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# QUANTIFYING “PERSISTENCE” IN THE CONTEXT OF FIND, FIX, FINISH

Dr. Roy Rice  
13-15 June 2007

# Outline

- Set the Stage
- Define the problem
- Probability derivation
- Limits
- Derivatives to show rate of change
- Persistent ISR Ratio (PIR)
- Examples

# Set the Stage

- *“Maintaining **persistent ISR** around the globe would allow the military to continue to function as a ‘strategically relevant, continental United States-based projection force,’ Bair said during the Defense News Media Group conference, *ISR Integration 2003: The Net-Centric Vision*, in Arlington, Va.” Ref[1]. Edward Bair is the Army program executive officer for intelligence, electronic warfare and sensors.*

## Set the Stage (cont.)

- *“We were trying to craft more of a story and a message that says we’re moving away from, say, the reconnaissance paradigm to that **persistent surveillance** paradigm and let’s look at what we’re buying and see if that really does accomplish where we’re trying to go.”* Ref [2] - Kevin Meiners; the director of intelligence strategies, technologies and assessments, Office of the Deputy Undersecretary of Defense for Intelligence and Warfighting Support.

## Set the Stage (cont.)

- “*We need long-term investment in **persistent ISR** capability with assured electromagnetic spectrum access utilizing up-to-date collection technologies to **find, track and interdict** mobile and technologically competent terrorist groups and platforms operating with the vast regions of Africa and Europe, including both air and maritime environments.*” Ref[3]– General James L. Jones, USMC, Commander United States European Command, before the House Armed Services Committee on 8 March 2006.

## Set the Stage (cont.)

- *“Precision operations are intelligence-driven. As noted above, we need to rebalance our ‘**find, fix, finish**’ targeting cycle.”* Ref[4] – General John P. Abizaid, USA, Commander, United States Central Command, before the House Armed Service Committee on 15 March 2006.



## Set the Stage (cont.)

- In an August 10, 2006, interview in Secretary Rumsfeld's office, the Secretary said, *"I was asked that when I was up at the confirmation hearings in January of '01, and I said intelligence. And if you think about this department, we have just enormous capability to **finish**. If you use the phrase "**find, fix and finish**," we can **finish** something if we can **find** it and **fix it in time** and location. The problem is **finding** it. And you can find big armies and big navies and big air forces, and we've gotten quite good at that in this department. It is a whale of a lot harder to deal with a network, with individuals, with people that don't wear uniforms, with people that mix among civilians and hide among innocent people."* Ref[5]

## Set the Stage (cont.)

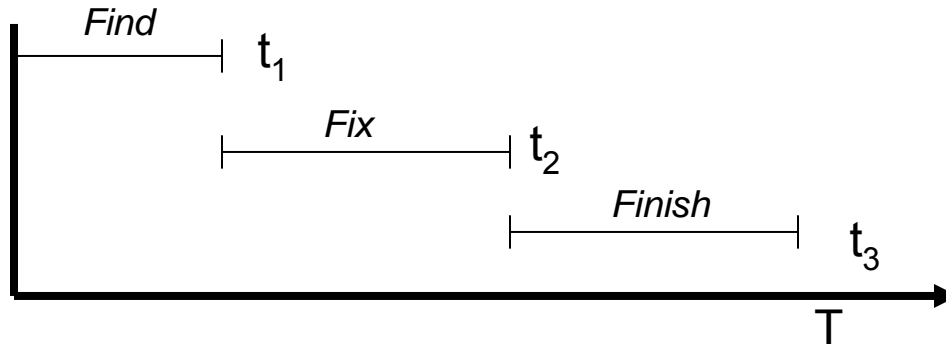
- And, finally, in a news briefing on 12 January 2006 with Secretary Rumsfeld and Chairman General Peter Pace, Secretary Rumsfeld said, *“And the reality that this department has responsibilities to **find and to fix and to finish** -- to use the phrase -- in terms of dealing with threats and enemies to this country, and a recognition that we have a great deal of ability to fix and -- correction, to **finish** -- in an operation and much less ability to **find** and **fix**, and the importance of seeing that our department over time recognizes that imbalance and does everything humanly possible to see that we find ways to link -- we are the biggest user of intelligence -- this department is -- and we need to see that there is an intimate relationship in proximity and time between intelligence and operations.”* Ref[6]

# Set the Stage

- Combatant Commands refer to Kill Chain as F-F-F (F3)
  - Find, fix, finish
  - Can be further decomposed into F2T2EA
    - Find, fix, track, target, engage, assess
- From OEG Report on *Search Theory*
  - Prob (event in time t) =  $1 - e^{-\lambda t}$ 
    - Where  $\lambda$  is the rate and t is the time to accomplish the event
    - Density function is  $f(t) = \lambda e^{-\lambda t}$
  - We'll assume this density applies to each of the events or phases – Find-Fix-Finish

# Problem Set-up

Borrowing from Reliability Theory of Redundant Systems and Bayes' Theorem:



$t_1$  = time to Find  
 $t_2$  = time to Fix  
 $t_3$  = time to Finish

- $P[(t_1 \leq T) \cap (t_2 \leq T - t_1) \cap (t_3 \leq T - t_1 - t_2)] = P(T)$

=

$$\int_0^T \lambda_1 e^{-\lambda_1 t_1} \int_0^{T-t_1} \lambda_2 e^{-\lambda_2 t_2} \int_0^{T-t_1-t_2} \lambda_3 e^{-\lambda_3 t_3} dt_3 dt_2 dt_1$$

\* Assume the  $\lambda_i$ s are unequal...otherwise it's trivial

$$= 1 - e^{-\lambda_1 T} \left( \frac{\lambda_2 \lambda_3}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} \right) - e^{-\lambda_2 T} \left( \frac{\lambda_1 \lambda_3}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} \right) - e^{-\lambda_3 T} \left( \frac{\lambda_1 \lambda_2}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} \right)$$

In general:

$$P(T) = 1 - \sum_j \frac{\left[ e^{-\lambda_j T} * \left( \prod_{i \neq j} \lambda_i \right) \right]}{\prod_{i \neq j} (\lambda_j - \lambda_i)}$$

# Problem Solution

- Let  $\theta_i = 1/\lambda_i$   
= mean-time-to-accomplish event  $i$
- Then,  $P(T) = P$

Can be rewritten as:

$$P(T) = 1 - e^{-\frac{T}{\theta_1}} \left( \frac{\theta_1^2}{(\theta_1 - \theta_2)(\theta_1 - \theta_3)} \right) - e^{-\frac{T}{\theta_2}} \left( \frac{\theta_2^2}{(\theta_2 - \theta_1)(\theta_2 - \theta_3)} \right) - e^{-\frac{T}{\theta_3}} \left( \frac{\theta_3^2}{(\theta_3 - \theta_1)(\theta_3 - \theta_2)} \right)$$

In general:

$$P(T) = 1 - \sum_j \frac{[e^{-T/\theta_j} * \theta_j^2]}{\prod_{i \neq j} (\theta_j - \theta_i)}$$

# Density Function for T

$$\boxed{dP/dT}$$

$$dP/dT = \left( \prod_i \lambda_i \right) * \sum_j \frac{[e^{-\lambda_j T}]}{\prod_{i \neq j} (\lambda_j - \lambda_i)}$$

...or...

$$dP/dT = \sum_j \frac{[e^{-T/\theta_j} * \theta_j]}{\prod_{i \neq j} (\theta_j - \theta_i)}$$



# Limits

$$\theta_1 \rightarrow 0$$

$$P = 1 - e^{-\frac{T}{\theta_2} \left( \frac{\theta_2}{\theta_2 - \theta_3} \right)} - e^{-\frac{T}{\theta_3} \left( \frac{\theta_3}{\theta_3 - \theta_2} \right)}$$

$$\theta_2 \rightarrow 0$$

$$P = 1 - e^{-\frac{T}{\theta_1} \left( \frac{\theta_1}{\theta_1 - \theta_3} \right)} - e^{-\frac{T}{\theta_3} \left( \frac{\theta_3}{\theta_3 - \theta_1} \right)}$$

$$\theta_3 \rightarrow 0$$

$$P = 1 - e^{-\frac{T}{\theta_1} \left( \frac{\theta_1}{\theta_1 - \theta_2} \right)} - e^{-\frac{T}{\theta_2} \left( \frac{\theta_2}{\theta_2 - \theta_1} \right)}$$

Limits are the  $P(T)$  calculations given “instantaneous” events

# SENSITIVITIES OF $\theta_i$ 's

- Interested in sensitivity of  $P(T)$  to changes in  $\theta_i$ 's
- Need to perform trade-off studies
  - Operations
  - Cost

# Derivatives of P

$$\frac{\partial P}{\partial \theta_1} = -e^{\frac{T}{\theta_1}} \left( \frac{\theta_1^2}{(\theta_3 - \theta_1)^2 (\theta_2 - \theta_1)} + \frac{\theta_1^2}{(\theta_2 - \theta_1)^2 (\theta_3 - \theta_1)} + \frac{T}{(\theta_3 - \theta_1)(\theta_2 - \theta_1)} + \frac{2\theta_1}{(\theta_3 - \theta_1)(\theta_2 - \theta_1)} \right) \\ + e^{\frac{T}{\theta_2}} \left( \frac{\theta_2^2}{(\theta_3 - \theta_2)(\theta_2 - \theta_1)^2} \right) - e^{\frac{T}{\theta_3}} \left( \frac{\theta_3^2}{(\theta_3 - \theta_2)(\theta_3 - \theta_1)^2} \right)$$

$$\frac{\partial P}{\partial \theta_2} = e^{\frac{T}{\theta_2}} \left( \frac{\theta_2^2}{(\theta_3 - \theta_2)^2 (\theta_2 - \theta_1)} - \frac{\theta_2^2}{(\theta_2 - \theta_1)^2 (\theta_3 - \theta_2)} + \frac{T}{(\theta_3 - \theta_2)(\theta_2 - \theta_1)} + \frac{2\theta_2}{(\theta_3 - \theta_2)(\theta_2 - \theta_1)} \right) \\ + e^{\frac{T}{\theta_1}} \left( \frac{\theta_1^2}{(\theta_3 - \theta_1)(\theta_2 - \theta_1)^2} \right) - e^{\frac{T}{\theta_3}} \left( \frac{\theta_3^2}{(\theta_3 - \theta_1)(\theta_3 - \theta_2)^2} \right)$$

