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14. ABSTRACT We have investigated high order finite difference weighted essentially non-oscillatory (WENO) schemes, finite volume WENO schemes and discontinuous Galerkin finite element methods, for solving partial differential equations with discontinuous or rapidly changing solutions. Algorithm development, analysis, implementation and applications have been carried out.  Research has been performed in all areas listed in the original proposal, and progress and results consistent with the original					
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## Report Title

Final Report: Algorithm Development and Application of High Order Numerical Methods for Shocked and Rapid Changing Solutions

### ABSTRACT

We have investigated high order finite difference weighted essentially non-oscillatory (WENO) schemes, finite volume WENO schemes and discontinuous Galerkin finite element methods, for solving partial differential equations with discontinuous or rapidly changing solutions. Algorithm development, analysis, implementation and applications have been carried out.

Research has been performed in all areas listed in the original proposal, and progress and results consistent with the original objectives have been obtained. There are 53 refereed journal publications (42 appeared, 11 accepted and to appear) resulting from this project.

These achievements have strengthened our objective to obtain powerful and reliable high order numerical algorithms and use them to solve convection dominated problems, especially those of army interest.

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**List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:**

**(a) Papers published in peer-reviewed journals (N/A for none)**

- [1] Y.-X. Liu and C.-W. Shu,  
Local discontinuous Galerkin methods for moment models  
in device simulations: formulation and one dimensional results,  
Journal of Computational Electronics, v3 (2004), pp.263-267.
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A high order ENO conservative Lagrangian type scheme for  
the compressible Euler equations,  
Journal of Computational Physics, v227 (2007), pp.1567-1596.

**Number of Papers published in peer-reviewed journals:** 42.00

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**(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)**

**Number of Papers published in non peer-reviewed journals:** 0.00

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**(c) Presentations**

**Number of Presentations:** 0.00

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**Non Peer-Reviewed Conference Proceeding publications (other than abstracts):**

**Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):** 0

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**Peer-Reviewed Conference Proceeding publications (other than abstracts):**

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**(d) Manuscripts**

- [1] J. Cheng and C.-W. Shu,  
A high order accurate conservative remapping method on staggered meshes,  
Applied Numerical Mathematics, to appear.
- [2] F. Li and C.-W. Shu,  
A local-structure-preserving local discontinuous Galerkin  
method for the Laplace equation,  
Methods and Applications of Analysis, to appear.
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partial differential equations with higher order derivatives,  
Mathematics of Computation, to appear.
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A local discontinuous Galerkin method for the Camassa-Holm  
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under large time steps,  
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- [6] Y.-J. Liu, C.-W. Shu, E. Tadmor and M. Zhang,  
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International Journal for Numerical Methods in Fluids, to appear.
- [8] Y. Lu, S.C. Wong, M. Zhang, C.-W. Shu and W. Chen,  
Explicit construction of entropy solutions for the Lighthill-Whitham-Richards  
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Transportation Research Part B, to appear.
- [9] Y. Liu, C.-W. Shu and M. Zhang,  
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convection dominated problems,  
SIAM Review, to appear.
- [12] L. Huang, S.C. Wong, M. Zhang, C.-W. Shu and W.H.K. Lam,  
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submitted to Transportation Research Part B.

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Convergence of high order finite volume weighted essentially non-oscillatory  
scheme and discontinuous Galerkin method for  
nonconvex conservation laws,  
submitted to SIAM Journal on Scientific Computing.

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 $L^2$  stability analysis of the central discontinuous Galerkin method  
and a comparison between the central and regular  
discontinuous Galerkin methods,  
submitted to ESAIM: Mathematical Modelling and Numerical Analysis ( $M^2AN$ ).

[15] Y. Lu, S.C. Wong, M. Zhang and C.-W. Shu,  
The entropy solutions for the  
Lighthill-Whitham-Richards traffic flow model with a discontinuous  
flow-density relationship,  
submitted to Transportation Science.

