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by Jesse A. Huguet, Sarah Reichwein, Stanley E. Jones, and William P. Walters

ARL-RP-199 **February 2008**

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Jesse A. Huguet, Sarah Reichwein, and Stanley E. Jones
University of Alabama

William P. Walters
Weapons and Materials Research Directorate, ARL

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14. ABSTRACT
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In a recent paper, Walters et al [3] showed that all of the important effects of the classic Tate Theory could be captured by a regular perturbation solution of the fundamental equations. The small parameter that they used was $\varepsilon = Y_p/\rho_p v_0^2$, where Y_p is the penetrator yield strength, ρ_p is the penetrator density and v_0 is the impact speed.
In 1987, Jones et al [4] modified the equation of motion of the undeformed section to include mass loss and mushrooming at the interface with the target. The changes to the theory that result from these modifications bring the strengths of the target and penetrator into line with laboratory levels while achieving reasonable penetration depths. In this report, we will show that the regular perturbation analysis used by Walters et al [3] can be extended to the modified system of equations from Jones et al [4] using the same small parameter mentioned in the previous paragraph. The perturbation process is carried out to terms of first order and an approximate analytical solution is found. This solution is then used to repeat the reduction of data given by Wilson et al [5] for Aluminum and Steel alloy penetrators normally impacting Aluminum and Steel targets. Another case involving heavy metal penetrators impacting Rolled Hard Armor (RHA) targets is also presented.

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Jesse A. Huguet, Sarah Reichwein, Stanley E. Jones
College of Engineering
University of Alabama
Tuscaloosa, Alabama 35487-0280

and

William P. Walters
Army Research Laboratory (ARL)
Terminal Effects Division (TED)
Aberdeen 21005-5066

ABSTRACT

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NOMENCLATURE

ρ_t ≡ density of target
ρ ≡ density of penetrator
R_T ≡ strength of target
Y_P ≡ strength of penetrator
u ≡ instantaneous velocity of the deformed rod section
v ≡ instantaneous velocity of the undeformed rod section
ψ ≡ instantaneous length of the undeformed rod section
t ≡ time
A_0 ≡ initial cross-sectional area of projectile
v_0 ≡ initial velocity of the penetrator
ℓ_0 ≡ initial length of the penetrator
A ≡ cross-sectional area of crater
z ≡ penetration depth
e = mean strain in the penetrator mushroom

INTRODUCTION

In a 1967 paper, Tate [1] introduced his one-dimensional theory of long rod penetration that has been used to predict the penetration depth of long rod penetrators for decades. The equations he reported to govern such impacts are as follows:

$$\frac{1}{2}\rho_t u^2 + R_t = \frac{1}{2}\rho(v-u)^2 + Y_p \tag{1}$$

$$Y_p = -\rho\ell\frac{dv}{dt} \tag{2}$$

$$\frac{d\ell}{dt} = -(v-u) \tag{3}$$

$$z = \int u\,dt \tag{4}$$

Simultaneous solution of Equations 1-4 leads to a prediction for the penetration depth, z, requiring only the impact velocity of the penetrator, along with the other physical parameters in the model. These equations show that target and penetrator strengths and densities along with impact velocity are the determining factors when estimating penetration depth. In a second publication in 1969 [2], Tate detailed two distinct cases that are possible when comparing target and penetrator strengths. Case 1 is applicable when the target strength is greater than the penetrator strength. Case 2 involves penetrator strengths greater than the target's. Though not developed in Tate's publication, a third case exists when both the penetrator and the target are of equal strength. It is also true that these cases can be further subdivided by comparing the target and penetrator densities.

Recently, Walters et al [3] have developed, via regular perturbation techniques, an analytical solution to the Tate equations. This approach required normalization of the Tate equations using the following change of variables:

