Mathematical Perspectives on the Federal Thrift Savings Plan (TSP)


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# Mathematical Perspectives on the Federal Thrift Savings Plan (TSP)

## Operations Research Center
U.S. Military Academy at West Point

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Disclaimer

• **You will not receive any personal financial advice during this talk, as I am not officially qualified or certified to do so.**

• **However, my presentation is intended to get you to think mathematically about one of the retirement savings options available to many of you.**
Questions for Consideration

• Why might I be more risk tolerant than I currently believe?

• What are the L (Lifecycle) funds? How are they constructed? Why might they be of interest (or not) to me?

• What if stock and index fund returns are not normally distributed, as is commonly assumed?

• How does the choice of reward and risk measures affect optimal TSP portfolios?
“There is no scholarship for retirement!”

-Unknown

- Spouse’s 401(k) with matching funds
- Roth IRA (for Soldier/civilian and spouse)
- Thrift Savings Plan (TSP)
- Spouse’s 401(k) without matching funds
- Coverdale Educational Savings Accounts
- 529 Tuition Plans (prepaid or savings)

(ordering of these depends on tax considerations)
Thrift Savings Plan (TSP) Overview

• Largest defined contribution retirement savings and investment plan
  – 3.7 million participants
  – $210 billion in assets
• 401(k) equivalent for government employees and uniformed service members
• 5 non-traded core funds
• Can rebalance daily with no direct costs
# Core TSP Funds

<table>
<thead>
<tr>
<th>Fund</th>
<th>Description</th>
<th>Assets*</th>
<th>Mean Return#</th>
<th>Standard Deviation#</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>short-term, specially issued Treasury securities</td>
<td>$66.6B (39.2%)</td>
<td>6.4%</td>
<td>1.5%</td>
</tr>
<tr>
<td>F</td>
<td>tracks Lehman Brothers U.S. Aggregate (LBA) Index</td>
<td>$10.2B (6.0%)</td>
<td>7.3%</td>
<td>5.6%</td>
</tr>
<tr>
<td>C</td>
<td>tracks S&amp;P 500 Index</td>
<td>$66.7B (39.3%)</td>
<td>13.0%</td>
<td>17.9%</td>
</tr>
<tr>
<td>S</td>
<td>tracks Dow Jones Wilshire 4500 Completion Index</td>
<td>$13.7B (8.1%)</td>
<td>13.3%</td>
<td>19.9%</td>
</tr>
<tr>
<td>I</td>
<td>tracks MSCI EAFE (Europe, Australia, Far East) Index</td>
<td>$12.6B (7.4%)</td>
<td>7.8%</td>
<td>18.7%</td>
</tr>
</tbody>
</table>

* As of Dec 31, 2005  # For the period 1988-2005
Returns / Investment Horizon

• Returns
  – Arithmetic:
  \[ r_{i,t} = \frac{S_{i,t} - S_{i,t-1}}{S_{i,t-1}} \]
  – Log :
  \[ r_{i,t} = \ln(S_{i,t}) - \ln(S_{i,t-1}) = \ln\left(\frac{S_{i,t}}{S_{i,t-1}}\right) \]

• Investment Horizon: 20 years
  – Point of military (not ultimate) retirement
  – System encourages 20 year careers
  – Employment options vary greatly
  – Can move TSP assets to other plans
Daily Returns Time Series

G Fund

F Fund

C Fund

S Fund

I Fund

Daily Returns Distributions

- G fund appears approximately Gaussian
- C, S, I, and F funds are more peaked with heavy tails
- Goodness of Fit testing at common levels of significance rejects Normal for F, C, and S funds, even with batched means
Mean-Variance Portfolio Optimization
(\textit{Markowitz, 1952})

Let $X_i$ = fraction of funds invested in asset $i$

$\bar{R}_i$ = expected return of asset $i$

$\bar{R}_p$ = expected return of portfolio $p$

$\sigma_i^2$ = variance of return of asset $i$

$\sigma_{jk}$ = covariance of return of asset $j$ with asset $k$

Minimize

$$\sigma_p^2 = \sum_{j=1}^{N} X_j^2 \sigma_j^2 + \sum_{j=1}^{N} \sum_{k=1 \atop k \neq j}^{N} X_j X_k \sigma_{jk}$$

Subject to:

$$\sum_{i=1}^{N} X_i = 1$$

$$\bar{R}_p = \sum_{i=1}^{N} X_i \bar{R}_i$$

$$X_i \geq 0, \quad i = 1, \ldots, N$$
**L (Lifecycle) Funds**

- Invest in 5 core TSP funds based on time horizon to provide highest possible rate of return for risk taken.

