Formation Flying through Geodesic Motion and the Different Geometrical Requirements

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ABSTRACT

We investigated the possibility of preserving a preset plane flying formation through geodesic (free) motion around an attractive centre. After a series of approximations, we obtained a first order solution for this problem (the small parameter of the approximation series is set by the characteristic length of the formation and by an eccentricity-like factor, which is related to the trajectory of the geometrical centre of the formation). Such a solution consists in a collection of Keplerian solutions with the same period. The free movement of these bodies creates the impression of a united motion (like in a rigid body motion), at least in the first order of approximation. The instantaneous plane described by the formation experience a precessional motion with respect to a fixed direction, in the same way we know from the kinematics of the LISA mission. We calculated the relative errors for a regular hexagonal formation, for different side lengths, for one period of time.

1.0 INTRODUCTION

In the last decades, a powerful concept has entered in the domain of space exploration, the multi-module mission: several modules working together in order to achieve the purposes of the mission. Although this type of mission has been used from the early times of the space era (we mention the Moon-landing missions of the Apollo program - there were two modules, one acting as a station orbiting the Moon, and the second, doing some complicated manoeuvres to and from the Moon surface, also the ESA CLUSTER mission, which involved four identical spacecraft flying in a tetrahedral formation), the true force of this idea was revealed by the development of the global positioning systems and by the constellations of telecommunication satellites. In a multi-modular mission the tasks are distributed and some activities are made in parallel, in order to increase the fiability and the security of the mission. A perfect example of using multi-modular architecture is given by the LISA mission project, where the geometry of the trajectories will play an essential role in reaching the goals: actual measurement of gravitational waves.

Report Documentation Page

Form Approved
OMB No. 0704-0188

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE **01 DEC 2006**	2. REPORT TYPE **N/A**	3. DATES COVERED **-**	
4. TITLE AND SUBTITLE **Formation Flying through Geodesic Motion and the Different Geometrical Requirements**		5a. CONTRACT NUMBER	
		5b. GRANT NUMBER	
		5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)		5d. PROJECT NUMBER	
		5e. TASK NUMBER	
		5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Institute for Space Sciences Comuna Măgurele, judeţul Ilfov, Str. Atomiştilor nr. 409, OP-CP MG-23, cod RO 077125 Romania**		8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR'S ACRONYM(S)	
		11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT **Approved for public release, distribution unlimited**			
13. SUPPLEMENTARY NOTES **See also ADM202419., The original document contains color images.**			
14. ABSTRACT			
15. SUBJECT TERMS			

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT **UU**	18. NUMBER OF PAGES **12**	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**			

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

In fact, there is nothing really new about this concept, but rather a rediscovery of a good, classical one; for thousand of years the human civilization was defined by missions and expeditions, made by groups of sea-ships or aircrafts...

Formation flying can be regarded as a particular type of multi-modular space mission. The specific elements are:

- *I.1. The mutual distances between modules are small or very small with respect to the length of the trajectory path or to the orbital characteristic length;*

- *I.2. The relative geometry is chosen to fulfil some conditions and some of its characteristics are preserved during the motion;*

- *I.3. We can ignore both the mutual gravitational interactions and their influence on the gravitational sources (i.e. we are able to separate the group movement problem into a collection of restricted one-body problems);*

We have an important number of specific problems:

- *The problem of choosing the optimal geometry for the particular needs of the mission;*

- *The problem of orbital/trajectory design;*

- *The problem of launching (getting into the desired orbit/trajectory);*

- *The problem of adapting the solution to the technologic resources and budget restrictions;*

- *The management of the tasks distribution over the modules;*

- *The problem of communication in between the modules and with the flight coordinating centre;*

- *The problem of orbital correction and mutual geometry correction;*

- *etc.*

It is clear that the flying formations are the perfect field for using and improving the WIRELESS and GRID technologies. But, in the sequel of this paper, we will be interested only in the mechanical aspects of the flight. More precisely, we will investigate the possibility of preserving an arbitrary plane flying formation, without propulsion, in Keplerian hypothesis (i.e. a restricted $N + 1$ bodies problem: N modules and *an* attractive centre).