$$V = \frac{v}{v_0} \tag{5}$$

$$U = \frac{u}{v_0} \tag{6}$$

$$\lambda = \frac{\ell}{\ell_0} \tag{7}$$

$$\mu^2 = \frac{\rho_t}{\rho} \tag{8}$$

$$\beta = \left(\frac{\mu}{1+\mu}\right)\frac{v_0}{\ell_0} \tag{9}$$

$$\tau = \beta t \tag{10}$$

$$\alpha = \frac{R_T - Y_P}{Y_P} \tag{11}$$

$$Z = \frac{z}{\ell_0} \tag{12}$$

$$\varepsilon = \frac{Y_P}{\rho v_0^2} \tag{13}$$

Equations 5-13 involve dimensionless groupings with the exception of β in Equation 9 which has the dimension of time^{-1}. The dimensionless grouping in Equation 13 is usually very small, as observed by Walters et al [3]. Thus, the small parameter, ε, is a candidate for the focus parameter in a regular perturbation expansion. Expanding each dimensionless variable into a series of powers of ε leads to:

$$V = V_0 + \varepsilon V_1 + \varepsilon^2 V_2 + \varepsilon^3 V_3 + \ldots \tag{14}$$

$$U = U_0 + \varepsilon U_1 + \varepsilon^2 U_2 + \varepsilon^3 U_3 + \ldots \tag{15}$$

$$\lambda = \lambda_0 + \varepsilon\lambda_1 + \varepsilon^2\lambda_2 + \varepsilon^3\lambda_3 + \ldots \tag{16}$$

$$Z = Z_0 + \varepsilon Z_1 + \varepsilon^2 Z_2 + \varepsilon^3 Z_3 + \ldots \tag{17}$$

In 1987 Jones et al [4] modified the equation of motion of the undeformed rod section. They accounted for instantaneous mass loss and mushrooming at the penetrator/target interface. This analysis formulated the new equation of motion:

$$\ell\frac{dv}{dt} + \frac{d\ell}{dt}(v-u) = \frac{-Y_P}{\rho(1+e)} \tag{18}$$

In this paper, we will apply the regular perturbation method used by Walters et al to the same set of equations governing long rod impact with the equation of motion developed by Jones et al, Equation 18 in place of Tate's, Equation 2.

THEORY

The parameters defined in Equations 5-13 are applied to the governing Equations 1, 3, 4 and 18 to give the following dimensionless equations:

$$(V-U)^2 - 2\alpha\varepsilon = \mu^2 U^2 \tag{19}$$

$$\frac{d\lambda}{d\tau} = -\left(\frac{1+\mu}{\mu}\right)(V-U) \tag{20}$$

$$Z = \frac{1+\mu}{\mu}\int U d\tau \tag{21}$$

$$\lambda\frac{dV}{d\tau} + \frac{d\lambda}{d\tau}(V-U) = \frac{-\varepsilon(1+\mu)}{(1+e)\mu} \tag{22}$$

By substituting the perturbation expansions, Equations 14-17, into these dimensionless equations, we can derive systems of equations for the zeroth, first, and higher order terms if necessary. The zeroth order system is:

$$(V_0 - U_0)^2 = \mu^2 U_0^2 \tag{23}$$

$$\frac{d\lambda_0}{d\tau} = -\left(\frac{1+\mu}{\mu}\right)(V_0 - U_0) \tag{24}$$

$$Z_0 = \frac{1+\mu}{\mu}\int U_0 d\tau \tag{25}$$

$$\lambda_0\frac{dV_0}{d\tau} + V_0\frac{d\lambda_0}{d\tau} - U_0\frac{d\lambda_0}{d\tau} = 0 \tag{26}$$

This system of four equations and four unknowns can be easily solved to provide:

$$V_0 = \lambda_0^{\frac{-\mu}{1+\mu}} \tag{27}$$

$$U_0 = \frac{V_0}{1+\mu} \tag{28}$$

$$\lambda_0 = \left(-\frac{1+2\mu}{1+\mu}\tau + 1\right)^{\frac{1+\mu}{1+2\mu}} \tag{29}$$

$$Z_0 = \frac{1}{\mu}\left[1 - \left(1 - \frac{1+2\mu}{1+\mu}\tau\right)^{\frac{1+\mu}{1+2\mu}}\right] \tag{30}$$

These variables are all explicit functions of the dimensionless time variable τ.

