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14. ABSTRACT

We have extended Wilcox's classical result to address an issue of acceptable approximation of the "ideal" ambiguity surface in the area of interest. We have considered a problem of constructing a waveform with minimal volume under the ambiguity surface in a certain given area. In case when the region of interest is a circle centered at the origin, we have proven that Hermite waveform is a solution to such optimization problem. We have developed software for numerical implementations for various choices of areas where ambiguity surface desired to be small.
We have also considered frequency stepping design, which is one of the known techniques employed by modern radars to

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Report Title

Final Report: A New Approach to Radar Waveform Design

ABSTRACT

We have extended Wilcox's classical result to address an issue of acceptable approximation of the "ideal" ambiguity surface in the area of interest. We have considered a problem of constructing a waveform with minimal volume under the ambiguity surface in a certain given area. In case when the region of interest is a circle centered at the origin, we have proven that Hermite waveform is a solution to such optimization problem. We have developed software for numerical implementations for various choices of areas where ambiguity surface desired to be small.

We have also considered frequency stepping design, which is one of the known techniques employed by modern radars to achieve high range resolution. We have developed an approach which allows us to suppress grating lobes below a desired threshold level in the case of appropriately chosen stepped frequency waveforms. We have introduced a multi-parametric generalization of a stepped frequency train, and by exploiting a factorization of the autocorrelation function, achieved a useful trade-off between competing properties of the factors by careful choices of relevant parameters.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

I.Gladkova, D.Chebanov, On a new extension of Wilcox's Method, WSEAS Transactions on Mathematics, Vol.3, p.244--249, April 2004.

I.Gladkova, A Family of Frequency Modulated Waveforms with Excellent Resolution Properties, IEEE Transactions on Aerospace and Electronic Systems, AES-40(1): p.355--359, January 2004.

Number of Papers published in peer-reviewed journals: 2.00

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I. Gladkova, A General Class of Stepped Frequency Trains, Proceedings of the 2006 IEEE International Radar Conference, April 24-27, 2006, 6 pp.

D. Chebanov, I. Gladkova, J. Weber, Family of Stepped-Frequency LFM Trains with Low Autocorrelation Sidelobes, The Second IASTED International Conference on Antennas, Radar, and Wave Propagation, July 19-21, 2005, Banff, Canada

D. Chebanov, I. Gladkova, Stepped-frequency LFM trains with low range sidelobes, Proceedings of International Radar Symposium, Sept. 2005, pp. 511-515

I. Gladkova, D. Chebanov, Suppression of Grating Lobes in Stepped-Frequency Train, Proceedings of the 2005 IEEE International Radar Conference, May 9-12, 2005, Arlington, VA, p.371-376

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I. Gladkova, D. Chebanov, On the synthesis problem for a waveform having a nearly ideal ambiguity surface, Proceedings of the International Conference RADAR 2004, 18-22 October 2004, Toulouse, France

I.Gladkova, D.Chebanov, On a new extension of Wilcox's Method invited paper, 5th WSEAS International Conference on Applied Mathematics, Miami, Florida, April 21-23, 2004

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts): 8

(d) Manuscripts

I. Gladkova, D. Chebanov, Grating Lobes Suppression in Stepped-Frequency Pulse Train, to appear in IEEE Transaction on Aerospace and Electronic Systems.

I. Gladkova, Analysis of Stepped Frequency Pulse Train Design, to appear in IEEE Transaction on Aerospace and Electronic Systems.

Number of Manuscripts: 2.00

Number of Inventions:

Graduate Students

NAME	PERCENT_SUPPORTED
Dmitry Chebanov	1.00
FTE Equivalent:	**1.00**
Total Number:	**1**

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Irina Gladkova	0.15	No
Leonid Roytman	0.15	No
FTE Equivalent:	**0.30**	
Total Number:	**2**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
John Weber	0.50
Marcel Kei	0.50
FTE Equivalent:	**1.00**
Total Number:	**2**

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

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Names of Personnel receiving masters degrees

NAME	
Marcel Kei	
Total Number:	**1**

Names of personnel receiving PHDs

NAME	
Total Number:	