$$\frac{\partial P}{\partial \theta_3} = e^{\frac{T}{\theta_3}} \left( \frac{\theta_3^2}{(\theta_3 - \theta_2)^2 (\theta_3 - \theta_1)} + \frac{\theta_3^2}{(\theta_3 - \theta_1)^2 (\theta_3 - \theta_2)} - \frac{T}{(\theta_3 - \theta_1)(\theta_3 - \theta_2)} - \frac{2\theta_3}{(\theta_3 - \theta_1)(\theta_3 - \theta_2)} \right) \\ + e^{\frac{T}{\theta_1}} \left( \frac{\theta_1^2}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)^2} \right) - e^{\frac{T}{\theta_2}} \left( \frac{\theta_2^2}{(\theta_2 - \theta_1)(\theta_3 - \theta_2)^2} \right)$$

# Derivatives of P

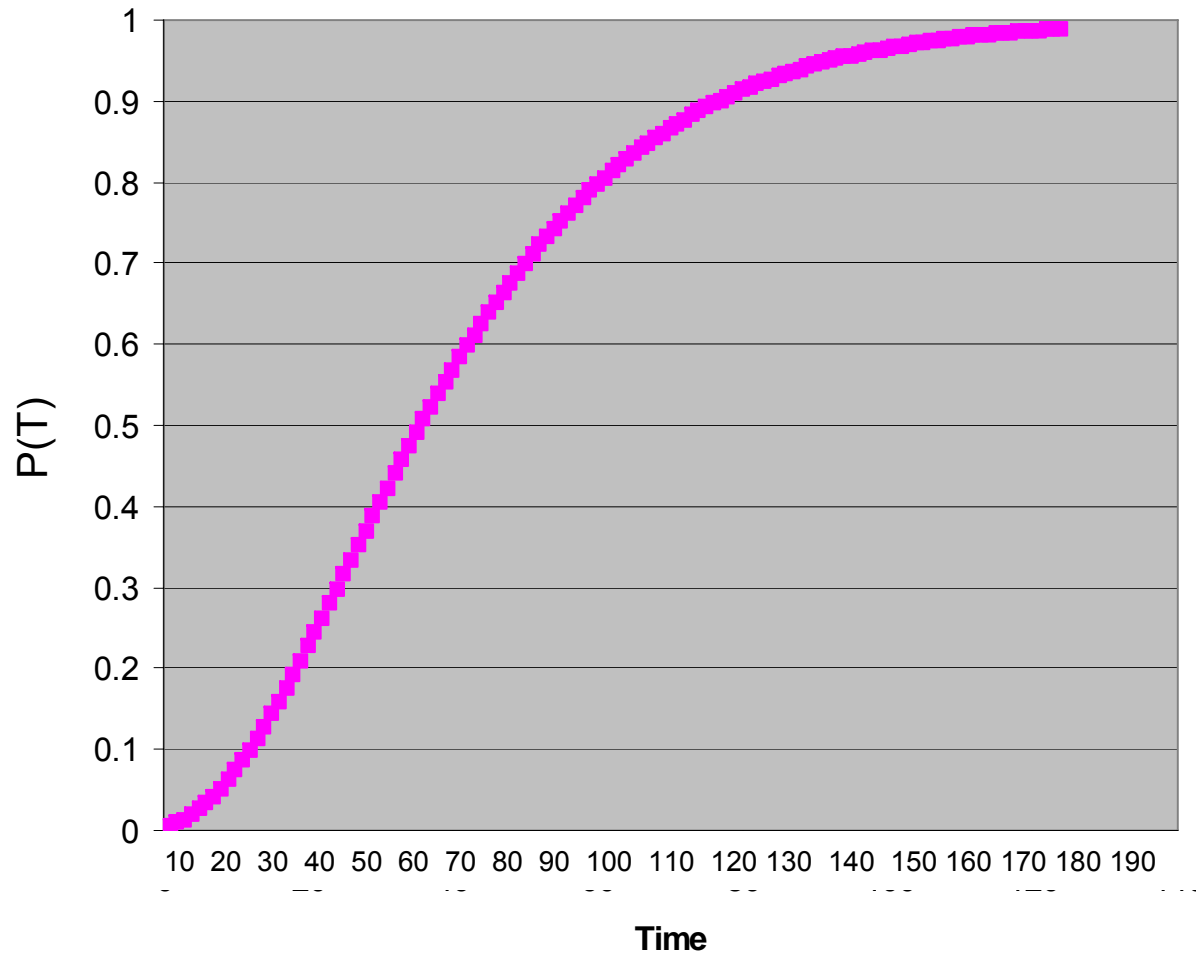
- Interesting to note (you can manipulate the previous equations to prove it):
  - If I swap  $\theta_i$  for  $\theta_j$ , then the “swapped” partials are equal and the third partial stays the same and  $P(T)$  remains the same.
  - If I swap all three  $\theta$ s, then all the “swapped” partials are equal and  $P(T)$  remains the same.
  - Example, let  $\theta_1 = 10$ ,  $\theta_2 = 15$ ,  $\theta_3 = 50$ , and  $T = 60$ . Then
    - $P(T) = 0.4845$
    - $\frac{\partial P}{\partial \theta_1} = -0.010007$
    - $\frac{\partial P}{\partial \theta_2} = -0.009737$
    - $\frac{\partial P}{\partial \theta_3} = -0.006251$
  - If I let  $\theta_1 = 50$ ,  $\theta_2 = 10$ ,  $\theta_3 = 15$ , and hold  $T = 60$ . Then
    - $P(T) = 0.4845$
    - $\frac{\partial P}{\partial \theta_1} = -0.006251$
    - $\frac{\partial P}{\partial \theta_2} = -0.010007$
    - $\frac{\partial P}{\partial \theta_3} = -0.009737$

