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13. ABSTRACT (Maximum 13 lines, use the attachment if necessary)
Proposals for adiabatic quantum computation generated renewed interest and questions about the adiabatic approximation. We presented a simple proof of the adiabatic theorem in which we showed that the first order correction has the expected dependence on an energy gap; however, determining the time scale needed to ensure a small error may require consideration of higher order terms. We also give a simple new proof of the key gap estimates needed to who that a quantum circuit can be approximated by adiabatic evolution in time polynomial in the number of gates; our methods also improve one of the estimates.

We obtained a number of new results about quantum channels, including several results about the conjugate channels obtained by reversing the roles of the system and environment. We considered several new classes of unital channels, one of which leads to the construction of new bound entangled states. We defined the concept of minimal conditional information of an channel and showed that it gives a measure of the extent to which channel breaks entanglement.

We also proved some mathematical results about norms of channels with implications for channel capacity and error correction.

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List of papers submitted or published that cite ARO support during this reporting period. List the papers, including journal references, in the following categories:

Number of Peer Reviewed Papers: 10

Papers published in peer-reviewed journals
M. Hayashi, H. Imai, K. Matsumoto, M.B. Ruskai and T. Shimono "Qubit Channels Which Require Four Inputs to Achieve Capacity: Implications for Additivity Conjectures" Quantum Inf. Comput. 5, 13–31 (2005). quant-ph/0403176

V. Giovannetti, S. Lloyd and M.B. Ruskai "Conditions for multiplicativity of maximal p-norms of channels with fixed integer p" J. Math. Phys. 46, 042105 (2005). quant-ph/0408103

C. King, M. Nathanson and M.B. Ruskai "Multiplicativity properties of entrywise positive maps" Lin. Alg. Appl. 404, 367--379 (2005) quant-ph/0409181

M.B. Ruskai, "Lieb's simple proof of concavity of $Tr\ A^p\ K^\dagger\ B^{(1-p)}\ K$ and remarks on related inequalities" International Jour. Quant. Info 3, 570--590 (2005). quant-ph/0404126

I. Devetak, M. Junge, C. King and M.B. Ruskai, "Multiplicativity of completely bounded p-norms implies a new additivity result" Commun. Math. Phys. 266, 37-64 (2006). quant-ph/0506196.

N. Datta and M.B. Ruskai, "Maximal output purity and capacity for asymmetric unital qudit quantum channels" J. Physics A 38, 9785-9802 (2005). quant-ph/0505048

C. King , K. Matsumoto, M. Nathanson and M.B. Ruskai, "Properties of conjugate channels with application to additivity and multiplicativity" Markov Process and Related Fields in press (2006). quant-ph/0509126.

D. Perez-Garcia, D. Petz, M.B. Ruskai and M. Wolf, "Contractivity of positive and trace preserving maps under Lp norms" J. Math. Phys. 47, 083506 (2006).
math-ph/0601063

M.B. Ruskai, "Comment on "Stronger subadditivity of entropy" Phys.Rev. A 74, 026303 (2006).
math-ph/0603022

P. Defit, M.B. Ruskai and W. Spitzer, "Improved gap estimates for simulating quantum circuits by adiabatic evolution" Quantum Information Processing in press (2006). quant-ph/0605156

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Non Peer Reviewed Papers:

Number of Presentations: 8

Presentations at conferences, but not published in conference proceedings
M.B. Ruskai "Recent progress on additivity and multiplicativity of of capacity of quantum channels" seminar during program on Quantum Information Science at the Isaac Newton Institute
(Cambridge, UK, Oct.-Nov., 2005).

M.B. Ruskai "Additivity of capacity of quantum channels"
invited talk at meeting of the Korean Physical Society (Seoul, 22 April 2005)

M.B. Ruskai "From Cauchy-Schwarz to quantum entropy",
invited talk at Workshop on Quantum Information and Robustness,
(University of Tokyo, 25 April 2005).

M.B. Ruskai "Completely bounded p-norms in quantum information theory"
invited talk in special session on Mathematical Aspects of Quantum Information
at the summer meeting of the Canadian Mathematical Society
(Waterloo, Ontario, 4-6 June 2005).

M.B. Ruskai "The complementary roles of POVM's and ensembles in the capacity of quantum channels" invited talk at J.T. Lewis Memorial Conference,
(Dublin, 13-17 June 2005)

M.B. Ruskai "A simple new proof of strong subadditivity of quantum entropy"
invited talk during workshop on Quantum Information Theory,
(Benasque, Spain, 20-30 June, 2005).