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Final Progress Report

A New Approach to Radar Waveform Design

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No. DAAD19-03-1-0329

Contents

1 Statement of the problem studied

1.1 Approximation of ideal ambiguity surface

The ambiguity function of a waveform $u(t)$, which is assumed to be a function of time with unit energy, is defined by

$$\chi_u(\tau,\nu) = \int_{-\infty}^{\infty} u\left(t-\frac{\tau}{2}\right)\overline{u\left(t+\frac{\tau}{2}\right)}\, e^{-j2\pi\nu t}\, dt.$$

A radar waveform with ideal range-doppler characteristics would produce an ambiguity surface that is zero everywhere except the origin. However, no finite energy signal gives rise to such surface [3]. Waveform synthesis has been an important problem in radar design since the publication of Woodward's book [20] , but despite numerous attempts to solve it, the search for practical solutions to the synthesis problem remains open. The elegant paper of Wilcox [19], presents a mathematically complete solution (using Hilbert space technique), provided that the desired ambiguity shape is given in analytical form, which is not the case in any practical radar applications. In practice, engineers have a general idea of acceptable shape rather than the formulas describing it, thus making Wilcox's algorithm not applicable. Moreover, in many situations, it is not even necessary to have a certain shape for all the values of time and doppler delays (the region where the ambiguity surface is desired to be small depends on the particular radar application), and Wilcox's algorithm does not treat the situations where only part of the ambiguity surface has to be approximated.

We have adapted Wilcox's method to the case of specified subregion of R^2 and have shown that this generalization enables us to construct many promising new waveforms with desired ambiguity profiles in the regions surrounding the main lobe.

The optimization over a subregion of R^2 generalizes Wilcox's approach, which optimizes over all of R^2. There are new subtleties that appear with this approach, since we can seek, for example, to make an ambiguity small over some region, which, if successful, will push the bulk of the function outside the region where we want it to be small. Obviously, this is not possible if the region is all of R^2, because of the volume property of the ambiguity function.

We should note, that one of the desired features of radar waveforms is the constant amplitude (due to some radar hardware limitations). This greatly complicates the design of waveform with prescribed ambiguity surface. One of the possibilities that allows the consideration of the waveforms with variable amplitude is to work with a pair of waveforms: the transmitted signal of constant amplitude and the reference signal of arbitrary amplitude that is used during the signal processing stage at the receiver. Thus we are interested in cross-ambiguity function

$$\chi_u(\tau,\nu) = \int_{-\infty}^{\infty} u_{\text{tr}}\left(t-\frac{\tau}{2}\right)\overline{u_{\text{ref}}\left(t+\frac{\tau}{2}\right)}\, e^{-j2\pi\nu t}\, dt,$$

where $u_{\text{tr}}(t) = u_0 e^{j\pi w(t)}$ is a transmitted frequency modulated signal and $u_{\text{ref}}(t) = \sigma(t)e^{j\pi w(t)}$ is a reference signal with varying envelope $\sigma(t)$.

We have considered the problem of sidelobe suppression of the cross-ambiguity surface over the given region of interest. We extended our approach to the case of designing a pair of transmitted/reference waveforms with desired characteristics.

1.2 Stepped-Frequency Waveforms

For purposes of radar, it is desirable to emit waveforms having wide bandwidth in order to enhance range resolution. If, for whatever reasons, e.g., hardware limitations, such emitted signals are not practical, the effect of a wide bandwidth signal can nevertheless in practice be achieved by the method of stringing together, in a suitable manner, narrow-band signals which collectively approximate a wideband emitted signal. Such narrowband pulse sequences are said [11, 15, 18] to comprise a ”synthetic wideband waveform” (specific variants are also known by the names ”stepped-frequency waveform,” ”frequency jump burst,” and ”frequency jump train”).

These synthetic wideband waveforms were introduced in the 1960’s [18]. As pointed out in [15], the S-band Tradex radar (located on the U.S. Army’s Kwajelein Missile Range facility) implemented waveforms of this type in 1974, and experiments using the Aegis SPY-1 radar, the Patriot radar, and RSTER have been performed since then.