- Over time, investments shift away from stocks and into bonds.

- L Funds are great, but …
Reward-Risk Profile of TSP Funds

Risk (Standard Deviation) vs. Reward (Annual Return) for various TSP funds: G Fund, F Fund, C Fund, S Fund, I Fund, with different percentage allocations (20% each) at various time points (L 2010, L 2020, L 2030, L 2040).
VG-ICA Factor Model

(Madan & Yen, 2004)

\[(R - \mu) = XB + \varepsilon\]

\[D = XB + \varepsilon\]

1. Use **Independent Component Analysis (ICA)** on asset returns \(D\) to identify underlying factors \(X\)

2. Fit the **Variance Gamma (VG)** distribution to each retained factor by MLE

3. Use Expected Utility to determine optimal portfolio of VG-ICA factors; convert back to optimal portfolio of assets (TSP funds)
Independent Component Analysis (ICA)

- Principal Component Analysis (PCA)
  - Focus on finding **uncorrelated** components in Gaussian data
  - Maximizes explained variance
  - Uses second-order statistics
- Factor Analysis
  - Essentially PCA with extra terms to model noise
- ICA
  - Focus on **independent** and non-Gaussian components
  - Maximizes non-Gaussianity (to maximize information)
  - Uses higher-order statistics
Another ICA Example

Original Signals (s)

Mixed Signals (x)

ICA source estimates (y)

A
(mixing Matrix)

W
(de-mixing Matrix)
ICA versus PCA

- **Principal Component Analysis (PCA) finds:**
  - directions of maximal variance in Gaussian data (second-order statistics).
  - directions of maximal variance in non-Gaussian data (second-order statistics).

- **Independent Component Analysis (ICA) finds**
  directions of maximal independence in non-Gaussian data (higher-order statistics).
How Many ICs to Keep?

"Scree" Plot

R² Values from Regression

<table>
<thead>
<tr>
<th># ICs Kept</th>
<th>F Fund</th>
<th>C Fund</th>
<th>S Fund</th>
<th>I Fund</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>.9980</td>
<td>.9994</td>
<td>.9998</td>
<td>.9949</td>
</tr>
<tr>
<td>3</td>
<td>.0611</td>
<td>.8983</td>
<td>.9652</td>
<td>.9869</td>
</tr>
<tr>
<td>2</td>
<td>.0611</td>
<td>.8962</td>
<td>.9629</td>
<td>.1276</td>
</tr>
</tbody>
</table>

- Dropping more than one IC reduces fit on at least one fund
- The first four have excess kurtosis
  → Keep 4 Independent Components (ICs)
VG Process and Distribution

• **Pure jump process** with two representations
  – Time-changed Brownian motion (Madan & Seneta, 1990)
    \[ X_{VG}(t; \nu, \theta, \sigma) = b(\gamma(t;1, \nu), \theta, \sigma) \]
  – Difference of 2 Gamma processes (Madan, Carr & Chang, 1998)
    \[ X_{VG}(t) = G_p(t) - G_n(t) \]

• **Parameters:**
  \( \sigma \) controls spread
  \( \nu \) affects kurtosis
  \( \theta \) impacts skewness

• **Density Function** (Madan, Carr & Chang, 1998)
  \[
  h(z) = \frac{2 \exp(\theta x / \sigma^2)}{\nu^{1/\nu}} \sqrt{\frac{1}{2\pi\sigma}} \Gamma\left(\frac{1}{\nu}\right) \left(\frac{x^2}{2\sigma^2 / \nu + \theta^2}\right)^{1/2 - 1/4} K_{1/2} \left(\frac{1}{\sigma^2} \sqrt{x^2 (2\sigma^2 / \nu + \theta^2)}\right)
  \]
  with \( x = z - \theta \) where \( z = \ln(S(t) / S(t-1)) \)
Examples of VG Distributions

Effect of $\nu$

Effect of $\theta$

LEGEND
- VG $\nu = 2$
- VG $\nu = 1$
- VG $\nu = 0.5$
- $N(0,1)$

LEGEND
- VG $\theta = 0$
- VG $\theta = 1$
- VG $\theta = -1$
- $N(0,1)$
Fitting VG by MLE

• Given observed IID data $X_1, X_2, \ldots, X_n$, define the likelihood function as:

$$ L(\theta) = f_\theta(X_1) f_\theta(X_2) \cdots f_\theta(X_n) $$

• The MLE (maximum likelihood estimator) $\hat{\theta}$ maximizes $L(\theta)$ over all permissible values of $\theta$.