# Persistent ISR Ratio (PIR)

- ISR is mainly concerned with “Find”; although it’s often part of other two.
- Focused on “Find”...but emphasizes others.
- Compares “rates of change” relative to other “F’s”
- If  $PIR > 1$ , then reductions in  $\theta_1$  result in larger increases in  $P(T)$

$$PIR = \min \left\{ \frac{\partial P / \partial \theta_1}{\partial P / \partial \theta_2}, \frac{\partial P / \partial \theta_1}{\partial P / \partial \theta_3} \right\}$$

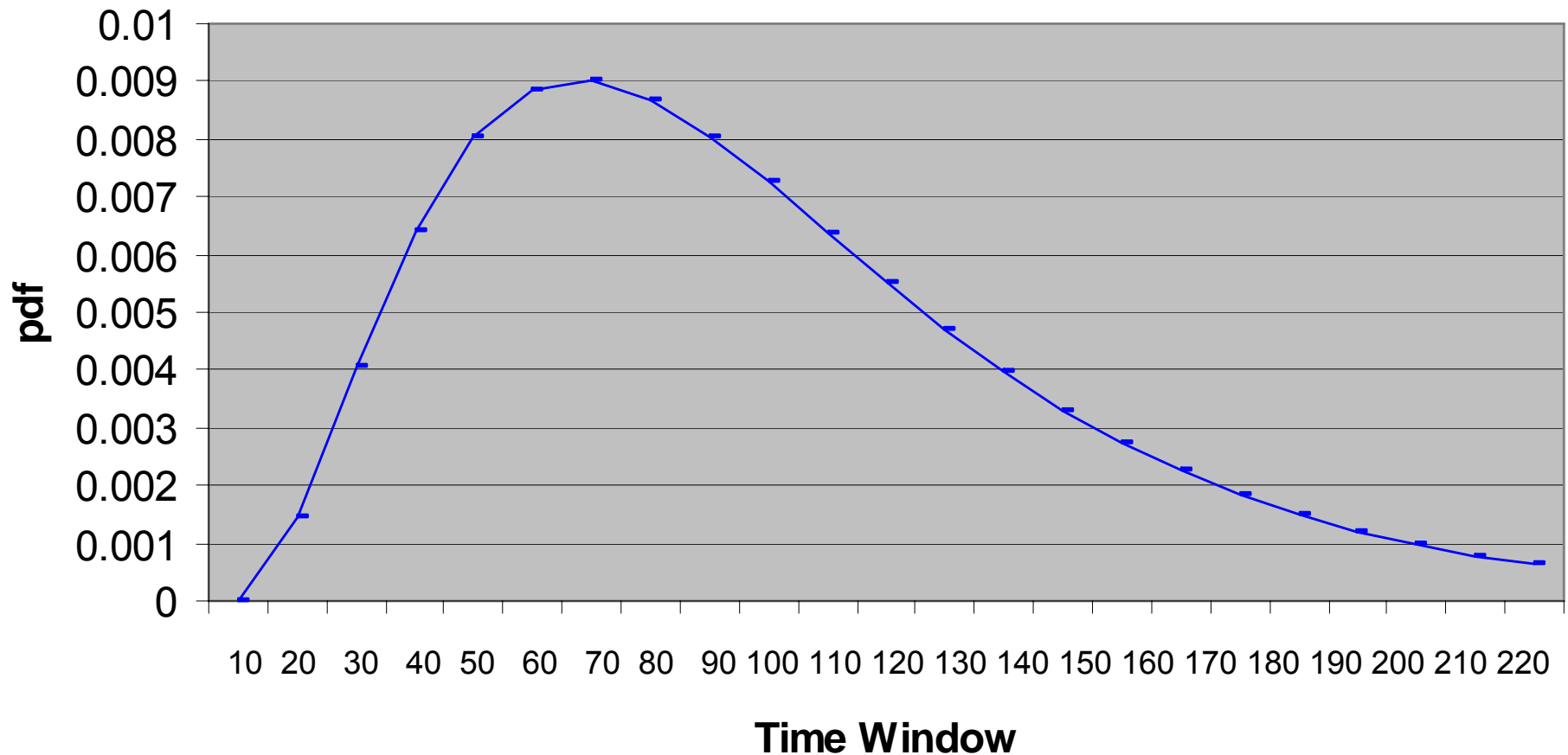
# $P(T)$ as a Function of Time Window



Given: Time varies (X axis) from 10 to 190 time units;  $\theta_i = 20, 30, 40$  respectively