The hardware required to produce these synthetic wideband waveforms is not particularly expensive. In practical implementations of the above approach, good range resolution is attained by transmitting a sequence of narrowband pulses (called ”bursts”), with the center frequencies of the bursts adjusted so that the string of bursts occupies an interval of the desired bandwidth, with time intervals between bursts adjusted so that a narrowband radar receiver can be tuned to receive target echoes from each pulse. Then the received pulses are appropriately combined in a way that effectively duplicates the result that would be obtained from a corresponding wide bandwidth pulse (see Figure 1).

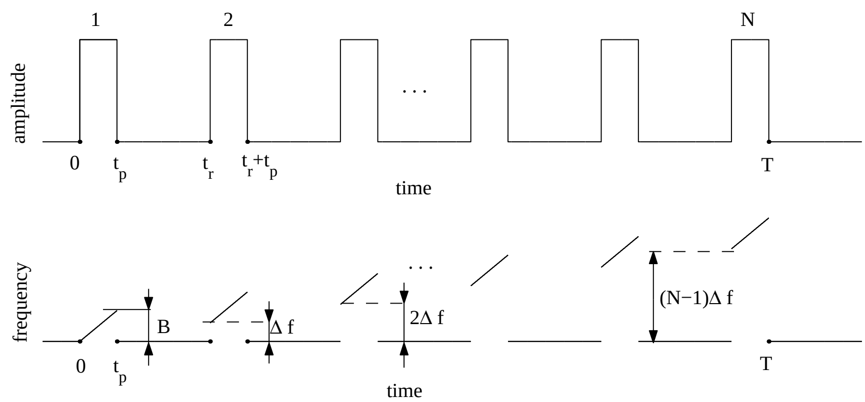

Figure 1: Stepped-frequency LFM pulse train.

It is well-known that the autocorrelation function of a stepped-frequency pulse train suffers from grating lobes that appear due to the presence of the constant frequency step Δf. These high spikes essentially reduce the waveform’s resolution capabilities and, hence, they are undesirable (see figure 2). Recently, publications [8, 12, 13, 15] have discussed different approaches leading to either acceptable suppression or complete elimination of the grating lobes. A new approach to generate nonlinear synthetic wideband waveforms is formulated in [15]. These waveforms distribute energy nonuniformly over the desired frequency band (that is, have nonconstant frequency step between consecutive pulses) and are shown to offer improved performance (i.e., lower range sidelobes, higher range resolution, and/or reduced grating lobes). In [12, 13] grating lobes are reduced by varying the pulsewidths, thus destroying periodicity. [8] contains an analysis that provides very simple relationships between the pulse bandwidth B, its time duration t_p and the frequency step Δf, that allows one to nullify several (or, sometimes, even all) grating lobes of an LFM pulse train. One of the conclusions that can be drawn from the relationship between t_pB and $t_p\Delta f$ obtained in [8] is that the overlap ratio $B/\Delta f$ is large for large values of t_pB. Therefore, in order to increase the bandwidth significantly (under the restriction that the first two grating lobes are nullified), the number of pulses N has to be large, i.e. the obtained relationship can only be applied to "Slow Burst" waveforms.