M.B. Ruskai, "Pauli diagonal channels constant on axes" invited plenary lecture at 38th Symposium on Mathematical Physics "Quantum Entanglement and Geometry" (Torun, Poland, 4-7 June 2006).

M.B. Ruskai, "Pauli diagonal channels constant on axes" invited plenary lecture at Conference on Theory and Technology in Quantum Information, Communication, Computation and Cryptography, (Abdus Salam ICTP, Trieste, Italy, 19-23 June 2006).

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts) 1

Peer-Reviewed Conference Proceeding publications (other than abstracts)
M.B. Ruskai, "Some connections between frames, mutually unbiased bases and POVM's in Quantum Information Theory" talk at AMS-SIAM special session on frames at annual joint meeting (San Antonio, Texas, January 2006) to appear in Contemporary Mathematics (published by AMS).

Number of Non-Peer-Reviewed Conference Proceeding publications (other than abstracts) 1

Non-Peer-Reviewed Conference Proceeding publications (other than abstracts)
A. Ambainas and M.B. Ruskai, Report of the workshop on "Mathematical aspects of adiabatic quantum computation" (Perimeter Institute, Waterloom, 9-11 February, 2006)
http://www.perimeterinstitute.ca/activities/scientific/PI-WORK-6/related_links.php

Number of Manuscripts: 2

Manuscripts submitted, but not published (N/A for none)
S. Jansen, M.B. Ruskai and R Seiler "Bounds for the adiabatic approximation with applications to quantum computations" quant-ph/0603175

M.B. Ruskai "Another short and elementary proof of strong subadditivity of quantum entropy" quant-ph/0604206

Number of Books: 1

(d) Books (N/A for none)
"Introduction to Quantum Information Theory" invited chapter in Handbook of Nanotechnology ed. by A. Lakhtakia, pp. 395–464 (SPIE Press and ASM, 2004)

Inventions	Inventors
Sub Contractors:	
Graduate Students	

Faculty	
Mary Beth Ruskai	0.08
TOTAL PERCENT: 0.08	
Post Doctorates	
Under Graduates	
Other Research Staff	
PHDs awarded	
Masters Awarded	
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1 Quantum computation by adiabatic evolution

In 2001, Fahri, et al [18, 19] proposed a method for adiabatic quantum optimization (AQO) and suggested on the basis of numerical simulations that it could solve 3-SAT, a well-known NP-complete problem, in polynomial time. The P.I. [65] proved the existence of a gap g between the energy of the ground state and the first excited state when the final Hamiltonian has a non-degenerate ground state. But for complexity issues this is not enough; one needs estimates on the size of the gap. Despite a great deal of interest and attention, the question of whether or not any form of adiabatic quantum computation (AQC) can solve hard problems remains unresolved.

However, this work generated renewed interest in the adiabatic approximation itself. In 2004, Marzlin and Sanders [46] muddied the waters with a claim to a counter-example to the adiabatic theorem. In fact, their example contains a rapidly oscillating term, and it is well-known to experts that the adiabatic theorem does not apply in such cases. They also claimed that their example satisfied the hypotheses in several well-known [5, 30] mathematically sophisticated papers on the subject. This resulted in considerable confusion, especially among computer scientists not familiar with the mathematical physics literature.

The conventional wisdom [43, 47] says that the time τ needed to make the error involved in using the adiabatic approximation less than some fixed δ satisifies $\tau > C/g^2$ where g denotes the minimum energy difference between the ground state and the first excited state. By contrast, most rigorous proofs [5, 24, 30, 29, 75] of the adiabatic theorem state the result as an asymptotic expansion for the error in approximating the exact ground state of the final Hamiltonian by the result of adiabatic evolution. The relation between these views was clarified by the P.I. and collaborators in [28] and during a workshop organized by the P.I at the Perimeter Institute [3]. The situation can be summarized as follows:

- The first order correction can be written in the form $\frac{C}{\tau g^2}$. However, this implies only that when t is sufficiently large, the error has this form. It does *not* imply that when $t > C\delta/g^2$, the error is less than δ!

