Non-Linear Rolling of Ships in Large Sea Waves

by

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Submitted to the Department of Mechanical Engineering in Partial Fulfillment of the Requirements for the Degrees of
Master of Science in Naval Architecture and Marine Engineering
and
Master of Science in Mechanical Engineering
at the
Massachusetts Institute of Technology

11 May 2007

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
XX-05-2007

2. REPORT TYPE
Master's Thesis

3. DATES COVERED *(From - To)*
JAN-JUN 2007

4. TITLE AND SUBTITLE
Non-Linear Rolling of Ships in Large Sea Waves

5a. CONTRACT NUMBER
N62271-97-G-0026

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Scott M. Vanden Berg

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Massachusetts Institute of Technology

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Naval Postgraduate School
Monterey, Ca 93943

10. SPONSOR/MONITOR'S ACRONYM(S)
NPS

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT
1. DISTRIBUTI9ON STATEMENT A. Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
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15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
					Sean Tibbitts, Educational Technician
a. REPORT	b. ABSTRACT	c. THIS PAGE	UU	66	19b. TELEPHONE NUMBER *(include area code)* (831) 656-2319 civins@nps.edu

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39.18

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The United States Navy has taken a new interest in tumblehome hulls. While the stealth characteristics of these hull forms make them attractive to the Navy, their sea keeping characteristics have proven to be problematic. Normal approximations of sea keeping characteristics using linear differential equations with constant coefficients predict a very stable platform, while observations in model tests show a ship that is prone to extreme roll transients. This thesis examines a simple method of producing a non-linear simulation of roll motion using a tumblehome hull provided by the Office of Naval Research. This research demonstrates the significant difference that a variable restoring coefficient introduces into a hull's seakeeping characteristics.

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Acknowledgements

The author would like to acknowledge the following organizations and individuals for their assistance. Without them this thesis would not have been possible.

- Professor Yuming Liu, for his aid in computer simulation schemes
- LT Andrew Gillespy (USN), and LT(jg) Kyriakos Avgouleas, who were fantastic sounding boards during my research
- CDR Joe Harbour for advisement as thesis readers
- Mr. William Belknap from Naval Surface Warfare Center, Carderock Division, for the use of the ONR tumblehome hull form
- Finally, Professor Jerry Milgram, for his advice and experience

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1 Introduction

1.1 Motivation for Research

Naval architecture in the United States Navy has come full circle. With the selection of a tumblehome hull for the DDG-1000 project, the United States Navy has come back to a design concept that was used on the oldest commissioned warship in their fleet, the *USS Constitution*. Tumblehome, the inward slope of the upper part of the hull of a ship, has come back in style due to the stealthy benefits it imparts upon its ship. However, with the use of this ‘new’ concept come interesting challenges in naval architecture.

The latest designs for the hulls of United States naval warships have sparked a controversy over their stability. In the April 2007 edition of the Navy Times, experts are quoted stating that the hull may be unstable in roll motion.[1] Programs that use linear sea keeping approximations predict good stability for tumblehome hulls. However, non-linear effects in the coupling of pitch and heave with roll motions can degrade the stability of a tumblehome ship. Programs that calculate the non-linear effects of this nature may require significant computing power making them costly and time consuming. A simpler approach could reduce computing requirements and allow naval architects to quickly estimate the stability of any new hull.

1.2 Objective and Outline of Thesis

The purpose of this thesis is to explore the non-linear effects that pitch, heave, and roll exert on a ship’s restoring moment. The linear seakeeping equations will be modified to allow the restoration force to be calculated as a function of roll, pitch, and heave. Time simulations of pitch and heave responses derived from linear seakeeping theory are used to provide a basis for numerical solution of the non-linear seakeeping equation for roll. Although the research presented is for one specific hull form and operating condition, the methods used are intended to applicable to any ship in any seakeeping situation. Chapter Two discusses the characteristics of the hull used in this research, including the operational scenario that is used for conducting the research. Chapter Three explains linear seakeeping theory, as well as the approximations used by the seakeeping program Maxsurf. Chapter Four discusses the method used to determine the excitation forces used in the non-linear simulation. Chapter Five describes the computer programs used to simulate the roll motion of the ship. Chapter Six presents a comparison of the linear and non-linear results, and Chapter Seven presents recommendations for future work and conclusions.

2 The ONR Tumblehome Hull

2.1 Hull Description

The hull form used for this thesis is the tumblehome hull used by the Office of Naval Research. This ship has a length of 154 meters and a beam of 18.5 meters. The displacement at a draft of 5.5 meters is approximately 8800 metric tons.[2] The hull has a vertical center of gravity assumed to be 7.5 meters above baseline, and the transverse metacentric height is 1.47 meters. The bow rakes aft instead of forward necessitated by the tumblehome sides. The profile and body plan of the ship is shown in Figure 2-1 and Figure 2-2:

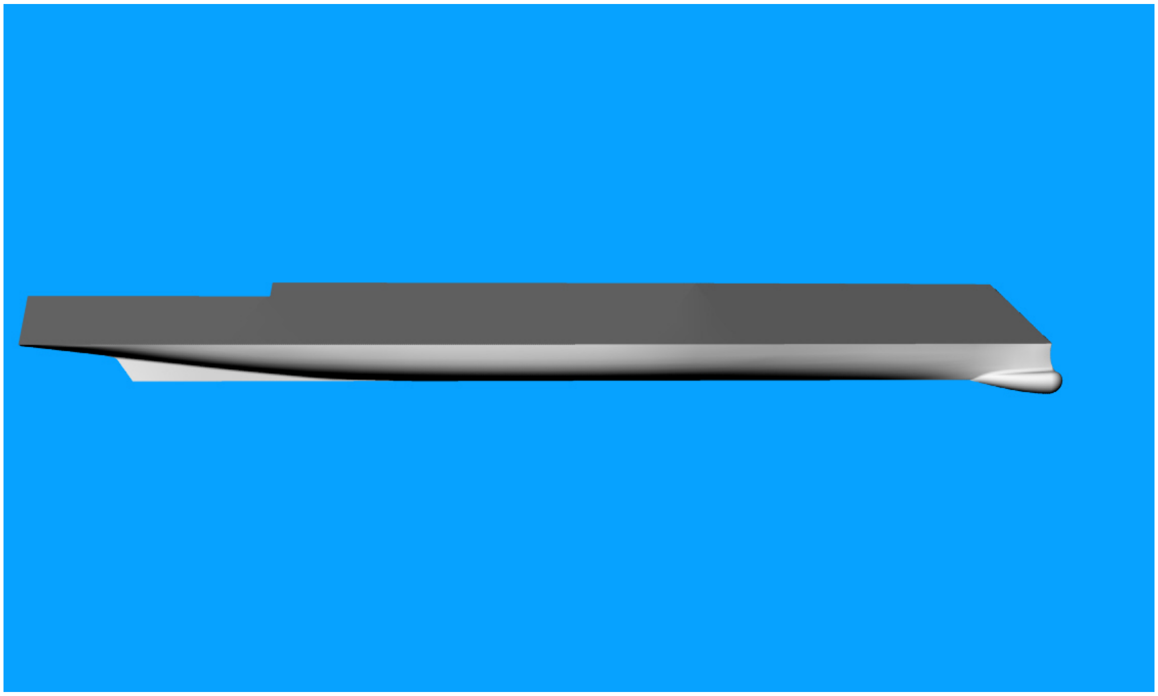


Figure 2-1: Profile View of the ONR Tumblehome Hull.

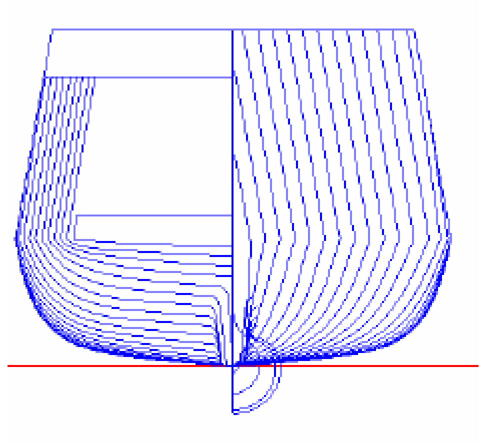


Figure 2-2: Body Plan of ONR Tumblehome Hull

The tumblehome hull has military advantages that make it attractive for use in surface combatants. The chief advantage comes from the fact that the sides of the hull are angled away from the waterline. This will tend to reflect radar energy that is directed towards the ship from another up into the atmosphere, and not back towards the radar receiver. This scattering of radar energy significantly reduces the radar cross section of the ship, making the ship harder to detect.

From a naval architecture standpoint, tumblehome hulls have some less than ideal properties. The decreasing beam above the waterline limits the deckhouse area, and reduces the damaged stability of the hull form compared to wall sided or flared hulls. In the area of stability, the hull suffers compared to traditional hull forms. As the ship lists, the tumblehome causes a relative reduction in the waterplane area of the ship, lowering the righting arm. The righting arm for zero pitch is shown in Figure 2-3, compared to ships with similar shapes, but with wall sides and flare instead of tumblehome.

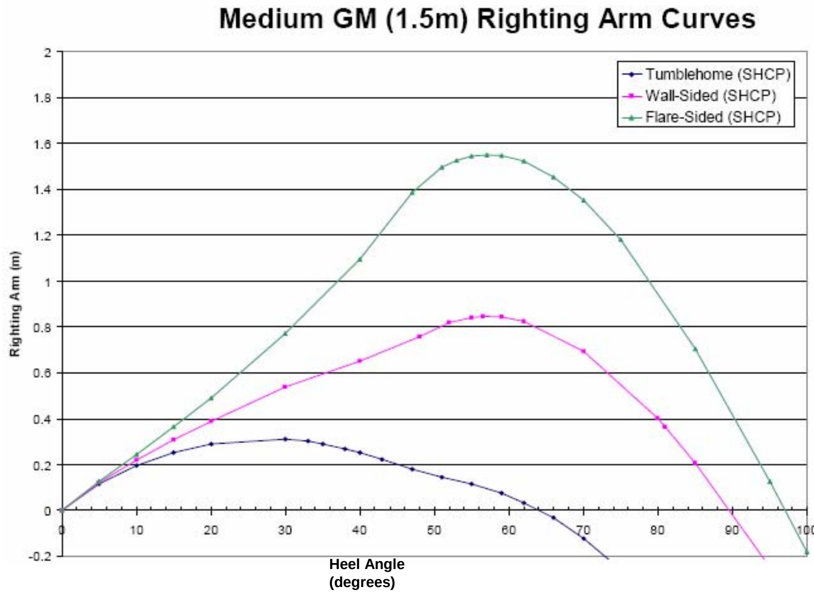


Figure 2-3: Righting Arm Curves for Different Hull Shapes

The wave piercing bow has its own military advantages. Because of the reduced buoyancy of the bow, the ship tends to pitch less as it rides through waves, giving the ship a more stable platform for its weapons systems. Additionally, the design may reduce the waves generated by the ship, lowering resistance of the hull. These advantages come at a price in the stability analysis. The ship loses waterplane area as it pitches down by the bow, resulting in a smaller transverse righting arm as the bow sinks into the water. In most operating cases, this effect can be neglected. In heavier seas where pitch is significant, the loss of righting arm can cause severe rolling transients.

2.2 Computer Modeling in Maxsurf and Matlab

The ONR tumblehome hull was provided by the Carderock Division of the Naval Surface Warfare Center. In order to conduct this research, it was modeled in both Maxsurf and Matlab. Offsets were generated in Maxsurf using a computer aided drawing of the hull. The offsets were used in three different parts of this research.

First, Maxsurf's Sea Keeper module used the offsets to determine a pitch, heave, and roll response using the program's linear strip theory calculations. These outputs were in the form of linear response amplitude operators (RAOs) for both magnitude and phase relative to the incident waves. As explained in Chapter Five, the RAOs were used to generate a time simulation of the ship's pitch, heave, and roll. The pitch and heave time simulations were used to determine the non-linear roll righting moment. The roll response was used to compare against the results of the non-linear simulation. The offsets were also used to model the ship in Matlab to determine excitation forces. The offsets were used as a set of nodes to compute the Froude-Krylov and diffraction excitation moments. Finally, the hull offsets were used in Maxsurf's Hydromax module to determine the ship's righting moment in roll as a function of pitch, heave, and roll. This will be explained in Section 2.3.

Mass distribution for the ship was chosen to approximate a ship with a relatively high center of gravity to allow for high weight, such as weapons systems on a naval vessel. The center of gravity was chosen to be on the centerline two meters above the still waterline. This was arbitrarily chosen so that the metacentric height would match the height shown in Figure 2-3. The longitudinal position was determined as directly above the center of buoyancy to give the ship no trim in pitch. In order to match Maxsurf approximations, an estimate of 40 percent of the ship's beam was used to approximate the ship's radius of gyration. Therefore the roll moment of inertia is given by the formula:

$$I_{44} = (0.4B)^2 \rho \forall \tag{2.1}$$

This mass distribution may be used to estimate the undamped natural period of the hull [5] through the use of the equation

$$T = 2\pi \sqrt{\frac{(I_{44} + A_{44})}{\rho \forall g GM_t}} \tag{2.2}$$

The added mass in roll, A_{44}, was assumed to be 0.3 times the mass moment of inertia, as discussed in Section 3.4. For the values given in this section, the natural period is approximately 13.9 seconds. This will be used to compare against results in Chapter Six.

2.3 Non-Linearities in Righting Moment

One of the unique aspects of a tumblehome hull is its roll restoring moment behavior with its movement in the three principal degrees of freedom. Because of the wave piercing bow, its buoyancy forward is much smaller than its buoyancy aft. This asymmetry leads to a non-linear behavior in restoring moment. As mentioned in the previous section, the offsets were used to determine the hydrostatic righting moment as a function of roll angle, pitch angle, and draft.

Righting moment varies significantly with both pitch and draft. In the linear simulations determined from Sea Keeper, pitch varied between 15 meters to -15 meters by the stern, and draft at amidships varied between 3 meters and 7.5 meters. This behavior is shown graphically in Figure 2-4 and Figure 2-5. Wave elevation and slope effects are not included in the restoring moment determination. These effects were left for objects of further study.