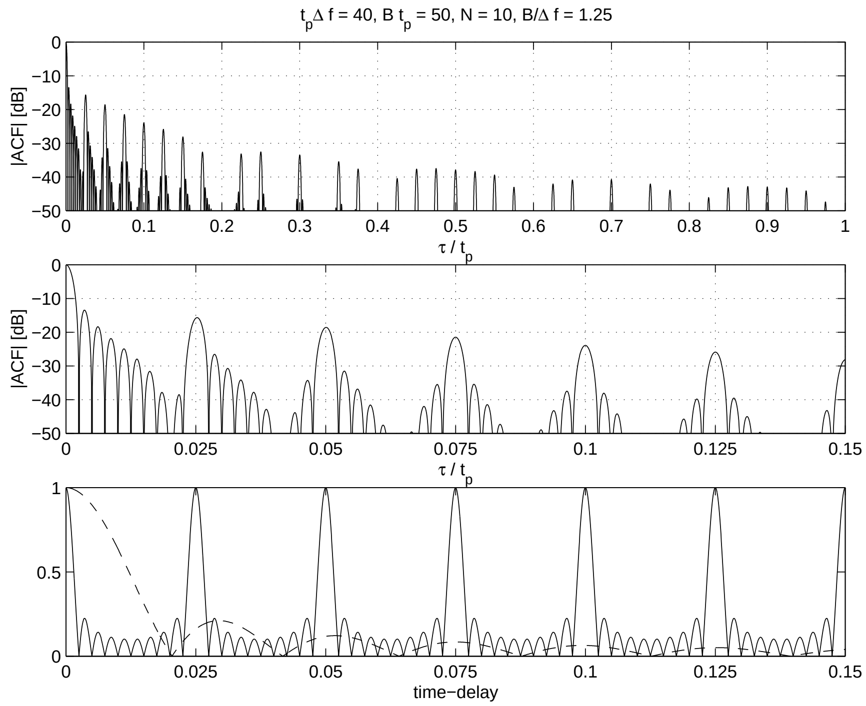

Figure 2: Stepped-frequency train of LFM pulses. Bottom: the first term $|R_1|$ (dashed line) and the second term $|R_2|$ (solid line). Top and Middle: Partial auto-correlation function.

We have developed a modified method for suppressing grating lobes in stepped-frequency pulse trains which allows "Quick Burst" (i.e when N is small) waveforms to be considered. Our modification of the grating lobes suppression problem leads to a significantly different relationship between t_pB and $t_p\Delta f$ than in [8] which allows us to find waveforms with large t_pB, small ratio $B/\Delta f$, and small (rather then zero) grating lobes.

Another issue related to a stepped-frequency LFM pulse train is the high sidelobes of its autocorrelation function appearing in the close vicinity of the main lobe. It is known that within the chosen construction of the stepped-frequency waveform with constant time duration t_p, constant bandwidth B, constant step Δf, and small ratio $B/\Delta f$, it is not possible to suppress these few sidelobes. We have considered non-linear FM pulses as well as linear FM pulses, in order to suppress the sidelobes near the main lobe as well as the grating lobes.

2 Summary of the most important results

Approximation of ideal ambiguity surface

1. We have considered a problem of constructing a waveform with globally optimal ambiguity surface properties in a circular region of an arbitrary size surrounding the main lobe. We have shown that under some, rather general assumption, Hermite waveform of certain order is a solution for this problem.

2. We have considered various aspects of our new formulation of the synthesis problem and have found examples of waveforms with the desired "clear area" under the ambiguity surface. In the considered examples, the height of the sidelobe peaks in substantial neighborhoods of the origin appears to be less than -35dB, whereas the nearest sidelobes outside of the area of interest attain the level of -15dB.

3. We have extended Wilcox's classical results to the case of subregions of R^2. This generalization enables us to construct many promising new waveforms with desired ambiguity profiles in the regions being considered.

4. We have developed algorithms and software that allows numerical simulations needed in our waveform design work. In, particular, we obtained a method of computing Hermite waveform of significantly large order. This algorithm is new and have desired features such as robustness and fast convergence.

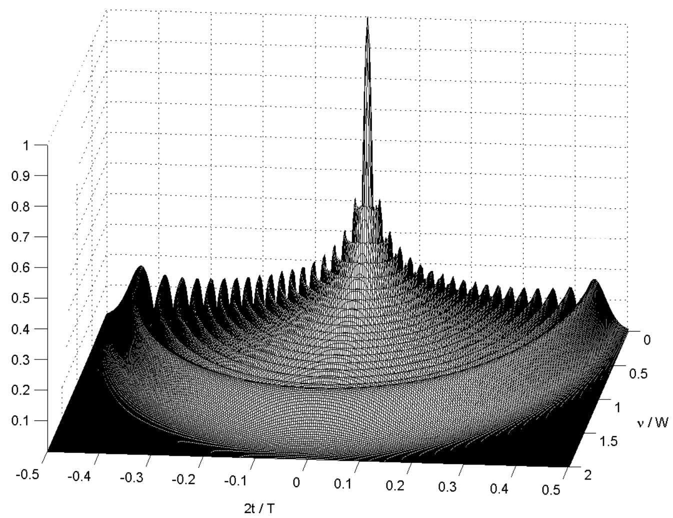

Figure 3: Partial ambiguity surface of Hermite waveform.