- For a particular time-dependent Hamiltonian $H(s)$, the asymptotic expansion begins

$$\frac{C_1(H)}{\tau g^2} + \frac{1}{\tau^2} C_2(H, g) \tag{1}$$

where we use notation which emphasizes that the coefficients in the expansion depend on the Hamiltonian and can even involve gap dependence. Indeed, standard estimates easily yield bounds [10, 28, 63] of the form

$$\frac{1}{\tau} \frac{C_1(H)}{g^2} + \frac{1}{\tau^2 g^6} \widetilde{C}_2(H) \tag{2}$$

which would require $\tau = O(g^{-3})$ to justify neglecting the second order term The bound (2) is not best possible because it estimates integrals by the worst case. However, improvement requires additional information and does **not** allow one to conclude that one can always make the error small by choosing $\tau = O(g^{-2})$. (Claims of rigorous proofs of $O(g^{-2})$ estimates are flawed. For details see [3].)

- Alex Elgart [16] sketched an argument, based on Nenciu's expansion, which shows that it suffices to choose $\tau = O\left(\frac{(\log g)^4}{g^2}\right)$. He used the explicit dependence of expansion coefficients on g to determine the number of terms to use to optimize the dependence of τ on g.

- None of these results allow one to directly translate the gap dependence for a family of interpolating Hamiltonians $\{H_{n,P}(s)\}$ depending on the number of qubits n and an instance P in some class of problems. To do this one would first need to know that expansion coefficients $C_k(H)$ are uniformly bounded in n and P. However, in AQO one expects the coefficients to grow with n.

It is worth emphasizing that the proof presented by the P.I. and collaborators in [28], like other approaches [5, 24, 30, 29, 75] , is not restricted to an energy gap between the ground state and the first excited state; an analogous result holds when there is a gap anywhere in the spectrum. This could be useful in analyzing AQO in situations where the final Hamiltonian has a degenerate ground state.

In a different direction, the AQC model has been shown to be equivalent to the standard circuit model of quantum computation. In particular, it was shown in [1] that for any quantum circuit with L gates, one can construct an interpolating Hamiltonian which will evolve to final state from which, with a certain probability, a measurement would give the same result as the final state of the circuit model. They stated polynomial time estimates based gap estimates of for two interpolating Hamiltonians. However, their proofs of these key estimates was extremely complicated. In [12] we used a simple variational argument to prove gap estimates for these Hamiltonians. In addition, we improved the second estimate by a factor of L so that both gaps are $O(L^{-2})$.

2 Quantum channels

The model for noise in the transmission of quantum information is called a quantum channel and is based on the mathematical concept known as a completely positive, trace preserving map. The P.I. has been working on various aspects of quantum channels. Although this work is primarily supported by the National Science Foundation and some is fairly mathematical, it is relevant to our broader understanding of decoherence and entanglement. I will

mention here some results of more direct relevance to the ARDA program and to quantum cryptography.

2.1   Minimal conditional information

Transmission of quantum information is known to be described by the coherent information which is non-additive, i.e., entanglement can increase the quantum capacity of a memoryless channel. The coherent information of a channel Φ can be written as

$$C_Q(\Phi) = \sup_\psi \left(S\big[\mathrm{Tr}_1 \left(\mathcal{I} \otimes \Phi\right)\big(|\psi\rangle\langle\psi|\big)\big] - S\big[\left(\mathcal{I} \otimes \Phi\right)\big(|\psi\rangle\langle\psi|\big)\big] \right). \qquad (3)$$

In [15] we consider instead the quantity

$$C_{CB}(\Phi) = -S_{CB}(\Phi) = \sup_\psi \left(S\big[\mathrm{Tr}_2 \left(\mathcal{I} \otimes \Phi\right)\big(|\psi\rangle\langle\psi|\big)\big] - S\big[\left(\mathcal{I} \otimes \Phi\right)\big(|\psi\rangle\langle\psi|\big)\big] \right). \qquad (4)$$

The very small change in the subspace over which the partial trace is taken yields a quantity which is not enhanced by entanglement, i.e.,

$$S_{CB}(\Phi \otimes \Phi) = 2S_{CB}(\Phi), \qquad (5)$$

and can be regarded as the minimal conditional information of a channel. When combined with recent work of Horodecki, Oppenheim and Winter [27], this gives a measure of the extent to which a channel breaks or preserves entanglement ,which is much more precise than the crude concept of entanglement-breaking channels. When $S_{CB}(\Phi)$ is negative (or $C_{CB}(\Phi) > 0$) it corresponds to the number of EPR pairs remaining for further use, after exchange of information; when it is positive (or $C_{CB}(\Phi) < 0$), it describes the number of EPR pairs needed to complete the transmission.