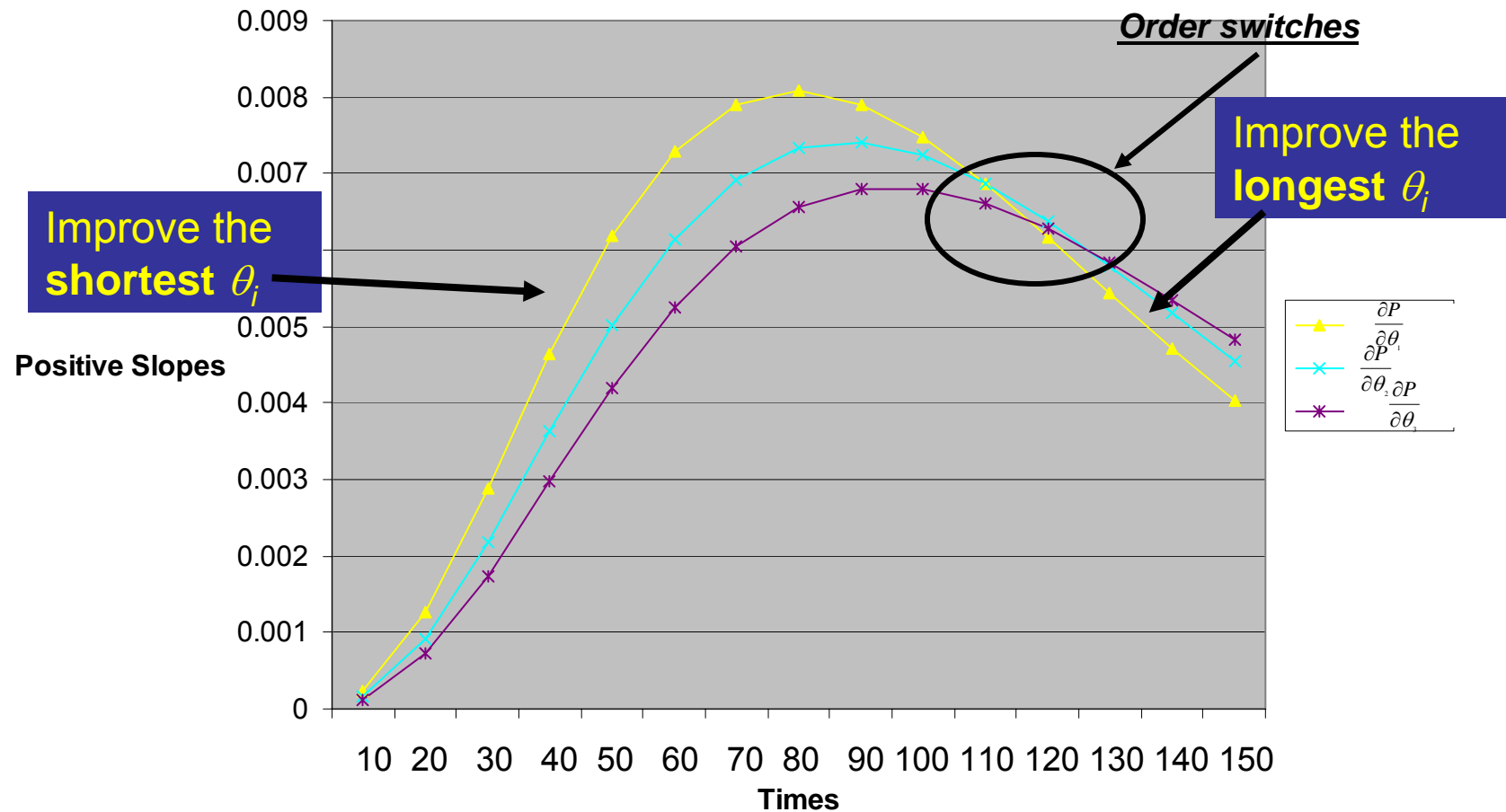
# Density Function

pdf



Given: Time varies (X axis) from 10 to 220 time units;  $\theta_i = 20, 30, 40$  respectively

# Derivatives



Given: Time varies (X axis) from 10 to 150 time units;  $\theta_i = 20, 30, 40$  respectively