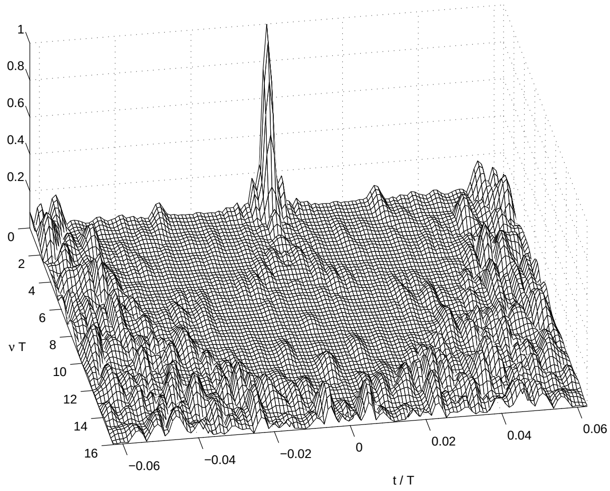

Figure 4: Partial ambiguity surface with suppressed sidelobes in the circular ring.

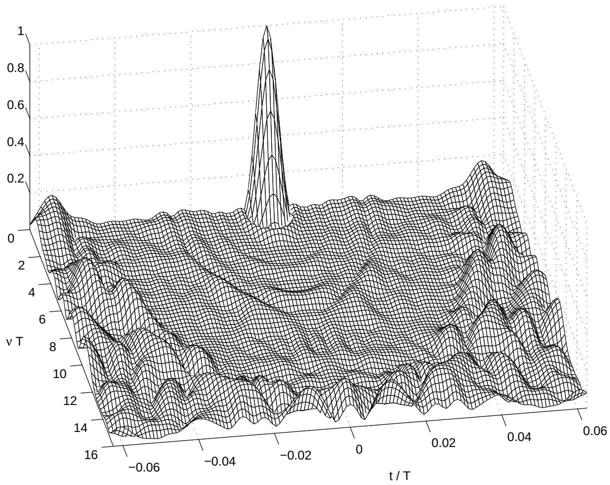

Figure 5: Partial ambiguity surface with suppressed sidelobes in the circular ring over Hermite basis

Stepped-Frequency Waveforms

5. We have developed an approach, which allows us to suppress grating lobes below a desired threshold level in the case of appropriately chosen, stepped frequency waveforms, i.e., sequences of narrowband pulses that span the desired bandwidth. We have developed methodology that allows to choose relevant parameters in order to produce such waveforms with small grating lobes, and give examples of waveforms with small overlap ratio.

6. We have considered the issue of high sidelobes in the vicinity of the main lobe, which are inevitable in a train of LFM waveforms, and have show that it is possible to suppress these, as well as the grating lobes, by means of phase modulation.

7. We have introduced a multiparametric generalization of a stepped frequency train, and by exploiting a factorization of the autocorrelation function, will be able to effect a useful tradeoff between competing properties of the factors by careful choices of relevant parameters. We have developed an appropriate software and applied our method to a representative data set.

8. We have considered several designs that allow us to construct a variety of stepped-frequency waveforms with non-uniform frequency steps producing an autocorrelation function whose sidelobes are suppressed below some predetermined level, and perform complete analysis of the set of parameter values that provide the desired sidelobe suppression. Such analysis allows us to perform a systematic search for stepped-frequency waveforms with desired characteristics.