• Actually, maximizing the log likelihood function $\ln(L(\theta))$ is easier

• For the VG distribution with three parameters, this becomes:

$$ l(\sigma, \nu, \theta) = \ln L(\sigma, \nu, \theta) = \sum_i f_{(\sigma, \nu, \theta)}(X_i) $$

(using pdf given before from Madan, Carr, and Chang, 1998)
Comparison of Fitted VG and Normal(0,1)
### Fitted VG Parameters / Chi-Square Statistics

<table>
<thead>
<tr>
<th>IC#</th>
<th>Fitted VG Parameters - Daily (Annualized)</th>
<th>$\chi^2$ Test Statistic (p-values) ($\chi^2_{0.01,17} = 33.41$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma$</td>
<td>$\nu$</td>
</tr>
<tr>
<td>IC1</td>
<td>0.933</td>
<td>0.969</td>
</tr>
<tr>
<td></td>
<td>(14.814)</td>
<td>(.00385)</td>
</tr>
<tr>
<td>IC2</td>
<td>0.980</td>
<td>0.820</td>
</tr>
<tr>
<td></td>
<td>(15.558)</td>
<td>(.00326)</td>
</tr>
<tr>
<td>IC3</td>
<td>0.989</td>
<td>0.586</td>
</tr>
<tr>
<td></td>
<td>(15.703)</td>
<td>(.00232)</td>
</tr>
<tr>
<td>IC4</td>
<td>0.991</td>
<td>0.468</td>
</tr>
<tr>
<td></td>
<td>(15.739)</td>
<td>(.00186)</td>
</tr>
</tbody>
</table>

- Some excess kurtosis and slight negative skewness in each IC
- VG fits much better than Normal distribution
Utility Theory & Risk Aversion

*Utility* - a measure of relative satisfaction obtained

*Risk Aversion* - concave utility function, as shown below
Aside on Risk Aversion/Tolerance
(Jennings & Reichenstein, 2001)

• Pensions considered when planning retirement income….. but NOT when calculating asset allocation
• Pensions and investment portfolio generate retirement funds; why not consider both in total portfolio?
• Many similarities between inflation-indexed Treasury bonds (TIPS) and military retirement
  – Linked to Consumer Price Index (CPI)
  – Backed by federal government
• Suggest treating after-tax present value as a “pseudo-bond” in total portfolio
• Discounting can be at recent TIPS rates (3%-5%) or higher personal discount rate (18+%)  
• Results in more aggressive (risk tolerant) portfolio in active investments than would otherwise result
NPV of Military Retirement

*(Jennings & Reichenstein, 2001)*

<table>
<thead>
<tr>
<th>Rank at Retirement</th>
<th>Years of Service</th>
<th>After-Tax NPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTC</td>
<td>20</td>
<td>$726,674</td>
</tr>
<tr>
<td>LTC</td>
<td>22</td>
<td>$802,690</td>
</tr>
<tr>
<td>COL</td>
<td>24</td>
<td>$994,468</td>
</tr>
<tr>
<td>COL</td>
<td>26</td>
<td>$1,096,490</td>
</tr>
<tr>
<td>COL</td>
<td>28</td>
<td>$1,166,125</td>
</tr>
<tr>
<td>COL</td>
<td>30</td>
<td>$1,205,255</td>
</tr>
</tbody>
</table>

Assumptions:

- Officer currently at 18 years of service
- 28% tax bracket
- 4% TIPS rate / inflation
“Pseudo-Bond” Example
(Nestler, 2007)

Desired/Current Financial Portfolio

- 40% Bonds
- 60% Stocks

Resulting Expanded Portfolio

- 75% Bonds and “Pseudo-Bonds”
- 25% Stocks

Military or Government Pension
Negative Exponential Utility

\[ U(w) = -e^{-cw}, \quad c > 0 \]

- Constant Absolute Risk Aversion (CARA) -- no “wealth effect”
- Computational tractability advantage over other (log, power) utility functions
- Analytical solution to maximization problem is available using Certainty Equivalent (CE)
- CE is well-known for Normal and given for VG-ICA (Madan and Yen, 2004)
Implied Risk Aversion Coefficient

Risk (SD) vs. Risk Aversion Coefficient

Risk (SD)