The system that results from retaining terms of first order of ε is:

$$(V_0 - U_0)(V_1 - U_1) - \alpha = \mu^2 U_0 U_1 \tag{31}$$

$$\frac{d\lambda_1}{d\tau} = -\left(\frac{1+\mu}{\mu}\right)(V_1 - U_1) \tag{32}$$

$$Z_1 = \frac{1+\mu}{\mu}\int U_1 d\tau \tag{33}$$

$$\lambda_1\frac{dV_0}{d\tau} + \lambda_0\frac{dV_1}{d\tau} + V_0\frac{d\lambda_1}{d\tau} + V_1\frac{d\lambda_0}{d\tau} - U_0\frac{d\lambda_1}{d\tau} - U_1\frac{d\lambda_0}{d\tau} = -\frac{1+\mu}{(1+e)\mu} \tag{34}$$

Equation 31 can be algebraically manipulated to solve for U_1 in terms of V_1 and Equation 32 algebraically renders V_1 in terms of λ_1.

$$U_1 = \frac{V_1}{1+\mu} - \frac{\alpha}{\mu V_0} \tag{35}$$

$$V_1 = -\left(\frac{1+\mu}{\mu^2}\right)\frac{\alpha}{V_0} - \frac{d\lambda_1}{d\tau} \tag{36}$$

Substituting Equation 36 into Equation 35 and eliminating V_1 provides:

$$U_1 = -\frac{(1+\mu)\alpha}{\mu^2 V_0} - \frac{1}{1+\mu}\frac{d\lambda_1}{d\tau} \tag{37}$$

Equations 36 and 37 can be substituted into Equation 34 which will then simplify to:

$$\lambda_0 \frac{d^2\lambda_1}{d\tau^2} - \frac{2\mu}{1+\mu}V_0\frac{d\lambda_1}{d\tau} - \lambda_1\frac{dV_0}{d\tau} = \frac{\alpha(1+\mu)}{\mu^2 V_0^2}\lambda_0\frac{dV_0}{d\tau} + \frac{(1+\mu)}{\mu(1+e)} \tag{38}$$

By changing the independent variable, τ, in this equation to ξ, as defined in Equation 39, we obtain Equation 40.

$$\xi = 1 - \frac{1+2\mu}{1+\mu}\tau \tag{39}$$

$$\left(\frac{1+2\mu}{1+\mu}\right)^2 \xi^{\frac{1+\mu}{1+2\mu}}\frac{d^2\lambda_1}{d\xi^2} + \frac{2\mu(1+2\mu)}{(1+\mu)^2}\xi^{\frac{-\mu}{1+2\mu}}\frac{d\lambda_1}{d\xi} - \frac{\mu}{1+\mu}\xi^{-\frac{1+3\mu}{1+2\mu}}\lambda_1 = \frac{1+\mu}{\mu(1+e)} + \frac{\alpha}{\mu} \tag{40}$$

Multiplying Equation 40 through by the factor

$$\xi^{\frac{1+3\mu}{1+2\mu}} \tag{41}$$

reduces this equation to

$$\left(\frac{1+2\mu}{1+\mu}\right)^2 \xi^2\frac{d^2\lambda_1}{d\xi^2} + \frac{2\mu(1+2\mu)}{(1+\mu)^2}\xi\frac{d\lambda_1}{d\xi} - \frac{\mu}{1+\mu}\lambda_1 = \left(\frac{1+\mu}{\mu(1+e)} + \frac{\alpha}{\mu}\right)\xi^{\frac{1+3\mu}{1+2\mu}} \tag{42}$$

which may be recognized as an Euler Equation. To solve this second order differential equation, the homogenous, or reduced, solution shown in Equation 43 is considered.

$$\xi^2\frac{d^2\lambda_{1C}}{d\xi^2} + \frac{2\mu}{1+2\mu}\xi\frac{d\lambda_{1C}}{d\xi} - \frac{\mu(1+\mu)}{(1+2\mu)^2}\lambda_{1C} = 0 \tag{43}$$

To solve this equation, the power solution shown in Equation 44 is assumed.

$$\lambda_{1C} = \xi^n \tag{44}$$

where n is a constant exponent to be determined and λ_{1C} is the solution to the reduced equation. Substituting Equation 44 into Equation 43 leads to the quadratic equation for n:

$$n^2 + \left(\frac{2\mu}{1+2\mu} - 1\right)n - \frac{\mu(1+\mu)}{(1+2\mu)^2} = 0 \tag{45}$$

The roots of this quadratic equation are n_1 and n_2 below.