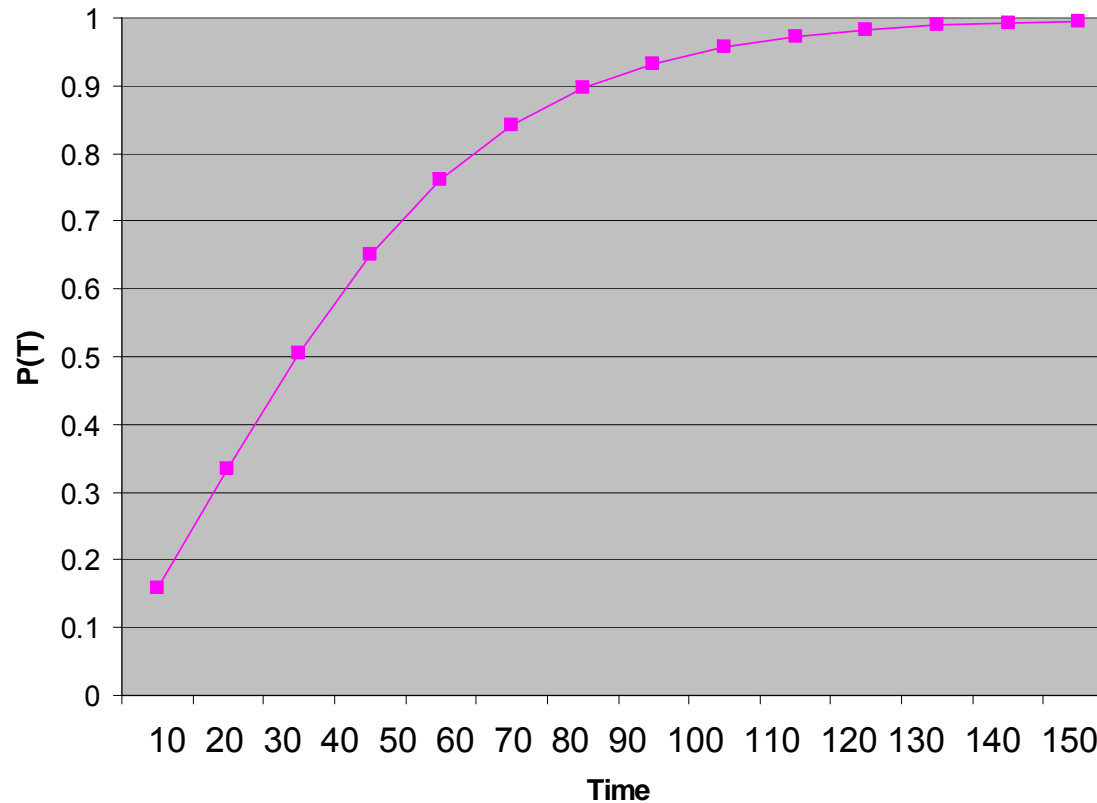


# Persistent ISR Ratio



Minimum of Derivative (slope) of "Find" to other Derivatives.