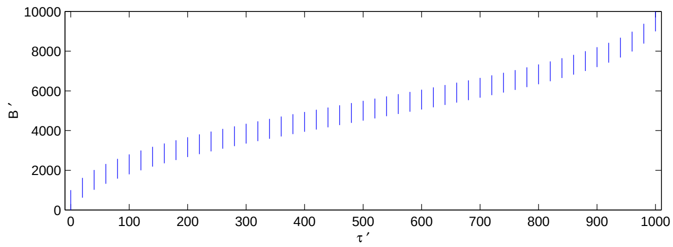

Figure 6. Frequency evolution of stepped frequency train.

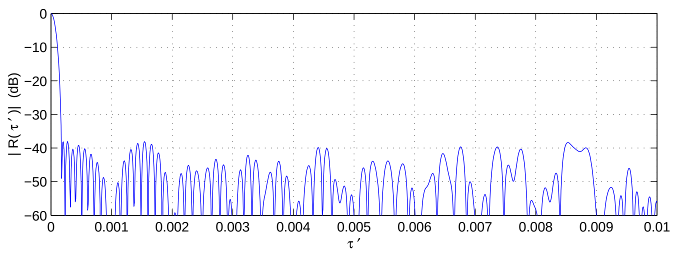

Figure 7. Partial autocorrelation function for $\tau' \in [0, 0.01]$.

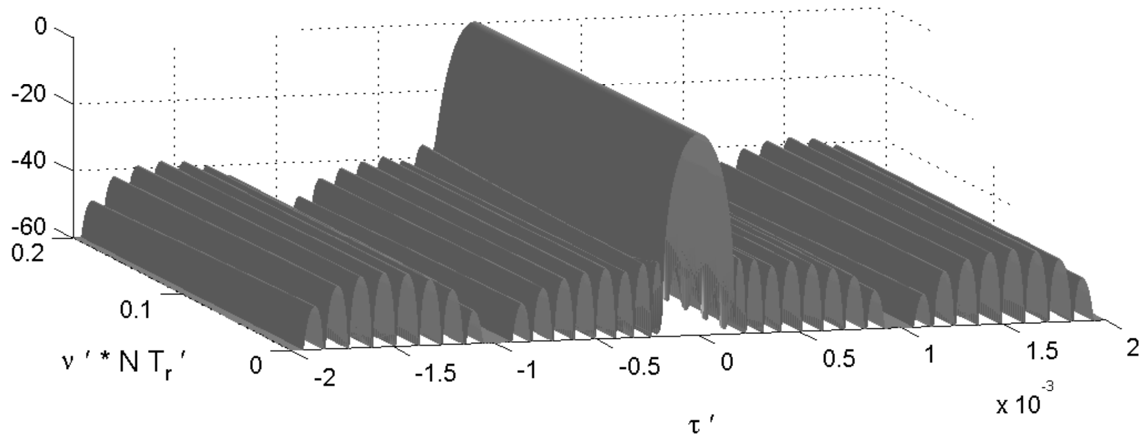

Figure 8. Partial ambiguity surface.

3 Listing of all publications supported under this grant

3.1 Papers published in peer-reviewed journals

1. I.Gladkova, D.Chebanov, On a new extension of Wilcox's Method, WSEAS Transactions on Mathematics, Vol.3, p.244–249, April 2004.

2. I.Gladkova, A Family of Frequency Modulated Waveforms with Excellent Resolution Properties, IEEE Transactions on Aerospace and Electronic Systems, AES-40(1): p.355–359, January 2004.

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2. D. Chebanov, I. Gladkova, J. Weber, Family of Stepped-Frequency LFM Trains with Low Autocorrelation Sidelobes, The Second IASTED International Conference on Antennas, Radar, and Wave Propagation, July 19-21, 2005, Banff, Canada

3. D. Chebanov, I. Gladkova, Stepped-frequency LFM trains with low range sidelobes, *Proc. of Intern. Radar Symposium*, Sept. 2005, pp. 511-515

4. I. Gladkova, D. Chebanov, Suppression of Grating Lobes in Stepped-Frequency Train, Proceedings of the 2005 IEEE International Radar Conference, May 9-12, 2005, Arlington, VA, p.371-376

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8. I.Gladkova, D.Chebanov, On a new extension of Wilcox's Method invited paper, 5th WSEAS International Conference on APPLIED MATHEMATICS, Miami, Florida, April 21-23, 2004

3.3 Manuscripts submitted, but not published

1. I. Gladkova, D. Chebanov, Grating Lobes Suppression in Stepped-Frequency Pulse Train, to appear in IEEE Transaction on Aerospace and Electronic Systems.

