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14. ABSTRACT

Our objective in this project was to derive a mathematically exact theory of unsteady fluid flow separation. We have obtained analytic formulae for the location and shape of separation profiles in terms of measurable, wall-based physical quantities. These formulae can now be used to design feedback controllers that alter, destroy, or create separation. Our main achievements are as follows:
(1) We have developed a mathematical theory of unsteady three-dimensional separation for flows with a steady mean component.
(2) We have developed a theory of moving unsteady separation for flows with a time-varying mean component.
(3) We have developed a theory of separation for two-dimensional flows with a slip boundary ((1)-(3) above cover no-slip boundaries)
(4) We also conducted experiments to prove existence of a new three-dimensional separation pattern (separation along a limit cycle of the wall-shear field) first predicted by our 3D steady separation theory.

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NONLINEAR DYNAMICS AND CONTROL
OF THREE-DIMENSIONAL SEPARATED FLOWS
Final Report

AFOSR GRANT NO. FA 9550-06-1-0101

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Summary

Our objective in this project was to derive a mathematically exact theory of unsteady fluid flow separation. We have obtained analytic formulae for the location and shape of separation profiles in terms of measurable, wall-based physical quantities. These formulae can now be used to design feedback controllers that alter, destroy, or create separation.

Our main achievements are as follows:

(1) We have developed a mathematical theory of unsteady three-dimensional separation for flows with a steady mean component.

(2) We have developed a theory of moving unsteady separation for flows with a time-varying mean component.

(3) We have developed a theory of separation for two-dimensional flows with a slip boundary (the results in (1)-(3) above were obtained for no-slip boundaries).

(4) We also conducted experiments to prove existence of a new three-dimensional separation pattern (separation along a limit cycle of the wall-shear field) first predicted by our 3D steady separation theory.

1. Fixed separation in 3D unsteady flows with a steady mean

In unsteady flows with a steady mean component, time-varying separation surfaces turn out to emanate from *fixed* lines located on the surface of the aerodynamic body. These fixed separation lines remain hidden in the instantaneous velocity and pressure fields; we find the fixed separation lines using a combination of averaging methods and invariant manifold techniques. We have obtained similar results for time-varying reattachment surfaces (stable manifolds) and the corresponding fixed reattachment lines.

As an example, we show in Fig. 1 separation from a flat surface in a three-dimensional unsteady swirling flow model.

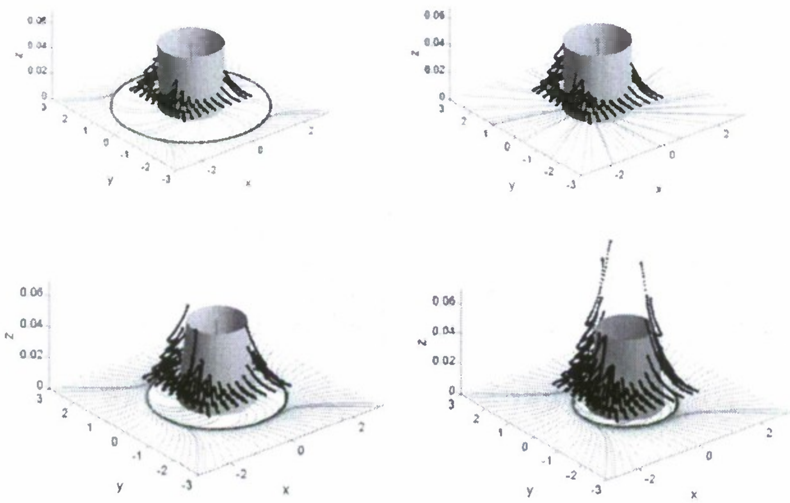


Figure 1: Separation surface (green) at four time instances, as predicted analytically from on-wall measurements in a 3D unsteady flow model (cf. [8] for details).

Note that the topology of the wall shear-field (red) in Fig. 1 changes in time, yet fluid trajectories (black) separate from the wall along a fixed limit cycle that we predict analytically. The time-varying separation surface emanating from this fixed limit cycle is also predicted up to first order (green surface) by our theory.

2. Moving separation in flows with a time-varying mean

We have proved in [5] that moving separation along a no-slip wall is due to the presence of *ghost manifolds*, i.e., non-hyperbolic unstable manifolds that emanate from a boundary layer near the critical manifold. The boundary layer is thin, hence the misleading perception that the unstable manifolds slide on the critical manifolds. Still, we showed that these ghost manifolds can be detected from wall-shear and wall-pressure measurements on the boundary. Thus a wall-based analysis is able to predict off-wall separation.

Figure 2 shows an application of our criteria to a moving separation bubble. The velocity field of the bubble has a time-varying mean component with stochastic oscillations superimposed on it. Numerical experiments show the formation of a sharp material spike that keeps moving to the left as time progresses. As we discussed above, such a moving spike cannot be explained by classical unstable manifolds familiar from nonlinear dynamics, because such manifold must remain anchored to the wall.
We use the analytical formulae developed in [5] to obtain the exact spike location and a leading-order approximation for the shape of the separation spike (see Fig. 2.)

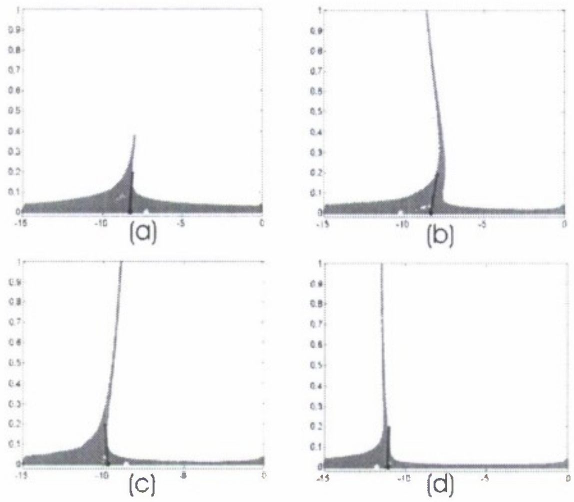


Figure 2. A close-up of a moving material spike in a separation bubble flow with stochastic oscillations superimposed on a time-varying mean. The red spike is formed out of particles released close to the wall. The blue line is a first prediction of the moving separation spike from a wall-based analytical criterion. Subfigures (a)-(c) correspond to increasing times.