$$n_1 = \frac{1+\mu}{1+2\mu} \tag{46}$$

$$n_2 = \frac{-\mu}{1+2\mu} \tag{47}$$

The general solution to Equation 42 now has the form:

$$\lambda_1 = c_1\xi^{n_1} + c_2\xi^{n_2} + \xi_p \tag{48}$$

where ξ_p is any particular solution to Equation 42 and c_1 and c_2 are arbitrary constants. To find ξ_p, assume a particular solution of the form:

$$\xi_p = K\xi^{\frac{1+3\mu}{1+2\mu}} \tag{49}$$

By substituting Equation 49 into Equation 42 and matching the coefficients, we find K to be the following:

$$K = \frac{(1+\mu)^2[(1+\mu)+\alpha(1+e)]}{2\mu^2(1+e)(4\mu+1)} \tag{50}$$

All that remains to be accomplished to complete the general solution of Equation 42 is to evaluate the arbitrary constants c_1 and c_2. The initial conditions required to solve for these constants occur at $\tau = 0$ ($\xi = 1$). The normalized rod velocity, V, and rod length, λ, are equal to one at impact. This stipulates that V_0 and λ_0 are one at $\tau = 0$. Equations 14 and 16 thus require that V_1 and λ_1 be zero at $\tau = 0$. The conditions, Equations 51 and 52, used to solve for c_1 and c_2 come directly from these relationships. The initial conditions imposed on U_0 and U_1 are determined by Equations 23 and 31. The concept of non-zero initial conditions for normalized penetration velocity is not inconsistent with common practice. The initial conditions are applied to those quantities which appear in the equations of motion, Equations 19 through 22, with derivatives. These equations do not contain any derivatives of U. While an order of magnitude analysis does apply to U, its initial conditions should be dictated by the other variables.

$$\lambda_1(1) = 0 \tag{51}$$

and

$$\frac{d\lambda_1}{d\xi}(1) = \frac{\alpha(1+\mu)^2}{\mu^2(1+2\mu)} \tag{52}$$

Applying Equations 51 and 52 to Equation 48 with the particular solution described by Equations 48-50 leads to values for the arbitrary constants c_1 and c_2 given below.

$$c_1 = -\frac{4\mu+1}{1+2\mu}K + \frac{\alpha(1+\mu)^2}{\mu^2(1+2\mu)} \tag{53}$$

$$c_2 = \frac{2\mu}{1+2\mu}K - \frac{\alpha(1+\mu)^2}{\mu^2(1+2\mu)} \tag{54}$$

Having determined the constants c_1 and c_2, the first order terms for the expansions of U_1 and V_1 can be computed directly using Equations 36 and 37. Then, using Equation 33, we can find the first order term for the penetration depth. Since all quantities have been expressed in terms of ξ, it is convenient to change the variables to integration with respect to ξ.

$$Z_1 = \frac{1+\mu}{\mu(1+2\mu)}\int_\xi^1 V_1(\xi)d\xi - \frac{\alpha(1+\mu)^2}{\mu^2(1+2\mu)}\int_\xi^1 \xi^{\frac{\mu}{1+\mu}}d\xi \tag{55}$$

Now, the penetration depths for all cases can be found. This will be done in the next section for two Aluminum alloys and two Steel alloys.

RESULTS

In order to test the theory, experimental results from Wilson et al [5] are examined. These results contain examples of all three cases of penetration: equal strength, target stronger than penetrator, and penetrator stronger than target. Wilson et al include crater diameters in their test reports, and this data was used to calculate mean strain using Equation 56.

$$e = \frac{A_0}{A} - 1 \tag{56}$$

Results for penetration depth are then compared to the experimental data. The properties of target and penetrator materials ([5], [6] and [7]), as well as reference numbers to be used for the remainder of this paper, are recorded in Table 1.