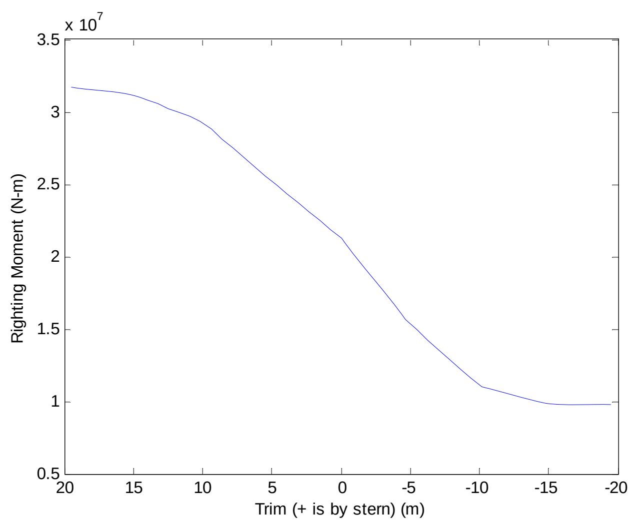


Figure 2-4: Righting Moment vs. Trim for Roll=10, Draft=5.5m

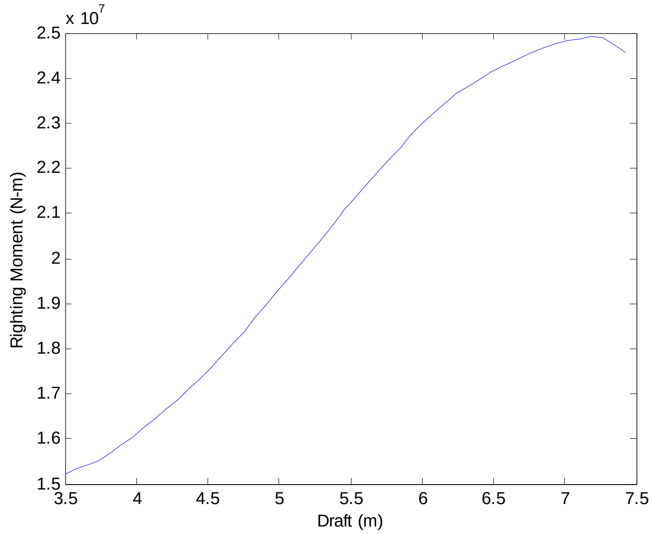


Figure 2-5: Righting Moment vs. Draft, Roll=10, No pitch

Trim effects were most drastic near zero trim. As the ship pitches by the bow, the righting moment decreases as expected. This leads to an obvious design technique for improved sea keeping of trimming the vessel by the stern. However, this method of improving the righting moment reduces the advantages of having a wave piercing bow in the first place. The draft effects are not as significant as the effects for pitch, but a low draft coupled with a significant pitch by the bow could lead to very small righting moments. In very extreme cases, the moment could turn to an overturning moment. Obviously, this situation should be avoided at all costs.

2.4 Seakeeping Scenario

The seakeeping scenario chosen is well outside a normal operating envelope for naval vessels. The conditions were chosen to excite significant motion in any ship. The ship was set to a speed of 25 knots, traveling in Sea State Eight with seas 30 degrees abaft of the beam. The significant wave height for Sea State Eight is 11.5 meters, with a modal period of 0.37 radians per second. The seas are assumed to be fully developed, long-crested, waves from a distant storm. A sketch of this scenario is shown in the figure below:

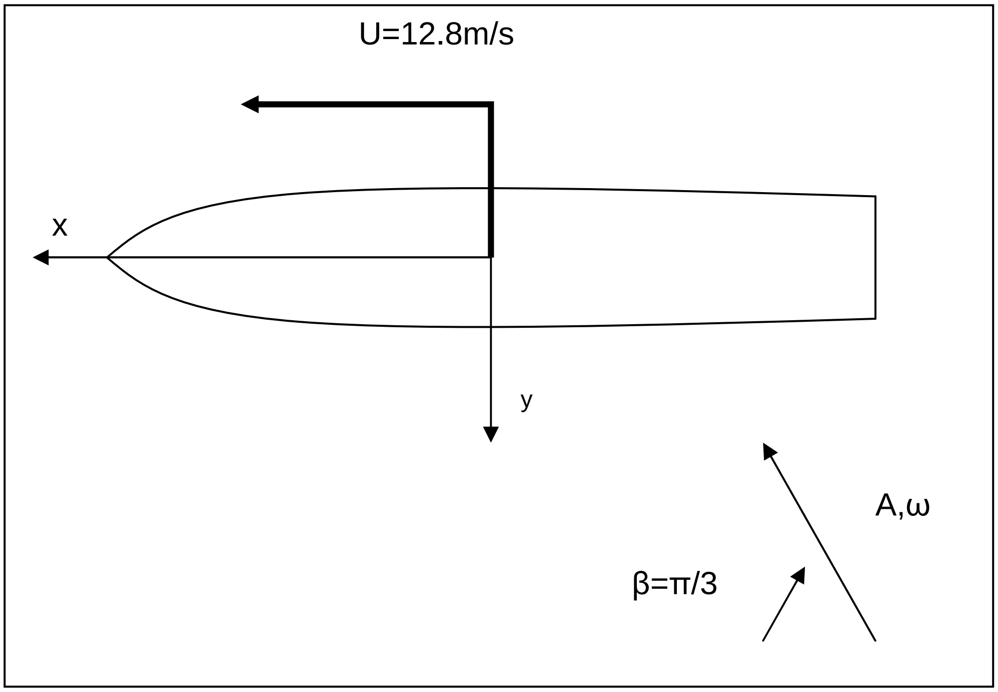


Figure 2-6: Seakeeping Scenario

A consistent coordinate system was chosen for all parts of the study. The coordinate system is the Cartesian coordinate system normally used for sea keeping problems. The x-axis is directed along the longitudinal center of the ship, and the z-axis is pointed so that positive z is up. Therefore, the y-axis is directed out the port side of the ship. The six degrees of freedom are therefore surge in the x-direction; sway in the y; heave in the z; roll about the x-axis; pitch about the y; and finally yaw about the z. The origin was selected as the centerline at amidships, at a height equal to that of the still waterline.

For the seakeeping part of the problem, roll was assumed to be decoupled from sway and yaw. While this is not the actual case as discussed in Section 3.3, it provides a simplification to the problem to get a basis for comparison with another seakeeping program.

3 Linear Seakeeping Theory

3.1 Linear Plane Progressive Waves

Inside the domain, random seas were simulated by unidirectional plane waves incident upon the hull at an angle β from the stern of the ship. The plane waves were assumed to be a superposition of sinusoidal waves at different frequencies. To treat the waves using potential theory, the following assumptions were made. First, the unsteady viscous forces on the hull are neglected. Next, the density of the seawater is constant and all flows were incompressible. Finally flow was approximated as being irrotational.

All flow potential functions must satisfy conservation of mass. With the assumption of incompressible flow, the conservation of mass states that the divergence of the velocity field must be zero. In mathematical terms, this means that the governing equation for all potentials must be:

$$\frac{\partial u'}{\partial x'}+\frac{\partial w'}{\partial z'}=\frac{\partial^2\Phi}{\partial x'^2}+\frac{\partial^2\Phi}{\partial z'^2}=\nabla^2\Phi=0 \qquad (3.1)$$

In equation(3.1), the primed quantities are with respect to a coordinate system with the waves propagating in the x' direction. The two boundary conditions for these come from the kinematics of the waves and the dynamic pressure of the wave's surface. The kinematic boundary condition is that along the wave surface, z=ζ where ζ (x,y,t) is the wave elevation as a function of position and time. By defining a function F=z- ζ , then the kinematic boundary condition may be written as:

$$\frac{DF}{Dt}=\frac{\partial F}{\partial t}+(\vec{V}\bullet\nabla)F=0 \text{ on } z=\zeta \qquad (3.2)$$

The dynamic free surface condition comes from the fact that the pressure is atmospheric along the surface of the wave. From Bernoulli's equation for potential flow,

$$\frac{\partial\Phi}{\partial t}+\frac{1}{2}\nabla\Phi\bullet\nabla\Phi+gz=-\frac{P_a}{\rho} \text{ on } z=\zeta \qquad (3.3)$$

In most case, the wave slopes encountered are small, so these equations may be treated by linearizing the boundary conditions. The linearized boundary conditions become:

$$\frac{\partial^2\Phi}{\partial t^2}+g\frac{\partial\Phi}{\partial z}=0 \qquad \& \qquad \zeta=-\frac{1}{g}\frac{\partial\Phi}{\partial t} \quad \text{on } z=0 \qquad (3.4)$$

The solution to this boundary value problem in deep water with x directed along the direction of the propagation of the waves in a stationary coordinate system is:

$$\Phi = \mathrm{Re}\left\{ \frac{iAg}{\omega} e^{-ikx+i\omega t} e^{kz} \right\} \tag{3.5}$$

with the dispersion relationship

$$\omega^2 = kg \tag{3.6}$$

Because the waves are traveling at an angle with respect to the longitudinal axis, the waves must undergo a coordinate transformation. It is convenient to work in complex space without the real notation, so the complex wave potential incident upon the ship's hull for a given frequency is:

$$\Phi_I = \frac{iAg}{\omega} e^{-ik(x\cos\beta - y\sin\beta)+i\omega t} e^{kz} \tag{3.7}$$

3.2 Wave Spectra

Because real seas are never monochromatic, a statistical approach is needed to simulate random seas. The approach used in this thesis assumes that the waves experienced are unidirectional, as if they all come from a distant storm. It also assumes that the surface elevation is a zero mean, ergodic collection with a Gaussian distribution. The energy of the storm may be contained in a spectrum of frequencies.

A common spectrum used is the Bretschneider Spectrum. This is the standard spectrum recognized by the International Towing Tank Convention, and will be used throughout this paper. The Bretschneider Spectrum has the form:

$$S_\zeta(\omega) = \frac{1.25}{4} \frac{\omega_m^4}{\omega^5} \mathrm{H}_{1/3}^2 \mathrm{e}^{-1.25*(\frac{\omega_m}{\omega})^4} \tag{3.8}$$

This spectrum requires two parameters, $H_{1/3}$, or significant wave height, and modal frequency, ω_m. The significant wave height is the average of the one third highest waves in the seas, and the modal frequency is the single frequency that is most likely in the distribution. For fully developed storms, the modal frequency is related to the significant wave height by the equation:

$$\omega_m = 0.4 * \sqrt{\frac{g}{H_{1/3}}} \tag{3.9}$$

With this relationship, the spectrum is called the Pierson-Moskowitz spectrum. In Sea State Eight, these values are $H_{1/3}$=11.5m, and ω_m=0.37 rad/s. A graph of the spectrum is shown below.

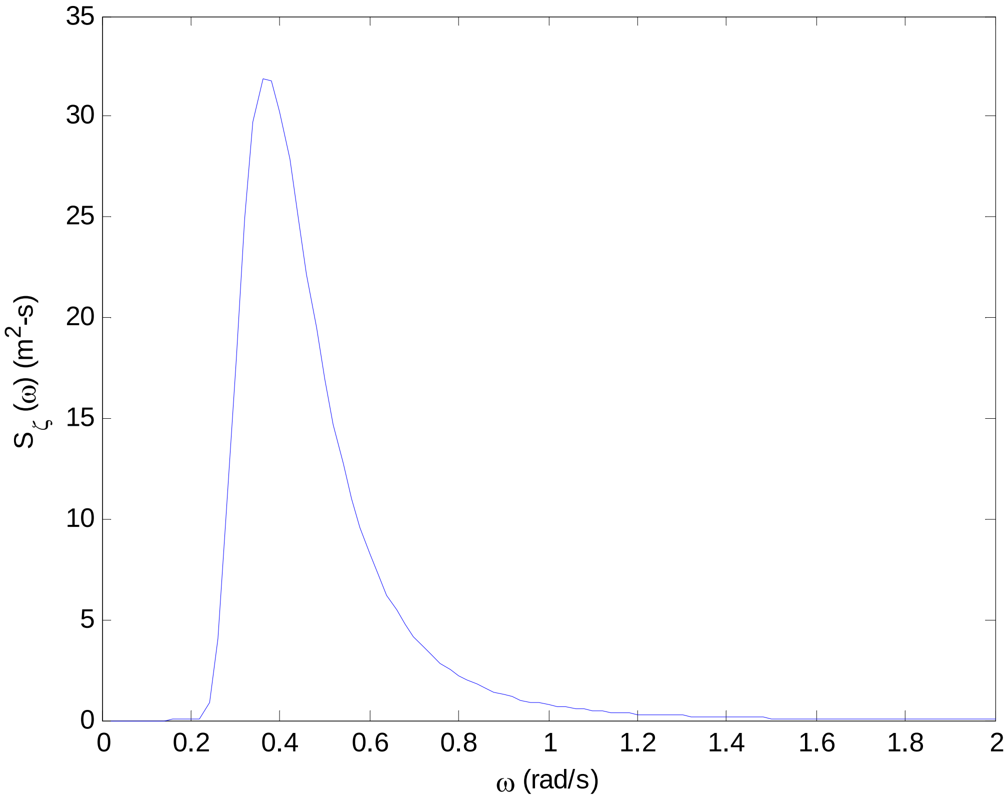


Figure 3-1: Fully Developed Bretschneider (Pierson-Moskowitz) Spectrum for Sea State Eight

When scaled by ρg, the area under the spectrum curve represents the energy per unit area of the sea state. Because of the assumption that the sea state is described by a collection of waves at distinct frequencies, this energy relationship can be used to simulate a random sea state in a discrete form. For a random sea, the energy density of the system may be expressed as

$$\mathrm{E} = \rho g \sum_{i=1}^{\#\,freqs} \frac{A_i^2}{2} = \rho g \int_0^{\infty} S_{\varsigma}(\omega)\, d\omega \tag{3.10}$$

By discretizing the integral in the above relationship, the wave amplitudes for discrete frequencies, separated by $\Delta\omega$, can be calculated as:

$$A_i = \sqrt{2 S_{\varsigma}(\omega_i)\Delta\omega} \tag{3.11}$$

Adding in a random phase angle, random seas may be simulated on a computer rather quickly using only the formula for the wave height spectrum. The surface elevation on a fixed (x,y) vertical line $\zeta(t)$ is simply

$$\zeta(t) = \sum_{i=1}^{\#\,freq} A_i \cos(\omega_i t + \psi_i) \tag{3.12}$$

An example of the record of a random sea in Sea State Eight is shown in figure 3-2. The sea state was generated using 500 discrete frequencies equally spaced from 0 to 2.5 radian/second, calculated every .01 seconds.