2.0 MATHEMATICAL MODEL

Let us consider an attractive centre of mass M and a family of N test bodies $(P_i)_{i=1,N}$, with masses $(m_i)_{i=1,N}$. Also, let's assume the above mentioned hypothesis **I.3**, which can be written

(1)
$$m_i << M, (\forall) i = 1, N$$

The problem decouples in N classic Keplerian [4],

(2)
$$\frac{d^2 \overrightarrow{OP_i}}{dt^2} = -\frac{\mu_i}{\left\|\overrightarrow{OP_i}\right\|^3} \overrightarrow{OP_i}, (\forall) i = 1, N$$

where $\mu_i := k^2 (M + m_i) \approx k^2 M =: \mu, (\forall) i = 1, N$

Problem statement: *Can we configure a plane arbitrary N-body flying formation (*see **Figure 1**)*, such that its geometry stays unchanged in the instantaneous mutual plane*?
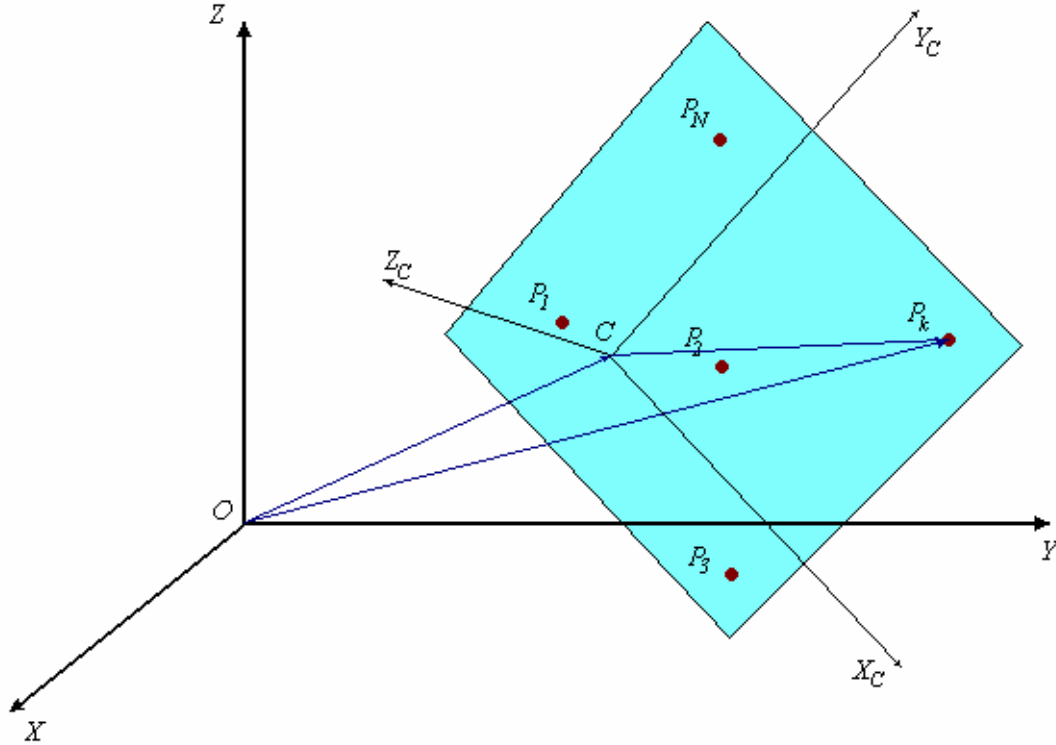

Figure1: The two referential systems: *OXYZ*, centred in the gravitational source and inertial, and the noninertial *CX$_C$Y$_C$Z$_C$*, with *C* as the geometrical centre of the formation. We can consider that the bodies are in the fundamental plane *CX$_C$Y$_C$* in some fixed points *P$_i$*.

We will be interested only in solutions with elliptic trajectories. Observe that in the point *C* (which is still ambiguous defined) we can place a massless (*N*+1)-th body. Using this observation, we can set the relation between the referential systems in the following way:

(3) $$\vec{r} = A(t) \cdot \vec{r}_C + \vec{K}_C(t),$$

where

- $\vec{r}$ is the position vector of an arbitrary point with respect to *OXYZ*,

- $\vec{r}_C$ is the position vector of the same point with respect to *CX$_C$Y$_C$Z$_C$*,

- *A(t)* is the instantaneous rotation matrix,

- $\vec{K}_C(t)$ is a Keplerian (elliptic) motion for the virtual body *C*, which can be interpreted as a translation of the origin of the mobile referential system.

We reduced the initial problem to a simpler one: finding the instantaneous rotation matrix $A(t)$. Taking into account (2) and (3) we obtain

(4)
$$\frac{d^2 A}{dt^2} \cdot \vec{r}_C + 2\frac{dA}{dt} \cdot \frac{d\vec{r}_C}{dt} + A \cdot \frac{d^2 \vec{r}_C}{dt^2} + \frac{d^2 \vec{K}_C}{dt^2} = -\frac{\mu}{\left\| A \cdot \vec{r}_C + \vec{K}_C \right\|^3} \left(A \cdot \vec{r}_C + \vec{K}_C \right).$$

Let's remind the hypothesis: $\vec{r}_C = ct.$, $\vec{K}_C$ is a solution for the problem (2) and **I.1** ($\left\| \vec{r}_C \right\| << \left\| \vec{K}_C \right\| = O(1)$).
We have:

(5)
$$\frac{d^2 A}{dt^2} \cdot \vec{r}_C = -\mu \frac{A \cdot \vec{r}_C}{\left\| \vec{K}_C \right\|^3} + 3\mu \frac{\vec{r}_C \cdot \vec{K}_C}{\left\| \vec{K}_C \right\|^5} \vec{K}_C + O\left(\left\| \vec{r}_C \right\|^2 \right)$$

Due to the mathematical complexity of the problem, we are forced to renounce to look for an exact solution and to consider a linear simplification. We must take the relation (5) as an equation in A, with a sufficiently small given $\vec{r}_C$. The equation also has a restriction, as A must be a rotation:

(6)
$$A(t) = \begin{pmatrix} \cos f_1(t) & -\sin f_1(t) & 0 \\ \sin f_1(t) & \cos f_1(t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos f_2(t) & 0 & -\sin f_2(t) \\ 0 & 1 & 0 \\ \sin f_2(t) & 0 & \cos f_2(t) \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos f_3(t) & -\sin f_3(t) \\ 0 & \sin f_3(t) & \cos f_3(t) \end{pmatrix},$$

where f_1, f_2, f_3 are arbitrary functions of time.

3.0   APPROXIMATE SOLUTIONS IN A *CLOHESSY-WILTSHIRE*-TYPE SYSTEM

Despite the assumed approximation, the simplified problem (5)+(6) remains complicated for an analytical approach. For a further simplification let us introduce a CW (*Clohessy-Wiltshire*) referential system [1], [3]. Consider that the trajectory of C is an ellipse (semi-major axis $a = 1$ and eccentricity e), contained in the plane *OXY*, with O as one of the focuses and *OX* as the pericentre line (see **Figure 2**).

Now, we can introduce the referential system $CX_1Y_1Z_1$ through the relations:

(7)
$$\vec{r} = A(f) \cdot \vec{r}_1 + \vec{R}(f),$$

where

(8)
$$A(f) = \begin{pmatrix} \cos f & -\sin f & 0 \\ \sin f & \cos f & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \vec{R}(f) = \rho(f) \begin{pmatrix} \cos f \\ \sin f \\ 0 \end{pmatrix}, \quad \rho(f) = \frac{1-e^2}{1+e\cos f}$$

and $f = f(t)$ is the true anomaly of the fictitious motion of C.

We get

(9)
$$\left(\frac{d^2 A}{df^2} \cdot \frac{G^2}{\rho^4} - \frac{dA}{df} \cdot \frac{G}{\rho^3} \cdot \frac{d\rho}{dt}\right) \cdot \vec{r_1} + 2\left(\frac{dA}{df} \cdot \frac{G}{\rho^2}\right) \cdot \frac{d\vec{r_1}}{dt} + A \cdot \frac{d^2\vec{r_1}}{dt^2} =$$
$$= -\mu \frac{A \cdot \vec{r_1}}{\rho^3} + 3\mu \frac{\vec{r_1} \cdot \vec{R}}{\rho^5} \vec{R} + O\left(\left\|\vec{r_1}\right\|^2\right)$$

where $G = \sqrt{\mu a\left(1 - e^2\right)}$ is the (constant) value of the kinetic momentum.
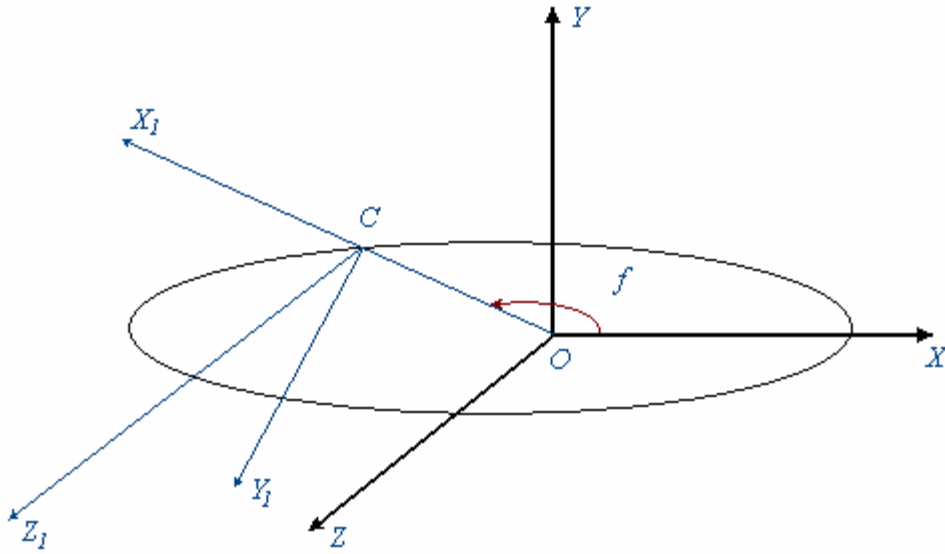
Figure 2: The *Clohessy-Wiltshire*-type referential system, *CX₁Y₁Z₁*.