2. I. Gladkova, Analysis of Stepped Frequency Pulse Train Design, to appear in IEEE Transaction on Aerospace and Electronic Systems.

4 List of all participating scientific personnel

Prof. Irina Gladkova, PI

Prof. Leonid Roytman, Co-PI

Dmitry Chebanov, Doctoral student (have passed the second exam – dissertation proposal, and advanced to the third level of the PhD program)

John Weber, undergraduate student

Marcel Kei, completed B.S. degree in C.Sc. and continued as graduate student

5 Bibliography

References

[1] R.E. Blahut: *"Theory of Remote Surveillance Algorithms"*, in *"Radar and Sonar"*, Part 1, Springer-Verlag, 1991.

[2] M.N. Cohen, An overview of high range resolution radar techniques, *Proc. of The National Telesystems Conference*, 1991, pp. 107-115.

[3] C.E. Cook, M. Bernfeld, *"Radar Signal: An Introduction to Theory and Applications"*, Academic Press Inc., 1967.

[4] J.P. Costas: *"A Study of a Class of Detection Waveforms Having Nearly Ideal Range-Doppler Ambiguity Properties"*, Proc. of the IEEE, Vol. 72, 1984.

[5] E.C. Farnet and G.H. Stevens, Pulse compression radar, *in* M.I. Skolnik, *"Radar handbook"*, McGraw-Hill, 1990, 2nd edn., Chapter 10.

[6] A. Kajiwara, Stepped-FM pulse radar for vehicular collision avoidance, *Electron. Commun. Jpn. 1, Commun.*, 1999, 82 (6), pp. 1-7.

[7] B.M. Keel, J.A. Saffold, M.R. Walbridge, and J. Chadwick, Nonlinear stepped chirp waveforms with subpulse processing for range sidelobe suppression, *Proc. SPIE*, Vol. 3395, 1998, p. 87-98.

[8] N. Levanon: *"Radar Principles"*, J. Wiley & Sons, 1988.

[9] N. Levanon, Stepped-frequency pulse-train radar signal, *IEE Proc. - Radar, Sonar, and Navigation*, Vol. 149, No. 6, December 2002, pp. 297-309.

[10] N. Levanon and E. Mozeson, Nullifying ACF Grating Lobes in Stepped-Frequncy Train of LFM Pulses, *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 39, No. 2, April 2003, pp. 694-703.

[11] N. Levanon and E. Mozeson, Radar Signals, Wiley, New York, 2004.

[12] D.E. Maron, Non-periodic frequncy-jumped burst waveforms, *Proc. of the IEE International Radar Conference*, 1987, pp. 484-488.

[13] D.E. Maron, Frequncy-jumped burst waveforms with stretch processing, *Proc. of the IEEE International Radar Conference*, 1990, pp. 274-279.

[14] M.I. Skolnik: *"Introduction to Radar Systems"*, McGraw Hill, 1980.

[15] D.J. Rabideau, Nonlinear synthetic wideband waveforms, *Proc. of the IEEE Radar Conference*, 2002, pp. 212-219.

[16] F. Riesz and B. Sz.-Nagy: *"Functional Analysis"*, F. Ungar Publishing Co., New York, 1955.

[17] A.W. Rihaczek, Principles of High-Resolution Radar, Artech House, 1996.

[18] D.R. Wehner, High-Resolution Radar, Artech House, 2nd edition, 1995.

[19] C.H. Wilcox: *"The synthesis problem for radar ambiguity functions"*, MRC Tech. Summary Report 157, US Army, Univ. of Wisconsin, Madison, Wisconsin, April 1960.

[20] P.M. Woodward: *"Probability and Information Theory, with Applications to Radar"*, Pergamon Press, 1953.