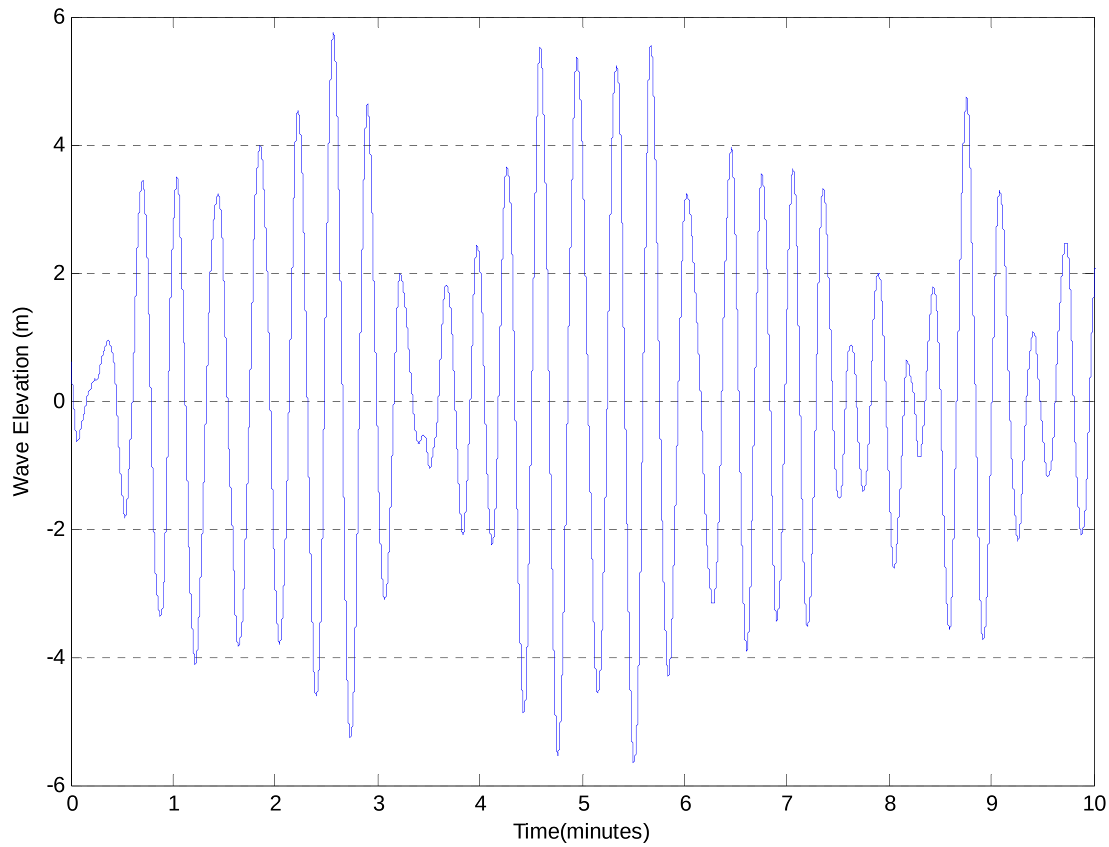


Figure 3-2: Simulated Wave Elevation Record for Sea State Eight

3.3 Equations of Motion & Linear Superposition

Most seakeeping analysis starts with the assumption that the motions of a ship are linear in nature [3]. This a reasonable due to the small wave slopes experienced during most normal operating scenarios. The linearized seakeeping equations of motion for the six degrees of freedom may be written in short as

$$\sum_{k=1}^{6}[(M_{jk}+A_{jk})\ddot{\eta}_k+B_{jk}\dot{\eta}_k+C_{jk}\eta]=F_j e^{i\omega t} \quad \text{j=1-6} \tag{3.13}$$

The matrices **M, A, B,** and **C** represent the generalized mass, added mass, linear damping, and hydrostatic restoration coefficients in each equation.

Because of the lateral symmetry of the ship, several terms can be set to zero. The result is that the generalized mass matrix has the form:

$$\begin{bmatrix} m & 0 & 0 & 0 & mz_c & 0 \\ 0 & m & 0 & -mz_c & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & -mz_c & 0 & I_{44} & 0 & -I_{46} \\ mz_c & 0 & 0 & 0 & I_{55} & 0 \\ 0 & 0 & 0 & -I_{46} & 0 & I_{66} \end{bmatrix}$$

The added mass matrix has the form:

$$\begin{bmatrix} A_{11} & 0 & A_{13} & 0 & A_{15} & 0 \\ 0 & A_{22} & 0 & A_{24} & 0 & A_{26} \\ A_{31} & 0 & A_{33} & 0 & A_{35} & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & A_{46} \\ A_{51} & 0 & A_{53} & 0 & A_{55} & 0 \\ 0 & A_{62} & 0 & A_{64} & 0 & A_{66} \end{bmatrix}$$

The damping coefficient matrix has the same shape as the added mass matrix. The damping coefficients are non-zero where the added mass coefficients are non-zero, and zero everywhere else.

Almost all of the restoring coefficients for ships on the ocean surface are zero. The only non-zero terms are C_{33}, C_{44}, C_{55}, C_{35}, and C_{53}. Expanding the equations with all coefficients in them shows a very interesting phenomenon. Because of the symmetrical zeroes in each matrix, the whole system divides into two sets of three coupled equations; one set consisting of surge, heave, and pitch, and the other set describing sway, roll, and yaw. This can further be simplified, as Salveson, et.al point out[4], by noting that the surge force and the surge response, are very small compared to the other forces and motions. The equations of motion for each fixed frequency are then:

$$
\begin{aligned}
&(M + A_{33})\ddot{\eta}_3 + B_{33}\dot{\eta}_3 + C_{33}\eta_3 + A_{35}\ddot{\eta}_5 + B_{35}\dot{\eta}_5 + C_{35}\eta_5 = F_3(t) \\
&A_{53}\ddot{\eta}_3 + B_{53}\dot{\eta}_3 + C_{53}\eta_3 + (I_{55} + A_{55})\ddot{\eta}_5 + B_{55}\dot{\eta}_5 + C_{55}\eta_5 = F_5(t)
\end{aligned}
\tag{3.14}
$$

and

$$
\begin{aligned}
&(M + A_{22})\ddot{\eta}_2 + B_{22}\dot{\eta}_2 + (A_{24} - Mz_c)\ddot{\eta}_4 + B_{24}\dot{\eta}_4 + A_{26}\ddot{\eta}_6 + B_{26}\dot{\eta}_6 = F_2(t) \\
&(A_{42} - Mz_c)\ddot{\eta}_2 + B_{42}\dot{\eta}_2 + (A_{44} + I_{44})\ddot{\eta}_4 + B_{44}\dot{\eta}_4 + C_{44}\eta_4 + (A_{46} - I_{46})\ddot{\eta}_6 + B_{46}\dot{\eta}_6 = F_4(t) \\
&(A_{62})\ddot{\eta}_2 + B_{62}\dot{\eta}_2 + (A_{64} - I_{64})\ddot{\eta}_4 + B_{64}\dot{\eta}_4 + (A_{66} + I_{66})\ddot{\eta}_6 + B_{66}\dot{\eta}_6 = F_6(t)
\end{aligned}
\tag{3.15}
$$

These five differential equations are the basis of linearized seakeeping theory in the time domain. Because of the assumption of linearity, the added mass and damping coefficients may be considered functions of frequency and forward speed only. The system of equations represents a linear time independent system, and may be worked in complex space by taking the Fourier transform of each term. This, in effect, reduces the coupled differential equations to algebraic equations. The solutions to the linear equations are in fact the transfer functions for the responses, and the response amplitude operator used in spectral analysis is the magnitude of the transfer function. Principles of Naval Architecture outlines the procedure for pitch and heave; the solutions provided are[5]:

$$
\begin{aligned}
\hat{\eta}_3 &= \frac{\hat{F}_3 S - \hat{F}_5 Q}{PS - QR} \\
\hat{\eta}_5 &= \frac{\hat{F}_5 P - \hat{F}_3 R}{PS - QR}
\end{aligned}
\tag{3.16}
$$

where

$$
\begin{aligned}
P &= C_{33} - \omega_e^2 (M + A_{33}) + i\omega B_{33} \\
Q &= C_{35} - \omega_e^2 A_{35} + i\omega B_{35} \\
R &= C_{53} - \omega_e^2 A_{53} + i\omega B_{53} \\
S &= C_{55} - \omega_e^2 (I_{55} + A_{55}) + i\omega B_{55}
\end{aligned}
$$

It should be noted that the transfer functions are complex operators, with only the real part of the product having physical meaning. An equivalent expression which is used in Maxsurf and later in the computer programming is the modulus and phase of the transfer function $H(\omega)$, as shown below:

$$H(\omega) = \frac{\hat{\eta}}{A} = |H(\omega)| e^{i\alpha(\omega)} \tag{3.17}$$

However, the transfer functions are only one third of the picture. The other two parts are the hydrodynamic coefficients in the equations and the excitation forces. These values are difficult to compute, and that difficulty is compounded by the fact that many of the coefficients depend on the frequency of the incident waves. Additionally, the excitation forces depend on both the frequency and amplitude of the incident waves. The amplitude of the waves is treated by non-dimensionalizing the response. Forces and displacements are considered proportional to the amplitude of the wave, so the non-dimensional response is η / A and the excitation for is per unit amplitude. Likewise, the non-dimensional angular displacements are η / kA.

As mentioned, the added mass and damping coefficients are dependent on both the frequency and forward speed of the ship in a seakeeping scenario. This thesis treats the added inertia and damping coefficients for roll as constants. This simplifies calculations and provides a basis for comparison with the results for Maxsurf.

For pitch and heave motion, the Sea Keeper module of Maxsurf was used to obtain both the hydrodynamic coefficients and the exciting forces. The resultant transfer functions and the discretized wave spectrumwere transformed into time series for both pitch and heave. The wave heights and phases at many discrete frequencies were determined as discussed in section 3.3. The response at time t becomes:

$$\eta(t) = \sum_{m=1}^{\# freq} A_m RAO(\omega_m) \cos(\omega_m t + \psi_m) \tag{3.18}$$

Examples of the pitch response and heave response of the ONR tumblehome hull are shown in the figures below. The pitch and heave responses were used as inputs to the righting arm function discussed in section 5.4.

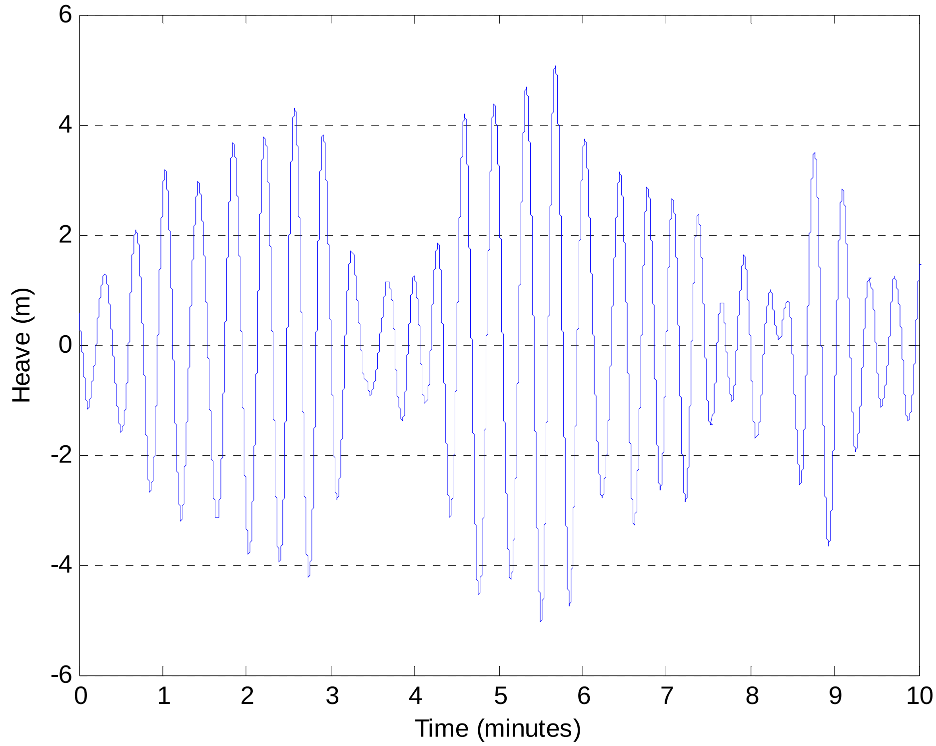


Figure 3-3: Heave Response in Sea State Eight Time Simulation from Sea Keeper

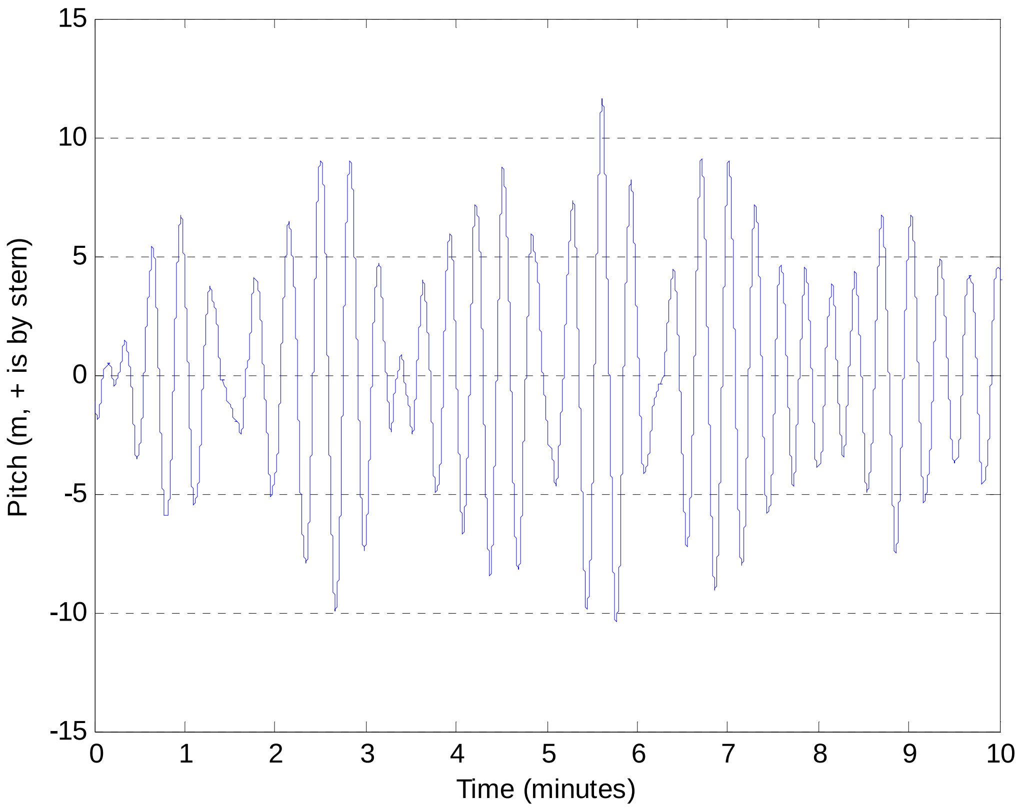


Figure 3-4: Pitch Response in Sea State Eight Time Simulation from Sea Keeper

The mean period for pitch is 17.3 seconds, with a root mean square average amplitude of 1.9 degrees. This translates to ±5.2 meters of pitch over the length of the ship. The heave period is 21 seconds, with an average amplitude of 2.4 meters.

