotron harmonic*, Subhash Antani & DJ Kaup (work in progress).