Table 1

Mechanical Properties of the Test Materials

Material (#)	Strength (MPa)	Density (kg/m^3)
4340 steel annealed (1)	1000	7850
4340 steel hardened (2)	1448	7880
7075-T6 aluminum (3)	465	2810
2024-T4 aluminum (4)	248	2780
Rolled Hard Armor (5)	879.6	7880
DU-3/4Ti (6)	1951	19613

First, the equal strength case is considered, specifically using a target and penetrator of material (2). In this case, μ is equal to one, and α is equal to zero, as they are defined in Equations 8 and 11. Penetration in the equal strength case will persist until penetrator velocity (V) equals zero. Therefore, the value of ξ corresponding to $V = 0$ determines the time for which the event concludes. The positive root of Equation 57, derived from Equations 14, 27, 36, and 39 determines this value of ξ.

$$\xi^{\frac{-1}{3}} + \varepsilon\left[\frac{4}{3(1+e)}\xi^{\frac{-1}{3}} - \frac{8}{30(1+e)}\xi^{\frac{-4}{3}} + \frac{8}{5(1+e)}\xi^{\frac{1}{3}}\right] = 0 \tag{57}$$

Total dimensionless penetration depth is then calculated in Equation 58, which is derived from Equations 17, 30, 39, and 55, using the ξ found from Equation 57.

$$Z = \left(1 - \xi^{\frac{2}{3}}\right) + \varepsilon\left[\frac{2}{3}\int_\xi^1 V_1(\xi)d\xi\right] \tag{58}$$

Using Equation 58 and the given length of the penetrator, Equation 12 is used to determine total penetration. Theoretical and experimental results are compared in Figure 1.

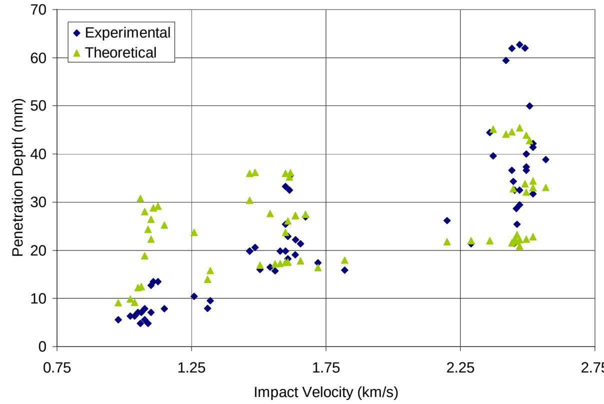

Figure 1. Penetration vs. velocity for material 2 target and penetrator.

Next, Tate's first case, target strength greater than penetrator strength, is considered. In this case, the value of α is positive. Two specific situations are examined: material 3 penetrating material 2 and material 1 penetrating material 2. In the first situation, the target significantly exceeds the penetrator in both strength and density. In the second situation, the target only slightly exceeds the penetrator in strength and density.

Penetration in this case will continue until penetration velocity (U) equals zero, as noted in Equation 59. Variable substitutions from Equations 36 and 39 are made to Equation 59 to solve for ξ which defines the duration of penetration.

$$\frac{1}{(1+\mu)}\xi^{\frac{-\mu}{(1+2\mu)}} + \varepsilon\left[\frac{V_1}{(1+\mu)} - \frac{\alpha}{\mu}\xi^{\frac{\mu}{(1+2\mu)}}\right] = 0 \tag{59}$$

After the time corresponding to ξ, as found with Equation 59, residual deformation of the penetrator occurs but penetration ceases.

Using Equations 12, 30, and 55, total penetration is again calculated with the value of ξ found with Equation 59. Theoretical and experimental results are compared in Figure 2.

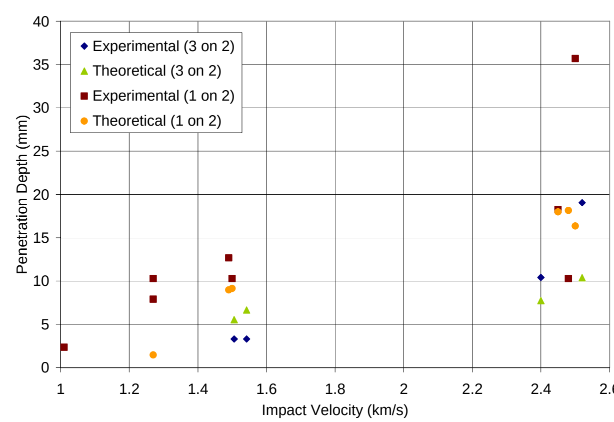

Figure 2. Penetration vs. velocity for cases where target strength exceeds penetrator strength.