3.4 Treatment of Roll in MAXSURF

Maxsurf uses the same linear sea keeping equations as described in Section 3.3 with three important differences.[4] First, roll is assumed to be uncoupled from sway and yaw. In other words, the roll degree of freedom is simulated as its own independent mass-spring-dashpot system. The reasoning for decoupling roll from sway and yaw is due to the large force and moment that the rudder places on a ship. Since this is often ignored in seakeeping, the relatively smaller wave force and moment are ignored. The roll equation with this assumption becomes

$$(I_{44} + A_{44})\ddot{\eta}_4 + B_{44}\dot{\eta}_4 + C_{44}\eta_4 = F_4 e^{i\omega_e t} \tag{3.19}$$

The second assumption that Maxsurf makes deals with the added mass and damping coefficients. In normal seakeeping theory, these coefficients are functions of excitation frequency and forward speed. This requires calculations in the frequency domain with an inverse Fourier transform (IFFT) to obtain a time domain solution. For Sea Keeper, they are assumed to be constants. The equation is then an ordinary linear differential equation in the time domain with constant coefficients, with the solution

$$\eta_4 = \frac{F_4}{\sqrt{(C_{44} - (I_{44} + A_{44})\omega_e^2)^2 + B_{44}^2 \omega_e^2}} \cos(\omega_e t + \alpha) \tag{3.20}$$

$$\tan(\alpha) = \frac{B_{44}\omega_e}{C_{44} - (I_{44} + A_{44})\omega_e^2} \tag{3.21}$$

The mass moment of inertia is calculated with a user-defined roll gyradius, as described in equation (2.1) . The added mass is set to 0.3 times the moment of inertia of the ship, and the damping coefficient is set by the user through the input of a damping ratio, β_{44}. For this research, the Maxsurf default value of 0.075 was used for all calculations. The damping coefficient is calculated according to the equation

$$B_{44} = 2\beta_{44}\sqrt{C_{44}(I_{44} + A_{44})} \tag{3.22}$$

Normally, the damping ratio is a function of frequency, making the equation true for each frequency separately. However, the frequency dependence is neglected by making β_{44} a constant. This permits a direct time domain solution.

Finally, Sea Keeper does not calculate the excitation moment using a rigorous method. It assumes that the moment may be approximated as the product of the wave slope and the hydrostatic righting moment, in phase with the wave slope. Therefore:

$$F_4 = kAC_{44} \tag{3.23}$$

where

$$C_{44} = \rho g \forall GM_t \tag{3.24}$$

Using all of the values from this section, the roll response amplitude operator for roll becomes

$$\frac{\eta_4}{kA} = RAO = \frac{C_{44}}{\sqrt{(C_{44} - (I_{44} + A_{44})\omega_e^2)^2 + \omega_e^2 B_{44}^2}} \tag{3.25}$$

The roll response amplitude operator may be used in conjunction with the wave height spectrum to produce a time series simulation just as in pitch and heave. An example of the time simulation for roll is shown below:

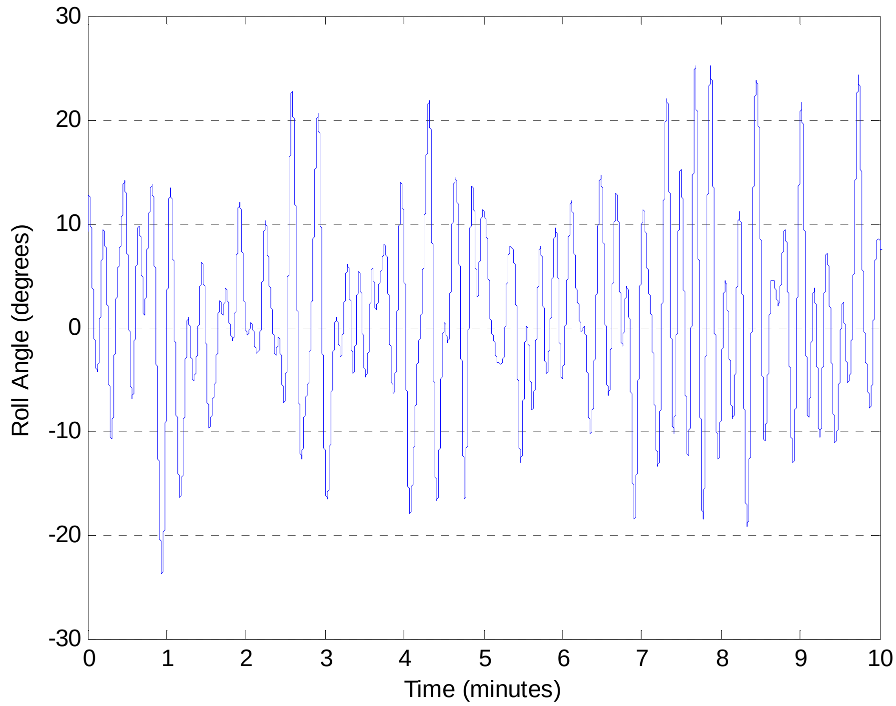


Figure 3-5: Roll Response Time Simulation from Sea Keeper

Although Sea Keeper's treatment of roll does not give the most accurate results, it does give a basis for comparison of the results of this research. Therefore, the new programs for making time simulations of roll angle utilized the same assumptions for moment of inertia, added inertia, and damping that Sea Keeper uses. The modifications used in this research are to the restoring coefficient C_{44} and the excitation moments. The non-linearities of the restoring coefficients have been discussed in section 2.3, and the treatment of the excitation forces will be discussed in the next section.

4 Roll Excitation Moment

Many programs provide a spectrum for the excitation roll moment due to incident waves. Sea Keeper's gives an order of magnitude estimation of the excitation roll moment. A better estimate can be obtained using the linearized potential due to incident waves. The water particles exert a dynamic pressure on the hull surface, which can be integrated over the surface of the ship to give a good approximation of the excitation moment. Typically, the moment is calculated in two parts. First, the moment due to the incident potential alone, called the Froude-Krylov moment, is calculated. The second part calculated is the moment due to diffraction of the wave as it interacts with the ship. Both the Froude-Krylov moment and the Diffraction moment will be discussed in this section.

4.1 Froude-Krylov Moment

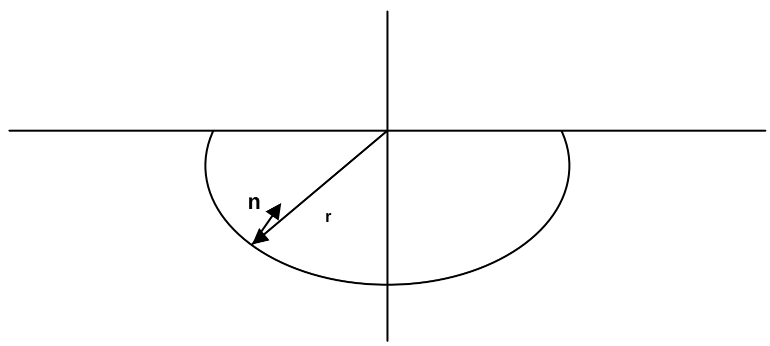
Linear plane progressive waves are assumed to impact the ship at an angle β to the stern of the ship. As discussed previously, the two dimensional incident wave potential is given by equation(3.7) . The moment resulting from the incident potential is given by the Froude-Krylov hypothesis, which is expressed as:

$$M_4^{FK} = -\rho \iint_C \frac{\partial \Phi_I}{\partial t} n_4 dS \quad (4.1)$$

In complex space, the Froude-Krylov hypothesis is written:

$$M_4^{FK} = -i\omega\rho \iint_C \Phi_I n_4 dS \quad (4.2)$$

In this case, the surface C is the wetted surface of the ship's hull in still water, and the normal vector component, n_4, is the cross product of the hull's two-dimensional normal vector with the position vector of the point in question, shown in Figure 4-1:



$n_4 = |\, \mathbf{r} \times \hat{\mathbf{n}} \,|$

Figure 4-1: Normal Vector for Moments

The method of numerical analysis becomes quite clear. For a given section, the normal and incident potential are known at each point in the table of offsets. The complex sectional Froude-Krylov force is expressed as:

$$f_4^{FK} = -i\omega\rho \sum_{n=1}^{\#points} \Phi_I(y_n, z_n) n_{4,n} \Delta\ell_n = \rho A g \sum_{n=1}^{\#points} e^{i(\omega t - k(x\cos(\beta) - y_n \sin(\beta)))} e^{kz_n} n_{4,n} \Delta\ell_n \quad (4.3)$$

And the overall moment can be calculated using a trapezoidal rule along the length of the ship:

$$F_4^{FK} = \sum_{m=1}^{\#stations-1} \frac{(f_{4,m} + f_{4,m+1})}{2}(x_m - x_{m+1}) \qquad (4.4)$$

4.2 Diffraction Moment

The excitation moment due to the diffraction of waves around the hull of the ship is more complicated. This thesis uses the calculation of the diffraction moment used by Milgram [6]. Like the Froude-Krylov moment, the diffraction moment may calculated as

$$M_4^D = -\rho \iint_C \frac{\partial \Phi_D}{\partial t} n_4 dS \qquad (4.5)$$

The problem is that the diffraction potential is not known, and more than likely can not be found as an explicit function of position and time. Therefore, numerical methods must be used to determine the diffraction potential before numerical integration can be implemented.

4.2.1 Diffraction Boundary Value Problem

Just as in the case of plane progressive waves, the diffraction potential must satisfy a fluid flow boundary value problem. The boundary value problem can be stated as follows: the total potential must satisfy mass conservation inside the fluid, with no flux along the hull of the ship or bottom boundary of the domain. The potential of the sides and free surface reflect the fact that the diffraction will produce waves radiating outward. Therefore, the diffraction potential satisfies the following equations:

$$\nabla^2 \phi_D = 0 \qquad (4.6) \qquad \text{(Field Condition)}$$

$$\frac{\partial \phi_D}{\partial n} + \frac{\partial \phi_I}{\partial n} = 0 \qquad (4.7) \qquad \text{(On the Hull)}$$

$$\frac{\partial \phi_D}{\partial z} = 0 \qquad (4.8) \qquad \text{(Bottom)}$$

$$-\omega^2 \phi_D + g \frac{\partial \phi_D}{\partial n} = 0 \qquad (4.9) \qquad \text{(Free Surface)}$$

$$\frac{\partial \phi_D}{\partial n} = -ik\phi_D \qquad (4.10) \qquad \text{(On the Sides)}$$

4.2.2 Numerical Determination of Diffraction Potential

Once the boundary conditions are known, the potential may be determined using a constant panel method using Green's theorem. The methd for determining the diffraction potential is modeled on that used by Milgram[6]. It is known that the potential obeys Laplace's equation in the field. Another function that satisfies LaPlace is required for Green's theorem. In this case, the Rankine Green's function was used. The Rankine Green's function takes the form

$$G(y,z,\eta,\varsigma) = -\ln\left(\sqrt{(y-\eta)^2 + (z-\varsigma)^2}\right) \tag{4.11}$$

This function therefore depends on the distance from the field point (y,z) and the source point (η,ς). Because both the diffraction potential and Green's function satisfy LaPlace's equation, Green's theorem states that the following equation is true along the boundary of the domain:

$$\oint_S \left(\Phi \frac{\partial G}{\partial n(\eta,\varsigma)} - G\frac{\partial \Phi(\eta,\varsigma)}{\partial n(\eta,\varsigma)}\right) ds = -\pi\Phi(y,z) \tag{4.12}$$

To solve this numerically, the domain must be divided into several panel line elements. Along each panel, the values of Φ and δΦ/δn in the equation above may be assumed to be constant. This means that the panels must be small enough that the potential does not change significantly across the panel. This becomes important is selecting the domain size, since more panels significantly increases the computation time for each section. The result becomes a summation throughout the domain, namely:

$$\sum_{i=1}^{N} \Phi_j \int_{L_j} \frac{\partial G_{ij}}{\partial n_j} d\ell_j + \pi\Phi_j\delta_{ij} = \sum_{j=1}^{N} \frac{\partial \Phi}{\partial n_j} \int_{L_j} G_{ij} d\ell_j \tag{4.13}$$

The integrals in the above equation can be computed numerically as well. The boundary conditions provide relationships to eliminate the derivative of the diffraction potential at each point. This reduces the system to a set of N linear equations for the diffraction potential.

For the problem, a two dimensional domain chosen for this problem is a box of ocean fluid with the hull in the center of the top, as shown in Figure 4-2:

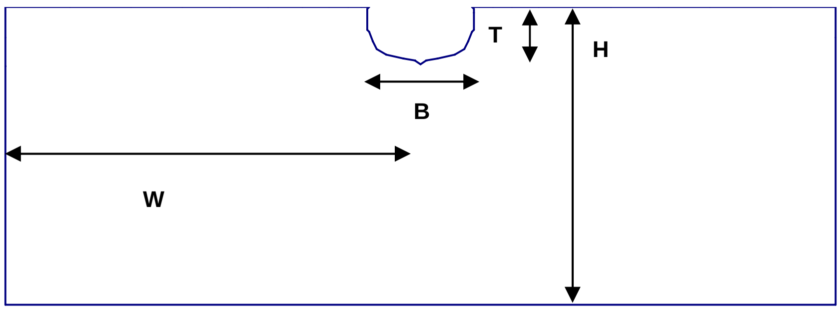


Figure 4-2: Domain for Diffraction Boundary Value Problem

The half-width of the domain W was set to ten ship beams. This width was determined to be acceptable for convergence by Milgram [6]. The depth H was set to one half the wavelength of the incident wave or three times the draft T, whichever was greater. This brings the domain to where the incident potential is near zero for long waves, and covers the bottom of the hull for short waves. The number of panels along the free surface balanced computation time against panel length. At 55 panels on either side of the ship, the panel length is approximately 1.5 meters. With a cutoff frequency of 2 radians per second, this means that each panel is less than one tenth of the wavelength even at the shortest wavelength.

5 Matlab Code Description

The objective for the computer programs is to obtain a reasonable estimate to the non-linear roll response using a desktop computer without requiring unreasonable computation time. For the most part, the objective is accomplished well. The programming language used is Matlab, release 14. The computer used was a Dell Optiplex GX620, with a 3.2 GHz Pentium 4 processor and 1GB of memory.

5.1 Files Required from Maxsurf

The computer codes written for this research require three files containing output from Maxsurf. First, a table of offsets for the hull in question must be generated for use in the force calculator program. This file is in Microsoft Excel format, with four columns. The first column is the station number of the offset point. The program is designed to handle an arbitrary number of stations. The next three columns are x-coordinates, z-coordinates, and y-coordinates, respectively. The reason for the switch in y- and z-coordinates is that the offsets may be cut and pasted directly from a table in Maxsurf. The title of the file must be ‘offsets.xls.’ Generation of this file in Maxsurf took approximately four hours. Most of this time was generating offsets from the imported surface.

The next file required for the programs is used in the linear response generator. This file contains the response amplitude operators for pitch, heave, and roll. Each RAO is defined as a 500 element row vector for amplitude and phase. Additionally, the corresponding frequencies, encounter frequencies, and wave amplitudes calculated as described in section 3.2 are included in the file. The file therefore contains nine row vectors with the following names: HRAO (heave response); Hph (heave phase); RRAO (roll response); Rph (roll phase); PRAO (pitch response); Pph (pitch phase); freqs (wave frequencies); efreq (encounter frequencies); and A (wave amplitudes). The name of the file is ‘raofunctions.mat.’ This file required about one hour to generate once the hull was saved as a Maxsurf file.

A complication to generating this file was the length of the vectors. The simulation included frequencies up to 2 radians per second, and 500 discrete frequencies were desired to provide adequate randomness for a 24 hour period. Since Maxsurf could only produce 132 frequencies in this range, Matlab’s interp1 function was used to interpolate values for the 500 elements of each variable. This did not significantly increase the preparation time for the file.