[16] W. Wang, X. Li and C.-W. Shu,  
The discontinuous Galerkin method for the multiscale modeling of dynamics of crystalline solids,  
submitted to Multiscale Modeling and Simulation: A SIAM Interdisciplinary Journal.

**Number of Manuscripts:** 16.00

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**Number of Inventions:**

---

**Graduate Students**

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
Yingda Cheng	0.50
Ling Yuan	0.90
<b>FTE Equivalent:</b>	<b>1.40</b>
<b>Total Number:</b>	<b>2</b>

**Names of Post Doctorates**

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
<b>FTE Equivalent:</b>	
<b>Total Number:</b>	

**Names of Faculty Supported**

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	National Academy Member
Chi-Wang Shu	0.10	No
<b>FTE Equivalent:</b>	<b>0.10</b>	
<b>Total Number:</b>	<b>1</b>	

**Names of Under Graduate students supported**

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
<b>FTE Equivalent:</b>	
<b>Total Number:</b>	



**Student Metrics**

This section only applies to graduating undergraduates supported by this agreement in this reporting period

- The number of undergraduates funded by this agreement who graduated during this period: ..... 0.00
- The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:..... 0.00
- The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:..... 0.00
- Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):..... 0.00
- Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:..... 0.00
- The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense ..... 0.00
- The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: ..... 0.00

**Names of Personnel receiving masters degrees**

<u>NAME</u>
<b>Total Number:</b>

**Names of personnel receiving PHDs**

<u>NAME</u>	
Shanqin Chen	
Zhengfu Xu	
Ching-Shan Chou	
Yulong Xing	
Ling Yuan	
Yingda Cheng	
Jingmei Qiu	
<b>Total Number:</b>	7

**Names of other research staff**

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
<b>FTE Equivalent:</b>	
<b>Total Number:</b>	

**Sub Contractors (DD882)**

**Inventions (DD882)**

## Final Report of ARO Grant W911NF0410291

### Algorithm Development and Application of High Order Numerical Methods for Shocked and Rapid Changing Solutions

Chi-Wang Shu  
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Providence, RI 02912  
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August 1, 2004 to July 31, 2007

#### 1. Foreword

We have investigated high order finite difference weighted essentially non-oscillatory (WENO) schemes, finite volume WENO schemes and discontinuous Galerkin finite element methods, for solving partial differential equations with discontinuous or rapidly changing solutions. Algorithm development, analysis, implementation and applications have been carried out. These achievements have strengthened our objective to obtain powerful and reliable high order numerical algorithms and use them to solve convection dominated problems, especially those of army interest.

#### 4. Statement of the Problem Studied

The problems studied in this project involve numerically solving partial differential equations with either discontinuous or rapidly changing solutions. These include the so-called “convection dominated” cases, such as the hyperbolic equations, parabolic equations with small viscosities (e.g. high Reynolds number Navier-Stokes equations), KdV type equations with small dispersions (small third derivative terms), time-dependent bi-harmonic equations and equations with hyper-viscosity (small fourth or higher even derivative terms). Furthermore, we are interested in long time solutions of such equations with waves traveling a long distance, such as sound waves in aeroacoustics, vortex in helicopter rotor motions, etc. The requirement on numerical methods would thus be high order accuracy, low dissipation/dispersion errors over long time, and a clean and sharp shock resolution. An emphasis during the proposed work period will be on a study of efficiency, especially parallel efficiency of the proposed methods for various physical problems with methods of different orders of accuracy, and efficient multi-domain and adaptive methods. Finite difference

weighted essentially non-oscillatory (WENO) schemes, finite volume WENO schemes and discontinuous Galerkin finite element methods have been considered.

## 5. Summary of the Most Important Results

Research has been performed in all areas listed in the original proposal, and progress and results consistent with the original objectives have been obtained. There are 53 refereed journal publications (42 appeared, 11 accepted and to appear) resulting from this project, see Section 6 for a list of them.