Neglecting the second order terms, equation (9) reads:

(10)
$$\begin{cases} F_1 \cos f - F_2 \sin f = 0, \\ F_2 \cos f + F_1 \sin f = 0, \\ F_3 = 0. \end{cases}$$

where

(11)
$$\begin{cases} F_1 = \frac{d^2 x_1}{dt^2} - 2\frac{G}{\rho^2}\frac{dy_1}{dt} - \left(\frac{G^2}{\rho^4} + 2\frac{\mu}{\rho^3}\right)x_1 + \frac{G}{\rho^3}\frac{d\rho}{dt}y_1, \\ F_2 = \frac{d^2 y_1}{dt^2} + 2\frac{G}{\rho^2}\frac{dx_1}{dt} - \left(\frac{G^2}{\rho^4} - \frac{\mu}{\rho^3}\right)y_1 - \frac{G}{\rho^3}\frac{d\rho}{dt}x_1, \\ F_3 = \frac{d^2 z_1}{dt^2} + \frac{\mu}{\rho^3}z_1. \end{cases}$$

Relations (10) take place if and only if $F_1 = F_2 = F_3 = 0$.

And now, the announced simplification:

(12)
$$e = O\left(\left\|\vec{r_1}\right\|^2\right).$$

Consequently, we can approximate the CW orbit with a circular one by neglecting the $O(e)$ terms. We obtain the following, explicitly integrable system:

(13)
$$\begin{cases} \dfrac{d^2 x_1}{dt^2} - 2G\dfrac{dy_1}{dt} - 3G^2 x_1 = 0, \\ \dfrac{d^2 y_1}{dt^2} + 2G\dfrac{dx_1}{dt} = 0_1, \\ \dfrac{d^2 z_1}{dt^2} + G^2 z_1 = 0. \end{cases}$$

4.0   OBTAINING AN ARBITRARY PLANE FLYING FORMATION

We introduce now what we will call the LISA referential system [2]. Its origin is in C, and the plane CX_LY_L intersects the plane CX_1Y_1 along CY_1, under an angle of 60°, rotating around the CZ_L axis with a constant angular speed G (see **Figure 3**).
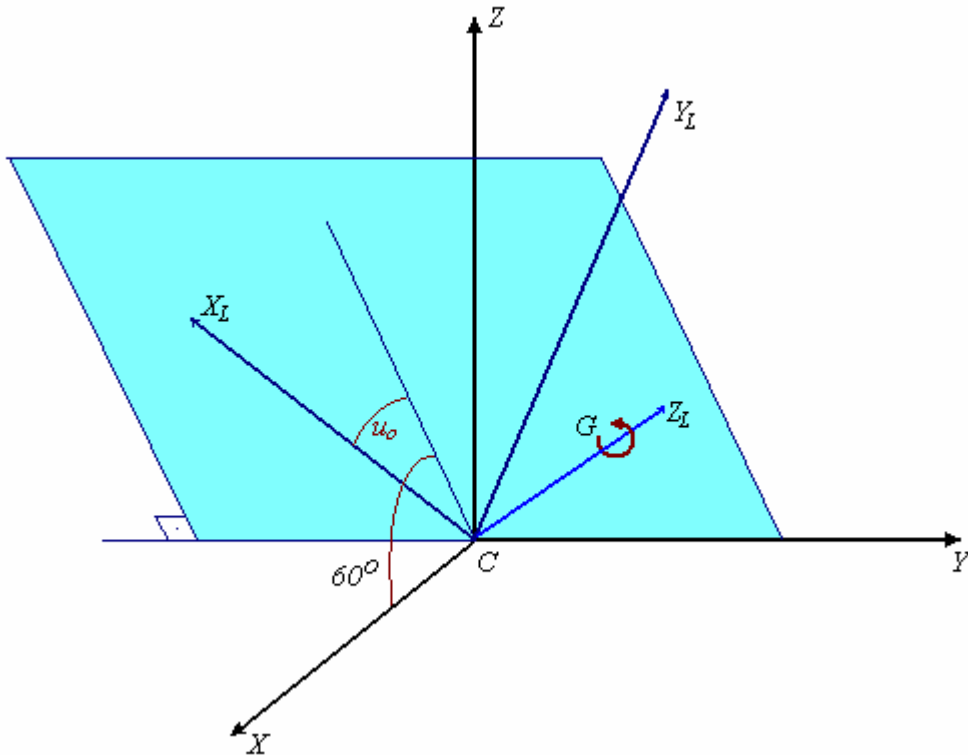
Figure 3: LISA referential system, $CX_LY_LZ_L$.