Finally, the righting moment lookup table is required. This is a three-dimensional lookup table with reference values for each index. The method for generating the lookup table utilized Maxsurf's Hydromax module. At each draft, pitch was fixed at values from 20m by the stern to 20m by the bow (+/- .133 radians. This ensures all reasonable values of pitch angle in the time simulation can be accommodated. The data was collated in a spreadsheet table in the format shown below:

Displ(kg)	LCG(m)	VCG(m)	T(m)				
1.6E+07	-10.8	7.5	9				
			Roll(degrees)				
Trim(m)			0	10	20	30	40
	20		0	3143262	6111013	8727739	10754106
	15		0	3079440	6126968	8951118	11216820
	10		0	2584815	5552565	8647961	11312554
	5		0	1850855	4244202	7690622	11137042
	0		0	1595565	3685754	6797106	10770062
	-5		0	1771077	4004867	7068352	10227570
	-10		0	2185924	4659049	7195997	9174497
	-15		0	2313569	4531404	6398214	7770400
	-20		0	2058278	3893178	5408964	6462037

Displ(kg)	LCG(m)	VCG(m)	T(m)				
1.5E+07	-10.625	7.5	8.75				
			Roll(degrees)				
Trim(m)			0	10	20	30	40
	20		0	3208086	6246513	8914777	10966101
	15		0	3146392	6261937	9146129	11459652
	10		0	2544876	5537033	8745118	11521346
	5		0	1804548	4148919	7557510	11197453
	0		0	1542349	3578249	6601253	10611360
	-5		0	1712007	3855872	6755488	9948150
	-10		0	2066748	4395694	6940570	8961047
	-15		0	2205559	4364847	6246513	7650051
	-20		0	2005054	3825025	5336527	6385324

Table 5-1: Righting Moment vs. Roll, Pitch, & Draft

The table contains restoring moment divided by the acceleration due to gravity (mass times righting arm) for each value of heave, pitch, and roll. The two-dimensional table of righting moment as a function of roll and pitch are a two-dimensional table for the i^{th} set of the variable RM. The index vectors contain the sequential values of trim, roll, and draft. The titles of these vectors are drafts, pitchs, and rolls. The pitch is in radians from high to low, while the roll is in radians from 0 to 1.74 radians. Drafts are in 0.25 meter increments from two meters to nine meters. This file is used in the non-linear response generator. The entire file for nine pitches, eleven roll angles, and 29 drafts took approximately eight hours to compile.

5.2 Excitation Force Calculator

The Matlab program ForceFinder calculates the linear excitation moment of a ship from offsets provided in an external file using the theory presented in Chapter Four. The program is designed to calculate the Froude-Krylov and Diffraction excitation moments for a user defined number of different frequencies. As mentioned, the program requires a table of offsets with points designated in meters from the origin. At each frequency, the program determines sectional moments per unit length of the ship at each section. These sectional moments are then integrated over the length of the ship to obtain the three dimensional excitation force. A block diagram of the program is shown in figure 5.1.

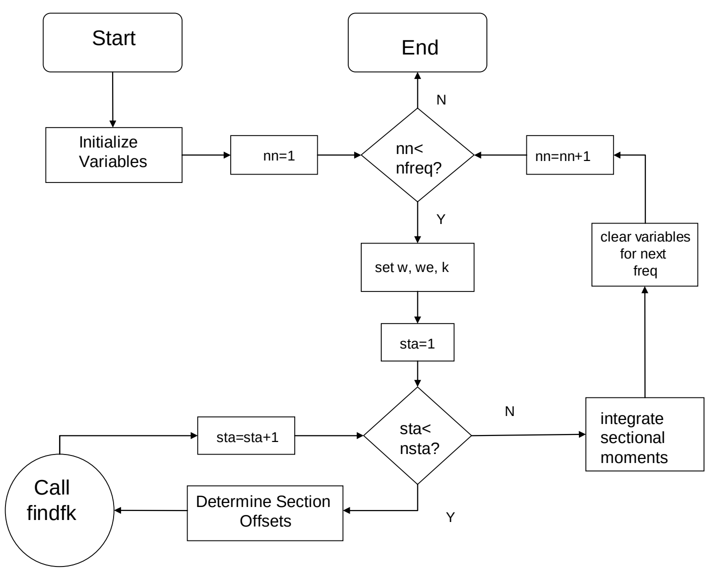


Figure 5-1: ForceFinder Block Diagram

ForceFinder uses several subroutines to determine the sectional moments. The main subroutine is 'findfk'. This subroutine takes a section's offsets, determines the sectional excitation moments for the frequency passed to it. First, it generates the domain shown in figure 4-2 using the subroutine 'setpans'. Next, the subroutine 'inflcoef' calculates the integrals of Green's function and its derivative at each point due to every other point. The subroutine 'findmatrix' uses these values and the boundary conditions to calculate the equations for determination of the diffraction potential. With the diffraction potential known, the subroutine calculates both the Froude-Krylov and diffraction moments through numerical integration.

Although this process is straightforward, a block diagram of findfk is presented in figure 5-2. The code for ForceFinder, findfk, and findmatrix are included in section 1 of Appendix A.

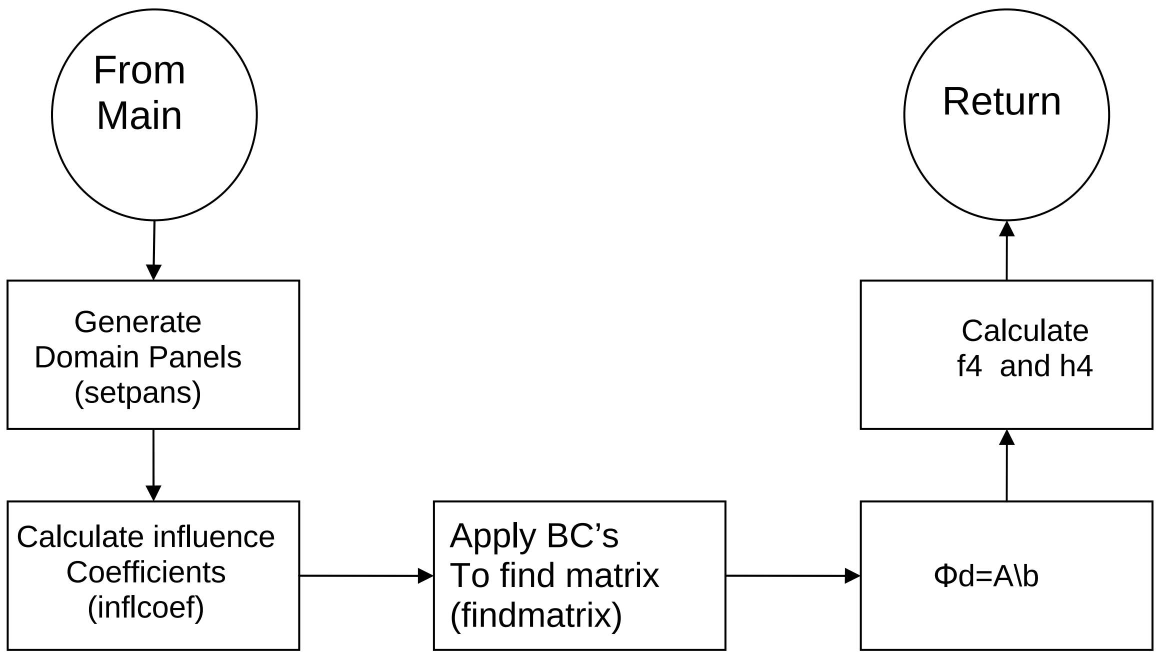


Figure 5-2: findfk Block Diagram

For the research program, the program was set to generate excitation moments at 500 frequencies equally divided from zero to 2 radians per second. This matches the number of wave amplitudes generated for the linear response spectrum. In this way, the random phasing in the linear response may be applied to the excitation moments as well. At each frequency, the moment and amplitude for a one meter wave was calculated. The result is a complex number, which may be interpreted as a modulus of moment and a phase relative to the wave height amidships. The output of the program was saved as a Matlab data file titled ‘forcedata500.mat’ for use in the non-linear time simulations. The excitation moment per unit wave amplitude for Sea State Eight is shown in figure 5-3:

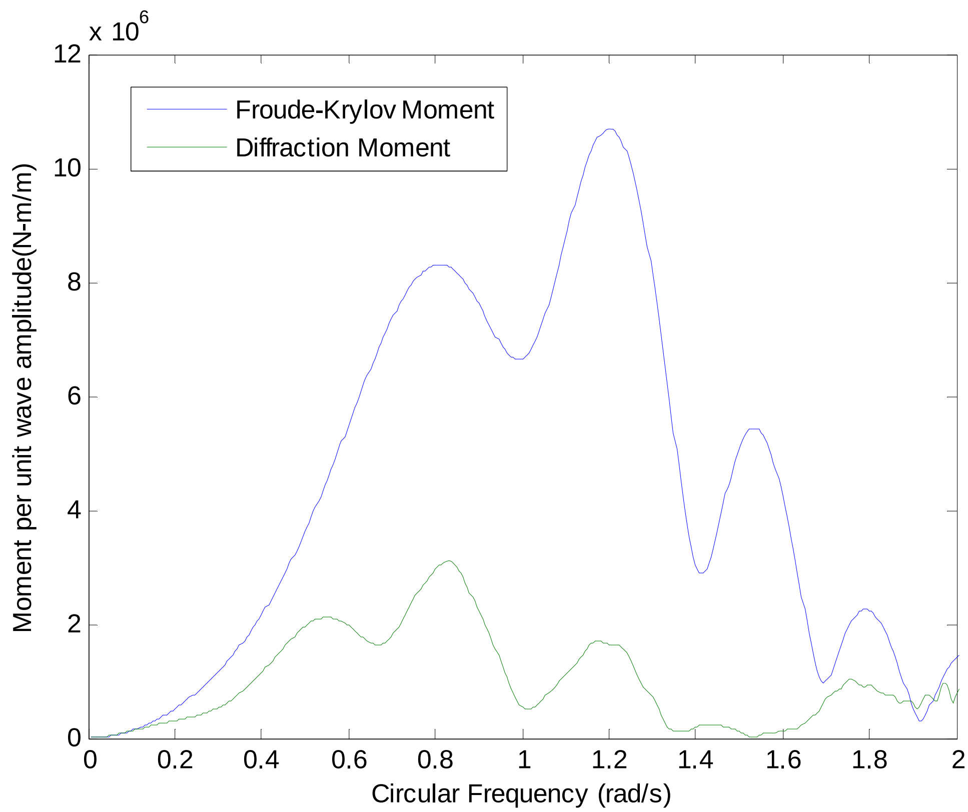


Figure 5-3: Excitation Moments for Seakeeping Problem

ForceFinder has the largest run time of the three programs described. Because of the large number of panels in the diffraction boundary value problem, the program requires approximately 50 seconds to find the total excitation force for each frequency. To match the 500 frequencies used in the spectrum response calculator, the program run time was 6.9 hours. The author acknowledges there are many programs available to evaluate the excitation force that are faster. These may be used as long as the output is in the form of a Matlab vector labeled F4 containing complex force magnitudes and is saved as ‘forcedata500’.

5.3 Linear Response Generator

The name of the linear response program is LinearResponse. Although it does not need any subroutines to operate, it does need the file ‘raofunctions.mat’ to generate a time simulation of heave, pitch, and roll. Although only the heave and pitch vs. time are required for use in the non-linear program, the roll was calculated for comparison to the time simulation generated in the non-linear roll simulation.

The linear response program and force program should be run with the same set of frequencies in their spectra. The linear response program calculates the wave amplitudes and random phases for each frequency in the simulation. The moments found in ForceFinder have a phase relative to the phase of their respective discrete wave system. If the frequencies are not matched prior to running the two programs, a certain amount of interpolation must happen to synchronize the frequencies before the final program can be run. This extra work is easily avoidable by using the same set of frequencies for the separate programs.

The linear response program is quite simple. For each time, the response may be expressed as a sum of the linear response at each frequency. This may be written as

$$\eta_j = \sum_{i=1}^{\#\,freq} A_i \left|H(\omega_i)\right| \cos(\omega_i t + \psi_i + \alpha_i) \quad \text{j=3,4,5} \tag{5.1}$$

The file raofunctions.mat described in section 5.1 contains the wave amplitudes, transfer function moduli, and transfer function phase angles for these calculations. Therefore, the program generates a random phase angle for each frequency, and performs the summation of discrete responses at each time to produce a wave height, pitch angle, roll angle, and heave.

The output of this program is the time series simulations of wave height, pitch, heave and roll. Also in the data set will be the frequencies, encounter frequencies, and wave amplitudes for the sea simulation. The output was saved as a file titled ‘linresp100.mat’ for use in the non-linear response simulation program. Sample graphs of the time simulations were shown previously in

Figure 3-3, Figure 3-4, and Figure 3-5. Run time for this program is approximately 40 seconds. The code is included in Appendix A.

5.4 Non-Linear Response Generator

The name of the non-linear response program is ‘rollintegration.m’. It uses one subroutine, a three dimensional lookup and interpolation routine to determine the righting moment as a function of pitch, roll, and heave. For this program to work properly, it requires the output of ForceFinder.m saved as a file titled ‘forcedata500.mat’, the output of LinearResponse.m saved as ‘linresp100.mat’, and the righting moment lookup table saved as ‘Rmomentdata.mat’.

The program uses a forward Euler scheme to integrate the initial value problem defined by equation (3.19). Here, however, C_{44} at each time step is the non-linear function $C_{44}[\eta_3(t),\eta_4(t),\eta_5(t)]$. Since this equation is a second order differential equation, it must be treated as two first order equations to evaluate it numerically.[7] Therefore, using dummy variables q_1 and q_2, equation (3.19) becomes

$$\begin{aligned}\dot{q}_2 &= (M(t) - C(t,q_1) - B_{44} * q_2)/(I_{44} + A_{44}) \\ \dot{q}_1 &= q_2\end{aligned} \tag{5.2}$$

At each time step, the excitation moment is determined by the equation

$$M(t) = \sum_{n=1}^{\#\,freq} \left|F_4 + H_4\right| \cos(\omega_n t + \psi_n + \alpha_n) \tag{5.3}$$

Initial conditions for both roll angle and its first derivative are chosen to match the output from ‘linresp100.mat’. The second time derivative is evaluated for the present time step and then integrated to give the first derivative at the next time step. The first time derivative at the present time step is integrated to calculate the roll angle at the next time step. At any time step, if the non-linear roll angle grows to greater than ninety degrees, the counter for catastrophic roll events is advanced by one and the roll angle and speed are reset to the values of the linear integration at for that time. The block diagram for this program is shown in Figure 5-4.