An anti-diffusive flux correction for high order finite difference WENO schemes has been designed to sharpen contact discontinuities without affecting accuracy and has been applied to shallow water with transport of pollutant. This technique has also been generalized to Hamilton-Jacobi equations to sharpen corners.

High order finite difference and finite volume WENO schemes and discontinuous Galerkin methods with the exact conservation property for the shallow water equations and for a broader class of equations including the equations governing the chemosensitive movement have been developed. The more difficult situation of moving water steady state has also been addressed.

Recovery of high order accuracy in WENO computations of steady state hyperbolic systems has been studied.

A new smoothness indicator for the WENO schemes has been designed and its effect on the convergence to steady state solutions has been studied.

Multistage interaction of a shock wave and a strong vortex has been studied by using a high order accurate WENO solver.

Effects of shock waves on Rayleigh-Taylor instability have been studied using a very high order accurate WENO solver.

A weighted essentially non-oscillatory numerical scheme for a multi-class traffic flow model on an inhomogeneous highway has been developed.

Second order high resolution schemes and higher order WENO schemes for a hierarchical size-structured model have been developed and analyzed.

Computational study of shock mitigation and drag reduction by pulsed energy lines has been performed using a multi-domain WENO solver.

Two-dimensional semiconductor device simulations by WENO-Boltzmann schemes have been investigated in terms of efficiency, boundary conditions and comparison to Monte Carlo methods.

A Hermite type WENO scheme has been developed for solving Hamilton-Jacobi equations. Also for Hamilton-Jacobi equations, a reinterpretation and simplified implementation of a discontinuous Galerkin method have been given. A discontinuous Galerkin finite element method for directly solving the Hamilton-Jacobi equations has been designed and tested.

A high order residual distribution conservative finite difference WENO scheme

for steady state problems on non-smooth meshes has been developed, which has a smaller computational cost than finite volume schemes.

A non-oscillatory hierarchical reconstruction for central and finite volume schemes has been designed.

A high order ENO conservative Lagrangian type scheme for the compressible Euler equations has been developed.

Stable local discontinuous Galerkin methods have been designed for solving the nonlinear Schrödinger equations, the Kadomtsev-Petviashvili equation, the Zakharov-Kuznetsov equation, the Kuramoto-Sivashinsky equations, the Ito-type coupled KdV equations, and the Cahn-Hilliard type equations. Error estimates for some of these methods have been given.

A discontinuous Galerkin method based on non-polynomial approximation spaces has been developed and analyzed.

A comparison of troubled-cell indicators for Runge-Kutta discontinuous Galerkin methods using weighted essentially nonoscillatory limiters has been performed.

A numerical study for the performance of the Runge-Kutta discontinuous Galerkin method based on different numerical fluxes has been performed.

The heterogeneous multiscale method based on the discontinuous Galerkin method for hyperbolic and parabolic problems has been developed. A discontinuous Galerkin implementation of a domain decomposition method for kinetic-hydrodynamic coupling multiscale problems in gas dynamics and device simulations has been addressed.

A unified local discontinuous Galerkin method for moment models for semi-conductor device simulations has been developed, which is suitable for adaptive computations.

Efficient time discretization for local discontinuous Galerkin methods has been investigated.

Analysis of the relativistic Vlasov-Maxwell model in an interval has been performed.

## **6. Bibliography: refereed journal papers (appeared or accepted) which have quoted partial support by this grant**

1. M.J. Caceres, J.A. Carrillo, I. Gamba, A. Majorana and C.-W. Shu, DSMC versus WENO-BTE: a double gate MOSFET example, *Journal of Computational Electronics*, v5 (2006), pp.471-474.
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  9. C.-S. Chou and C.-W. Shu, High order residual distribution conservative finite difference WENO schemes for steady state problems on non-smooth meshes, *Journal of Computational Physics*, v214 (2006), pp.698-724.
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15. K. Kremeyer, K. Sebastian and C.-W. Shu, Computational study of shock mitigation and drag reduction by pulsed energy lines, *AIAA Journal*, v44 (2006), pp.1720-1731.
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