Risk Aversion Coefficient

L 2040
L 2030
L 2020
L 2010
L Income

2 - 3
## Portfolios for Comparison

<table>
<thead>
<tr>
<th>Model</th>
<th>G Fund</th>
<th>F Fund</th>
<th>C Fund</th>
<th>S Fund</th>
<th>I Fund</th>
</tr>
</thead>
<tbody>
<tr>
<td>VG-ICA (Daily)</td>
<td>0%</td>
<td>1%</td>
<td>43%</td>
<td>30%</td>
<td>26%</td>
</tr>
<tr>
<td>Riskless</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>TSP “Market Portfolio”</td>
<td>39%</td>
<td>6%</td>
<td>39%</td>
<td>8%</td>
<td>8%</td>
</tr>
<tr>
<td>L 2030</td>
<td>16%</td>
<td>9%</td>
<td>38%</td>
<td>16%</td>
<td>21%</td>
</tr>
<tr>
<td>L 2040</td>
<td>5%</td>
<td>10%</td>
<td>42%</td>
<td>18%</td>
<td>25%</td>
</tr>
</tbody>
</table>

**NOTE:** These portfolios are created with returns assumed to be Normally distributed.
Stochastic Dominance

• Generalizes utility theory; don’t need a specific utility function

• First-Order Stochastic Dominance (FOSD)
  – Assumes only monotonicity; strongest result
  – A FOSD B IFF \( F_B(x) \geq F_A(x), \forall x \)

• Second-Order Stochastic Dominance (SOSD)
  – Also assumes risk aversion
  – A SOSD B IFF \( \int_{-\infty}^{x} [F_B(u) - F_A(u)]du \geq 0, \forall x \)

• Easy to test with empirical data
Traditional Risk Measures

• Dispersion Measures
  – Variance (or Standard Deviation)
    • Treats gains and losses equally
  – Semi-Variance
    • Only considers observations below mean
  – Mean Absolute Deviation (MAD)
    • Average absolute deviation from the mean

• “Safety Risk” Measures
  – Value-at-Risk (VaR)
  – Expected Tail Loss (ETL)
Value-at-Risk (VaR)

– “Expected maximum loss over a fixed horizon for a given confidence level”

\[ P(X \geq \text{VaR}_\lambda (X)) = \lambda \]

– Standard risk measure for past 12 years
– Does not reward diversification
– Addresses size but not shape of tail
Coherent Measures of Risk  
(Artzner, Delbaen, Eber, & Heath, 1999)

• Axioms for coherency:
  – Translation invariance \( \rho(X - \alpha) = \rho(X) - \alpha \)
  – Monotonicity \( X > Y \implies \rho(X) > \rho(Y) \)
  – Sub-additivity \( \rho(X + Y) \leq \rho(X) + \rho(Y) \)
  – Positive homogeneity \( \rho(\lambda X) = \lambda \rho(X) \)

• Variance: not monotonic or translation invariant
• VaR: not sub-additive in non-Gaussian world
• Other measures that are coherent exist.
Conditional VaR

- “Expected value of all losses greater than VaR for a specified $\lambda$.”

$$CVaR_{\lambda}(X) = E[X | X > VaR_{\lambda}(X)]$$

- Also known as Expected Shortfall (Rockafellar & Uryasev, 2001) and Tail VaR (Acerbi, Nordio, et al., 2001)

- Accounts for size and shape of left tail but ignores rest of distribution
Classes of Weighted VaR
(Cherny, 2006; Cherny & Madan, 2007)

\[ WVaR_\mu (X) = \int_{[0,1]} CVaR_\lambda (X) \mu(dx) \]

- **Beta VaR(\(\alpha, \beta\))**
  
  \[ \mu_{\alpha,\beta}(dx) = B(\beta + 1, \alpha - \beta)^{-1} x^\beta (1-x)^{\alpha-\beta-1} dx, \quad x \in [0,1] \]
  
  - Expectation of average of the \(\beta\) biggest of \(\alpha\) independent copies of portfolio loss
  - Faster to estimate than CVaR

- **Alpha VaR(\(\alpha\))**
  
  - Essentially Beta VaR with \(\beta=1\)
  - Expectation of biggest of \(\alpha\) copies of portfolio loss
Effect of Alpha and Beta

Can allow for more risk by decreasing $\alpha$ or increasing $\beta$
Performance (Reward-Risk) Measures

- Sharpe Ratio
  \[ SR = \frac{E(X)}{\sigma_X} \]