Observe that for each fixed point in the plane CX_LY_L there is a corresponding solution of (13). Indeed, if we take a fixed point in the fundamental LISA plane: $x_L = R\cos u$, $y_L = R\sin u$, $z_L = 0$; in the CW system, this point will describe a circular trajectory, with the following kinematics:

(14)
$$\begin{cases} x_1(t) = \dfrac{R}{2}\cos(u_0 - u - Gt), \\ y_1(t) = R\sin(u_0 - u - Gt), \\ z_1(t) = \dfrac{\sqrt{3}R}{2}\cos(u_0 - u - Gt). \end{cases}$$

One can easily verify that the functions (14) fulfil the equations (13) and, so, they are approximations for some real motions. Of course, because of the approximations we made, we have the necessary condition $R \ll 1$. It is convenient to consider that the trajectory of the fictitious body C lays in the OXY plane. In this way we can identify the Keplerian orbits approximated by (14). In terms of the orbital elements, the family of orbits (14) reads:

(15)
$$\begin{cases} a = 1 + O(R^2), \\ e = R/2 + O(R^2), \\ I = \arctan\dfrac{\sqrt{3}R}{2} + O(R^2), \\ \Omega \in [0, 2\pi], \\ \omega \in [0, 2\pi], \\ Gt_0 + \Omega + \omega = \Lambda + O(R), \ \Lambda = \text{const.} \end{cases}$$

We are able now to construct a flight formation by simply choosing an arbitrary number of (15)-type orbits, all sharing the same value of Λ; in this way we get a spatial grouping of the bodies, into a flight formation whose relative geometry will be preserved during the motion, in the $O(R^2)$–approximation.

Figure 4 presents a succession of eight intermediate positions from the motion of a 7-body flying formation. The values of R for the seven bodies vary in between 0 and 0.3, in order to allow visualization. For each freeze-frame we show:

- **On the left**: the image of the flying formation seen from outside the system of $N+1$ bodies (bodies are represented as differently coloured disks), together with their trajectories (the blue ellipses) and the motion plane of the geometrical centre (suggested through the black square and its normal),