Finally, Tate's second case, penetrator strength exceeds that of the target, is examined. In this case, α is negative. There are six combinations of penetrator and target materials where this is true. Wilson et al provide experimental data for all six of these combinations.

In all variations of this case, the penetration occurs in two stages: initial impact and rigid body penetration. The duration of the first stage is defined by the value of ξ that occurs when penetration velocity (U) equals penetrator velocity (V), as defined in Equation 60.

$$\frac{-\mu}{1+\mu}\xi^{\frac{-\mu}{1+2\mu}}+\varepsilon\left[\frac{-\mu}{1+\mu}V_1-\frac{\alpha}{\mu}\xi^{\frac{\mu}{1+2\mu}}\right]=0 \tag{60}$$

At this value of ξ, the initial impact stage is completed, and the penetrator begins to act as a rigid body. Equation 58 can be used to calculate the value of Z for this initial impact stage. To analyze the penetration depth of the rigid body penetration stage, the dimensionless length and velocity terms must be determined at the value of ξ from Equation 60. The value of λ at this value of ξ is obtained by substituting Equations 29 and 48 into the perturbation sequence, Equation 16. Similarly, the value V is obtained by substituting Equations 27 and 36 into Equation 14.

The equation of motion for this stage of rigid body penetration, Equation 61, is derived by substituting the modified Bernoulli equation, Equation 1, into the Jones et al equation of motion, Equation 18. It is assumed that during rigid body penetration, the penetrator has no mushroom, e = 0.

$$\rho\ell\frac{dv}{dt}=-\frac{1}{2}\rho\mu^2v^2+R_T \tag{61}$$

The rod length and velocity correspond to the values when rigid body penetration begins.

Equation 61 is modified using the dimensionless parameters defined by Equations 5-13 to yield the form in Equation 62, which can be used to calculate Z during the rigid body stage of the penetration.

$$Z_{rigid}=\frac{\lambda}{\mu^2}\ln\left(1+\frac{\rho\mu^2v_0^2V^2}{2R_T}\right) \tag{62}$$

The values of Z determined by Equations 58 and 62 can be combined to yield a final value of Z. This value and Equation 12 can be used to determine total penetration depth. Theoretical and experimental results for each of the six combinations are compared in Figures 3 through 8.

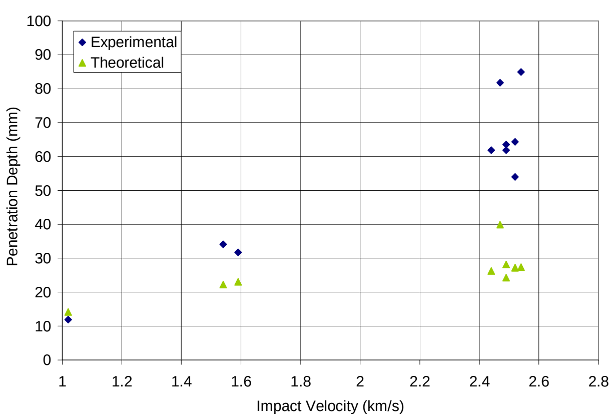

Figure 3. Penetration vs. velocity for material 2 penetrating 3.

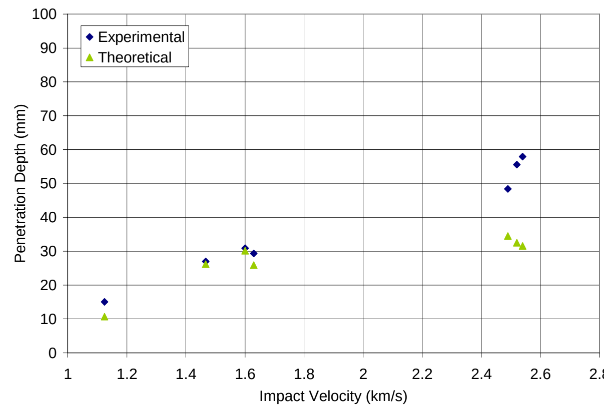

Figure 4. Penetration vs. velocity for material 1 penetrating 3.