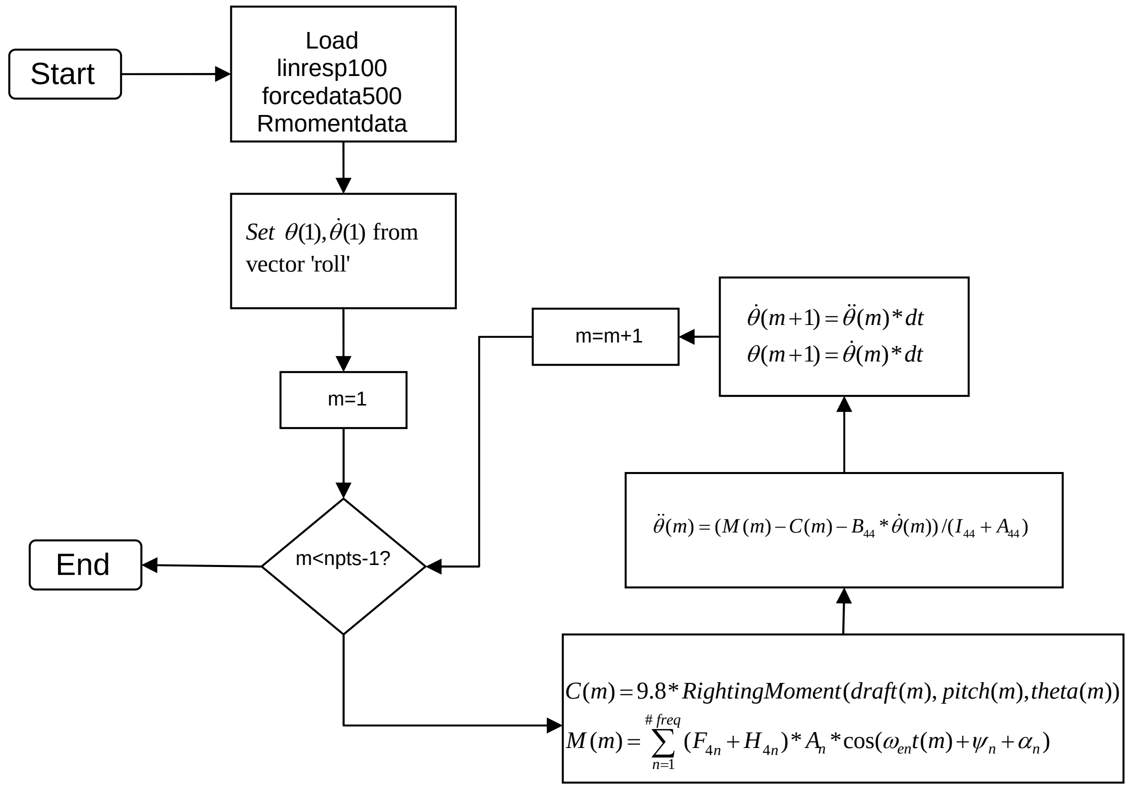


Figure 5-4: Block Diagram for Non-Linear Roll Integration

An additional capability written into the program is to conduct the integration using the same linear restoring coefficient that Maxsurf's Sea Keeper module uses, using equation(3.24). The linear and non-linear integration occur in the same loop, so little extra computation time is required for this addition. The programs as written in Appendix A are designed to use 500 frequencies up to a value of 2 radians per second to integrate the roll equation for one hour using a time step of 0.01 seconds. The run time for rollintegration is approximately one minute. The small time step is required for numerical convergence. For such a small time step, the Forward Euler method gives very nearly the same results as more complicated integration schemes such as Fourth Order Runge-Kutta. Convergence with a larger time step for implicit integration rules remains to be investigated for solving the roll differential equation.

6 Simulation Results

6.1 Sea Keeper and Linear Integration Simulations

The first comparison that can be made using the output of the integration program is between the Sea Keeper module of Maxsurf and the integration using the linear forces and a linear restoring coefficient. As noted in section 3.4, Sea Keeper uses an approximation of wave slope times the hydrostatic restoring coefficient for its excitation force. To compare the two excitation forces, the output from the ForceFinder program was normalized by the value of the Sea Keeper excitation force (kAC_{44}). This comparison is shown in Figure 6-1. The figure shows the normalized modulus of the excitation force vs frequency, along with the constant value of kAC_{44} that Sea Keeper uses as its excitation force. The largest discrete wave heights will be near the modal frequency, which is also shown in the figure.

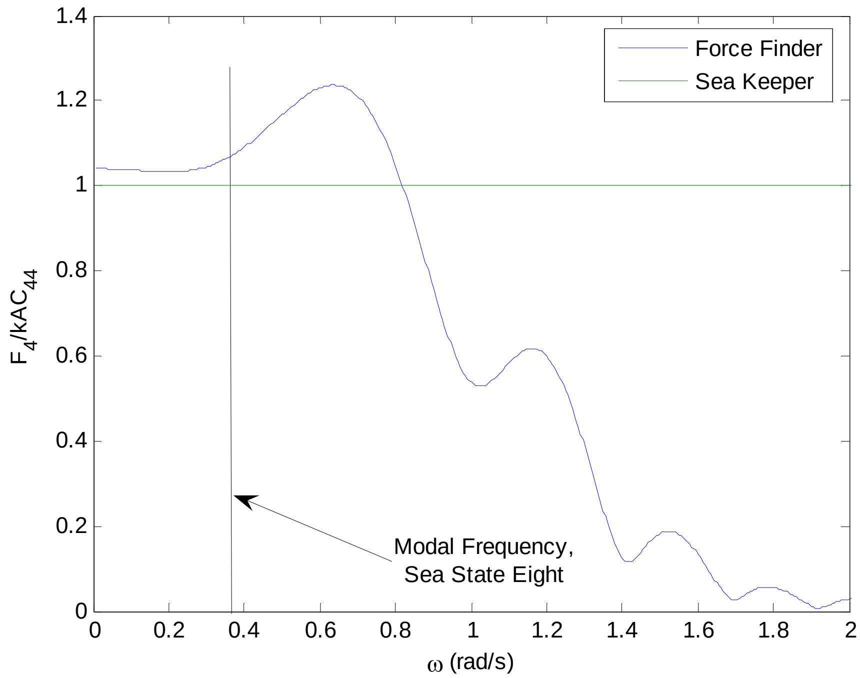


Figure 6-1: Modulus of Non-dimensional Excitation Force vs. Frequency

The phase angles of the excitation forces are not shown. Since the calculated excitation moment is computed as a complex number, the phase varies with the frequency of the wave. On the other hand, the Sea Keeper excitation moments do not include a phase shift, assuming that the excitation force always acts in phase with the wave height. While this assumption may be valid for longer wavelengths, it does not hold true for shorter waves. This difference does not significantly affect the amplitude of the response.

The modulus of the calculated force is larger in the area of the largest discrete wave heights. Therefore, the overall calculated force should be larger than the estimations used by Sea Keeper. The linear integration uses the same value of C_{44} as the Sea Keeper program. Therefore, the only difference of the time series for roll between the integration scheme and Sea Keeper is the excitation forces. The larger forces in the integration should result in a larger average amplitude for roll response. This is in fact the case. The average amplitude was determined by calculating the standard deviation of the time series of the two different methods. The amplitude of the Sea Keeper time series was 8.8 degrees, while the linear integration average amplitude was 11 degrees. A plot of the first five minutes is shown in Figure 6-2 for comparison of the two.

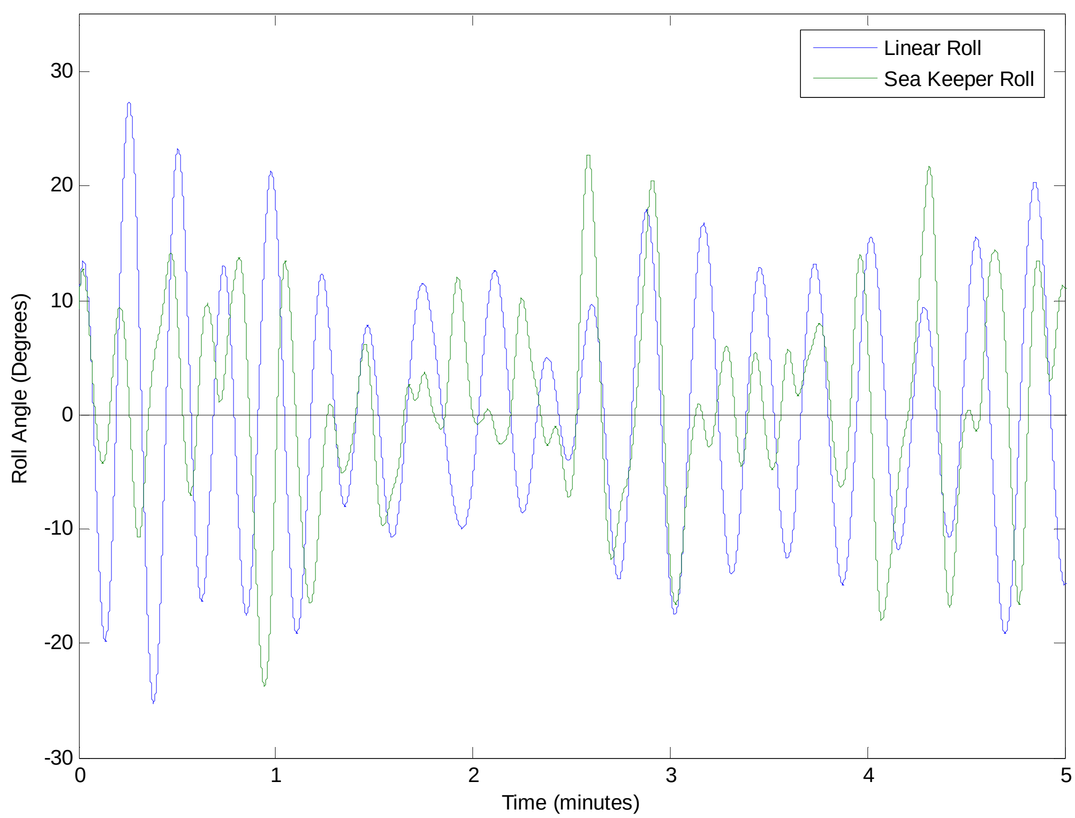


Figure 6-2: Roll Angle vs Time from Linear Integration and Response Spectrum

The period can be approximated by dividing the time period by the number of zero crossings in the simulation. If this is done, the roll period for the linear integration is 17.3 seconds, while the period of the Sea Keeper simulation is 14.1 seconds. Both of these agree with the natural period estimated section 2.2. The difference in them reflects the concentration of roll excitation at different frequencies due to the different functions of the roll excitation moment. The results of the two different methods are rather close, showing that the force calculation program appears to give results that are valid. The roll period is long because of the forward speed effects in stern quartering seas. The forward speed concentrates the sea state's excitation forces at lower frequencies, causing a longer roll period.

6.2 Linear Integration and Non-Linear Integration

The linear time simulation and the non-linear time simulation have only one difference. The restoring force in the non-linear case is a function of pitch and heave, as well as a function of roll. As noted in section 2.3, the restoring coefficient could change by up to a factor of three due to pitch about its still waterline. The response should have approximately the same frequency behavior, but there should be periods where the amplitude of the linear model is larger than that of the non-linear model, and periods where the reverse is true.

Surprisingly, very few cases could be found where the linear response had larger amplitude. In Figure 6-3 below, three such occurrences happen in five minutes. As you can see, the larger linear response is not much larger than the non-linear response. Conversely, Figure 6-4 shows the much more frequent case of the larger non-linear response. The non-linear response peaks tend to be much larger. This indicates that the non-linear effect is a net loss in stability for the ship as it pitches and heaves in stern quartering seas.

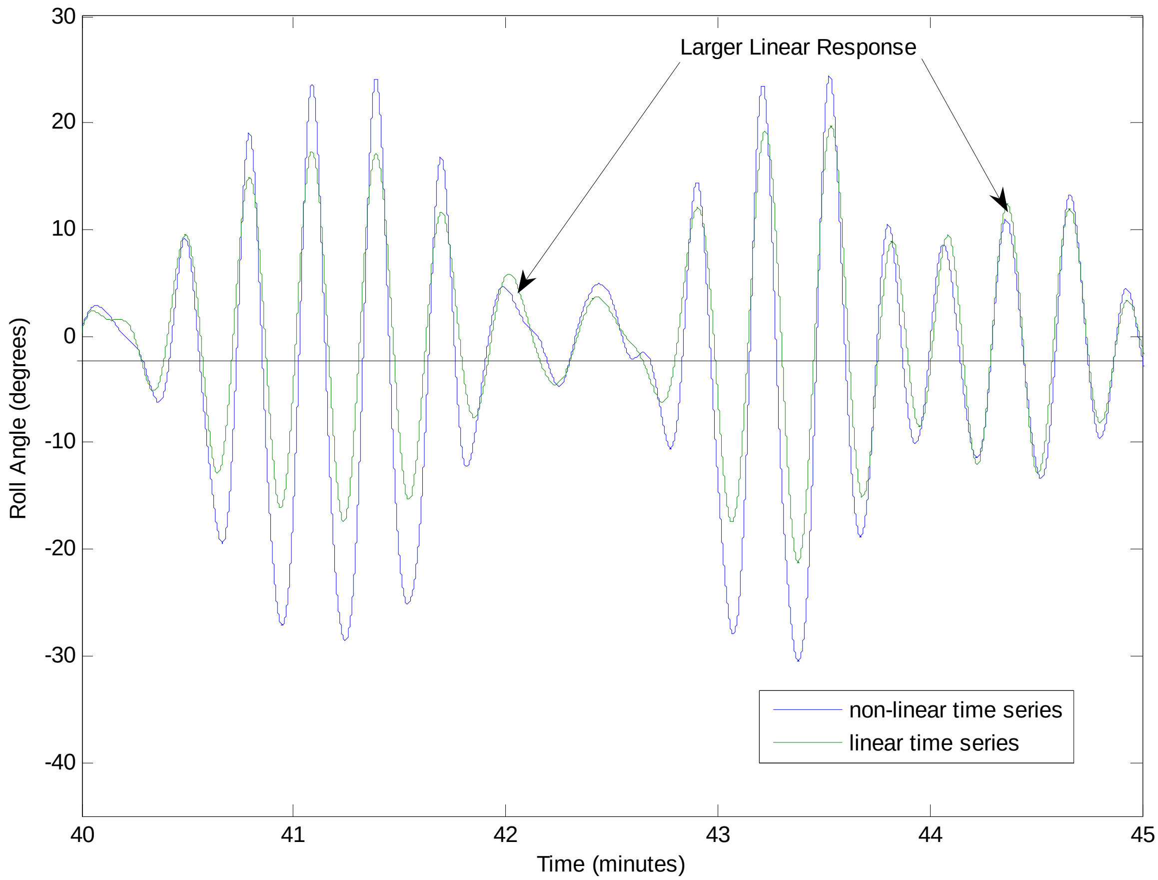


Figure 6-3: Linear Simulation vs. Non-linear Simulation with Larger Rolls in the Linear Case

The period of the two integration schemes were very similar. While the linear integration period was 17.3 seconds, the non-linear time series showed a period of 17.0 seconds. The amplitudes of the responses were quite different, however. As noted before, the standard deviation of the linear series was 11 degrees. The non-linear roll had a standard deviation of 16.9 degrees, which is an increase of more than fifty percent. With larger roll amplitude and the same period, this implies much greater roll speed and acceleration. Therefore not only is the hull less stable when treated in a non-linear fashion, it would be less comfortable for crew as well.