- **On the right**: just the apparent geometry of the formation, as it appears in projection for the eyes of an observer placed in the attractive centre, tracking the motion of the geometrical centre C.
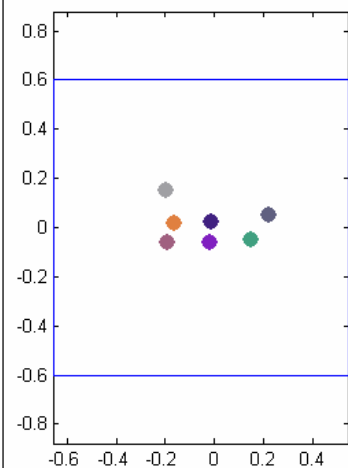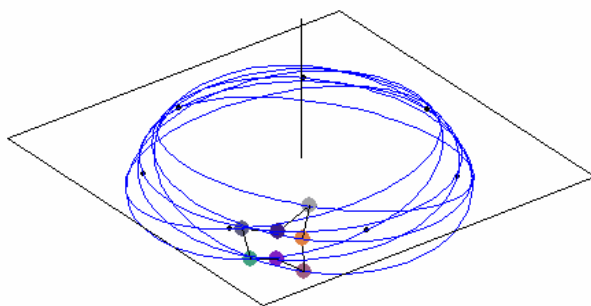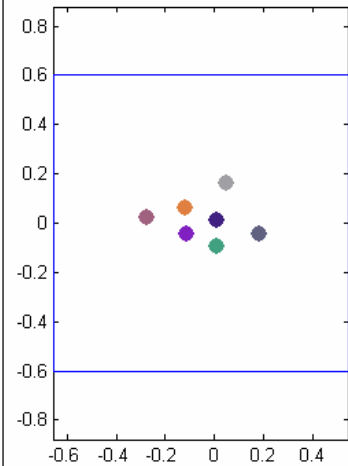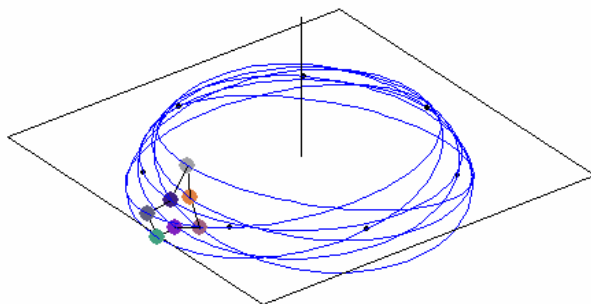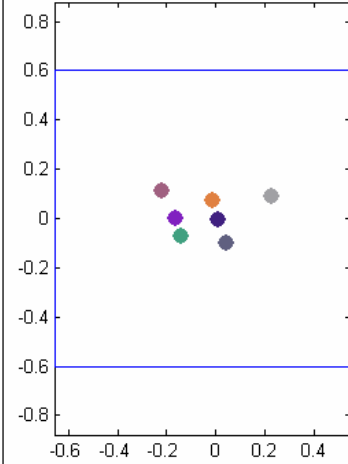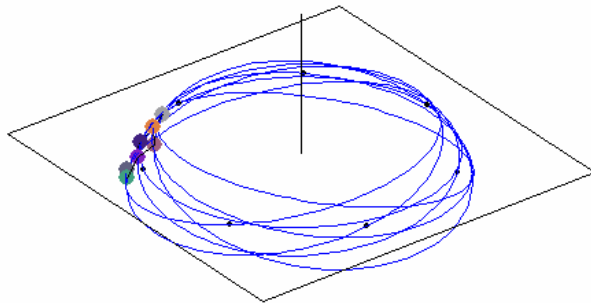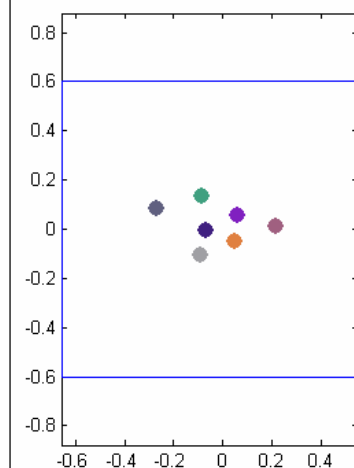