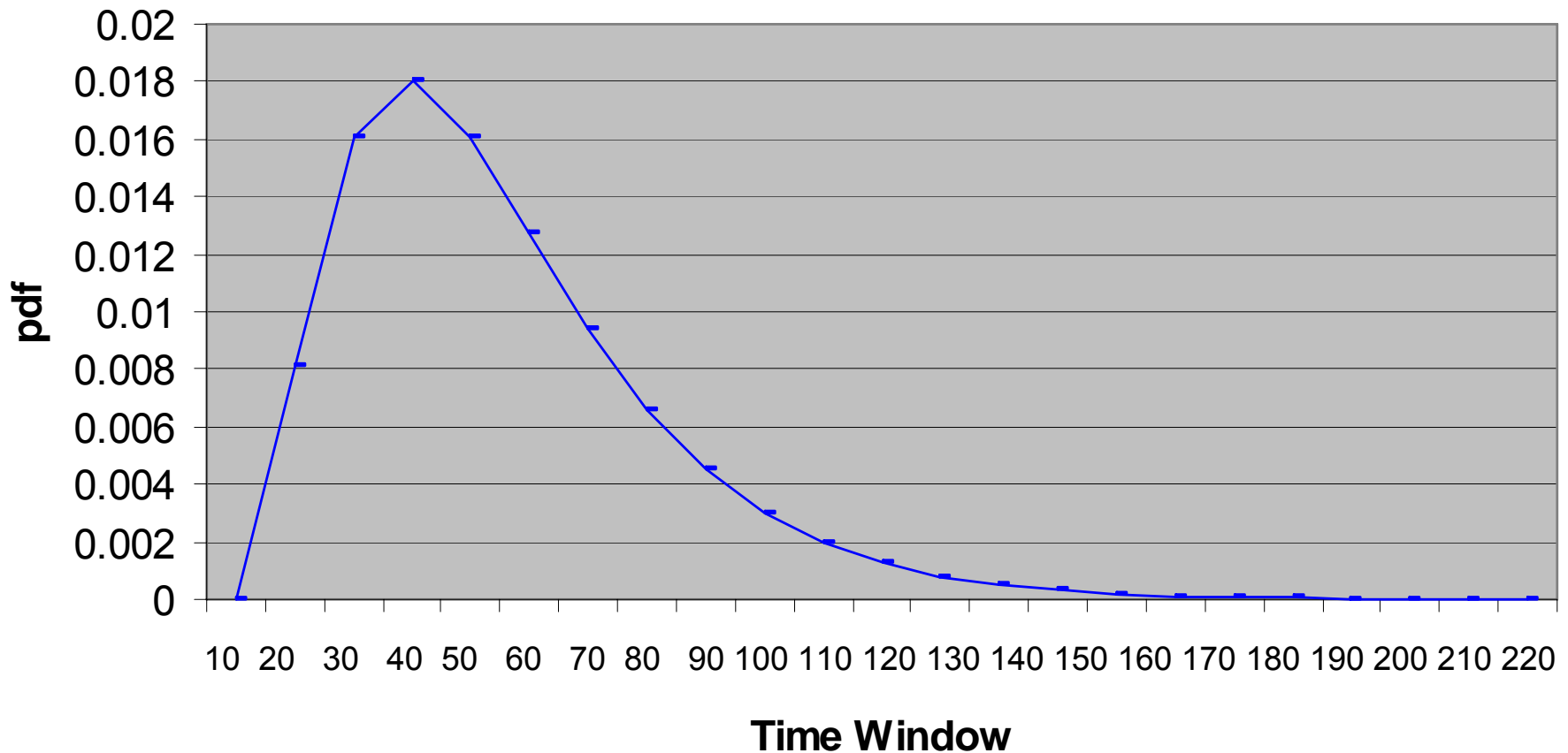
# $P(T)$ as a Function of Time Window



Given: Time varies (X axis) from 10 to 150 time units;  $\theta_i = 20, 15, 10$  respectively

# Density Function

pdf

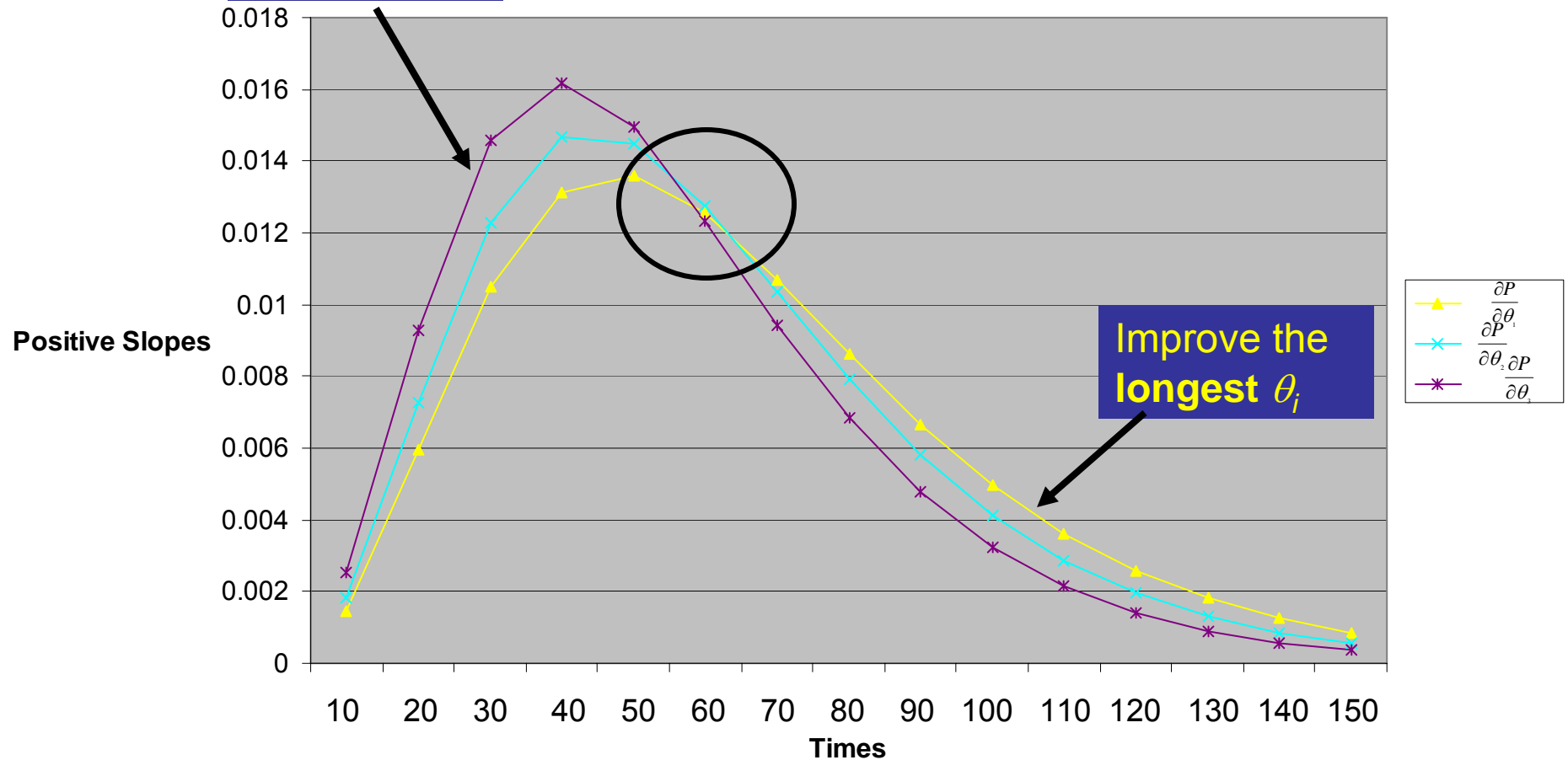


Given: Time varies (X axis) from 10 to 220 time units;  $\theta_i = 20, 15, 10$  respectively