This result is part of a much broader investigation into applications of operator spaces in quantum information. Although we subsequently found a much simpler proof of (5), it is unlikely that the result would have been discovered without the "completely bounded" investigations. Roughly speaking, "completely bounded" refers to a sequence of norms associated with $\mathcal{I}_d \otimes \Phi$ rather than Φ itself in a manner similar to the notion of completely positive. This is part of the subject of operator spaces which also include concepts like "complete isometry", etc. It is a very natural framework for quantum information. Indeed, it appears implicitly in work on fault tolerance and one special case is sometimes called the diamond $\diamond$ norm [41]. However, much of the literature on operator spaces is very abstract and mathematically technical.

However, the importance of CB norms is becoming increasingly evident. For example, Kretschmann, Schlingemann and R. F. Werner [39] recently used continuity estimates for

CB norms to prove an information-disturbance tradeoff with implications for quantum cryptography. In February, 2007, D. Kribs and the P.I. are organizing a workshop in Banff which will bring together mathematicians and quantum information scientists to explore the role of operator structure in quantum information theory.

2.2 Conjugate channels

The underlying model of noise in a quantum system regards the original system (Alice) as a subsystem of a larger system which includes both the original system and the environment (Bob). Either system can be described at a later time by taking a partial trace over the other. Typically, the unitary interaction entangles the two systems so that each subsystem is in a mixed state. In the most common scenario, Alice can prepare a variety of different states, but Bob always uses the same state $|\phi\rangle\langle\phi|$. The map which takes Alice's state $|\psi\rangle\langle\psi|$ to $\text{Tr}_B U(t)|\psi \otimes \phi\rangle\langle\psi \otimes \phi|U(t)^\dagger \equiv \Phi(|\psi\rangle\langle\psi|)$ at a fixed time t gives the usual notion of a channel Φ. Bob might like to know what Alice is up to even though he never bothers to change his initial state. Taking $\text{Tr}_A U(t)|\psi \otimes \phi\rangle\langle\psi \otimes \phi|U(t)^\dagger$ defines a map Φ^C whose output is a state $\Phi^C(|\psi\rangle\langle\psi|)$ which describes the information available to Bob at the same fixed time t. We call this map the *conjugate channel.* Together with C. King, K. Matsumoto and M. Nathanson, the P.I. has made an extensive study [33] of conjugate channels.

We have shown that the conjugate of the completely noisy map acting on a pure state contains all the information in the original state, i.e., when the noise completely destroys Alice's state, Bob can recover it. We have also shown that the output of a channel or its conjugate acting on a pure state input are identical. This allows one to obtain results about a class of channels by studying their conjugates and vice versa. Some of our results were obtained independently by Holevo [25], who called them "complementary.

This terminology was used earlier by Devetak and Shor [13] in their work on the quantum capacity of a channel, but they only used it to define degradable channels, which means that the composition with another channel yields the environment. Although channels are rarely degradable, their quantum capacity is readily computed [9, 22, 79, 80]. Working with Graeme Smith, the P.I. obtained a different proof [71] of the main result in [80], namely, that a qubit channel is degradable if and only if it is extreme. This work [71] is based on [70] and also shows that the composition of any two extreme qubit channels is also extreme.

2.3 Special unital channels

In 2003, Shor showed that additivity of minimal output entropy was equivalent to several other important conjectures, including some about entanglement of formation. More recently, Fukuda [21] showed that it would suffice to prove these conjecture for unital channels. This gives additional importance to understanding the properties of unital channels,

which can be much more complicated in higher dimension than for the qubit case $d = 2$.

A channel can be regarded as a model of decoherence. Instead of directly examining the decoherence of a specific physical system we have looked at the different types of channel behavior that can arise from the mathematical definition. In recent work with N. Datta [14] we showed that in higher dimensions $d > 2$ channels which are convex combinations of unitary conjugations can exhibit behavior normally associated with non-unital channels, such as the amplitude damping channel.