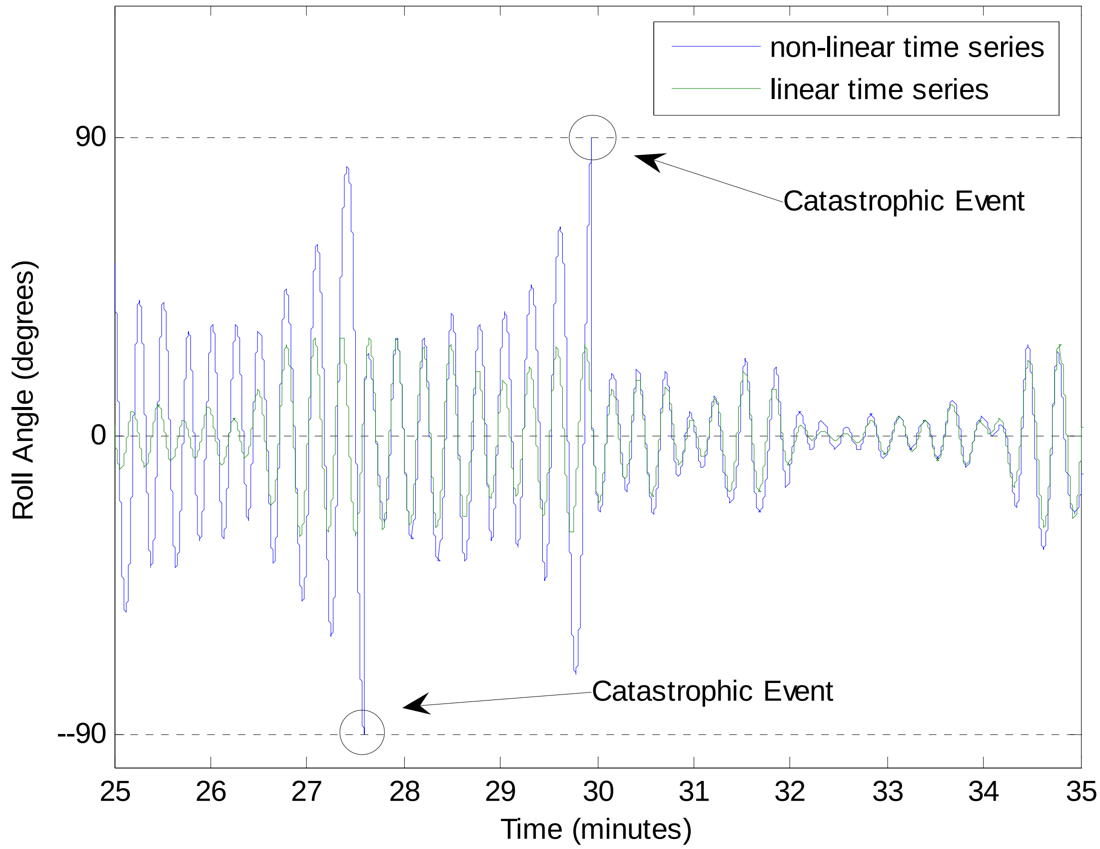


Figure 6-4:Linear Simulation vs. Non-linear Simulation with Larger Rolls in the Non-linear Case

Examination of Figure 6-4 shows a phenomenon that is unique to the non-linear case. The roll angle exceeds ninety degrees. This large of a roll angle exceeds the ability of the program to determine the righting moment, and can be called a catastrophic roll event. This hull form may have a positive righting moment at roll angles of more than ninety degrees, but a roll of more than ninety degrees could be catastrophic nonetheless for two reasons. First, anything inside the ship that is not tied down will undoubtedly move towards the furthermost hull. Among other problems this represents, the change in the center of gravity will cause an upsetting moment within the ship. The second undesirable effect would be downflooding. The downflood angles of this hull design are not designated, but immersing half of the main deck and superstructure should almost certainly cause down flooding even for roll angles less than 90 degrees.

Overall, the ship experienced two catastrophic roll events in the one hour simulation during the non-linear case. Neither linear simulation contained a similar event. A comparison of average period and amplitude for the three cases is shown in Table 6-1.

Simulation	Period(sec)	Amplitude(degrees)	Catastrophic Events
Sea Keeper	14.1	8.8	0
Linear Integration	17.3	11	0
Non-Linear Integration	17.1	16.9	2

Table 6-1:Comparison of Different Roll Response Calculations

One interesting use for the integration program was to evaluate the influence of initial trim on the stability of the program. A 0.5 meter trim by the stern was added to the pitch response before the time series for roll was generated. Even this small amount of pitch was beneficial. Trimming the ship reduced the mean amplitude of the roll by three degrees in the non-linear case and reduced the number of catastrophic events to one. Obviously, in a real situation, additional actions can be taken to prevent such large roll events. As stated in section 2.4, the high seas, speed of the ship, and angle of incidence of the incoming waves are all chosen to evoke a maximum roll response. Therefore, slowing the ship and maneuvering for bow seas would further improve the ship's seakeeping ability.

7 Future Work and Conclusion

7.1 Added Mass and Damping

As pointed out in section 3.3, the added mass and damping coefficients were treated as constants instead of functions of frequency. An addition to the excitation moment calculator program could be the solution to the radiation potential at each frequency, which will give the added mass and damping coefficients for use in the non-linear integration problem. The frequency dependent added mass and damping can be calculated by a numerical problem very similar to that used to find the diffraction force[6]. Only the boundary conditions on the ship surface are different.

7.2 Directional Seas

The waves that are simulated in this thesis are long-crested waves assumed to be from a distant storm. Simulation of multidirectional random seas may be included in this analysis through methods described in previous research from Mr. Sam Geiger[8]. The unidirectional spectrum could be replaced with a sum of several unidirectional seas with different headings, all adding up to the same energy as the single unidirectional sea state. This inclusion would provide a better estimate of the excitation forces involved in real seas.

7.3 Non-linear Excitation Moments

The Froude-Krylof and Diffraction excitation moments were calculated using linear strip theory along the still waterline of the ship's hull. In reality, the moments will depend on the actual waterline of the ship as the waves travel in space. The moment will also depend on the amount of the hull in the water as the ship is displaced in roll, heave, and pitch. Calculation of these effects is left as a topic of further research.

7.4 Wave Profile Effects on Righting Moment

The righting moment for the tumblehome hull was calculated using the draft of the ship at amidships, assuming no effects for wave height or wave slope. The draft at other positions along the length of the ship is a function of the longitudinal wave height and the pitch of the ship relative to the wave slope. Therefore, the righting moment may be refined by taking these two factors into consideration.

7.5 Conclusion

The non-linear roll integration program developed in this thesis can provide an estimate of the roll response for any hull, given its offsets and the righting moment data. Although the force calculation program takes some computation time, any available program that determines the excitation force may be used to give the roll integration program its input. The program could be used in any number of design applications, such as determination of a safe operating envelope for a ship, or the effects of different trim configurations on stability. Most importantly, it provides a non-linear simulation in a relatively short time using only a desktop computer. This fact makes the computer program a valuable design tool.

List of References

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Appendix A: Matlab Code for Finding Excitation Forces

1. Main Program: 'ForceFinder.m' – This program needs the following subroutines to function : findfk, findmatrix, setpans, inflcoef, rank2d, localize. Also needs a Microsoft Excel file with offsets provided as described in the program. The m-files inflcoef, rank2d, and localize were provided by Professor Jerome Milgram.

```
%  Finds Froude-Krylov and Diffraction Excitation Moments
%  for frequencies up to 2.5 rad/s, unit amplitude wave
%  Requires the following files to operate:
%  offsets.xls:  table of offsets in MS Excel with the following columns:
%   Col 1: station number (can handle any number of stations)
%   Col 2: x-values (m)
%   Col 3: z-values (m)
%   Col 4: y-values (m)
%  Subroutines: findfk, findmatrix, setpans, inflcoef, rank2d, localize
%  # of frequencies is set by nfreq
%  output is in the variables F4, H4
% for the research, set nfreq to 500, save data as forcedata500

clear all
close all
tic
timestampb=clock;
starttime=timestampb(4)*100+timestampb(5);
fprintf('start time =%1.0f\n',starttime)

% initialization of variables:
%   rho: density of seawater (kg/m^3)
%   nsta: # of stations (from offsets file)
%   B: beam of ship (m)
%   gr: acceleration due to gravity
%   npts: number of offsets
%   U: forward speed of the ship
%   nfreq: number of frequencies in interval 0:2.5
%   F4: Foude Krylov excitation moment
%   H4: Diffraction excitation moment
%   wo, we, k: stationary frequency, encounter frequency, wave number
%   T: draft
%   beta: angle of incidence of waves

rho=1025;
A= xlsread('offsets');
nsta=max(A(:,1));
B=2*max(A(:,3));
gr=9.8;
```

```
[npts,d]=size(A);
U=12.8;
nfreq=500;
T=5.5;
beta=pi/3;
F4=zeros(nfreq,1);
H4=zeros(nfreq,1);

for nn=1:nfreq
   wo=2.5*nn/nfreq;
   k=wo^2/gr;
   we=wo-k*U*cos(beta);

   % set up p (position matrix) for findfk
   for sta=1:nsta
      clear p
      q=1;
      for r=1:npts
         if A(r,1) == sta
            p(1,q)=A(r,2);
            p(2,q)=A(r,4);
            p(3,q)=A(r,3);
            q=q+1;
            x(sta)=A(r,2);
         end
      end
    % get sectional FK, Diffraction Moments
   [f4(sta),h4(sta)]=findfk(p, wo, we, k,T, beta,B,sta);
   end

   % Trapezoidal Rule Integration of Moments
   F4(nn)=0;
   H4(nn)=0;
   for sta=1:nsta-1
      dx=x(sta)-x(sta+1);
      F4(nn)=F4(nn)+dx*(f4(sta)+f4(sta+1))/2;
      H4(nn)=H4(nn)+dx*(h4(sta)+h4(sta+1))/2;
   end
   F4(nn)=rho*gr*F4(nn);
   H4(nn)=rho*H4(nn);

   % reset for next frequency
   clear f4
   clear h4
   % counter for status
   if nn/10 == round(nn/10)
      fprintf('n= %1.0f\n',nn)
      toc
   end
```

```
end
toc
```

1.a. Subroutine: 'findfk.m'

```
function [f4, h4] = findfk(p, w, we, k, T, beta,B,sta)
% Finds sectional Froude Krylov & Diffraction Excitation Moments for
% ForceFinder
% p is matrix of offsets, a is incremental waveheight, psi is random phase angle,
w,k are freq & wave nr
%KG is height of cg above baseline T is still water draft
%posit(1,:) are y-values, posit(2,:) are z values in the section plane
%x is the longitudinal position of the section
%beta is the angle of incidence of the incoming wave, measured in radians from
the stern
%eta1 & eta2 are the actual waterlines of the ship after translation and
application of the wave
%r & q are the point indices corresponding to eta1 & eta2
%f2 & f3 are the two-d F-K forces for the section
%posit is a matrix of y,z ordered pairs
posit(1,:)=p(2,:);
posit(2,:)=p(3,:);
x=p(1,1);
[d,npts]=size(posit);
posit(3,:)=zeros(1,npts);
gr=9.8;
c1=gr/w;
c2=k*x*cos(beta);
c3=k*posit(2,:)-T;

% This block of code calculates the 2dimensional diffraction potential for the
section
npv=10;
nph=55;
npb=10;
% defines domain for constant panel method
[np,npanels,yvert,zvert,ybv,zbv,ycontrol,zcontrol,lgth,ny,nz] =
setpans(posit(1,:),posit(2,:)-T,npv,nph,npb,B,k,T);
% finds G, dG/dn for each panel
[g, dgdn] = inflcoef(npanels,yvert,zvert,ycontrol,zcontrol,lgth);
% finds A & b in discrete BVP A(phi)=b
[Amatrix,b] =
findmatrix(np,nph,npv,npb,w,we,k,beta,x,ycontrol,zcontrol,ny,nz,g,dgdn);
phid=Amatrix\b;

% determines total disturbance potential
for m=1:np-1
   phiI=c1*exp(i*(c2+k.*sin(beta)*posit(2,m)))*exp(c3(m));
   PSI4(m)=phiI+phid(np+nph-1+m);
```

```
    phil=c1*exp(i*(c2-k.*sin(beta)*posit(2,m)))*exp(c3(m));
    PSI4left(m)=phil+phid(np+nph-m);
end

%Translation to origin, which is still water WL
posit(2,:)=posit(2,:)-T;
WL=0;
f4=0;
h4=0;

% q is number of points below waterline
if min(posit(2,:))>WL
    fprintf('station is above water\n')
    return
end
if max(posit(2,:))>WL
    q=1;
    while posit(2,q)<WL
        q=q+1;
    end
    q=q-1;
    fprintf('%1.2f out of %1.2f points\n',q,npts-1)
else
    q=npts-1;
    fprintf('station is below water\n')
end
r=q;

aa=zeros(3,npts);
%find forces on 'the right half' of the hull
for j=1:q

    % compute normals
    aa(:,j)=cross(posit(:,j+1)-posit(:,j),[0;0;-1]);
    amag=sqrt(aa(1,j)^2+aa(2,j)^2);
    n3(j)=aa(1,j)/amag;
    n2(j)=aa(2,j)/amag;
    mid=(posit(:,j)+posit(:,j+1))/2;
    n=[n2(j) n3(j) 0];
    n4v=cross(mid,n);
    n4(j)=n4v(3);

    z1=posit(2,j);
    z2=posit(2,j+1);
    dl=sqrt((posit(1,j)-posit(1,j+1))^2+(posit(2,j)-posit(2,j+1))^2);

    % calculate the incremental excitation moment
    pt1=exp(k*posit(1,j)*sin(beta)*i)*exp(k*z1)*exp(-i*k.*x*cos(beta));
    pt2=exp(k*posit(1,j+1)*sin(beta)*i)*exp(k*z2)*exp(-i*k.*x*cos(beta));
```

```
    f4=f4+n4(j)*(pt1+pt2)/2*dl;
    h4=h4-i*we*(n4(j))*dl*PSI4(j);
end
%find forces on 'the left half'

%first, change the y-values to negative
posit(1,:)=-posit(1,:);

clear n3
clear n2
for j=1:r

    aa(:,j)=cross(posit(:,j+1)-posit(:,j),[0;0;-1]);
    amag=sqrt(aa(1,j)^2+aa(2,j)^2);
    n3(j)=aa(1,j)/amag;
    n2(j)=aa(2,j)/amag;
    mid=(posit(:,j)+posit(:,j+1))/2;
    n=[n2(j) n3(j) 0];
    n4v=cross(mid,n);
    n4(j)=n4v(3);

    z1=posit(2,j);
    z2=posit(2,j+1);
    dl=sqrt((posit(1,j)-posit(1,j+1))^2+(posit(2,j)-posit(2,j+1))^2);

    % the next two blocks calculate the sum of discretized waves for
    % exp(-ikxcosb)*exp(ikysinb)*e(kz),a=1

    pt1=exp(k*posit(1,j)*sin(beta)*i)*exp(k*z1)*exp(-i*k.*x*cos(beta));
    pt2=exp(k*posit(1,j+1)*sin(beta)*i)*exp(k*z2)*exp(-i*k.*x*cos(beta));
    f4=f4+n4(j)*(pt1+pt2)/2*dl;
    h4=h4-i*we*(n4(j))*dl*PSI4left(j);
end
```