# Derivatives

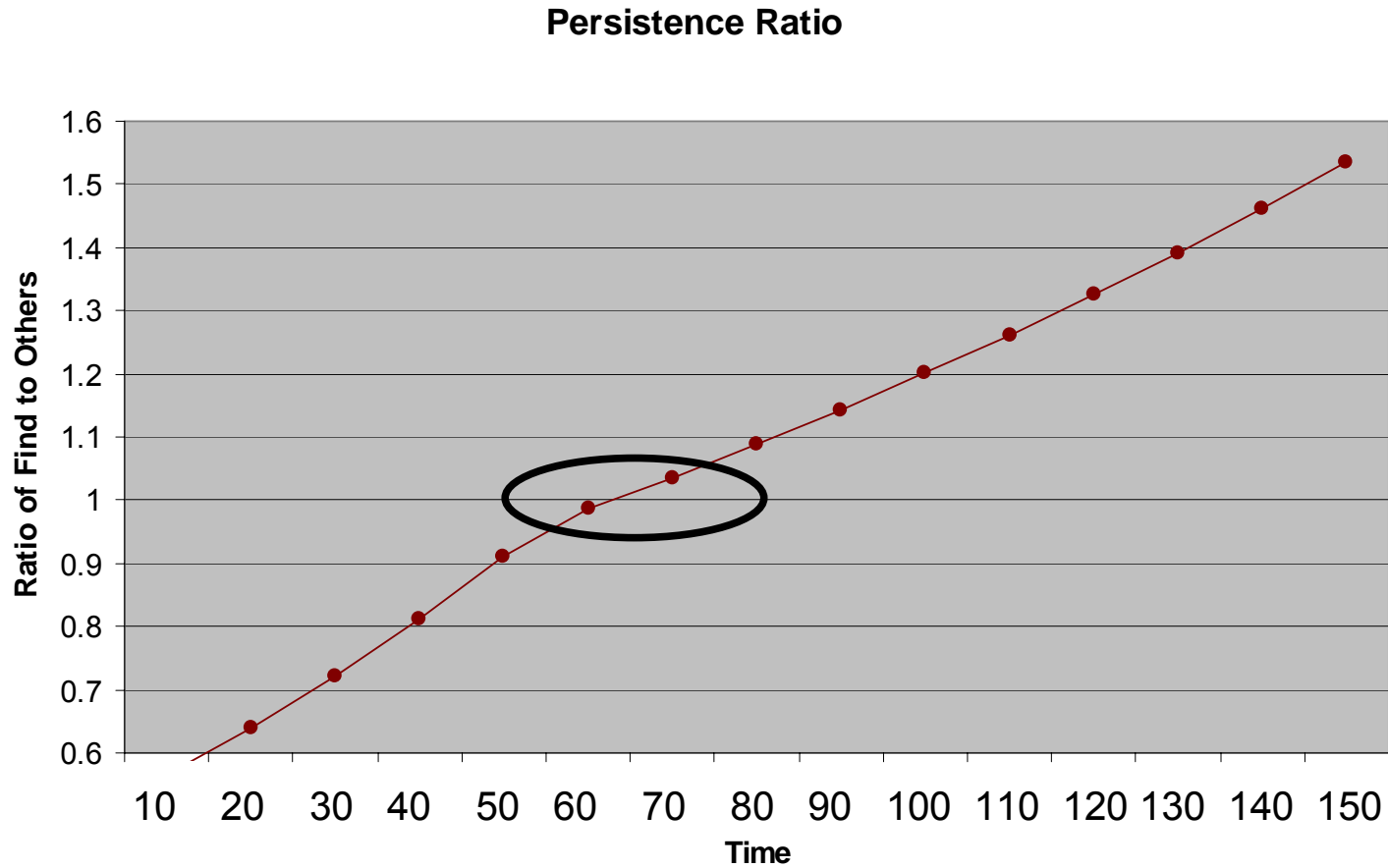
Improve the shortest  $\theta_i$

Improve the longest  $\theta_i$



Given: Time varies (X axis) from 10 to 150 time units;  $\theta_i = 20, 15, 10$  respectively

# Persistent ISR Ratio



Minimum of Derivative (slope) of “Find” to other Derivatives.

## Another Application: Solving for Window of Opportunity

- Find the minimum window of opportunity given a set of  $\theta_i$ s.
  - That is, if we know our  $\theta_i$ s and we want no less than a given  $P(T)$ , we can solve  $P(T)$  equation for  $T$ .
  - For example, if  $\theta_1 = 10$  minutes,  $\theta_2 = 15$  minutes, and  $\theta_3 = 5$  minutes and we want no less than a  $P(\text{FFF}) = 0.80$ , we solve  $P(T)$  equation for  $T$  which yields  $T = 43.19$  minutes.
  - This tells us that, given these  $\theta_i$ s, if we have a window of opportunity of less than 43.19 minutes, we have a  $P(T) < 0.80$ .

Now, we can perform cost-benefit trade studies

# Summary

- We've derived a closed form equation for  $P(T)$
- We've derived closed form solutions for partial derivatives to study driving factors of  $P(T)$
- We've looked at various applications
- We've developed a method for conducting cost-benefit studies and trade-offs.

# EXAMPLES



Microsoft Excel  
Worksheet