During the past year, the P.I. has been working [49] on a class of channels which generalize the unital qubit channels. For $d = p^m$ a prime power, one can use mutually unbiased bases to define a notion of axis, similar to one of the the three Bloch sphere axes for qubits. A unital qubit channels can be written as

$$\Phi : \tfrac{1}{2}[I + \sum_k w_k \sigma_k] \longmapsto \tfrac{1}{2}[I + \sum_k \lambda_k w_k \sigma_k] \qquad (6)$$

For inputs $\rho = \tfrac{1}{2}[I \pm \sigma_k]$ on the k-th axis, the channel Φ has the same effect as the depolarizing channel $\rho \mapsto \lambda\rho + (1 - \lambda)\tfrac{1}{d}I$ with $\lambda = \lambda_k$. In higher dimensions, the channels we study are also described by $d + 1$ "multipliers" $[\lambda_1, \lambda_2, \ldots \lambda_{d+1}]$, and the effect of Φ on inputs on the J-th axis is identical to that of a depolarizing channel with $\lambda = \lambda_J$. However, these channels exhibit properties not seen for unital qubit channels. Roughly speaking, they behave like unital qubit channels only when all multipliers are positive. Our work on these channels has led to the construction of new bound entangled states [26] in dimension $d = 3$.

3   Other work

3.1   Quantum Error Correction

Quantum error correction is now well-developed in the case of so-called stabilizer codes, which arise as invariant subspaces of Abelian subgroups of the Pauli group. These codes generalize some classical ideas, such as Hamming distance, to quantum settings and seem best suited to situations in which all one-bit errors are equally likely and the noise is uncorrelated. Unfortunately, their use in full-scale fault tolerant computation involves concatenations requiring a large number of physical qubits for each logical unit.

The P.I. has been collaborating with H. Pollatsek (Mt. Holyoke College), on the generalization of stabilizer codes using the action of non-Abelian groups. In 2002, we found a pair of new 7-bit codes and a large class of 9-bit codes which can correct all 1-bit errors [59]. However, we also showed that the 9-bit codes could not correct all double errors of a particular type (e.g., $Z_r Z_s$) as we had hoped.

Our codes differ from the "Clifford codes" associated with "nice error bases" as proposed by Knill [38] and developed by Klappenecker and Rötteler [32, 34, 35]. The most significant difference is that the KKR approach does not yield new binary codes. We also require that the code be a subspace spanned by bases for the trivial representation of a group, and reserve the other irreducible representation for error correction.

In 2004, we began to study codes associated with the dihedral group, which one can describe by considering 4,5, 8 or 9 qubits arranged in a square array of the form

$$
\begin{array}{ccc}
1 & \quad 2 \\
& 5 \\
4 & \quad 3
\end{array}
\qquad
\begin{array}{ccc}
1 & 6 & 2 \\
9 & 5 & 7 \\
4 & 8 & 3
\end{array}
\tag{7}
$$

with the middle qubit labelled "5" omitted in the case of 4-bit and 8-bit codes. The dihedral group is the symmetry group generated by reflections across the 4 symmetry planes of a square. (The arrays above give different, but equivalent, realizations of this group.) Although the group actions do not affect the center qubit, it can be useful for achieving orthogonality.

We constructed a large class of new 4-bit and 5-bit error detection codes which are invariant under the dihedral group. (These are different from the non-additive 5-bit code of Rains, et al [62], which is invariant under a much larger non-Abelan group.) Unfortunately, we were unable to construct 4-bit or 5-bit codes which can also correct one type of error (e.g., all single bit flips). This was disappointing because we had hoped to use the 4-bit and 5-bit codes to build up new 9-bit codes

One intriguing possibility remains. We had originally regarded the arrays above as vehicles for defining the symmetry, but did not require that the qubits form a lattice. However, we found some codes which could correct certain errors on nearest neighbor sites in a lattice. This may eventually be worth pursuing as a method of generating special codes for particular physical implementations.

3.2   Norm contraction effects of noise

In [58] we considered the question of when the p-norm of the output of a quantum channel is less than that of the input. This question has arisen in several contexts in physics, including quantum information theory [23, 61, 77]. We proved that the p-norm is decreasing if and only if the channel is unital. This has an interesting corollary with implications for decoherence [6]. It is natural to ask if one can always remove the "non-unital" part of a channel in a way that yields a unital map which is a valid quantum operation. We showed that this is possible for qubit channels, but that for $d > 2$ the resulting map might not be completely positive.

3.3 Expository papers

The P.I. has written a number of expository articles including [66, 67, 68, 69]. Although the grant did not support graduate students and the P.I.'s position makes it difficult for her to have any, she has contributed to graduate training in other ways. She assisted in the supervision of M. Nathanson, a recent student of C. King at Northeastern University. She has written several expository article on quantum entropy [67, 68, 69] intended to make the proofs of important entropy inequalities accessible to graduate students and less mathematically sophisticated scientists working in quantum information theory. In addition, the P.I. wrote an introduction [66] to quantum information theory for engineers working in nanotechnology.