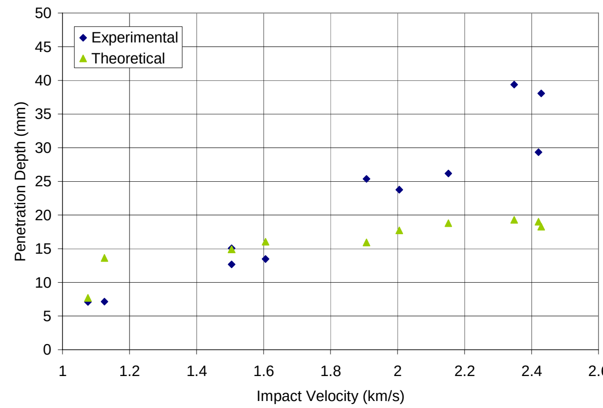

Figure 5. Penetration vs. velocity for material 2 penetrating 1.

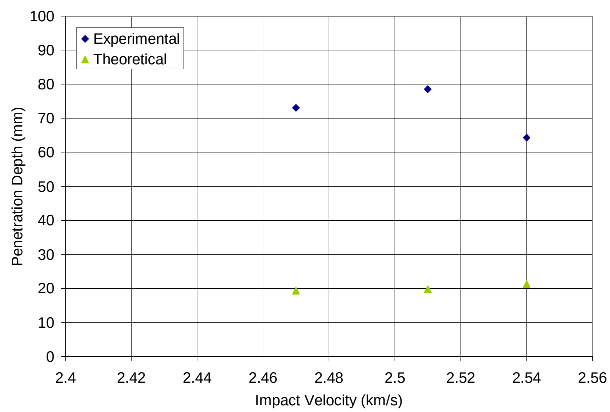

Figure 6. Penetration vs. velocity for material 2 penetrating 4.

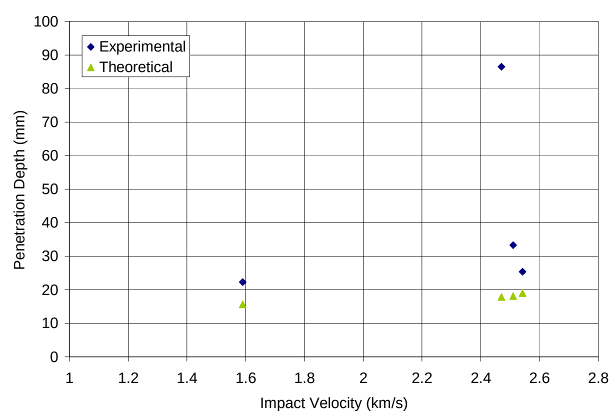

Figure 7. Penetration vs. velocity for material 3 penetrating 4.

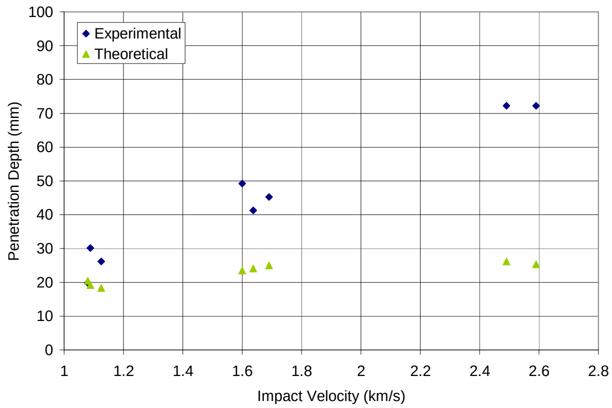

Figure 8. Penetration vs. velocity for material 1 penetrating 4.

As a final example, consider the penetration of a Rolled Homogenous Armor, RHA, target by a heavy metal projectile, DU-3/4Ti (Depleted uranium-70% Titanium). The density of the DU-3/4Ti is given in Table 1 and the dynamic strength is taken from Taylor cylinder test data repeated by Jones et al [6]. Penetration depth estimates require the same methods and equations used to find the theoretical penetration depths reported in Figures 3-8, as the penetrator strength is greater than the target strength. Figure 9 shows the theoretical penetration depths, as calculated with this method, compared with the experimental depths reported by Keele et al [7]. The agreement is reasonable, although the theoretical estimates are always less than the experimental results.