- STARR Ratio
  \[ STARR = \frac{E(X)}{CVaR_\lambda(X)} \]

- R-Ratio (Rachev)
  \[ R = \frac{CVaR_{\lambda_1}(-X)}{CVaR_{\lambda_2}(X)} \]
New Portfolio Performance Measures
(Nestler, 2007b)

- Similar to R-Ratio but use Alpha-VaR and Beta-VaR in place of CVaR

- AVaR-Ratio:
  \[ AVR = \frac{AVaR_{\alpha_1}(-X)}{AVaR_{\alpha_2}(X)} \]

- BVaR-Ratio:
  \[ BVR = \frac{BVaR_{\alpha_1, \beta_1}(-X)}{BVaR_{\alpha_2, \beta_2}(X)} \]
Monthly Contribution

• Assumes saving 10% of base pay each month (median for TSP)

TOTAL CONTRIBUTIONS: $170K
Realistic Scenario: Portfolio Value
(5000 sample paths)

Expected Value:
- VG-ICA: $418,381
- L2040: $318,840
- L2030: $313,936
- TSP MP: $310,247
- Riskless: $259,642

Upside Potential:
- VG-ICA: $1,992,133
- L2040: $1,127,069
- L2030: $1,093,643
- TSP MP: $1,064,902

Total Contrib. $170,505
PDF of Discounted Portfolio Value

Legend:
- VG-ICA
- TSP
- MP
- L2030
- L2040
Realistic Scenario: CDF Comparison

NOTE:
No SD.
Realistic Scenario: Zoomed CDF Comparison

Portfolio Gain/Loss

LEGEND
- VG-ICA
- TSP MP
- L 2030
- L2040

95% VaR

95% Value at Risk

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<table>
<thead>
<tr>
<th>Risk Measure (↓ better)</th>
<th>VG-ICA</th>
<th>TSP MP</th>
<th>L 2030</th>
<th>L 2040</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std Dev</td>
<td>$168,885</td>
<td>$80,890</td>
<td>$94,515</td>
<td>$105,525</td>
</tr>
<tr>
<td>95% VaR</td>
<td>$43,382</td>
<td>$44,146</td>
<td>$60,378</td>
<td>$66,910</td>
</tr>
<tr>
<td>95% CVaR</td>
<td>$68,056</td>
<td>$54,789</td>
<td>$74,684</td>
<td>$82,783</td>
</tr>
<tr>
<td>Alpha VaR(50)</td>
<td>$77,575</td>
<td>$59,250</td>
<td>$81,352</td>
<td>$87,754</td>
</tr>
<tr>
<td>Beta VaR(50,5)</td>
<td>$43,938</td>
<td>$44,010</td>
<td>$60,203</td>
<td>$67,757</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Performance Measure (↑ better)</th>
<th>VG-ICA</th>
<th>TSP MP</th>
<th>L 2030</th>
<th>L 2040</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharpe Ratio</td>
<td>0.94</td>
<td>0.63</td>
<td>0.57</td>
<td>0.54</td>
</tr>
<tr>
<td>STARR Ratio</td>
<td>2.33</td>
<td>2.90</td>
<td>0.72</td>
<td>0.69</td>
</tr>
<tr>
<td>R-Ratio(.05,.05)</td>
<td>8.87</td>
<td>2.82</td>
<td>4.10</td>
<td>4.07</td>
</tr>
<tr>
<td>AVR</td>
<td>9.35</td>
<td>5.41</td>
<td>4.62</td>
<td>4.67</td>
</tr>
<tr>
<td>BVR</td>
<td>11.60</td>
<td>5.05</td>
<td>4.24</td>
<td>4.12</td>
</tr>
</tbody>
</table>
Traditional Reward-Risk Profile
Some Possible Answers

• Why might I be more risk tolerant than I currently believe?
  – Counting military or government pension as “pseudo-bonds” could change the target stock-bond asset mix.

• What are the L (Lifecycle) funds? How are they constructed? Why might they be of interest (or not) to me?
  – “Set it and forget it” funds built using mean-variance optimization with returns assumed to be distributed Normally.
  – Depends on an individual’s level of interest and involvement.

• What if stock and index fund returns are not normally distributed, as is commonly assumed?
  – Possible to take advantage of information contained in higher moments.

• How does choosing reward-risk measures affect optimal TSP portfolios?
  – Ability to capture information from entire distribution is useful.
  – Need to do further work on optimizing performance measures instead of using expected utility.
QUESTIONS?