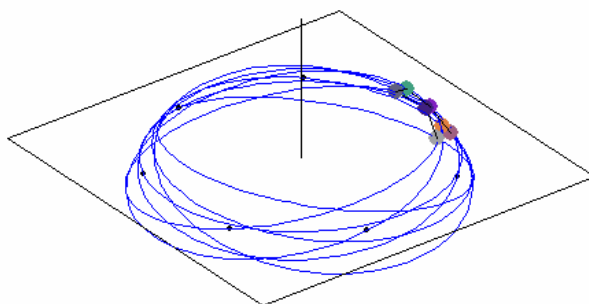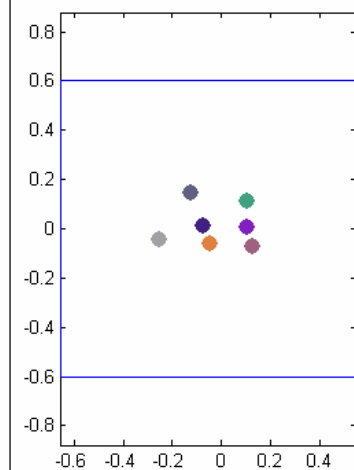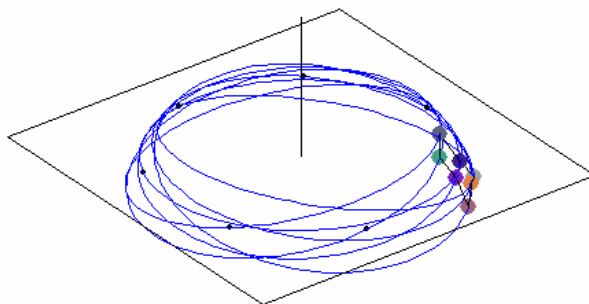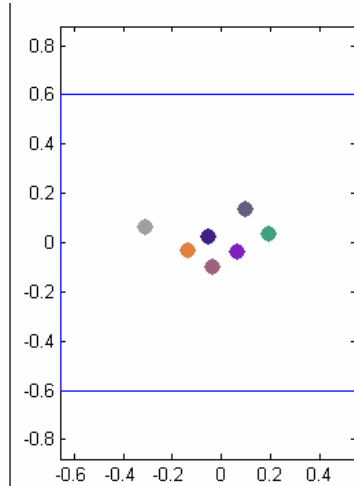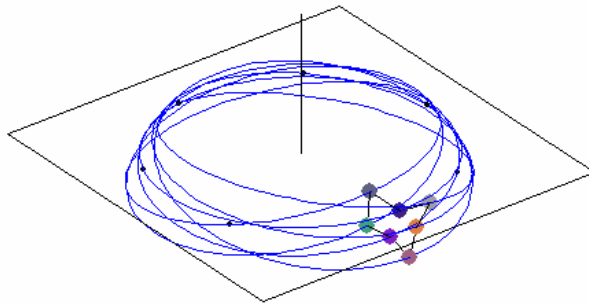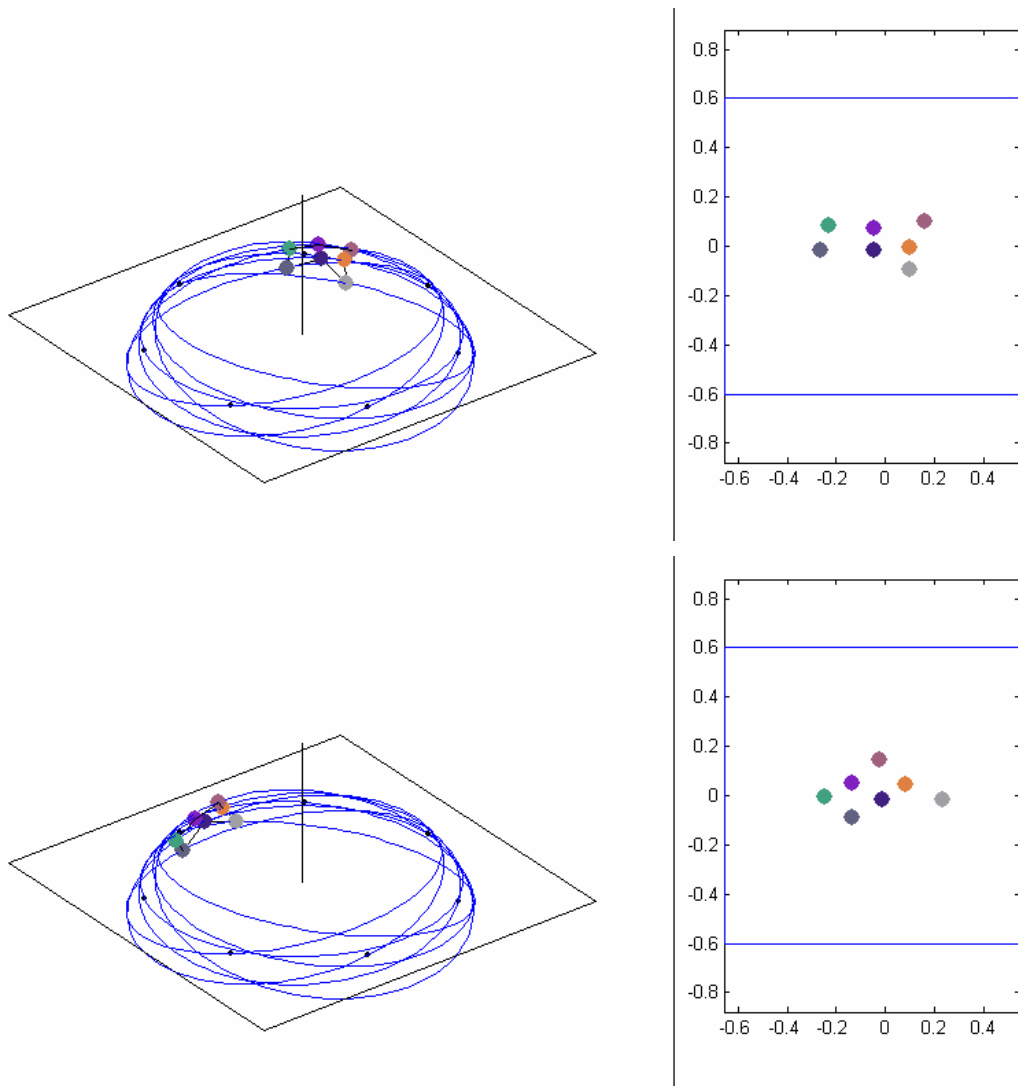
Figure 4: Arbitrary 7 bodies formation, $R_i \in (0,0.3)$, in 8 successive moments of a period. On the LHS we have a view from outside the system, on the RHS a simultaneous view from the attractive centre.