References

[1] D. Aharonov, W. van Dam, J. Kempe, Z. Landau, S. Lloyd and O. Regev "Adiabatic Quantum Computation is Equivalent to Standard Quantum Computation" quant-ph/0405098

[2] A. Ambainis and O. Regev "An Elementary Proof of the Quantum Adiabatic Theorem" quant-ph/0411152

[3] A. Ambainas and M.B. Ruskai, Report of the workshop on "Mathematical aspects of adiabatic quantum computation" (Perimeter Institute, Waterloom, 9-11 February, 2006) http://www.perimeterinstitute.ca/activities/scientific/PI-WORK-6/related_links.php

[4] J. E. Avron and A. Elgart "Adiabatic Theorem without a Gap Condition" *Commun. Math. Phys.* **203**, 445–463 (1999).

[5] J.E. Avron, R. Seiler and L. G. Yaffe, "Adiabatic theorems and applications to the quantum Hall effect" *Comm. Math. Phys.* **110** 33–49 (1987); erratum **156**, 649–650 (1993).

[6] R. Blume-Kohout and L. Viola, private communication

[7] M. Born and V. Fock: "Beweis des Adiabatensatzes" *Z.. Physi* **51**, 165–169 (1928)

[8] M. Born andR. Oppenheimer: *Zur Quantentheorie der Molekeln.* Ann. Phys. 84, 457-484 (1927)

[9] F. Caruso, V. Giovannetti, A. S. Holevo "One-mode Bosonic Gaussian channels: a full weak-degradability classification" quant-ph/0609013

[10] A. Childs, private communication about work based on [75].

[11] W. van Dam, M. Mosca and U. Vazirani, "How powerful is adiabatic quantum computation" 42nd IEEE Symposium on Foundations of Computer Science (Las Vegas, NV, 2001), 279–287; quant-ph/0206003

[12] P. Defit, M.B. Ruskai and W. Spitzer, "Improved gap estimates for simulating quantum circuits by adiabatic evolution" *Quantum Information Processing* in press (2006). quant-ph/0605156

[13] I. Devetak and P. W. Shor "The capacity of a quantum channel for simultaneous transmission of classical and quantum information" *Commun. Math. Phys.* **256**, 287–303 (2005). quant-ph/0311131

[14] N. Datta and M.B. Ruskai , "Maximal output purity and capacity for asymmetric unital qudit channels" quant-ph/0505048

[15] I. Devetak, M. Junge, C. King and M.B. Ruskai "Multiplicativity of completely bounded p-norms implies a new additivity result" quant-ph/0506196

[16] A. Elgart, Lecture at the workshop on "Mathematical aspects of adiabatic quantum computation" (Perimeter Institute, Waterloom, 9-11 February, 2006). Available on streaming video.

[17] E. Farhi and S. Gutmann "An Analog Analogue of a Digital Quantum Computation" *Phys. Rev. A* **57**, 2403-2406 (1998). quant-ph/9612026

[18] E. Farhi, J. Goldstone, S. Gutmann and M. Spiser "Quantum Computation by Adiabatic Evolution" quant-ph/0001106

[19] E. Farhi, et al "A Quantum Adiabatic Evolution Algorithm Applied to Random Instances of an NP-complete Problem" *Science* **292**, 472–474 (20 April 2001); see also quant-ph/0104129

[20] E. Farhi, J. Goldstone, S. Gutmann "Quantum Adiabatic Evolution Algorithms with Different Paths" quant-ph/0208135

[21] M. Fukuda, "Extending additivity from symmetric to asymmetric channels" *J. Phys. A* **38**, L753–58 (2005). quant-ph/0505022.

[22] V. Giovannetti and R. Fazio, "Information-capacity description of spin-chain correlations" *Phys. Rev. A* **71**, 032314 (2005).

[23] A. Harrow, private communication

[24] G.A. Hagedorn and A. Joye, "Elementary exponential error estimates for the adiabatic approximation" *J. Math. Anal. Appl.* **267**, 235–246 (2002).

[25] A. S. Holevo, "On complementary channels and the additivity problem" quant-ph/0509101

[26] M. Horodecki, P. Horodecki, and R. Horodecki "Mixed-State Entanglement and Distillation: Is there a Bound Entanglement in Nature?" *Phys. Rev. Lett.* **80**, 5239–5242 (1998).