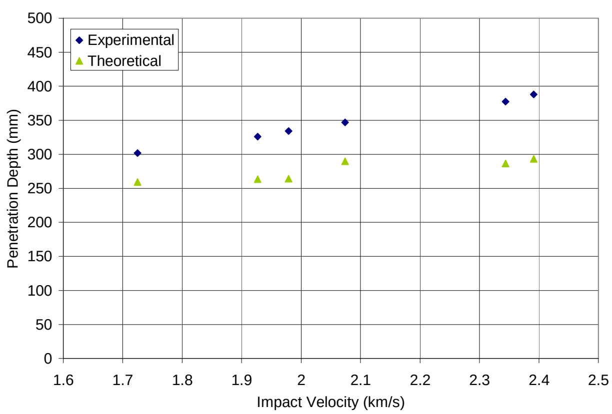

Figure 9. Penetration vs. velocity for material 2 target and penetrator.

CONCLUSIONS

In this paper, we have presented a small parameter analysis of the modified Tate equations. It is well known that to achieve reasonable correlations with experimental observations using the Tate theory, unusually high strength values must be assumed for the target and penetrator materials. For this reason, the equation of motion of the undeformed section of the projectile was modified by Jones et al [4] to include mass loss and mushrooming during the penetration process. These changes have allowed laboratory strength estimates to produce reasonable correlation with experiments. However, like the Tate theory, the equations for anything beyond the simplest cases required numerical integration. The small parameter analysis employed by Walters et al [3] overcame this difficulty and produced an accurate approximate solution for the Tate equations. The small parameter in Equation 13 applies to almost every reasonable case. This same small parameter is used in this paper to extend the results to the modified Tate equations. The result is a set of approximate solutions with which penetration depth or any of the other relevant physical parameters, such as residual rod length, can be estimated using laboratory strength data. To estimate the mean mushroom strain, a key parameter in the modified Tate equations, measurements of actual craters in targets were used.

The crater geometry employed here is cylindrical with the diameter estimated from the "profile hole diameter" in the target (Wilson et al [5]). For the case of penetrator strength exceeding target strength, the crater geometry consists of two cylinders. The second cylinder, which captures rigid body penetration, has the original diameter of the penetrator. It is possible to make the strain estimates a priori using laboratory strengths, Cinnamon et al [8]. There is a period of initial transient behavior at impact which is dominated by the shock. A narrow opening is created by the projectile followed by mushrooming of the nose, during which there is negligible deceleration of the undeformed section (Cinnamon et al [8]). Including this behavior in the analysis will modify the crater geometry and increase the penetration depth, which is in line with the comparison between predicted and observed penetration depths. Further study of the effects of the transient behavior will be considered in subsequent work.

REFERENCES

1. A. Tate, "A Theory for the Deceleration of Long Rods after Impact," J. Mech. Phys. Solids, 15, pp. 387-399 (1967).

2. A. Tate, "Further Results in the Theory of Long Rod Penetration," J. Mech. Phys. Solids, 17, pp. 141-150 (1969).

3. W. Walters, C. Williams, and M. Normandia, "An Explicit Solution of the Alekseevski-Tate Penetration Equations," Int. J. Impact Engng, Vol. 33 pp. 837-846, Dec. 2006.

4. S.E. Jones, P.P. Gillis, and J. C. Foster, Jr., "On the Penetration of Semi-Infinite Targets by Long Rods," J. Mech. Phys. Solids, 35, 1, pp. 121-131 (1987).

5. L.L. Wilson, J.C. Foster, Jr., S.E. Jones, and P.P. Gillis, "Experimental Rod Impact Results," Int. J. Impact Engng, 8, 1, pp. 15-25 (1989).

6. S. E. Jones and Peter P. Gillis, "On the Equation of Montion of the Undeformed Section of a Taylor Impact Specimen," J. Appl. Phys. 61 (2), 15 January 1987.

7. M. Keele, E. Rapacki, Jr., and W. Bruchey, Jr., "High Velocity Ballistic Performance of a Uranium Alloy Long Rod Penetrator," Proceedings of the 12th International Symposium on Ballistics, Oct. 1990.

8. John D. Cinnamon, S. E. Jones, Joel W. House, and L. L. Wilson, "A One-dimensional Analysis of Rod Penetration," Int. J. Impact Engng, 12, 2, pp. 145-166, 1992.

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