Of course, during one period, the distances between bodies experience small oscillations around an average value. In **Figure 5** we represented these relative oscillations for the six sides of a "regular intended" hexagon.

To increase the precision in preserving the geometry of the flight formation one has to impose smaller values for *R*. In **Figure 6** we have, this time in absolute value, the amplitude of the oscillation vs. the value of *R*, for hexagonal regular formations.
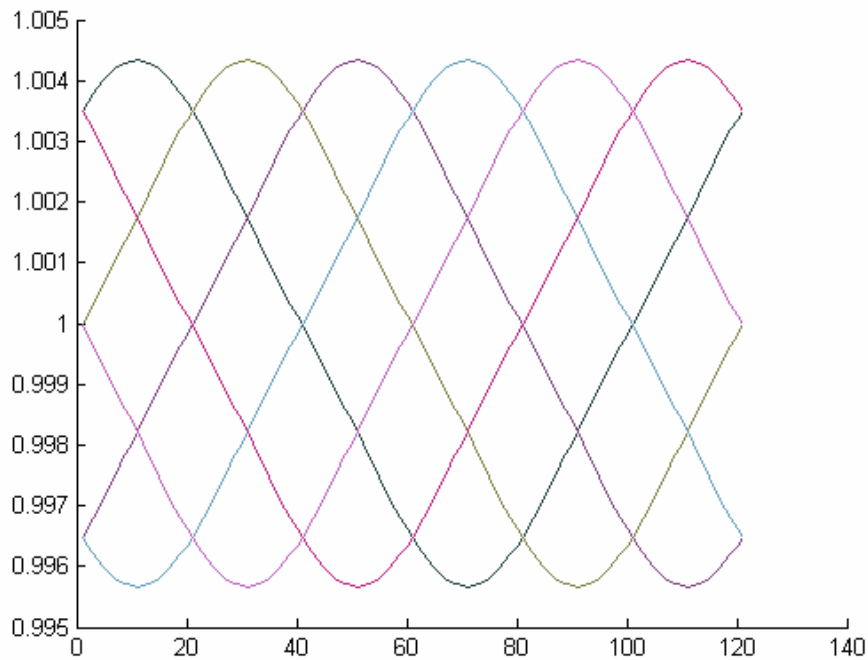
Figure 5: Regular 6 bodies formation, $R_i = 0.01$. We represented, with different colours, the oscillations of the six sides during one period (normalized with their average).
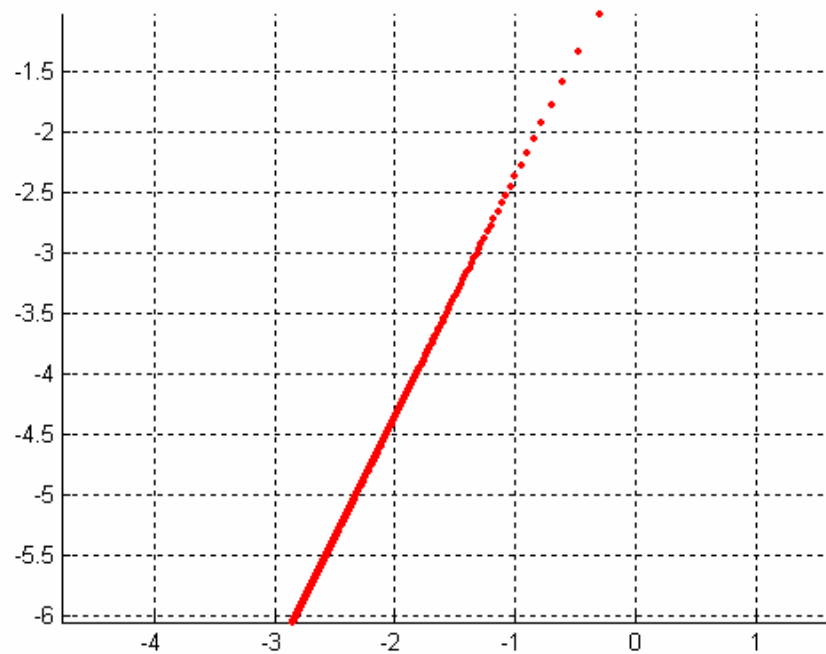

Figure 6: The absolute value of the oscillation of the sides (on the ordinate, in a 10-base logarithmic scale) vs. the value of *R* (on the abscissa, in a 10-base logarithmic scale), for hexagonal regular formations.

5.0   FINAL REMARKS

We proved that the technique used for designing the LISA mission can be extended for arbitrary, but plane, flying formations. The main advantage of using this technique consists in low cost for maintaining the trajectories; the motion is geodesic and the orbital manoeuvres are only for eliminating the effects of the perturbations. Also, we remind again that there are no restrictions other than keeping a small value for the ratio of the mutual distances to the characteristic length of the orbits (semi-major axis for example).

We did not discuss the problem of stability for these geodesic flying formations. First, because we do not have a clear definition for this and, second, because the stability of the formation (in any way you want to consider it) is strictly determined by the stability of the Keplerian orbits, which is a well-known subject.

6.0   REFERENCES

[1]   Clohessy, W. H., Wiltshire, R. S.: **1960**, Journal of Aerospace Sciences, 653-658.

[2]   http://lisa.jpl.nasa.gov.

[3]   Dhurandhar, S.V., Rajesh Nayak, K., Koshti, S., Vinet, J-Y. : **2005**, Class. Quant. Grav. 22, 481-488.

[4]   Brouwer, D., Clemence, G.M.: **1961**, Methods of Celestial Mechanics, Academic Press New York and London.