[27] M.Horodecki, J. Oppenheim and Andreas Winter "Quantum information can be negative" quant-ph/0505062

[28] S. Jansen, M.B. Ruskai and R. Seiler, "Bounds for the adiabatic approximation with applications to quantum computation" quant-ph/0603175

[29] A. Joye and C. Pfister "Full asymptotic expansion of transition probabilities in the adiabatic limit" *J. Phys. A* **24**, 753–766 (1991) .

[30] T. Kato, "On the adiabatic theorem of quantum mechanics" *J. Phys. Soc. Japan* **5**, 435–439 (1950).

[31] J. Kempe, A. Kitaev and O. Regev, "The Complexity of the Local Hamiltonian Problem" quant-ph/0406180

[32] A. Ketkar, A. Klappenecker, S. Kumar, P.K. Sarvepalli, "Nonbinary stabilizer codes over finite fields" quant-ph/0508070

[33] C. King, K. Matsumoto, M. Nathanson and M. B. Ruskai, "Properties of Conjugate Channels with Applications to Additivity and Multiplicativity" (quant-ph/0509126).

[34] A. Klappenecker and M. Rötteler, "Beyond Stabilizer Codes I: Nice Error Bases; II : Clifford Codes" *IEEE Trans. Info. Theory* **48**, 2392–2399 (2002).

[35] A. Klappenecker, "Clifford Codes" pp. 253–273 in *Mathematics of Quantum Computation* ed R. Brylinski and G. Chen (CRC Press, 2002).

[36] A. Klappenecker and M. Rötteler, "Remarks on Clifford codes" quant-ph/0312228

[37] M. Klein and R. Seiler "Power-Law Corrections to the Kubo Formula Vanish in Quantum Hall Systems" *Commun. Math. Phys.* **128**, 141-160 (1990).

[38] E. Knill, "Group Representations, Error Bases and Quantum Codes" quant-ph/9608048

[39] D. Kretschmann, D. Schlingemann, R. F. Werner, " The Information-Disturbance Tradeoff and the Continuity of Stinespring's Representation" quant-ph/0605009

[40] D. Kribs, R. Laflamme, D. Poulin and M. Lesosky, "Operator Quantum Error Correction" quant-ph/0504189

[41] A. Kitaev, *Classical and Quantum Computation* (AMS, 2002).
Authors:

[42] D. Kribs, R. Laflamme, D. Poulin, "A Unified and Generalized Approach to Quantum Error Correction" *Phys. Rev. Lett.* **94**, 180501 (2005). quant-ph/0412076

[43] L.D. Landau and E.M. Lifshitz *Quantum mechanics : non-relativistic theory* (Pergamon Press, 1977).

[44] R. MacKenzie, E. Marcotte, H. Paquette "A new perturbative approach to the adiabatic approximation" quant-ph/0510024

[45] A. Mizel, D. A. Lidar, M. Mitchell "Simple proof of equivalence between adiabatic quantum computation and the circuit model" quant-ph/0609067

[46] Karl-Peter Marzlin, Barry C. Sanders, "Inconsistency in the application of the adiabatic theorem" *Phys. Rev. Lett.* **93**, 160408 (2004). quant-ph/0404022

[47] Messiah *Quantum Mechanics, Vol. II.* North-Holland Publishing Company (1962).

[48] D. R. Mitchell, C. Adami, W. Lue, C. P. Williams "Random Matrix Model of Adiabatic Quantum Computing" *Phys. Rev.A* **71**, 052324 (2005). quant-ph/0409088

[49] M. Nathanson and M. B. Ruskai, "Pauli diagonal channels constant on axes" unpublished manuscript to be posted and submitted by 31 October 2006.

[50] G. Nenciu, "On the adiabatic theorem of quantum mechanics" *J. Phys. A* **13**, L15–L18 (1980).

[51] G. Nenciu, "Linear adiabatic theory: exponential estimates" *Commun. Math. Phys.* **152**, 479–496 (1993).

[52] M. A. Nielsen and D. Poulin, "Algebraic and information-theoretic conditions for operator quantum error-correction" quant-ph/0506069

[53] Q. Niu and D. J. Thouless "Quantised adiabatic charge transport in the presence of substrate disorder and many body interactions" *J. Phys. A* **17**, 30–49

[54] V. Paulsen, *Completely Bounded Maps and Operator Algebras* (Cambridge University Press, 2002).