1.b. Subroutine: ‘setpans.m’

```
function [np,npanels,yvert,zvert,ybv,zbv,ycontrol,zcontrol,lgth,nx,ny] =
setpans(yp,zp,npv,nph,npb,B,k,T)


%
%points will be in two arrays, yp & zp.
%nph is # of horizontal panels
%npv is # of vertical panels
%np is # points in arrays yp & zp
%hw is half width of domain, h is depth of domain
%dl is length of top hor. panels, dv is side panels, dw is bottom hor panels.
if max(zp)>0
    np=1;
```

```
    d=0;
    while zp(np)<0
        np=np+1;
    end
    yp=yp(1:np);
    zp=zp(1:np);
    dy=yp(np)-yp(np-1);
    dz=zp(np)-zp(np-1);
    ynew=yp(np)-zp(np)*dy/dz;
    yp(np)=ynew;
    zp(np)=0;
end
if max(zp)<=0
    [d,np]=size(yp);
end

% sets the domain size: hw is half-width (10 beams)
% h is depth: half the wavelength or three times the draft,
% whichever is greater.
% for the ONR hull, B=18.8m ->hw=188m, nph=55, dl=1.54m

hw=10*B;
h1=3/k;
h2=3*T;
h=max(h1,h2);
dl=(hw-yp(np))/nph;
dv=h/npv;
dw=2*hw/npb;
yvert(1)=-hw;
zvert(1)=0;

% vertices for free surface
for m=1:nph
    % left top half
    yvert(m+1)=yvert(m)+dl;
    zvert(m+1)=0;

    % right top half
    trh=2*np+nph-1+m;
    yvert(trh)=yp(np)+dl*m;
    zvert(trh)=0;
end

% vertices for hull
counter=np;
for m=1:np
    yvert(m+nph)=-yp(counter);
    zvert(m+nph)=zp(counter);
    counter=counter-1;
```

```
   mid=m+nph+np-1;
   yvert(mid)=yp(m);
   zvert(mid)=zp(m);
end

%vertices for sides
for m=1:npv
   right=2*np+2*nph+m-1;
   left=right+npv+npb;
   yvert(right)=hw;
   zvert(right)=-m*dv;
   yvert(left)=-hw;
   zvert(left)=-h+m*dv;
end

% vertices for bottom
for m=1:npb
   bot=m+2*np+2*nph+npv-1;
   yvert(bot)=hw-dw*m;
   zvert(bot)=-h;
end

endpt=2*(np+npv+nph)+npb-1;
yvert(endpt)=yvert(1);
zvert(endpt)=zvert(1);
npanels=endpt-1;

% control points
for k=2 :endpt;
  ycontrol(k-1) = 0.5*(yvert(k-1)+yvert(k));
  zcontrol(k-1) = 0.5*(zvert(k-1)+zvert(k));
  lgth(k-1)=sqrt((yvert(k)-yvert(k-1))^2 ...
     +(zvert(k)-zvert(k-1))^2);
  nx(k-1)= -(zvert(k)-zvert(k-1))/lgth(k-1);
  ny(k-1) = (yvert(k)-yvert(k-1))/lgth(k-1);
end

nbv=endpt-(2*np-3);
ybv(1:nph+1)=yvert(1:nph+1);
zbv(1:nph+1)=zvert(1:nph+1);
ybv(nph+2:nbv)=yvert(2*np+nph-1:endpt);
zbv(nph+2:nbv)=zvert(2*np+nph-1:endpt);
```

1.c. Subroutine: ‘findmatrix.m’

```
function [Amatrix,b] =
findmatrix(np,nph,npv,npb,w,we,k,beta,x,yc,zc,ny,nz,g,dgdn)
% x is longitudinal position of station
```

```
% initialize variables
gr=9.81;
c1=i*gr/w;
c2=k*x*cos(beta);
npanels=2*nph+2*np+2*npv-2+npb;
b=zeros(npanels,1);
Amatrix=zeros(npanels,npanels);

for m=1:npanels
   for n=1:nph
      % top left surface; w^2phi-gdphi/dn=0;

      Amatrix(m,n)=dgdn(m,n)-w^2/gr*g(m,n);

      %top right surface; same boundary condition
      nn=n+2*np+nph-2;
      Amatrix(m,nn)=dgdn(m,nn)-w^2/gr*g(m,nn);
   end

   for n=nph+1:2*np+nph-2
      %ship's hull; dphi/dn=-dphiI/dn
      Amatrix(m,n)=dgdn(m,n);
      phiI=c1*exp(i*(c2-k*sin(beta)*yc(n-nph))+k*zc(n-nph));
      vv=-phiI*i*k*sin(beta);
      ww=k*phiI;
      vdotn=vv*ny(n-nph)+ww*nz(n-nph);
      b(m)=b(m)-g(m,n)*vdotn;
   end
   for n=1:npv
      %right side dphi/dn=-ikphi
      nn=n+2*nph+2*np-2;
      Amatrix(m,nn)=dgdn(m,nn)+i*k*g(m,nn);
      %left side same boundary condition
      nn=n+2*nph+2*np-2+npv+npb;
      Amatrix(m,nn)=dgdn(m,nn)+i*k*g(m,nn);
   end
   for n=1:npb
      %bottom dphi/dn=0
      nn=n+2*nph+2*np-2+npv;
      Amatrix(m,nn)=dgdn(m,nn);
   end
   Amatrix(m,m)=Amatrix(m,m)+pi;
end
```

2. Main Program: LinearResponse This program only needs the raofunctions.mat file to run.

```
        % Calculates Time simulations for heave, pitch, and roll
```

```
% Needs raofuncations.mat to run
% raofunctions has the following row vectors:
% freqs (wave frequencies of interest)
% efreq (encounter frequencies at each wave frequency)
% A (wave amplitudes at each wave frequency)
% RRAO (roll RAO)
% Rph (roll phase in degrees)
% HRAO (heave RAO)
% Hph (heave phase in degrees)
% PRAO (pitch RAO)
% Pph (pitch phase in degrees)
% save output as linresp100

clear
close all
load raofunctions

tic

[b,a]=size(Hph);
Hph=Hph*pi/180;
Pph=Pph*pi/180;
Rph=Pph*pi/180;

k=freqs.^2/9.8;
we=efreq;
% allow for more than one run for statistics
runs=1;
tinc=100;
dt=1/tinc;
% previous two lines set dt to 1/tinc seconds

npts=3600*tinc;
```

```
t=(1:npts)*dt;
heave=zeros(npts,1);
pitch=zeros(npts,1);
roll=zeros(npts,1);
wh=zeros(npts,1);
for n=1:runs
   % set new random phase angles for discretized waves
   rand('state',sum(100*clock));
   psi=2*pi*rand(length(A),1)';
for m=1:npts

   wh(m)=sum(A.*cos(efreq*t(m)+psi));
   heave(m)=sum(A.*HRAO.*cos(abs(efreq)*t(m)+psi+Hph));
   pitch(m)=sum(k.*A.*PRAO.*cos(abs(efreq)*t(m)+psi+Pph));
   roll(m)=sum(k.*A.*RRAO.*cos(abs(efreq)*t(m)+psi+Rph));
   if m/tinc==round(m/tinc)
      time=m/tinc
      toc
   end
end
draft=5.5+wh-heave;
maxdraft(n)=max(draft);
mindraft(n)=min(draft);
n
toc
end
```

3. Main Program: rollintegration:

```
% Integrates exciting forces over time to produce a time sim of roll angle
% Needs RightingMoment.m to run
% Needs the following files to run:
% Rmomentdata.mat:
%   Contains data to conduct the lookup table function for righting moment
% forcedata500.mat: Output of ForceFinder, saved as a .mat file
```

```
% linresp100.mat: Output of LinearResponse, saved as .mat file

clear all
close all

global RM
global rolls
global drafts
global pitchs
tic
load Rmomentdata
load forcedata500
load linresp100

% sets the time simulation to npts*dt seconds
npts=360000;
dt=.01;

% Sets the moment of inertia, added inertia, damping
% and linear restoring coefficients
mass=8402180;
Iactual=mass*(.4*B)^2;
A44=.3*Iactual;
I=Iactual+A44;
b=.075;
c44=1.476*mass*gr;
B44=b*sqrt(c44*I);

% initialize the catastrophic roll event counter
count=0;

theta=zeros(npts,1);
thdot=zeros(npts,1);
thddot=zeros(npts,1);
theta2=zeros(npts,1);
thdot2=zeros(npts,1);
thddot2=zeros(npts,1);
M=zeros(npts,1);
C=zeros(npts,1);
C2=zeros(npts,1);
% provide the option to trim by the stern
pitch=pitch;

% set IC's to match linresp100
theta(1)=roll(1);
thdot(1)=(roll(2)-roll(1))/dt;
theta2(1)=theta(1);
thdot2(1)=thdot(1);
```

```
%Set magnitude and phase of excitation forces
exmoment=abs((F4+H4)'.*A);
mphase=angle(F4+H4)';

for m=1:npts-1

   M(m)=sum(exmoment.*cos(we*t(m)+psi+mphase));
   % protect for high pitch values
   if pitch(m)>max(pitchs)
      pitch(m)=max(pitchs);
   end
   % check for catastrophic roll event
   if theta(m)>pi/2
      theta(m)=theta2(m);
      thdot(m)=thdot2(m);
      count=count+1
   end
   if theta(m)<-pi/2
      theta(m)=theta2(m);
      thdot(m)=thdot2(m);
      count=count+1
   end
   % Find non-linear restoring moment
   C(m)=9.8*RightingMoment(draft(m),pitch(m),abs(theta(m)));
   if theta(m)<0
      C(m)=-C(m);
   end
   % Find linear restoring moment
   C2(m)=c44*theta2(m);

   % Forward Euler integration
   thddot(m)=(M(m)-C(m)-B44*thdot(m))/I;
   thdot(m+1)=thdot(m)+thddot(m)*dt;
   theta(m+1)=theta(m)+thdot(m)*dt;

   thddot2(m)=(M(m)-C2(m)-B44*thdot2(m))/I;
   thdot2(m+1)=thdot2(m)+thddot2(m)*dt;
   theta2(m+1)=theta2(m)+thdot2(m)*dt;

end

t=t/60;
figure(1)
plot(t(1:m),theta2(1:m)*180/pi,t(1:m),roll(1:m)*180/pi)
figure(2)
plot(t(1:m),theta(1:m)*180/pi,t(1:m),theta2(1:m)*180/pi)

toc
```

```
 % function to determine non-linear roll moment
% variable RM is lookup table for righting moments.
% RM(:,p,r) is moment vs. draft
% RM(d,:,r) is moment vs. pitch
% RM(d,p,:) is moment vs. roll
% From an excel spreadsheet:
%
% roll:        0              .05     ...
% pitch
%       -.15    RM(d,-.15,0)    RM(d,-.15,.05)
%       -.1     RM(d,-.1,0)     RM(d,-.1,.05)
%       ...
```

3.a subroutine: 'RightingMoment'

```
function moment = RightingMoment(heave, pangle, rangle)

global RM
global drafts
global pitchs
global rolls

% protects from high pitch/heave
if heave<2
   heave=2
end
if pangle<-0.1337
   pangle=-.1337
end

rangle=abs(rangle);

% find heave index
d=1;
while drafts(d)<=heave
   d=d+1;
end
% find pitch index
p=1;
while pitchs(p)>=pangle
   p=p+1;
end
% find roll index
r=1;
while rolls(r)<=rangle
   r=r+1;
end

% 3-D interpolation
```

```
HMRA=RM(d,p,r)-(rolls(r)-rangle)/(rolls(r)-rolls(r-1))*(RM(d,p,r)-RM(d,p,r-1));
LMRA=RM(d,p-1,r)-(rolls(r)-rangle)/(rolls(r)-rolls(r-1))*(RM(d,p-1,r)-RM(d,p-1,r-
1));
HRA=HMRA-(pitchs(p)-pangle)/(pitchs(p)-pitchs(p-1))*(HMRA-LMRA);
HMRA=RM(d-1,p,r)-(rolls(r)-rangle)/(rolls(r)-rolls(r-1))*(RM(d-1,p,r)-RM(d-1,p,r-
1));
LMRA=RM(d-1,p-1,r)-(rolls(r)-rangle)/(rolls(r)-rolls(r-1))*(RM(d-1,p-1,r)-RM(d-1,p-
1,r-1));
LRA=HMRA-(pitchs(p)-pangle)/(pitchs(p)-pitchs(p-1))*(HMRA-LMRA);

moment=HRA-(drafts(d)-heave)/(drafts(d)-drafts(d-1))*(HRA-LRA);
return
```

Appendix B: List of Symbols

Variable	Meaning
A	Wave Amplitude
A_{jk}	Added Mass
B	Beam
B_{jk}	Damping Coefficient
C_{jk}	Restoring Coefficient
f	Section Froude-Krylov Moment
F_j	Force In The j Direction
g	Acceleration Of Gravity
G	Green’s Function
h	Sectional Diffraction Moment
H	Transfer Function
H_4	Diffraction Moment
$H_{1/3}$	Significant Wave Height
I_{44}	Roll Moment Of Inertia
k	Wave Number
k_{44}	Roll Gyradius
ℓ	Length
M_{jk}	Generalized Mass Term
n_i	Normal Vector Component In J Direction
P	Pressure
RAO	Response Amplitude Operator
S	Wave Spectral Density
t	Time
u	Speed In The X-Direction

Variable	Meaning
v	Speed In The Y-Direction
$\vec{V}$	Velocity Vector
$\forall$	Volume
w	Speed In The Z-Direction
x	Longitudinal Direction
x_{44}	Non-dimensional Added mass coefficient
y	Transverse Direction
z	Vertical Direction
α	Phase Angle For Transfer Function or Moment
β	Wave Angle Of Incidence
β_{44}	Non-Dimensional Damping Term
Δ	Displacement
ζ	Wave Elevation
η	Response Value
θ	Roll Angle
λ	Tuning Coefficient Or Wavelength
ρ	Density
Φ	Velocity Potential
Φ_D	Diffraction Potential
Φ_I	Incident Potential
ψ	Wave Phase Angle
ω	Circular Frequency
ω_e	Encounter Frequency
ω_m	Modal Frequency
ω_o	Wave Frequency