We have also validated these results in laboratory experiments on two-dimensional unsteady separation. As we show in Fig. 3, our theory successfully identified the location of moving separation spiked from on-wall measurements of wall shear.

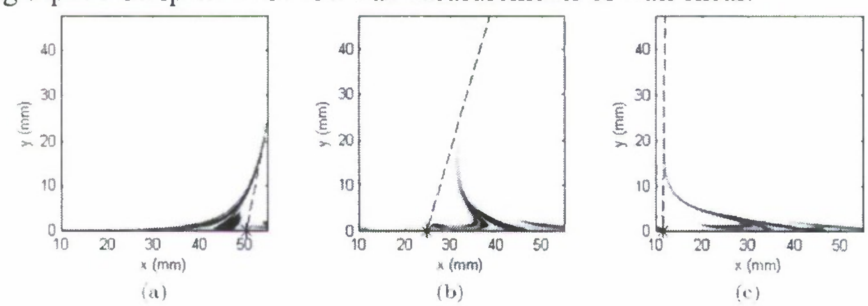


Figure 3. Ghost manifolds in an oscillating rotor experiment [7] with a time-varying mean flow. The material spikes at three different times are obtained from flow visualization. Star denotes the moving separation point obtained from our ghost manifold theory.

3. Unsteady separation along slip boundaries

In his seminal work on two-dimensional steady flows with no-slip boundaries, L. Prandtl showed that separation takes place at points of zero wall shear and negative wall-shear gradient. An open question has been how his criterion could be generalized to unsteady flows with slip boundaries. Such flows enjoy increasing interest due to their importance in geophysical models and micro-fluidic devices.

In joint work with F. Lekien (U. Brussels), we have developed a mathematically exact criterion for the separation location and angle on a two-dimensional slip boundary [6]. Figure 3 shows an application of our slip-separation criterion to a two-dimensional incompressible Rayleigh-Bernard flow model with random time dependence. The velocity field of this flow has two horizontal slip boundaries; material ejection near convection-cell boundaries can be viewed as flow separation. Our theory predicts the time-dependent location and angle of this separation from fluid motion on the cell boundaries.

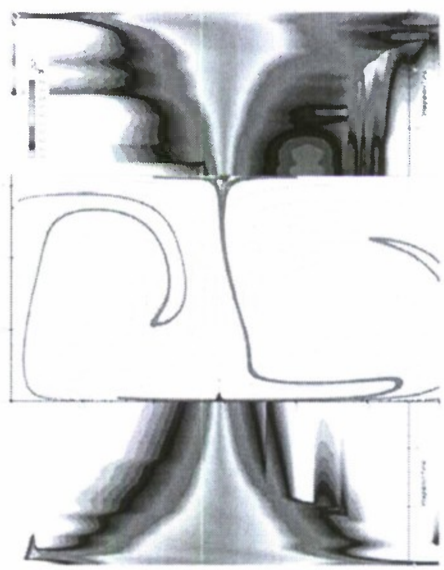


Figure 3. Unsteady separation and reattachment on the horizontal cell boundaries in a Rayleigh-Bernard convection model with random time dependence.

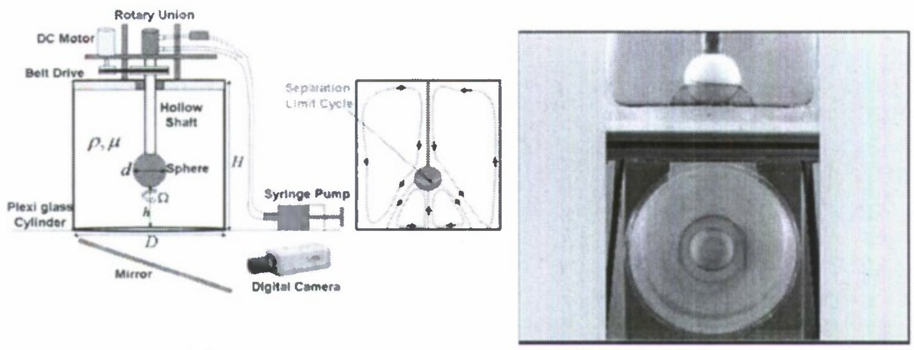
Further applications to boundary-current separation in Monterey Bay, California are described in [6].

4. Three-dimensional steady separation experiments

We have collaborated with T. Peacock (MIT) on an experimental verification (cf. Fig. 4) of complex separation patterns that are predicted to exist by our 3D steady theory [2,4], but have not yet been identified in practice.

We show the basic experimental set-up in Fig. 4(a). The experiments involve a rotating sphere from which the flow separates due to the geometry of the circulation patterns around the sphere. This geometry should give rise to separation along a limit cycle of the wall-shear field. The possibility of such a pattern was predicted in [2,4], but we are unaware of any prior experiments or simulations suggesting this pattern.

The result of our flow-visualization experiment is shown in Fig. 4(b). The dye surface emanating from a circle around the sphere is the separation surface emanating from a limit cycle of the wall-shear field.



(a) (b)

Figure 4. (a) Experimental set-up for flow past a rotating sphere (b) Dye visualization of the separation pattern: upper image is a side view, lower image is bottom view.

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