[55] G. Pisier, *Non-Commutative Vector Valued L_p-spaces and Completely p-summing Maps* (American Mathematical Society, 1996).

[56] G. Pisier, *The Operator Hilbert Space OH, Complex Interpolation and Tensor Norms* (Société Mathématique de France, 1998).

[57] G. Pisier, *Introduction to Operator Space Theory* (Cambridge University Press, 2003).

[58] D. Perez-Garcia, M. Wolf, D. Petz and M.B. Ruskai "Contractivity of positive and trace preserving maps under L_p norms" *J. Math. Phys.* in press (2006). math-ph/0601063

[59] H. Pollatsek and M. B. Ruskai, "Permutationally Invariant Codes for Quantum Error Correction" *Lin. Alg. Appl.* in press (2004). (quant-ph/0304153)

[60] D. Poulin, "Stabilizer Formalism for Operator Quantum Error Correction" quant-ph/0508131

[61] M. Raginsky, "Entropy production rates of bistochastic strictly contractive quantum channels on a matrix algebra" *J. Phys. A: Math. Gen.* **35**, 585-590 (2002).

[62] E.M. Rains, R. H. Hardin, P. W. Shor, and N. J. A. Sloane "A nonadditive quantum code" *Phys.Rev.Lett.* **79**, 953–954 (1997). (quant-ph/9703002)

[63] B. Reichardt "The quantum adiabatic optimization algorithm and local minima" STOC 2004.

[64] J. Roland and N. J. Cerf, "Quantum Search by Local Adiabatic Evolution" *Phys. Rev. A* **65**, 042308 (2002), quant-ph/0107015.

[65] M.B. Ruskai "Comments on Adiabatic Quantum Computation" in *Mathematical Results in Quantum Mechanics* ed by R. Weder, P. Exner, and B. Grbert *Contemporary Mathematics* **307**, 265-274 (AMS Press, 2002). (quant-ph/0203127)

[66] M.B. Ruskai, "Introduction to Quantum Information Theory" invited chapter in *Handbook of Nanotechnology* ed. by A. Lakhtakia, pp. 395464 (SPIE Press and ASM, 2004)

[67] M.B. Ruskai, "Lieb's simple proof of concavity of $\operatorname{Tr} A^p K^\dagger B^{1-p} K$ and remarks on related inequalities" *International Jour. Quant. Info.* 3, 570–590 (2005). quant-ph/0404126

[68] M.B. Ruskai, "Comment on "Stronger subadditivity of entropy" *Phys.Rev. A* **74**, 026303 (2006). math-ph/0603022

[69] M.B. Ruskai, "Another short and elementary proof of strong subadditivity of quantum entropy" quant-ph/0604206

[70] M. B. Ruskai, S. Szarek, E. Werner, "An analysis of completely positive trace-preserving maps M_2" *Lin. Alg. Appl.* **347**, 159 (2002).

[71] M.B. Ruskai and G. Smith, unpublished manuscript.

[72] G. Schaller, S. Mostame, R. Schützhold: "A Note on Adiabatic Quantum Computing." quant-ph/0510183

[73] P. W. Shor, "Equivalence of Additivity Questions in Quantum Information Theory", *Commun. Math. Phys.* **246**, 453– 472 (2004). quant-ph/0305035

[74] M. S. Siu "From quantum circuits to adiabatic algorithms" *Phys. Rev. A* **71**, 062314 (2005). quant-ph/0409024

[75] S. Teufel, *Adiabatic perturbation theory in quantum dynamics* (Springer, 2003).

[76] D. M. Tong, K. Singh, L. C. Kwek, C. H. Oh "Quantitative conditions do not guarantee the validity of the adiabatic approximation" *Phys. Rev. Lett.* **95**, 110407 (2005). quant-ph/0509073

[77] V. Vedral, M. B. Plenio, M.A. Rippin and P.L. Knight, *Phys. Rev. Lett.* **78** 2275 (1997).

[78] R. Verch and R. Werner, "Distillability and positivity of partial transposes in general quantum field systems" *Rev.Math.Phys.* **17**, 545–576 (2005). quant-ph/0403089

[79] M. M. Wolf, D. Perez-Garcia and G. Giedke, "Quantum Capacities of Bosonic Channels" quant-ph/0606132

[80] M. M. Wolf and D. Perez-Garcia "Quantum Capacities of Channels with small Environment" quant-ph/0607070